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# Open form and two combinatorial musical models: The cases of *Domaines* and *Duel*

Benny Sluchin (EIC, IRCAM), Mikhail Malt (IRCAM, MINT –Paris IV)

**Abstract.** Two “open” works, composed within a two-year period by Boulez and Xenakis, could be seen as based on a square matrix of order six and share several properties. Their combinatorial attributes, the theory and the practice of their performances are studied and compared. Our main aim will be to establish a relationship between the properties of the mathematical model and the use of it by Boulez and Xenakis in *Domaines* and *Duel*.

**Keywords:** Open Form, Matrix, Combinatory, Permutations, Game Theory, Computational Musicology, Boulez, Xenakis, *Domaines*, *Duel*.

## 1 Introduction

The interest in the “open work” that started in the fifties, led to a confusing variety of definitions. To express the different degrees of variance offered by this musical approach (from smallest to largest), to form and to all levels of composition, one could use the terms: “controlled randomness”, “mobile forms”, “open works” and/or “indetermination”. However, we know these terms answer to different, and sometimes opposed aesthetic concerns.

In this article, we propose two examples of this very diverse repertory. These works do not share a common aesthetic, but they do have many similarities. We shall describe the mathematical bases and the compositional preoccupations, which led to each of those works being recognized as open forms, thus showing their differences and similarities.

### 1.1 The Common Structure

Let  $R$  be a commutative field and  $M_n(R)$  the algebra of a square matrix of order  $n$  with coefficients in  $R$ . This structure plays a fundamental role in the history of mathematics. We shall study two cases where elements from  $M_6(\mathbb{Z})$  ( $\mathbb{Z}$  being the ensemble of relative integers) allow those works’ form to be “mobile” or open. In both cases, we don’t have “one work” but a class of works standing for the generic work.

## 1.2 Methodology

The starting point of this work was the pragmatic problems and issues connected with the performance of open works. We mainly focus on the study of the models than on the music. Our main goal will be to try to establish a bridge between the matrix properties and the structure generated by these “open works”. To study the actual dynamics arising from Boulez and Xenakis’ formalisms, we used computer simulation<sup>1</sup>, and specific interfaces for performance. In what follows, we only present the computer interfaces used for simulation<sup>2</sup> and study.

## 2 *Domaines* (1961–1968) by Pierre Boulez

### 2.1 Description

*Domaines* is a work in which the strictness of the composer’s writing contrasts with some formal indeterminacy.

The ensemble [2] is divided into six groups named *A* to *F* as follows:

- A* A trombone quartet (one alto, two tenors and one bass)
- B* A string sextet (two violins, two altos and two cellos)
- C* A duo (marimba and double bass)
- D* A quintet (flute, trumpet, saxophone alto, bassoon and harp)
- E* A trio (oboe, horn and guitar)
- F* A bass clarinet

The six groups are placed on the apexes of a hexagon surrounding the conductor (Fig. 1). The sound sources are thus clearly distinct and the role of space is well defined.

In terms of musical performance, *Domaines* is an alternation between a clarinet solo and one of the six instrumental groups. The work consists of two parts: the *Original* and the *Miroir*, each made of six sections in an order to be determined.

For the *Original*, the clarinet player fixes the order of performance of the six *cahiers* (the clarinet part), titled from *A* to *F*, and starts playing in the chosen order. After each clarinet solo, the corresponding instrumental group plays its sequence of *Original*. For example, if the soloist played *cahier B*, the *B* group (string sextet) will play its own sequence, and so on. At the end of the sixth sequence of *Original*, the process continues with the parts of *Miroir*. For this second part, the conductor fixes the order, and the clarinet player now has to play the corresponding *cahier* after the instrumental groups. The succession of parts is played without interruption. The work will start and finish with the clarinet solo.

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<sup>1</sup> We used the Max/Msp (© cycling ’74) and OpenMusic (© Ircam) environments.

<sup>2</sup> For a detailed presentation of *Duel* performance interfaces see: Xenakis International Symposium 2011, London, Benny Sluchin & Mikhail Malt “Play and game in *Duel* and *Strategy*”, 2011, April 2. A computer interface for Boulez’s *Domaines* is planned to be used in May 2011.

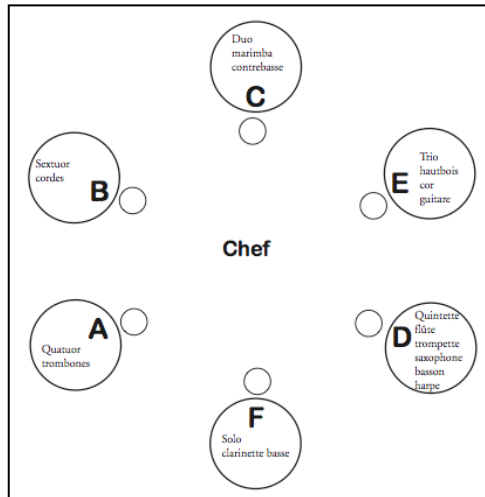


Fig. 1: Disposition of instrumental groups in *Domaines*

## 2.2 Combinatorics

From the standpoint of drama, the work is played by two agents: the clarinet player and the conductor, each having to determine a permutation of  $S_6$  (the symmetric group of order 6, subgroup of  $M_6(\mathbb{Z})$ ) thus fix the order of the *Original* and the *Miroir* correspondingly. A version of *Domaines* is not only the succession of the parts, but also a physical path of the sound, a spatialization, rendered by the soloist, which must meet the various groups, in agreement with the chosen permutation and the position of each of the 6 instrumental groups. This special feature creates a sound architecture, a motion of sound in space, which results from the combinatorial qualities of the work.

In this “open form” work, two permutations give birth to  $(6!)^2 = 720^2 = 518\,400$  possibilities for *Domaines*. This only accounts for the order of the parts, not any other non-fixed elements. Both agents have to determine the dynamics, the vertical or horizontal reading in the solo parts (see 2.5), the order of the inserts for the string sextet. The agents play their parts alternately, with only a short superposition for continuity.

## 2.3 Permutations

A permutation of  $S_6$  may be represented by a square permutation matrix with elements 0 or 1, only one 1 per line and only one 1 per column, or by a  $\square$  vector. Thus, a permutation,  $\square = (1\ 4\ 6\ 5\ 2\ 3)$  is also represented by a  $M(\square)$  matrix (Fig. 2)

1	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	1
0	1	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0

Fig. 2: Permutation matrix

The square matrices of order  $n$  are in bijection with  $S_n$  (the permutations of the ensemble  $\{1, 2, \dots, n\}$ ). One could represent a "version" by two permutations  $\square, \square$ , of  $S_6$  which will give us the order of the parts (Fig. 3)  $\square$  being the permutation related to the *Original* and  $\square$  being the permutation related to the *Miroir*.

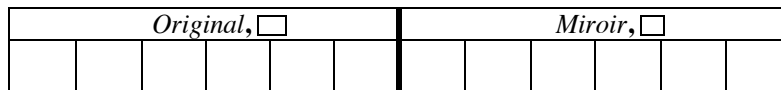


Fig. 3: Organization of the two parts in *Domaines*

In what follows, we will also use the graphic representation of Figure 4 to write in each of the boxes, *A* to *F* representing the group (above) or the corresponding clarinet (below).

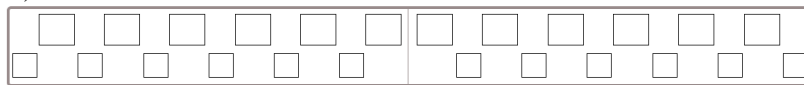


Fig. 4: Representation of the basic structure of the instrumental groups and of the clarinet

To each permutation  $\sigma$  we will associate two numerical values:

$\square$  – The density variation. For each step an integer (1 to 5), representing the difference in group size, is associated.

$$\square \quad (1)$$

where  $\square$  indicates the number of instruments in group  $\square$ . For each step, an integer (1 to 5) representing the difference in group size is associated.

$\square$  – The total angular distance. For each step is associated an angular position (between  $0^\circ$  and  $300^\circ$ ) corresponding to a group position.

$$\square \quad (2)$$

where  $\square$  indicates the angular position of the instruments of group  $\square$ .

## 2.4 Some special cases

It is useful to study a few specific versions, especially in the case of performances. A permutation is usually seen as a static process. However, the interest of the analysis of some *Domaines* performances shows that each permutation brings an internal dynamic movement concerning the structure of groups, its density and angular position.

1.  $\square = (1\ 2\ 3\ 4\ 5\ 6)$  the identical permutation represented by the diagonal matrix (Fig. 5).  
 $\square = 13$ ,  $\square = 420^\circ$ .

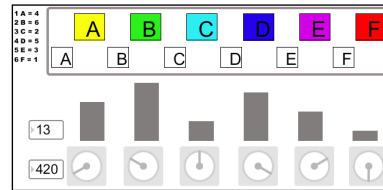


Fig. 5:  $\square = (1\ 2\ 3\ 4\ 5\ 6)$

We will notice that the groups are not placed in alphabetical order (inversion of D and E on the apexes of the hexagon).

2. Minimal movements path,  
 $\square = (1\ 2\ 3\ 5\ 4\ 6)$  (Fig. 6).  
 $\square = 13$ ,  $\square = 300^\circ$ .

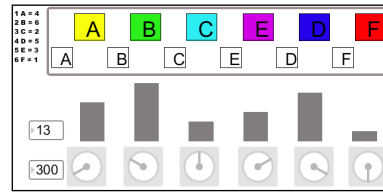


Fig. 6:  $\square = (1\ 2\ 3\ 5\ 4\ 6)$

Obviously, circular transpositions or inverted paths will be also of the same nature (Fig. 7).  $\square = 14$ ,  $\square = 420^\circ$ .

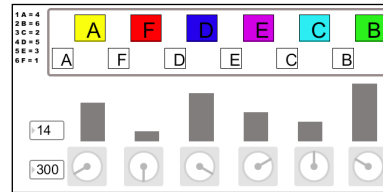


Fig. 7:  $\square = (1\ 6\ 4\ 5\ 3\ 2)$

3. Maximal movements path. Maximal movements are obtained by doing a maximum number of  $180^\circ$  paths (Fig. 8).  
 $\square = 9$ ,  $\square = 780^\circ$ .

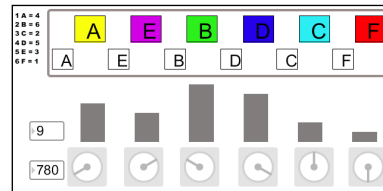


Fig. 8:  $\square = (1\ 5\ 2\ 4\ 3\ 6)$

4. Instrumental density in ascending order (Fig. 9).  
 $\square = 5$ ,  $\square = 720^\circ$ .

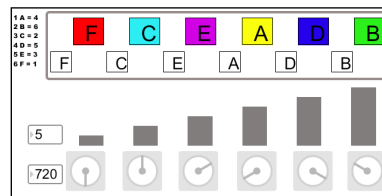


Fig. 9:  $\square = (6\ 3\ 5\ 1\ 4\ 2)$

5. Instrumental density in descending order (Fig. 10).  
 $\square = 5$ ,  $\square = 720^\circ$ .

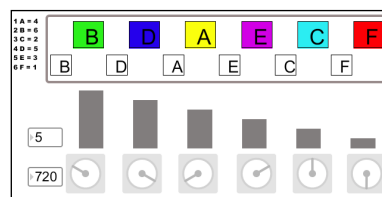


Fig. 10:  $\square = (2\ 4\ 1\ 5\ 3\ 6)$

In those last two cases, the values of spatial movement are high ( $\square = 720^\circ$ ). One can infer that the instrumental groups' positioning has been made to encourage that.

6. The path of "Boulez\_Damiens" (Munich 2005)

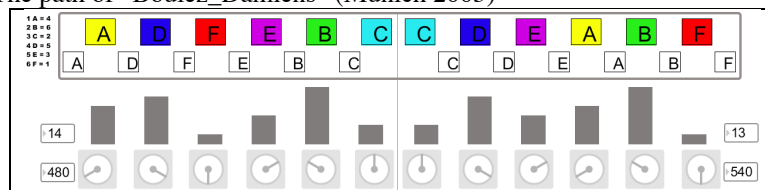


Fig. 11: *Original*:  $\square = (1\ 4\ 6\ 5\ 2\ 3)$ , *Miroir*:  $\square = (3\ 4\ 5\ 1\ 2\ 6)$ .

This version is a reference, as in this specific case; the composer is also the conductor. According to Boulez: "rehearsal time is lacking to test the possibilities. Some path sequences function better than others"<sup>3</sup>. Starting with an homogeneous group, like A (four trombones), introduces the work well, and concluding with F (the bass clarinet), a "solo" group playing with its back to the audience, influences the choice of the two permutations. Notice that the last element of  $\square$  is also the first of  $\square$ . This leads to other considerations<sup>4</sup>.  $\square = 14$ ,  $\square = 480^\circ$ , for the *Original*, and  $\square = 13$ ,  $\square = 540^\circ$ , for the *Miroir*.

<sup>3</sup> Personnal communication, Decmbre 2010.

<sup>4</sup> In order to rework the score of group F (marimba, double bass), the *Fm* part (the mirror part of F) has been removed from *Domaines*. It has been modified and augmented. This part is now at the Sacher Foundation in Basel. Having only *Fo* (the original part of F) to disposal, it has been divided into two parts (which share the same character). In this particular version, the part is played in full between the *Original* and the *Miroir*.

### 7. Masson\_Portal path

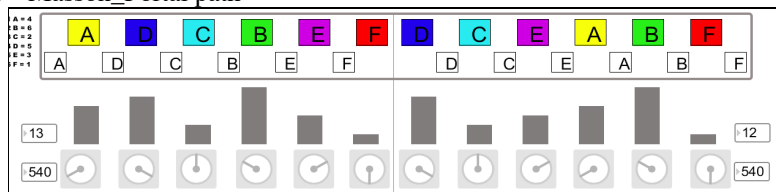


Fig. 12:  $\square = (1\ 4\ 3\ 2\ 5\ 6)$ ,  $\square = (4\ 3\ 5\ 1\ 2\ 6)$

This version is the only one recorded on a general public media (CD). From a 1971 vinyl [3], it allowed many analyses [1, 82-90], [5, 418-425]. Notice the resemblance with the preceding version (two inversions in the *Original*, and only one in the *Miroir*).  $\square = 13$ ,  $\square = 540^\circ$ , for the *Original*, and  $\square = 12$ ,  $\square = 540^\circ$ , for the *Miroir*.

### 8. Eötvös\_Damiens path (Paris, 1981)

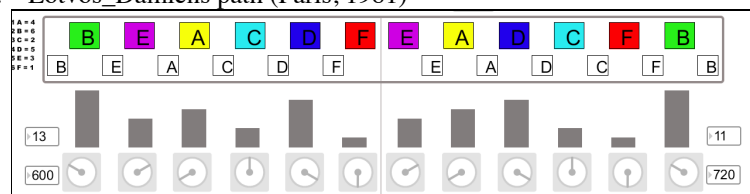


Fig. 13:  $\square = (2\ 5\ 1\ 3\ 4\ 6)$ ,  $\square = (5\ 1\ 4\ 3\ 6\ 2)$

Notice the high values for spatial movements ( $\square = 600^\circ$ ), as well as the sum of differences of contrast of instrumental density ( $\square = 13$ ) and the placement of B in first position in the *Original* and as last section in the *Miroir*.  $\square = 11$ ,  $\square = 720^\circ$ , for the *Miroir*.

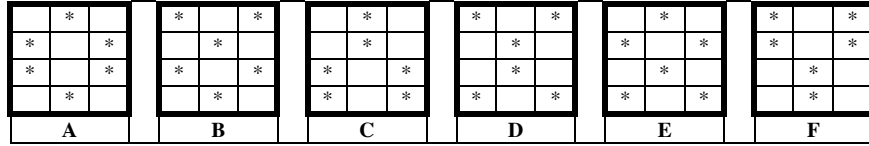
### 9. Special relationship between $\square$ and $\square$ .

The choice of permutations  $\square$  and  $\square$  are in principle independent. The two agents of *Domaines* are allowed to agree on a path to define their versions. The permutation  $\square$  may be chosen in relation to  $\square$ . It may have mathematical (e.g. inverse or transpose) or musical properties. However, as we saw, the permutations chosen can influence the dynamics of *Domaines*, giving more or less spatial movement, or creating a density evolution in the sequence of the instrumental groups.

## 2.5 Other elements of $S_6$

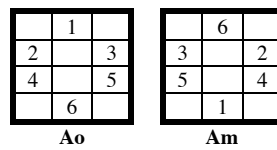
The clarinet sheets show us other permutations, playing a part in the composition and in the performance of *Domaines*. Each of the *cahiers* in the *Original*,  $\square$  from A to F, is made of six cells. They are placed on the sheet following the model of Figure 14.





**Fig. 14:** The six dispositions of cells of the clarinet solo

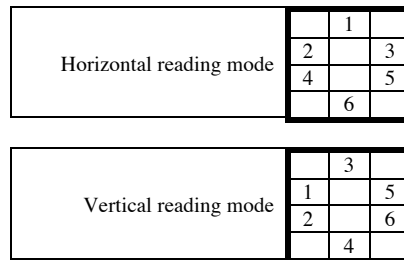
Each cell gives birth to a mirror form. They are placed on the corresponding sheet according with a permutation  $\square$ . So  $\square = (6\ 5\ 4\ 3\ 2\ 1)$  because in regard of the *Original* ( $Ao$ ), the mirror cells are placed on the *Miroir* part ( $Am$ ) following the model of Figure 15.



**Fig. 15:** The relationship between cells in the *Original* and in the *Miroir cahiers A*

These permutations are characterized by three transpositions (exchange of two elements). We have  $\square = (4\ 5\ 6\ 1\ 2\ 3)$ ,  $\square = (2\ 1\ 6\ 5\ 4\ 3)$ , and so on.

For the performance, the composer uses a horizontal reading mode and a vertical reading mode. For example, for A, both reading modes are shown in Figure 16.



**Fig. 16:** The two reading modes for the clarinet

Which means that the vertical reading is a permutation  $\square = (2\ 4\ 1\ 6\ 3\ 5)$  of the horizontal one. We have  $\square = (1\ 4\ 3\ 6\ 2\ 5)$ ,  $\square = (1\ 2\ 3\ 5\ 4\ 6)$ , ... If one chooses the horizontal reading mode for the *Original*, one will have to choose the vertical reading mode for the *Miroir* and vice-versa<sup>5</sup>. Naturally those permutations can be applied to the preceding structures.

## 2.6 Practical Problems of Performance

The “opening” of *Domaines* requires special practice. One can define the path after reflection (solitary or by consensus), but to test it, one needs concert conditions and

<sup>5</sup> See the score explanation pages [4].

time. One must also be able to swiftly reorganize one's scores and to memorize the path. That is why the musicians do not wait for the moment of performance to decide on those matters. The paths are determined beforehand and rehearsed with the soloist and the musicians. The transitions are delicate points to settle<sup>6</sup>.

A computer interface would give the conductor swift ordering, and computer display would make playing this piece, and expounding its combinatorial possibilities, easier. In [7] we have discussed the limits of paper scores and the benefits that could bring computer interfaces in improvisation and open works performances.

### 3 *Duel* (1959) by Iannis Xenakis

#### 3.1 Description

*Duel* (1959-60) by Iannis Xenakis is a musical game, for two conductors and two orchestras commissioned by the ORTF (Office de Radiodiffusion-Télévision Française). It was created in Hilversum in 1971 by Diego Masson and Fernand Tuby.

Each orchestra is made of three groups:

1. Winds: 1 piccolo, 1 oboe, 1 Eb clarinet, 1 Bb bass clarinet, 1 bassoon, 1 contrabassoon, 2 trumpets, 1 trombone
2. Percussions: 2 bongos, 3 congas, 1 snare drum, 1 drum.
3. Strings: 6 first violins, 6 second violins, 4 cellos, 2 double basses.

The two orchestras are positioned to the left and to the right of the stage, with the two conductors standing back to back, or on two opposed stages. Each conductor has at his disposal six sound constructs (that we will also call tactics), numbered *I* to *VI* in the score, which are stochastic structures<sup>7</sup>.

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<sup>6</sup> In personal communication, Peter Eötvös stated that: "Public, ignoring the multitude of possibilities, needs a solid performance where all the transitions and order of parts have been fixed and carefully rehearsed" (March 2011).

<sup>7</sup> In what follows we distinguish between the discussion in *Musiques Formelles* [9, 9], and the score [10], in which the three first tactics are indeed group combinations:

A, Strings (pointillism (I) or held (II) or crossed glissandi (III))

B, A + percussion or A + winds or percussion and winds

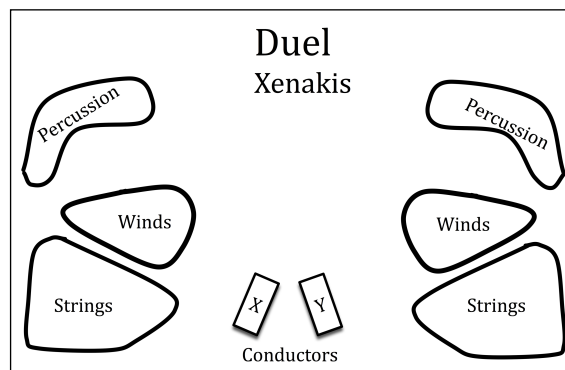
C, A + percussion + winds

The game matrix is the same.

<i>I</i>	<i>A cluster of sonic grains</i>
<i>II</i>	<i>Parallel sustained strings with fluctuations</i>
<i>III</i>	<i>Networks of intertwined string glissandi</i>
<i>IV</i>	<i>Stochastic percussion sounds</i>
<i>V</i>	<i>Stochastic wind instrument sounds</i>
<i>VI</i>	<i>Silence</i>

**Fig 17:** The six fundamental tactics

The on-stage disposition is two times three groups (Figure 18), that is, six groups and the two conductors. The two agents are at variance, acting and creating a sound mixture placed at their disposal by the composer. There are thus six tactics, which each conductor can have his orchestra perform.



**Fig. 18:** The diagram of the two orchestras on stage (simplified from the score)

### 3.2 Game theory

Game theory is a formal study of interactions between agents (from cooperation to conflict). Xenakis uses a special case: zero-sum two-player games [6, 169]. A matrix gives the gains and losses of the agents (playing lines or columns). The minmax theorem (Nash) [8] insures the existence of a unique solution (defining the value of the game). The choices of the two conductors are defined by a game matrix (in  $M_6(\mathbb{Z})$ ) given by the composer<sup>8</sup> (Fig. 19). This matrix is the *game matrix*, which defines a zero-sum two players game, a special case of the mathematical games theory.

<sup>8</sup> In chapter III (or chapter IV in [11]) of *Musique Formelles* [10, 140 - 150] Xenakis describes the method to create this matrix from subjective judgements about simultaneous sound event couples.

	I	II	III	IV	V	VI
I	-1	+1	+3	-1	+1	-1
II	+1	-1	-1	-1	+1	-1
III	+3	-1	-3	+5	+1	-3
IV	-1	+3	+3	-1	-1	-1
V	+1	-1	+1	+1	-1	-1
VI	-1	-1	-3	-1	-1	+3

Fig. 19: *Duel* game matrix

Each of the 36 elements of this matrix is a “gain” corresponding to a sound combination. The musical game *Duel* is generated by a choice of a tactic (Figure 17) by conductor *X* (playing lines) while conductor *Y* chooses one of the columns (Figure 19). Positive values are gains for conductor *X* and negative values are gains for conductor *Y*. The element of the matrix resulting from those choices gives *X*’s gain (a positive or negative integer) equal to the loss of *Y*. The goal of the game is to win a maximum of points at the end of the performance. Winning is not a matter of beauty or good musical choices. The composer itself states that the winner conductor is that follows the matrix rules better than his opponent, and the musical result is the composer’s responsibility<sup>9</sup>.

### 3.3 Practical Problems of Performance

The performance of *Duel* is not without difficulties and requires a complex preparation from both agents.

Some preliminary decisions are necessary before performing *Duel*.

1. The attribution of lines and columns (by flipping a coin between the two conductors)
2. The determination of the duration (by choosing an arbitrary duration *m* in minutes, the number of moves or the maximum number of points to obtain)
3. The decision of who will go first.

Other elements are at the agents’ discretion.

1. Where in the score to start each tactic.
2. Each move’s duration.

About the conductors’ choices of tactics, Xenakis names two main possibilities:

1. The “degenerate” fashion. Either by the beforehand determination by both agents of a preordained succession, or by arbitrarily following their intuition. It is easy to understand that the composer does not think highly of this possibility.

<sup>9</sup> “Le gagnant a gagné parce que il a simplement mieux suivi les règles du jeu imposées par le compositeur qui, par conséquent, revendique la responsabilité du "beau" ou du "laid" de sa musique.” See score instruction notes [11, 5].

2. Using the matrix and its values, either by choosing on the basis of the potential gain, or by drawing from an urn a ball numbered 1 to 6, with proportional quantities according to probabilities given by Xenakis. For this is a musical game, and winning is not without influence on the conflict<sup>10</sup>.

### 3.4 Special cases

#### 3.4.1 Random choices of each agent

After 500 moves, with random choices, thus uniformly distributed of the two agents, the dynamics runs through all 36 cells of the game matrix. The game value ends at “-0.03” and the morphology of the paths doesn’t seem to generate special patterns.

#### 3.4.2 Random choices of each agent with weighting

In this second simulation, we used random choices weighted by the probabilities calculated by the composer [9, 148]. The game value reaches “-0.12”<sup>11</sup> and the game runs through all 36 cells, thus the proposed 36 combinations. The morphology of the paths is dependent on the relative weight of lines and columns and doesn’t show special patterns.

#### 3.4.3 Choices of each agent with strategy of maximum gain

In this third simulation, each agent chooses a line or a column where he hopes to have a maximum gain. This strategy is a simplification of the “minmax” tactic. As a result, the game only runs through 23 cells of the matrix<sup>12</sup> (See Figure 20). Agent *X* never chooses tactic *V*, as he always has a better alternative (see Figure 19). Concerning the trajectory, it is possible to detect a group of irregularly repeating patterns (Figure 21): {21 → 24 → 36 → 33 → 21}, {16 → 15 → 21 → 23}, {3 → 1 → 13 → 18 → 36 → 33}, {16 → 18 → 36 → 33 → 3 → 4}, ...

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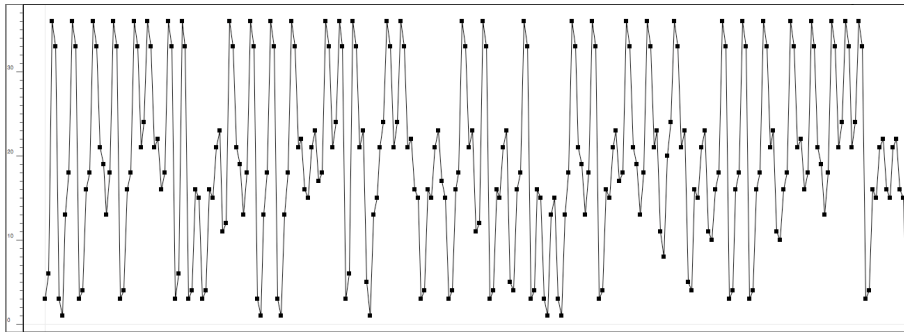
<sup>10</sup> « La seule (façon) valable, la seule qui apporte quelque chose de nouveau, dans le cas de plusieurs orchestres, est celle qui est sanctionnée par des gains et des pertes, par des victoires et des défaites. » [11, 4]

<sup>11</sup> This value is different from the one calculated by Xenakis (-0.07). This point needs further research in original sketches.

<sup>12</sup> In this simulation the resulting cells are: {1 3 4 5 6 8 9 10 11 12 13 15 16 17 18 19 20 21 22 23 24 33 36}.

	I	II	III	IV	V	VI
I	1	2	3	4	5	6
II	7	8	9	10	11	12
III	13	14	15	16	17	18
IV	19	20	21	22	23	24
V	25	26	27	28	29	30
VI	31	32	33	34	35	36

**Fig. 20:** Cells run through (in grey) by a simulation with maximum gain



**Fig. 21:** Game dynamics resulting from the simulation with random choice with gain. X axis, cells 1 to 36. Y axis, moves.

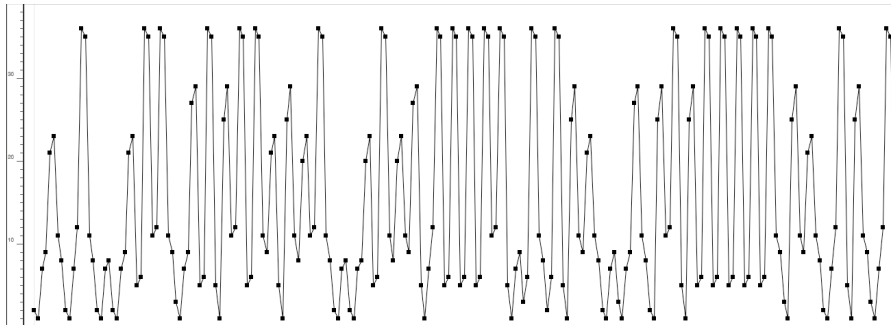
### 3.4.4 Choice of each agent using the “minmax” strategy

In this simulation, each agent chooses a line or a column, not only aiming towards a value that is in his favor, but also tries to choose a line or a column that will minimize his opponent’s gain. As a result, the game only runs through 18 cells<sup>13</sup> (Figure 22). One line (*III*) and one column (*IV*) are never chosen. As each agent tries to minimize his adversary’s gain, conductor *Y* never chooses column *IV* to prevent conductor *X* from choosing line *III* and getting that maximum gain (5 points). Concerning the trajectory, it is possible to detect a group of irregularly repeating patterns (Figure 23):  $\{36 \rightarrow 35 \rightarrow 5 \rightarrow 6 \rightarrow 36\}$ ,  $\{11 \rightarrow 12 \rightarrow 36 \rightarrow 35 \rightarrow 11\}$ ,  $\{1 \rightarrow 25 \rightarrow 29 \rightarrow 11 \rightarrow 9 \rightarrow 21 \rightarrow 23 \rightarrow 11 \rightarrow 8 \rightarrow 2 \rightarrow 1\}$ ,  $\{11 \rightarrow 8 \rightarrow 20 \rightarrow 23 \rightarrow 11 \rightarrow 12 \rightarrow 36 \rightarrow 35\}$ ,  $\{11 \rightarrow 9 \rightarrow 21 \rightarrow 23\}$ ,  $\{5 \rightarrow 1 \rightarrow 7 \rightarrow 8 \rightarrow 20 \rightarrow 23\}$ , ...

<sup>13</sup> In this simulation the resulting cells are:  $\{1\ 2\ 3\ 5\ 6\ 7\ 8\ 9\ 11\ 12\ 20\ 21\ 23\ 25\ 27\ 29\ 35\ 36\}$ .

	I	II	III	IV	V	VI
I	1	2	3	4	5	6
II	7	8	9	10	11	12
III	13	14	15	16	17	18
IV	19	20	21	22	23	24
V	25	26	27	28	29	30
VI	31	32	33	34	35	36

**Fig. 22:** Cells run through (in gray) by a simulation with a “minmax” strategy



**Fig. 23:** Game dynamics resulting from the simulation with a “minmax” strategy. X axis, cells 1 to 36. Y axis, moves.

### 3.5 Practical Problems in Performance

*Duel* is a challenge to perform according to the composer’s wishes. Both conductors have to communicate with their orchestras to let them know which tactic is next and where they should start. One should avoid pauses resulting from hesitation or communication, as silence is a choice, and corresponds to the tactics VI. A preliminary observation could conclude that, according to the conductors’ wishes, the strategy chosen leads to structural or formal issue. A completely random strategy and a weighted random strategy lead to a random structure. A “minmax”, with or without gain, strategy leads to a game generating cycles and more or less repetitive patterns. A strategy, being a combination of weighted and “minmax”, may generate a less rigid structure. This means that the conductors should have a tool to switch between strategies, avoiding “degenerate” ways of performing (see 3.3). For this reason *Duel* requires an interface to display the game’s results and to communicate. Xenakis had conceived an electrical interface [11, 1] based on the use of switches, relays and brackets with electrical bulbs of various colors. The need to change score swiftly

according to the choice of tactics, leads to CAP (Computer Aided Performance) interface to display the chosen score.

#### 4. Conclusion

While trying to compare both works and their respective requirements, we can find some similarities and some differences. They are both musical discourse with two agents. Two *open* works, that is, two works where the final form is not pre-established, or set by the composer. However, the performers combine pre-established musical elements according to specific rules. Both pieces contain detailed instructions for smooth going, and require a specific preparation for rehearsal and performance. Both pieces are based on the manipulation of six ensembles, either instrumental ensembles, as is the case in *Domaines*, or ensembles of timbres as is the case in *Duel*.

On the other hand, in *Domaines* the ordering of the parts is determined by a combinatorial model based on permutations. In *Duel*, the evolution of the piece is based on a model originating in games theory. In *Domaines*, the order of the sequences is predetermined, while in *Duel*, the composer insists that sequences must be decided upon during the performance of the piece. In *Domaines* Boulez allows (or does not oppose to) a choice of permutations based on musical considerations, accepting the fact that “some permutations work better than others”. In *Duel*, Xenakis demands that conductors do not bother with these musical considerations. He wants them to play according to the rules of the game matrix, and takes full responsibility for the resulting sound. *Domaines* is a sequential discourse with minimal superposition of events. The superposition is just a short overlapping between sections, which provides fluidity to the musical discourse. In *Duel* the superposition is the fundamental element. Finally, *Domaines* has a fixed length, with a final form that can be seen as a variation on the rondo. As for *Duel*, it has an indeterminate length with a form based on repeating sequences and on cycles resulting from the dynamics of the game matrix.

The  $M_6(\mathbb{Z})$  algebra is, in various ways, at the foundation of both works. However, each composer uses this structure differently. In *Domaines*, we could see the underlying matrix structure as an array “temporal positions  $\times$  musical groups”, while in Xenakis’ *Duel*, the matrix is an array “musical groups  $\times$  musical groups”, where the final sonorities are issues by a Cartesian product. In *Domaines*, the permutation matrix gives directly the structure of a section, while in *Duel*, the final structure is derived from the game dynamics.

The epistemological and historical matrix status shows us that this structure is an important element in the unification process of mathematical knowledge [4, 41] that characterizes the beginning of the 20<sup>th</sup> century. It is due mainly to the fact that a matrix can represent different practices and concepts, with the same representation and operatory mode. The openness of the matrix concept is probably what, in these two cases, made it appropriate to represent two different musical esthetics based on different mathematical models, in an “open work” context.



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