Fault-Driven Structural Diagnosis Approach in a Distributed Context

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Abstract: Distributed diagnosis is important for complex systems as a way to reduce computational costs or for large systems that require minimizing data transfer. This paper presents a distributed diagnosis method for continuous systems that only requires the knowledge of local models and limited knowledge of their neighboring subsystems. The notion of Fault-Driven Minimal Structurally Overdetermined (FMSO) set is used as the corner stone of the design of residual generators. We show that all the FMSO sets of the global system can be obtained in a distributed manner from so-called shared FMSO sets and shared CMSO sets that are computed along a structural approach for every local site.

Keywords: Model Based Diagnosis, Structural Analysis, Distributed systems.

1. INTRODUCTION

For complex systems with constraints such as communication bandwidth or large geographic distribution, it is more appropriate to use distributed approaches. In some cases, this is even the only viable solution given structural, computational and robustness issues.

Researchers have developed several decentralized and distributed diagnosis schemes in the past, mostly in the discrete event framework. Distributed schemes, Su and Wonham (2005), unlike decentralized schemes, do not make use of the global system model; instead, they use subsystem models for diagnosis, and the local diagnosers (LDs) for each them, communicate their diagnosis results to each other to obtain the global solution. Distributed diagnosis methods have been proposed recently for continuous systems, Bregon et al. (2014) presents a distributed diagnosis framework for physical systems with continuous behavior using structural model decomposition and employing Possible Conflicts approach. The global system model is decomposed into submodels that contain sufficient analytical redundancy to perform fault detection. However this is done ignoring pre-existing constraints that may be functional, geographical or privacy-based. Khorasgani et al. (2015) presents a distributed approach that provides a set of diagnosers that are as local as possible by extending local models with their neighboring subsystem's until maximal isolability is achieved.

Our approach considers pre-existing constraints mandatory and therefore considers predefined subsystems and in that context is designed in a distributed framework. It does not require a coordinator online, and there is no exchange of diagnosis information among the LDs, only exchange of measurements. Besides, this method introduces important properties of Fault-Driven Minimal Structurally Overdetermined (FMSO) sets Pérez et al. (2015), that allow us to establish the relation between FMSO sets for the subsystems and FMSO sets for the global system. This properties are key to demonstrate that all global FMSO sets can be generated from computations only at the level of the subsystems, hence achieving a truly distributed architecture. We use the structural framework that has shown to be a flexible and efficient tool for fault diagnosis and fault-tolerant control design, Krysander et al. (2010). FMSO sets are used to ensure the minimal redundancy of residual generators in order to optimize LDs. Each subsystem is monitored by a LD using the information provided by measured local variables and, when necessary, by a minimal amount of measurements from neighboring subsystems. In a large industrial systems due to the large number of components it is quite unrealistic to rely on a global model of such systems, accordingly we assume the non-availability of a global system model. The algorithm achieves the same results as a global diagnoser by extending local models as least as possible when it is required.

This paper is structured as follows: in section 2, some concepts of the structural approach are presented and the notion of FMSO set is introduced. Section 3 presents some fault distributed diagnosis concepts and properties for the structural approach. Section 4 explains how to design the set of LDs so that they achieve the same detectability and diagnosability as a centralized diagnoser. A four tanks example illustrates the approach in section 5. Finally, a conclusion and future work end the paper.

2. BACKGROUND THEORY

2.1 Analytical Redundancy via Structural Analysis

Let the system description consist of a set of n_e equations involving a set of variables partitioned into a set Z of n_Z

known (or measured) variables and a set X of n_X unknown (or unmeasured) variables. We refer to the vector of known variables as z and the vector of unknown variables as x. The system may be impacted by the presence of n_f faults that appear as parameters in the equations. F is the set of faults and we refer to the vector of faults as $\mathtt{f}$.

Definition 1. (System). A system, denoted $\Sigma(z, x, \mathtt{f})$ or Σ for short, is any set of equations relating z, x and $\mathtt{f}$. The equations $e_i(z, x) \subseteq \Sigma(z, x, \mathtt{f})$, $i = 1, \ldots, n_e$, are assumed to be differential or algebraic in z and x.

A four tank system (Fig. 1) is used to show the concepts throughout this paper. Its model $\Sigma(z, x, \mathtt{f})$ is composed of 20 equations e_1 to e_{20} relating the known variables $Z = \{u_1, u_2, y_1, y_2, y_3, y_4, y_5, y_6\}$, the unknown variables $X = \{\dot{p}_1, p_1, \dot{p}_2, p_2, \dot{p}_3, p_3, \dot{p}_4, p_4, q_{in1}, q_{in2}, q_1, q_2, q_3, q_4\}$ and the set of system faults $F = \{f_1, f_2, f_3, f_4, f_5, f_6\}$. All the equations are given in Khorasgani et al. (2015).

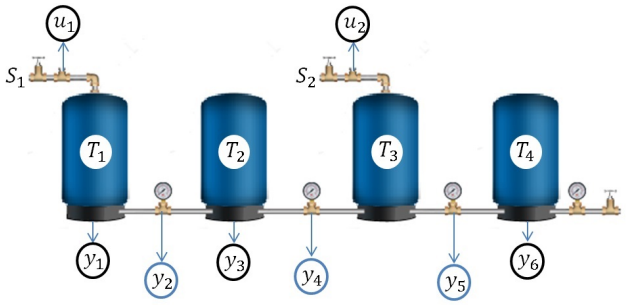

Fig. 1. Four Tank System.

Definition 2. (ARR for $\Sigma(z, x, \mathtt{f})$). Let $\Sigma(z, x, \mathtt{f})$ be a system. Then, a relation $r(z, \dot{z}, \ddot{z}, ...) = 0$ is an Analytical Redundancy Relation (ARR) for $\Sigma(z, x, \mathtt{f})$ if for each z consistent with $\Sigma(z, x, \mathtt{f})$ the relation is fulfilled.

Definition 3. (Residual Generator for $\Sigma(z, x, \mathtt{f})$). A system taking a subset of the variables z as input, and generating a scalar signal r as output, is a *residual generator for the model* $\Sigma(z, x, \mathtt{f})$ if, for all z consistent with $\Sigma(z, x, \mathtt{f})$, it holds that $\lim_{t\to\infty} r(t) = 0$.

ARRs can be used to check if the measured variables z are consistent with the system model and as the basis of residual generators used for diagnosis purposes.

The *structural model* of the system $\Sigma(z, x, \mathtt{f})$, also denoted with some abuse by $\Sigma(z, x, \mathtt{f})$ or Σ in the following, can be obtained abstracting the functional relations. It only retains a representation of which variables are involved in the equations. This abstraction leads to a biadjacency matrix representation between the set of equations of the system and the set X of its unknown variables.

Obtaining ARRs for a system $\Sigma(z, x, \mathtt{f})$ involves the elimination of unknown variables, which can be inferred from structural analysis, Travé-Massuyès et al. (2006). One should notice that results obtained in a structural framework are a best case scenario: causality considerations, algebraic and differential loops, etc. ultimately define which structural redundancies can be used for the design of actual residual generators.

2.2 Focused Residual Generation

A key tool for analyzing a structural model is the Dulmage-Mendelson (DM) canonical decomposition.

It results in a partition of the system model Σ into three parts: the *structurally overdetermined* (SO) part Σ^+ with more equations than unknown variables; the *structurally just determined* part Σ^0, and the *structurally underdetermined* part Σ^- with more unknown variables than equations (Blanke et al., 2006).

Definition 4. (Structural redundancy). The structural redundancy $\rho_{\Sigma'}$ of a set of equations $\Sigma' \subseteq \Sigma$ is defined as the difference between the number of equations and the number of unknown variables.

Proposition 1. Consider two sets of equations $\Sigma' \subseteq \Sigma$ and $\Sigma'' \subseteq \Sigma$, then $\rho_{\Sigma' \cup \Sigma''} = \rho_{\Sigma'} + \rho_{\Sigma''} + |X_{\Sigma'} \cap X_{\Sigma''}|$.

Definition 5. (PSO and MSO sets). A set of equations Σ is proper structurally overdetermined (PSO) if $\Sigma = \Sigma^+$ and minimally structurally overdetermined (MSO) if no proper subset of Σ is overdetermined , Krysander et al. (2010).

Since PSO and MSO sets have more equations than variables, they can be used to generate ARRs and residuals. MSO sets are of special interest since they are just overdetermined, i.e. they have structural redundancy 1. However, not all MSO sets are interesting to construct residual generators, in particular those that are not impacted by faults. Hence it is desirable to consider a fault-focused concept. The concept of *test equation support* (TES) has been introduced in Krysander et al. (2010). A TES is a set of equations expressing redundancy specific to a set of considered faults, known as the *test support* (TS) or as the *fault support*, term that we use in this paper. A minimal TES (MTES) is such that no proper subset is a TES.

It is necessary to notice that, whereas an MSO set is just overdeterminated and hence has redundancy 1, an MTES may have higher redundancy. This may be an advantage to develop more powerful tests; however, for the distribution problem, the aim is to minimize the information shared by subsystems, hence the concept of *Fault-Driven Minimal Structurally Overdetermined* set defined below is preferable, Pérez et al. (2015).

A Fault-Driven Minimal Structurally Overdetermined (FMSO) set can be defined as an MSO set of $\Sigma(z, x, \mathtt{f})$ whose fault support is not empty. In particular, an MTES of structural redundancy 1 is an FMSO set.

Let us define $Z_\varphi \subseteq Z$, $X_\varphi \subseteq X$, and $F_\varphi \subseteq F$ as the set of known variables, unknown variables involved in the FMSO set φ, and the set of faults in its fault support, respectively. We then have the following formal definition.

Definition 6. (FMSO set). A subset of equations $\varphi \subseteq \Sigma(z, x, \mathtt{f})$ is an FMSO set of $\Sigma(z, x, \mathtt{f})$ if (1) $F_\varphi \neq \emptyset$ and $\rho_\varphi = 1$ that means $|\varphi| = |X_\varphi| + 1$, (2) no proper subset of φ is overdeterminated.

We also define the concept of *Clear Minimal Structurally Overdetermined* (CMSO) set as an MSO set of $\Sigma(z, x, \mathtt{f})$ whose fault support is empty.

Definition 7. (CMSO set). A subset of equations $\Lambda \subseteq \Sigma(z, x, \mathtt{f})$ is a CMSO set of $\Sigma(z, x, \mathtt{f})$ if (1) $F_\Lambda = \emptyset$ and

$\rho_\Lambda = 1$ that means $|\Lambda| = |X_\Lambda| + 1$, (2) no proper subset of Λ is overdeterminated.

To illustrate these concepts, we consider an academic example with: $\Sigma = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, $X = \{x_1, x_2, x_3, x_4\}$ and $F = \{f_1, f_2\}$ as shown in Fig. 2.

Eq	Unknown				Faults	
	x_1	x_2	x_3	x_4	f_1	f_2
e_1	X					
e_2	X	X			X	
e_3		X				
e_4		X				
e_5			X			
e_6	X		X	X		X

Fig. 2. Academic example.

If we consider the fault f_1 and use the algorithm proposed in Krysander et al. (2010), there exists an *MTES* focused in fault f_1, $\Sigma_1 = \{e_1, e_2, e_3, e_4\}$ with redundancy $\rho_{\Sigma_1} = 2$. In the other hand, using our approach we can find minimal redundancy by two FMSO sets: $\varphi_1 = \{e_1, e_2, e_3\}$ and $\varphi_2 = \{e_1, e_2, e_4\}$ both focused on fault f_1 which is more efficient for distribution.

3. DISTRIBUTED DIAGNOSIS

This section reconsiders the concept of FMSO set in the distributed case. We establish properties on relations between FMSO sets for the subsystems and FMSO sets for the global system. This properties are key to demonstrate that all global FMSO sets can be generated from computations only at the level of the subsystems, hence achieving a truly distributed architecture.

3.1 Distribution and Related Notions

Let us consider the system Σ and define the following:

Definition 8. (Global FMSO set). A global FMSO set is an FMSO set of $\Sigma(z, x, \mathtt{f})$. The set of global FMSO sets is denoted by Φ.

A decomposition of the system Σ, into several subsystems Σ_i is defined as a partition of its equations. Let $\Sigma = \{\Sigma_1, \Sigma_2, ..., \Sigma_n\}$ with $\Sigma_i \subseteq \Sigma$, $\bigcup_{i=1}^{n} \Sigma_i = \Sigma$, $\Sigma_i \neq \emptyset$ and $\Sigma_i \cap \Sigma_j = \emptyset$ if $i \neq j$.

This decomposition leads to n subsystems denoted $\Sigma_i(z_i, x_i, \mathtt{f}_i)$, with $i = 1, ..., n$, where z_i is the vector of known variables in Σ_i, x_i the vector of unknown variables in Σ_i and $\mathtt{f}_i$ the vector of faults in Σ_i. The set of variables and faults of the ith subsystem Σ_i, denoted as X_i, Z_i, and F_i respectively, are defined as the subset of variables of X, Z, and F respectively, that are involved in the subsystem Σ_i.

For the four tanks system example, we consider that each tank and the outlet pipe to its right, constitute a subsystem. It is for example possible to define $\Sigma_1(z_1, x_1, \mathtt{f}_1)$ as: $\Sigma_1 = \{e_1, e_2, e_3, e_4, e_5, e_6\}$; $F_1 = \{f_1, f_2\}$; $X_1 = \{\dot{p}_1, p_1, p_2, q_{in1}, q_1\}$ and $Z_1 = \{u_1, y_1, y_2\}$.

Definition 9. (Local variables). The set of local variables of the ith subsystem, denoted X_i^l, is defined as the subset of variables of X_i that are only involved in the subsystem Σ_i.

Definition 10. (Shared Variables). The set of shared variables of the ith subsystem, denoted as X_i^s, is defined as:

$$X_i^s = \bigcup_{j=1, j\neq i}^{n} (X_i \cap X_j) = X_i \setminus X_i^l \tag{1}$$

The set of shared variables of the whole system Σ is denoted by X^s.

Without loss of generality, we consider that all known variables of Z_i are local to the subsystem Σ_i, for $i = 1, \ldots, n$. If the same input was applied to several subsystems, it could be artificially replicated.

3.2 Distributed FMSO sets

Definition 11. (Local FMSO set). φ is a local FMSO set of $\Sigma_i(z_i, x_i, \mathtt{f}_i)$ if φ is an MFSO set of $\Sigma(z, x, \mathtt{f})$ and if $\varphi \subseteq \Sigma_i$, $X_\varphi \subseteq X_i$ and $Z_\varphi \subseteq Z_i^l$. The set of local FMSO sets of Σ_i is denoted by Φ_i^l. The set of all local FMSO sets is denoted by $\Phi^l = \bigcup_{i=1}^{n} \Phi_i^l$.

Obviously, a local FMSO set for any subsystem Σ_i is also an FMSO set of Σ, hence a global FMSO set.

For the four tanks example, a local FMSO set $\varphi_1 = \{e_1, e_3, e_4, e_5, e_6\}$ is obtained for Σ_1. These equations include local and shared variables of Σ_1 and only involve the fault f_1. It can be deduced that to achieve detectability of fault f_1, only the equations included in φ_1 are required.

We now define *shared FMSO sets* for a subsystem Σ_i by considering shared variables as known variables and computing FMSO sets. FMSO sets including equations with shared variables are called *shared FMSO sets*.

Definition 12. (Shared FMSO set). φ is a shared FMSO set of subsystem $\Sigma_i(z_i, x_i, \mathtt{f}_i)$ if φ is an FMSO set of $\tilde{\Sigma}_i(\tilde{z}_i, \tilde{x}_i, \tilde{\mathtt{f}}_i)$, where $\tilde{z}_i$ is the vector of variables in $\tilde{Z}_i = Z_i \cup X_i^s$, $\tilde{x}_i$ is the vector of variables in $\tilde{X}_i = X_i^l$, and $\tilde{\mathtt{f}}_i = \mathtt{f}_i$. The set of shared FMSO sets for Σ_i is denoted by Φ_i^s. The set of all shared FMSO sets is denoted by $\Phi^s = \bigcup_{i=1}^{n} \Phi_i^s$.

From the above definition, a shared FMSO set φ for subsystem $\Sigma_i(z_i, x_i, \mathtt{f}_i)$ is such that $\varphi \subseteq \Sigma_i$, $X_\varphi \subseteq X_i^l$, $Z_\varphi \cap X_i^s \neq \emptyset$, and $Z_\varphi \subseteq (Z_i \cup X_i^s)$.

For the four tank example, the set of shared FMSO sets for Σ_1 is Φ_1^s is $\{\varphi_1, \varphi_2, \varphi_3\}$, where $\varphi_1 = \{e_2, e_5\}$, $X_{\varphi_1} = \{p_1\}$, $Z_{\varphi_1} = \{q_1, p_2, y_1, y_2\}$, $F_{\varphi_1} = \{f_2\}$.

Definitions 11 and 12 can also be applied to CMSO sets to define *local CMSO sets* Λ_i^l and *shared CMSO sets* Λ_i^s. The set of all shared CMSO sets is denoted by Λ^s.

Definition 13. (Compound FMSO set). A global FMSO set φ that includes at least one shared FMSO set $\varphi' \in \Phi_i^s$ is called a compound FMSO set. The set of compound FMSO sets of Σ_i is denoted by Φ_i^c. The set of all compound FMSO sets is denoted by $\Phi^c = \bigcup_{i=1}^{n} \Phi_i^c$.

Definition 14. (Root FMSO set). If a compound FMSO set $\varphi \in \Phi^c$ includes a shared FMSO set $\varphi' \in \Phi_i^s$, then φ' is a root FMSO set of φ with respect to system Σ_i.

Definition 15. (Locally detectable fault). $f \in F_i$ is locally detectable in the subsystem $\Sigma_i(z_i, x_i, \mathtt{f}_i)$ if there is an FMSO set $\varphi \in \Phi_i^l$ such that $f \in F_\varphi$.

Definition 16. (Locally isolable fault). Given two locally detectable faults f_j and f_k of F_i, $j \neq k$, f_j is locally isolable from f_k if there exists an FMSO set $\varphi \in \Phi_i^l$ such that $f_j \in F_\varphi$ and $f_k \notin F_\varphi$.

3.3 Properties of FMSO sets

This section aims at stating the properties of locally computed FMSO sets, i.e. local FMSO sets and shared FMSO sets, with regards to the generation of global FMSO sets. Interestingly, these properties allow us to prove that the whole set of global FMSO sets Φ can be obtained from the set of locally computed FMSO sets.

Property 1. A local FMSO set $\varphi \in \Phi^l$ is also a global FMSO set.

Property 2. A global FMSO set $\varphi \in \Phi$ for which $\exists! i \in 1, \ldots, n$ s.t. $X_\varphi \subseteq X_i^l$ is also a local FMSO set of Σ_i.

In the following, we show that global FMSO sets can be obtained from locally computed FMSO sets only, by forming compound FMSO sets with shared FMSO sets and shared CMSO sets.

Begin with a simple reasoning. Consider a shared FMSO set $\varphi \in \Phi_i^s$. The particularity of shared FMSO sets is that they are computed hypothesizing that the shared variables they include are known (cf. Definition 12). Actually, this hypothesis is just a trick that allows us to account locally for the FMSO sets that can possibly be generated if equations of other subsystems, indicated by the shared variables, are introduced. However, shared variables are actually unknown so we can define $X_\varphi^s = Z_\varphi \cap X^s$. The shared FMSO set φ can give rise to a global FMSO set if it can be supplemented with sets of equations from other subsystems (more precisely shared FMSO or CMSO sets) to balance the number of shared variables X_φ^s of φ and achieve structural redundancy 1. Let us notice that φ has a structural redundancy of $1 - |X_\varphi^s|$. As a matter of fact, every shared variable $x^s \in X_\varphi^s$ decreases the structural redundancy of φ by 1. Consider a shared FMSO set $\varphi' \in \Phi_j^s, j \neq i$ for which x^s is also a shared variable, i.e. $x^s \in X_{\varphi'}^s$. By Proposition 1, unioning φ' to φ potentially balances the structural redundancy deficiency for one shared variable, say x^s, in φ. However, if φ' introduces new shared variables, these also need to be balanced, each by an additional shared FMSO set. In addition, if x^s is not the only shared variable of φ, the other shared variables each require unioning a different shared FMSO set. The same reasoning also holds if φ' is a shared CMSO set. This leads to the following proposition.

Proposition 2. Let $G(\mathbb{X}, \Gamma)$ be a bipartite graph such that $\mathbb{X} = \mathbb{X}_1 \cup \mathbb{X}_2$ where: $\mathbb{X}_1 = \Phi^s \cup \Lambda^s$, $\mathbb{X}_2 = X^s$ and $\Gamma : \mathbb{X}_1 \longrightarrow 2^{\mathbb{X}_2}$ is a function that gives the set of successors of each $\varphi \in \mathbb{X}_1$. Let $\varphi \in \mathbb{X}_1$ and $x \in \mathbb{X}_2$ then (φ, x) belongs to the edges of G if $x \in X_\varphi$.

A compound FMSO set $\mathbb{X}_1'$ is built by a subgraph $G_s(\mathbb{X}', \Gamma')$ of $G(\mathbb{X}, \Gamma)$, where $\mathbb{X}' = \mathbb{X}_1' \cup \mathbb{X}_2'$, $\mathbb{X}_1' \subset \mathbb{X}_1$, $\mathbb{X}_2' \subset \mathbb{X}_2$ if: (i) $G_s(\mathbb{X}', \Gamma')$ contains no cycles.(ii) $\forall \varphi \in \mathbb{X}_1', \Gamma(\varphi) \subset \mathbb{X}_2'$ and $\forall x \in \mathbb{X}_2' \; \exists \varphi \in \mathbb{X}_1'$ such that $\Gamma'(\varphi) = x$. (iii) The terminal nodes of the graph belong to $\mathbb{X}_1'$.

Proposition 2 states the conditions for which a union of shared FMSO/CMOS sets originating from different subsystems forms a compound FMSO set. Condition (ii) guarantees that if an FMSO set belongs to the subgraph, then all shared variables are in this subgraph and for all shared variables there exists one shared FMSO/CMSO set that belongs to a subsystem different from any subsystem at the above level. Conditions (i) and (iii) guarantee that the structural redundancy of $\mathbb{X}_1'$ is equal to one and that $\mathbb{X}_1' = \varphi^c$ is a compound FMSO set.

Equivalently to Proposition 2 and in accordance with Chanthery et al. (2015) (Proposition 1 and its proof), compound FMSO can be characterized as sets of FMSO/CMSO that are MSOs with respect to shared variables. Compound FMSO sets can hence be found by running the FMSO generation algorithm (the algorithm run for every subsystem) considering FMSO/CMSO sets as equations and shared variables as unknown variables. Proposition 2 is stated in a form that makes the optimization problem aiming at only generating the compound FMSO sets that guarantee maximal diagnosability while minimizing shared information easier to formulate as a search problem.

Lemma 1. The subgraph $G_s(\mathbb{X}', \Gamma')$ corresponding to a compound FMSO set has a specific AND/OR tree structure (Fig. 3).

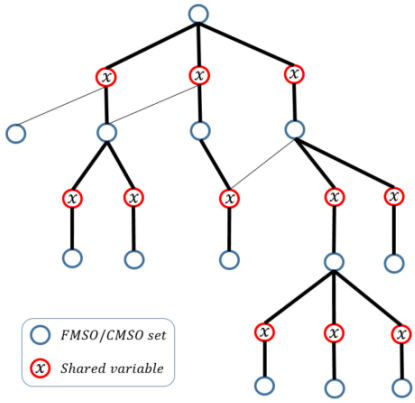

Fig. 3. AND/OR tree structure of a compound FMSO set.

The FMSO set at the top of Fig. 3 is considered as the root FMSO set. Its set of shared variables is then included in the structure. For each of them, only one FMSO set is chosen among the FMSO/CMSO sets that include the shared variable. For each chosen FMSO/CMSO set, the shared variables are included in the structure. This property repeats down the graph levels until there is no additional shared variable to include in the structure. We talk of an *iterative matching procedure*. It can be proved that all the global FMSO sets can be obtained from locally computed FMSO sets.

Proposition 3. The set of global FMSO sets Φ is given by the union of the set of local FMSO sets Φ^l and the set of compound FMSO sets Φ^c.

$$\Phi = \Phi^l \cup \Phi^c \tag{2}$$

Algorithm 1. Generation of the set of global FMSO sets

$\Phi = \emptyset$;
for i=1...n **do**
  $\Phi_i^l \leftarrow$ Calculate local FMSO sets of Σ_i;
  $\Phi_i^s \leftarrow$ Calculate shared FMSO sets of Σ_i;
  $\Lambda_i^s \leftarrow$ Calculate shared CMSO sets of Σ_i;
  for each shared FMSO set $\varphi \in \Phi_i^s$ **do**
    Label φ as root FMSO: $\varphi_r \leftarrow \varphi$;
    Let $X^s_{\varphi_r}$ be the set of shared variables of φ_r;
    while *it is possible to find a set* $\varphi^c \supseteq \varphi_r$ *that can be a set* $\mathbb{X}_1'$ *in Proposition* 2 *and such that* φ^c *is not included in* Φ **do**
      Store the global FMSO set φ^c:
      $\Phi \leftarrow \Phi \cup \varphi^c$;
    end while
  end for
  $\Phi \leftarrow \Phi \cup \Phi_i^l$;
end for
Return Φ;

4. OPERATIONAL PROCEDURE FOR DISTRIBUTED DIAGNOSIS

4.1 Distributed generation of all global FMSO sets

Our approach assumes the non-availability of a global system model. However, if needed, Algorithm 1 implements the procedure for computing the set of global FMSO sets starting with the local stage. As Khorasgani et al. (2015), our approach guarantees maximal diagnosability, i.e. the same diagnosability as a centralized approach. For each shared FMSO set computed at a local level, the iterative matching procedure is used to cover all the shared variables. The procedure is repeated for the new sets of shared variables that come with newly introduced shared FMSO sets. Iterations stop when no new shared variables are introduced. The computational complexity of the search problem increases with the number of shared variables. However, in practice, subsystems are generally designed so that their links are quite weak, hence sharing few variables. This makes the proposed approach applicable to complex dynamic systems made up of several subsystems.

4.2 Distributed generation of an optimized set of global FMSO sets

If the residuals corresponding to all the global FMSO sets were generated and used on-line to monitor the system, they would obviously achieve maximal detectability and isolability. However, all of them are not necessary and it is more efficient to minimize their number while maintaining the same property. The aim of this section is to obtain a set of distributed LD that together make the entire system completely diagnosable through local and compound FMSO sets. These LDs are designed to achieve maximal diagnosability with minimal communication between subsystems. First, local FMSO sets are determined for every subsystem Σ_i. If these are not sufficient to detect and isolate all of the faults in F_i, then a set of compound FMSO sets is determined to achieve full diagnosability, considering constraints of distance and amount of communication between subsystems.

Algorithm 2. Generation of LDs.

for i=1...n **do**
  $\Phi_i = \emptyset$;
  $\Phi_i^l \leftarrow$ Calculate local FMSO sets of Σ_i;
  if *there is any fault* $f \in F_i$ *not locally detectable or not locally isolable with the set of local FMSO sets* Φ_i^l **then**
    $\Phi_i^s \leftarrow$ Calculate shared FMSO sets of Σ_i;
    $\Lambda_i^s \leftarrow$ Calculate shared CMSO sets of Σ_i;
  end if
  while *it exists* $f \in F_i$ *that is not detectable or isolable* **do**
    Let $\varphi^* \in \Phi_i^s$ such that $f \in F_{\varphi^*}$ be the 'best' (not already selected) shared FMSO set of Φ_i^s;
    Label φ^* as root FMSO set: $\varphi_r \leftarrow \varphi^*$;
    Let $X^s_{\varphi_r}$ be the set of shared variables of φ_r;
    $\Phi_i^{c*} \leftarrow$ *Find a 'good' compound FMSO set including* φ^* *by always selecting the 'best' shared FMSO sets to cover newly introduced* X_i^s
    $\Phi_i \leftarrow \Phi_i \cup \Phi_i^{c*}$;
    $\Phi_i^{l*} \leftarrow$ *Find a minimal cardinality set of local FMSO sets achieving the same diagnosability as all local FMSO sets;*
    $\Phi_i \leftarrow \Phi_i \cup \Phi_i^{l*}$;
  end while
end for

The diagnosers design is done off-line with Algorithm 2, performed for each subsystem Σ_i, $i = 1...n$. The procedure to compute 'good' compound FMSO sets starting with φ^* as a root FMSO set makes use of an optimization heuristic based on the number of shared variables. In Algorithm 2, the term 'best' is hence used in the sense of this heuristic. Further work must be performed to assess the properties of the heuristic in terms of optimality.

5. APPLICATION TO THE FOUR TANKS SYSTEM

5.1 Finding of Global FMSO sets

Running Algorithm 1 on the four tanks system, we calculate local FMSO sets Φ_i^l, shared FMSO sets Φ_i^s and shared CMSO sets Λ_i^s of each subsystem ($i = 1..4$) as shown in Table 1. Then with each shared FMSO set as root FMSO set, we found all compound FMSO sets $\varphi \in \Phi^c$.For example, in Σ_1, considering the shared FMSO set φ_1 as a root FMSO set, a compound FMSO set is computed iteratively by the set $\varphi^c = \cup_{k=1}^{c} \varphi_k = \varphi_1 \cup \varphi_5 \cup \varphi_6 \cup \lambda_3 \cup \varphi_7 \cup \lambda_4 \cup \lambda_6$, with $\cup_{k=1}^{c} X^s_{\varphi_k} = \{q_1, p_2, q_2, p_3, q_3, p_4\}$, where each shared variable x^s is covered by two shared FMSO/CMSO sets as it is shown in the subgraph (Fig. 4). As a result, the compound FMSO set φ' obtained is $\{e_2, e_5, e_7, e_8, e_9, e_{11}, e_{13}, e_{16}, e_{20}\}$. Considering all possible φ^c, 164 compound FMSO sets are computed for this system. Added to $\varphi_4 = \{e_1, e_3, e_4, e_5, e_6\} \in \Phi_1^l$, we found 165 global FMSO sets in Φ.

5.2 Distributed Diagnosis

Given a set of faults, measurements and local models for every subsystem, we construct diagnosers that together make the entire system completely diagnosable. Computing the set of local FMSO sets Φ_i^l , $i = 1..4$ and adding

Table 1. local FMSO sets Φ_i^l, shared FMSO sets Φ_i^s and shared CMSO sets: Λ_i^s, $(i = 1..4)$.

$\Phi_1^s = \{\varphi_1, \varphi_2, \varphi_3\}$, $\Phi_1^l = \{\varphi_4\}$, $\Lambda_1^s = \{\lambda_1\}$,
$\Phi_2^s = \{\varphi_5, \varphi_6\}$, $\Phi_2^l = \emptyset$, $\Lambda_2^s = \{\lambda_2, \lambda_3\}$,
$\Phi_3^s = \{\varphi_7\}$, $\Phi_3^l = \emptyset$, $\Lambda_3^s = \{\lambda_4, \lambda_5\}$,
$\Phi_4^s = \{\varphi_8\}$, $\Phi_4^l = \emptyset$, $\Lambda_4^s = \{\lambda_6\}$,

Φ_i	X^s						F_i
Σ_1	q_1	p_2	q_2	p_3	q_3	p_4	F_1
$\varphi_1 = \{e_2, e_5\}$	X	X					$\{f_2\}$
$\varphi_2 = \{e_1, e_3, e_4, e_5\}$	X						$\{f_1\}$
$\varphi_3 = \{e_1, e_2, e_3, e_4\}$	X	X					$\{f_1, f_2\}$
$\varphi_4 = \{e_1, e_3, ..., e_6\}$	X	X					$\{f_1\}$
$\lambda_1 = \{e_6\}$	X						

Σ_2	q_1	p_2	q_2	p_3	q_3	p_4	F_2
$\varphi_5 = \{e_8\}$		X	X	X			$\{f_4\}$
$\varphi_6 = \{e_7, e_9\}$	X	X	X				$\{f_3\}$
$\lambda_2 = \{e_{10}\}$		X					
$\lambda_3 = \{e_{11}\}$			X				

Σ_3	q_1	p_2	q_2	p_3	q_3	p_4	F_3
$\varphi_7 = \{e_{13}\}$				X	X	X	$\{f_5\}$
$\lambda_4 = \{e_{16}\}$					X		
$\lambda_5 = \{e_{12}, e_{14}, e_{15}\}$			X	X	X		

Σ_4	q_1	p_2	q_2	p_3	q_3	p_4	F_4
$\varphi_8 = \{e_{17}, e_{18}, e_{19}\}$					X	X	$\{f_6\}$
$\lambda_6 = \{e_{20}\}$						X	

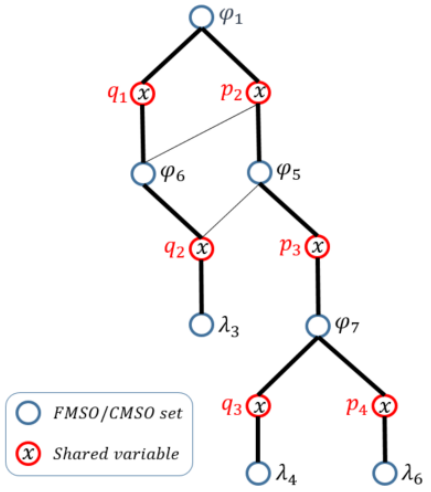

Fig. 4. Subgraph of φ'.

subsets of shared variables to found the set of shared FMSO sets Φ_i^s for each subsystem $i = 1..4$, we found FMSO sets for all faults.

The results demonstrate that all considered faults can be detected and isolated, e.g. in Σ_1: detectability is achieved for f_1 using $\varphi_4 \in \Phi_1^l$ of Table 1 (not additional measurement is needed). For f_2, detectability is achieved obtaining a compound FMSO set φ_9 lumping $\varphi_1 \in \Phi_1^s$ with $\lambda_1 \in \Lambda_1^s$ and $\lambda_2 \in \Lambda_2^s$. Fig. 5 shows a scheme of the proposed model based diagnosis for this system: the four subsystems with their physical interactions are represented on the left. On the right, each LD_i is rendered as a rectangle with selected FMSO sets. The arrows from the corresponding subsystem symbolize the direct measurement of local variables by the

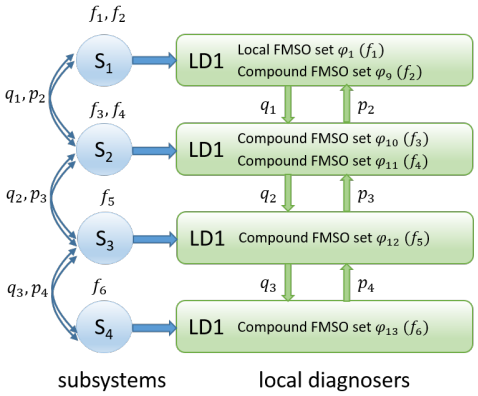

Fig. 5. Scheme of the distributed diagnosis designed.

LD, while the arrows between the LDs account for shared information necessary to complete local diagnosis.

6. CONCLUSION

In this paper, a distributed fault-driven structural diagnosis method is presented. FMSO sets are used in a distributed context to derive local tests and to build relevant tests minimizing shared information between subsystems. The operational procedures are presented. Future work will emphasize the optimization process.

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