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STATISTICAL THEORY OF HEAT

TIAN MA AND SHOUHONG WANG

ABSTRACT. In this paper, we present a new statistical theory of heat based on the statistical theory of thermodynamics and on the recent developments of quantum physics. One motivation of the theory is the lack of physical carriers of heat in classical thermodynamics. Another motivation is the recent discovered photon cloud structure of electrons. This leads to a natural conjugate relation between electrons and photons, reminiscent of the conjugate relation between temperature and entropy. The new theory in this paper provides a natural connection between these two conjugate relations: at the equilibrium of absorption and radiation, the average energy level of the system maintains unchanged, and represents the temperature of the system; at the same time, the number (density) of photons in the sea of photons represents the entropy (entropy density) of the system. The theory contains four parts: 1) the photon number formula of entropy, 2) the energy level formula of temperature, 3) the temperature theorem, and 3) the thermal energy formula. In particular, the photon number entropy formula is equivalent to the Boltzmann entropy formula, and, however, possesses new physical meaning that the physical carrier of heat is the photons.

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1. INTRODUCTION

The main objective of this paper is to establish a new theory of heat, based on the statistical theory of thermodynamics and on the recent developments of quantum physics. The theory contains four closely related parts:

- (1) photon number formula of entropy,
- (2) energy level formula of temperature,
- (3) a temperature theorem, and
- (4) thermal energy formula.

Hereafter first we address the main motivations and main results of the theory, and we recapitulate the needed theoretical foundations on statistical physics and quantum physics.

Motivations.

The main motivations of the new theory are three-fold.

First, as we know the current accepted theory of heat is also called the mechanical theory of heat, which related the heat with mechanical work. The theory was first introduced in 1798 by Benjamin Thompson, and further developed by such great scientists as Sadi Carnot, Rudolf Clausius, and James Clerk Maxwell. This is further developed into the modern theory of thermodynamics and statistical physics; see among many others [18, 22, 2, 19, 8].

However one of the remaining puzzling question is how the heat is transferred. As we know heat transfer is classified into various mechanisms, such as thermal conduction, thermal convection, thermal radiation, and transfer of energy by phase changes. However, the nature of heat transfer is still not fully understood.

To be precise, by classical thermodynamics, for a thermodynamic system, the thermal energy Q_0 is given by

$$(1.1) \quad Q_0 = ST.$$

Here T is the temperature of the system and S is the entropy given by the Boltzmann formula:

$$(1.2) \quad S = k \ln W,$$

where k is the Boltzmann constant, and W is the number of microscopic configurations of the system. It is then clear that

$$(1.3) \quad \begin{array}{l} \textit{in modern thermodynamics, there is simply no physical heat} \\ \textit{carrier in both the temperature } T \textit{ and the entropy } S, \textit{ and} \\ \textit{hence there is no physical carrier for thermal energy } Q_0 = \\ \textit{ST.} \end{array}$$

Historically, the caloric theory of heat was developed for such a purpose, and becomes now an obsolete theory. Basically, the caloric theory says that heat is made up of a fluid called caloric that is massless and flows from hotter bodies to colder bodies. It was considered that caloric was a massless gas that exists in all matter, and is conserved. However the transfer between heat and mechanical work makes the caloric theory obsolete.

Known physical facts show that the essence of thermal radiation is the absorption and radiation of photons, and as we shall see below, the new statistical theory of heat makes precise that photons are indeed the needed physical carrier of heat.

The second motivation of the study is the recent developments in particle physics, which reveal the photon cloud structure of electrons; see Section 3.2, as well as [13] for details. It is shown that there is an attracting shell region of weak interaction between the naked electron and photons as in Figure 1.1. Since photons carry weak charges, they are attached to the electron in the attracting shell region forming a cloud of photons.

The attracting shell region in Figure 1.1 of an electron results in the ability for the electron to attract and emit photons. A macroscopic system is immersed in a sea of photons (mediators). When a photon enters the attracting shell region of an electron, it will be absorbed by the electron. An electron emits photons as its velocity changes, which

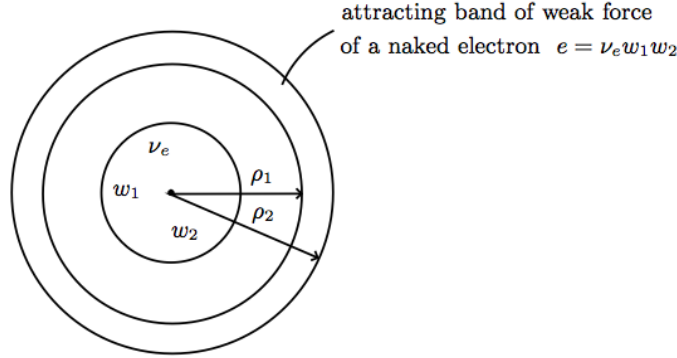


FIGURE 1.1. A cloud of photons forms in the region $\rho_1 < r < \rho_2$ near the naked electron.

is called the bremsstrahlung. Also, when the orbiting electron jumps from higher energy level to a lower energy level, it radiates photons. Hence electrons in the system are constantly in a state of absorbing and emitting photons, resulting changes on their energy levels. In other words, we reach the following physical conclusion:

(1.4) *in the microscopic world, electrons and photons form a natural conjugate pair of physical carriers for emission and absorption.*

This physical conclusion leads to the *third motivation* of the theory of heat developed in this paper. Namely, in view of (1.1), heat is attributed to the conjugate relation between temperature and entropy. Hence by (1.4), a theory of heat has to make connections between the two conjugate relations:

$$(1.5) \quad \begin{array}{c} \text{conjugation between electrons and photons} \\ \Updownarrow \\ \text{conjugation between temperature and entropy.} \end{array}$$

The new theory in this paper provides precisely such a connection: at the equilibrium of absorption and radiation, the average energy level of the system maintains unchanged, and represents the temperature of the system; at the same time, the number (density) of photons in the sea of photons represents the entropy (entropy density) of the system.

Main results

We now ready to address briefly the four parts of the statistical theory of heat developed in this paper.

1. *Energy level formula of temperature.* In view of the relation (1.5), temperature must be associated with the energy levels of electrons,

since it is an intensive physical quantity measuring certain strength of heat, reminiscent of the basic characteristic of energy levels of electrons. Also notice that there are abundant orbiting electrons and energy levels of electrons in atoms and molecules. Hence the energy levels of orbiting electrons, together with the kinetic energy of the system particles, provide a truthful representation of the system particles.

We derive the following energy level formula of temperature using the well-known Maxwell-Boltzmann, the Fermi-Dirac, and the Bose-Einstein distributions:

$$(1.6) \quad kT = \begin{cases} \sum_n \left(1 - \frac{a_n}{N}\right) \frac{a_n \varepsilon_n}{N(1 + \beta_n \ln \varepsilon_n)} & \text{for classical systems,} \\ \sum_n \left(1 + \frac{a_n}{g_n}\right) \frac{a_n \varepsilon_n}{N(1 + \beta_n \ln \varepsilon_n)} & \text{for Bose systems,} \\ \sum_n \left(1 - \frac{a_n}{g_n}\right) \frac{a_n \varepsilon_n}{N(1 + \beta_n \ln \varepsilon_n)} & \text{for Fermi systems.} \end{cases}$$

Here ε_n are the energy levels of the system particles, N is the total number of particles, g_n are the degeneracy factors (allowed quantum states) of the energy level ε_n , and a_n are the distributions, representing the number of particles on the energy level ε_n .

If we view

$$\bar{\varepsilon} = \frac{1}{N} \sum_n a_n \varepsilon_n$$

as the average energy level for the thermodynamical system, then the above formulas amount to saying that temperature is simply the (weighted) average energy level of the system.

Formulas (1.6) enable us to have a better understanding on the nature of temperature.

In summary, the nature of temperature T is the (weighted) average energy level. Also, the temperature T is a function of distributions $\{a_n\}$ and the energy levels $\{\varepsilon_n\}$ with the parameters $\{\beta_n\}$ reflecting the property of the material.

From the temperature formula, we can easily see that for a thermodynamic system with temperature at absolute zero, all particles fill the lowest energy levels. Also, it is not hard to see from the temperature formula the existence of highest temperature.

2. *Photon number formula of entropy.* In view of (1.5), since entropy S is an extensive variable, we need to characterize entropy as the number of photons in the photon gas between system particles, or

the photon density of the photon gas in the system. Also, photons are Bosons and obey the Bose-Einstein distribution.

Then we can make a connection between entropy and the number of photons and derive

$$(1.7) \quad S = kN_0 \left[1 + \frac{1}{kT} \sum_n \frac{\varepsilon_n a_n}{N_0} \right],$$

where ε_n are the energy levels of photons¹, and a_n are the distribution of photons at energy level ε_n , $N_0 = \sum_n a_n$ is the total number of photons between particles in the system, and $\sum_n \frac{\varepsilon_n}{kT} a_n$ represents the number of photons in the sense of average energy level.

It is worth mentioning that this new entropy formula is equivalent to the Boltzmann entropy formula (1.2). However, their physical meanings have changed: the new formula (1.7) provides explicitly that

$$(1.8) \quad \textit{the physical carrier of heat is the photons.}$$

3. *Temperature theorem.* By the temperature and the entropy formulas (1.6) and (1.7), we arrive immediately at the following results of temperature, stated in Theorem 5.2 as the law of temperature:

- (1) *There are minimum and maximum values of temperature with $T_{\min} = 0$ and T_{\max} being given by (4.33);*
- (2) *When the number of photons in the system is zero, the temperature is at absolute zero; namely, the absence of photons in the system is the physical reason causing absolute zero temperature;*
- (3) (Nernst Theorem) *With temperature at absolute zero, the entropy of the system is zero;*
- (4) *With temperature at absolute zero, all particles fills all lowest energy levels.*

4. *Thermal energy formula.* Thanks to the entropy formula (1.7), we derive immediately the following thermal energy formula:

$$(1.9) \quad Q_0 = ST = E_0 + kN_0T,$$

where $E_0 = \sum_n a_n \varepsilon_n$ is the total energy of *photons* in the system, ε_n are the energy levels of photons, and a_n are the distribution of photons at energy level ε_n , and N_0 is the number of photons in the system.

Statistical physics and quantum physics foundations

¹We emphasize here that for brevity we use the same ε_n to denote, respectively, the energy levels for photons in (1.7), and the energy levels for system particles (electrons) in (1.6).

The theory of heat presented in this paper is established based on physical theories on fundamental interactions, the photon cloud model of electrons, the first law of thermodynamics, statistical theory of thermodynamics, radiation mechanism of photons, and energy level theory of micro-particles. The theory utilizes rigorous mathematics to reveal the physical essence of temperature, entropy and heat.

1. *Statistical physics foundation.* In deriving the temperature and the entropy formulas (1.6) and (1.7), we make a direct use of the Maxwell-Boltzmann (MB), the Fermi-Dirac (FD), and the Bose-Einstein (BE) distributions. These distributions are respectively for classical systems, the Fermi systems, and the Bose systems.

In a recent paper [17], the authors postulated the potential-descending principle (PDP). We show that PDP is a more fundamental principle than the first and second laws in thermodynamics, gives rise to dynamical equations for non-equilibrium systems, and serves as the first principle to describe irreversibility of all thermodynamic systems. Also, together with the Boltzmann entropy formula (1.2) and the classical principle of equal probability stated in Principle 2.2, PDP leads to all three distributions. Hence for the new theory of heat, the needed statistical physics foundation is the Boltzmann entropy formula, principle of equal probability and the potential descending principle.

2. *Quantum physics foundation.* It is clear that a theory of heat depends on the quantum behavior of basic microscopic constituents of matter. We refer interested readers to [24, 5, 4, 6, 13] for modern theory of quantum mechanics and particle physics.

The following recent developments in quantum physics play a crucial role for our study in this paper:

- (1) weak interaction force formula,
- (2) weakton model of elementary particles,
- (3) photon cloud structure of electrons,
- (4) photon absorption and radiation mechanism of electrons, and
- (5) energy levels of micro-particles.

These quantum physics foundations are recapitulated in Section 3, and we also refer the interested readers to [13] and the references therein for more details.

The paper is organized as follows. Section 2 recalls the potential-descending principle (PDP) and the three basic statistics: the Maxwell-Boltzmann distribution, the Fermi-Dirac distribution and the Bose-Einstein distribution. Section 3 recapitulates quantum physics basis needed for the theory of heat. In Sections 4 and 5 we establish the

main components of the theory, and Section 6 addresses the nature of heat based on the new theory.

2. PRINCIPLES OF STATISTICAL PHYSICS

In this section, we introduce some basic principles for statistical physics, serving as the statistical foundation of the statistical theory of heat to be developed in Sections 4-6. We refer interested readers to [20, 21, 9, 12, 11, 7, 3, 23, 1] among many others for classical theories of statistical physics. The potential-descending principle is introduced by the authors in [17].

2.1. Potential-descending principle. For a given thermodynamic system, the order parameters (state functions) $u = (u_1, \dots, u_N)$, the control parameters λ , and the thermodynamic potential (or potential in short) F are well-defined quantities, fully describing the system. The potential is a functional of the order parameters, and is used to represent the thermodynamic state of the system. There are four commonly used thermodynamic potentials: the internal energy, the Helmholtz free energy, the Gibbs free energy, and the enthalpy.

After a thorough examination of thermodynamics, we discovered in [17] that the following Potential-Descending Principle (PDP) is a fundamental principle in statistical physics.

Principle 2.1 (Potential-Descending Principle). *For each thermodynamic system, there are order parameters $u = (u_1, \dots, u_N)$, control parameters λ , and the thermodynamic potential functional $F(u; \lambda)$. For a non-equilibrium state $u(t; u_0)$ of the system with initial state $u(0, u_0) = u_0$, we have the following properties:*

- 1) *the potential $F(u(t; u_0); \lambda)$ is decreasing:*

$$\frac{d}{dt}F(u(t; u_0); \lambda) < 0 \quad \forall t > 0;$$

- 2) *the order parameters $u(t; u_0)$ have a limit*

$$\lim_{t \rightarrow \infty} u(t; u_0) = \bar{u};$$

- 3) *there is an open and dense set \mathcal{O} of initial data in the space of state functions, such that for any $u_0 \in \mathcal{O}$, the corresponding \bar{u} is a minimum of F , which is called an equilibrium of the thermodynamic system:*

$$\delta F(\bar{u}; \lambda) = 0.$$

We have shown that PDP is a more fundamental principle than the first and second laws, and provides the first principle for describing irreversibility, leads all three distributions: the Maxwell-Boltzmann distribution, the Fermi-Dirac distribution and the Bose-Einstein distribution in statistical physics. Consequently, the potential-descending principle is the first principle of statistical physics.

Also importantly, based on PDP, the dynamic equation of a thermodynamic system in a non-equilibrium state takes the form

$$(2.1) \quad \frac{du}{dt} = -A\delta F(u, \lambda) \quad \text{for isolated systems,}$$

$$(2.2) \quad \begin{cases} \frac{du}{dt} = -A\delta F(u, \lambda) + B(u, \lambda), \\ \int A\delta F(u, \lambda) \cdot B(u, \lambda) = 0 \end{cases} \quad \text{for coupled systems,}$$

where δ is the derivative operator, B represents coupling operators, and A is a symmetric and positive definite matrix of coefficients. We refer interested readers to [17] for details.

2.2. Statistical distributions. For a thermodynamic system, a main component of statistical theory is to study the probability distribution of particles in different energy levels of the system in equilibrium. In this section, we aim to derive all three distributions: the Maxwell-Boltzmann distribution, the Fermi-Dirac distribution and the Bose-Einstein distribution, based only on 1) the principle of equal *a priori* probabilities, and 2) the potential-descending principle (PDP). The latter was stated in the last section, and the principle of equal *a priori* probabilities can be stated as follows:

Principle 2.2 (Principle of Equal Probability (PEP)). *An equilibrium thermodynamic system has an equal probability of being in any microstate that is consistent with its current macrostate.*

2.2.1. Maxwell-Boltzmann statistics. Now consider an isolated classical thermodynamic system, the total energy E and the total number of particles N are constants. Each particle in the system must be situated on a energy level ε , and the total energy is finite. Since the system is in equilibrium, we have

$$(2.3) \quad a_n \stackrel{\text{def}}{=} [\text{number of particles on } \varepsilon_n] = [\text{probability at } \varepsilon_n] \times N.$$

The PEP ensures the time-independence of a_n . We have then the following arrangement of number of particles on different energy levels:

$$\begin{aligned}
(2.4) \quad & \varepsilon_1 < \varepsilon_2 < \cdots < \varepsilon_{N_E}, \\
& g_1 & g_2 & \cdots & g_{N_E}, \\
& a_1 & a_2 & \cdots & a_{N_E},
\end{aligned}$$

where g_n represents the degeneracy factor (allowed quantum states) of the energy level ε_n . For an isolated system,

$$(2.5) \quad N = \sum_n a_n = \text{constant}, \quad E = \sum_n a_n \varepsilon_n = \text{constant}.$$

It is clear that the multiplicity function W for the distribution (2.4) is given by:

$$(2.6) \quad W = W(a_1, \dots, a_{N_E}).$$

The aim is then to find the relations between ε_n and a_n under constraint (2.5):

$$(2.7) \quad a_n = f(\varepsilon_n, T) \quad \text{for } 1 \leq n \leq N_E.$$

For an isolated thermodynamic system, the temperature T is a control parameter, and consequently, its thermodynamic potential functional is

$$(2.8) \quad F = E - ST,$$

where E is the total energy, and S is the entropy. Classically, the entropy is given by the famous Boltzmann formula:

$$(2.9) \quad S = k \ln W,$$

where $k = 1.381 \times 10^{-23} J/K$ is the Boltzmann constant.

By the potential-descending principle, Principle 2.1, the distribution $\{a_n\}$ at the thermodynamic equilibrium solves the following minimal potential variational equations of the potential functional (2.8):

$$(2.10) \quad \frac{\delta}{\delta a_n} \left[-kT \ln W + \alpha_0 \sum_n a_n + \beta_0 \sum_n a_n \varepsilon_n \right] = 0,$$

where α_0 and β_0 are the Lagrangian multipliers of constraints (2.5).

In view of the distribution of particles (2.4), the multiplicity function W is given by

$$(2.11) \quad W = \frac{N!}{\prod_n a_n!} \prod_n g_n^{a_n},$$

By the Stirling formula

$$k! = k^k e^{-k} \sqrt{2\pi k},$$

and using $\ln(2\pi k) \ll k$, we have

$$\ln W = N \ln N - N - \sum_n a_n \ln a_n + \sum_n a_n + \sum_n a_n \ln g_n.$$

Since $\sum_n a_n = N$, we arrive at

$$\ln W = N \ln N - \sum_n a_n \ln \frac{a_n}{g_n},$$

which implies from (2.10) that

$$\ln \frac{a_n}{g_n} + \alpha + \beta \varepsilon_n = 0 \quad \text{for } 1 \leq n \leq N_E.$$

This gives rise to the famous Maxwell-Boltzmann distribution:

$$(2.12) \quad a_n = \frac{N}{Z} g_n e^{-\varepsilon_n/kT}.$$

Here Z is the partition function defined by

$$(2.13) \quad Z = \sum_n g_n e^{-\beta \varepsilon_n},$$

Also, direct computation shows that the MB distribution is indeed the minimal point of the potential functional (2.8) under the constraints (2.5).

The partition function Z defined by (2.13) is another important thermodynamical quantity in statistical mechanics. In fact, once we know the detailed expression of the partition function, we can derive other related thermodynamical quantities as follows:

$$(2.14) \quad \begin{aligned} U &= -N \frac{\partial}{\partial \beta} \ln Z && \text{internal energy,} \\ S &= Nk \left(\ln Z - \beta \frac{\partial}{\partial \beta} \ln Z \right) && \text{entropy,} \\ f &= -\frac{N}{\beta} \frac{\partial}{\partial X} \ln Z && \text{generalized force,} \\ F &= -NkT \ln Z && \text{potential functional.} \end{aligned}$$

2.2.2. Bose-Einstein distribution. The Maxwell-Boltzmann statistics is for classical systems of particles. For systems where quantum behavior is prominent, quantum statistics is then ultimately needed. Quantum statistics consists of the Bose-Einstein (BE) statistics for systems of bosonic particles, and the Fermi-Dirac statistics for systems of fermionic particles.

The goal here is the same as the statistics for classical particle systems: to find the relations (2.7) between ε_n and a_n under constraint (2.5).

For a quantum system of bosonic particles, the multiplicity function associated with (2.4) is

$$(2.15) \quad W_{\text{BE}} = \prod_n \frac{g_n + a_n - 1)!}{a_n!(g_n - 1)!}.$$

As for the Maxwell-Boltzmann distribution, solving (2.10) leads to the following Bose-Einstein distribution:

$$(2.16) \quad a_n = \frac{g_n}{e^{(\varepsilon_n - \mu)/kT} - 1},$$

where μ is the chemical potential.

For a quantum system of bosonic particles, the partition function is given by

$$(2.17) \quad Z = \prod_n (1 - e^{-\alpha - \beta \varepsilon_n})^{-g_n}, \quad \alpha = -\frac{\mu}{kT}, \quad \beta = \frac{1}{kT}.$$

Also we can derive other related thermodynamical quantities, the total number of particles N , the total energy E , the generalized force f , the pressure p , the entropy S , the free energy F , and the Gibbs energy G , in terms of the partition function Z as follows:

$$(2.18) \quad \begin{aligned} N &= -\frac{\partial}{\partial \alpha} \ln Z, \\ E &= -\frac{\partial}{\partial \beta} \ln Z, \\ f &= -\frac{1}{\beta} \frac{\partial}{\partial X} \ln Z, \\ p &= \frac{1}{\beta} \frac{\partial}{\partial V} \ln Z, \\ S &= k \left[\ln Z - \left(\alpha \frac{\partial}{\partial \alpha} + \beta \frac{\partial}{\partial \beta} \right) \ln Z \right], \\ F &= E - ST = -kT \left(\ln Z - \alpha \frac{\partial}{\partial \alpha} \ln Z \right), \\ G &= -NkT\alpha = kT\alpha \frac{\partial}{\partial \alpha} \ln Z. \end{aligned}$$

2.2.3. Fermi-Dirac distribution. Fermions obey the Pauli exclusion principle. Hence for a quantum system of fermions, the multiplicity function is given by

$$(2.19) \quad W_{\text{FD}} = \prod_n \frac{g_n!}{a_n!(g_n - a_n)!}.$$

Then it is easy to find the following Fermi-Dirac distribution (2.7) between ε_n and a_n under constraint (2.5):

$$(2.20) \quad a_n = \frac{g_n}{e^{(\varepsilon_n - \mu)/kT} + 1},$$

where μ is the chemical potential.

Also the partition function for a Fermi system is given by

$$(2.21) \quad Z = \prod_n (1 + e^{-\alpha - \beta \varepsilon_n})^{g_n}, \quad \alpha = -\frac{\mu}{kT}, \quad \beta = \frac{1}{kT}.$$

The relations between the partition function and other thermodynamical quantities are given by (2.18) as well.

3. QUANTUM PHYSICS FOUNDATIONS

We introduce in this section some recent developments in quantum physics, which serve as the quantum physics foundations for the statistical theory of heat that we introduce in this paper. For a more detailed account of these recent developments, see [13].

3.1. Weakton model of elementary particle. The weakton model of elementary particle was first introduced by the authors [14, 13]. This theory proposes six elementary particles, which we call weaktons, and their anti-particles:

$$(3.1) \quad \begin{array}{cccccc} w^*, & w_1, & w_2, & \nu_e, & \nu_\mu, & \nu_\tau, \\ \bar{w}^*, & \bar{w}_1, & \bar{w}_2, & \bar{\nu}_e, & \bar{\nu}_\mu, & \bar{\nu}_\tau, \end{array}$$

where ν_e, ν_μ, ν_τ are the three generation neutrinos, and w^*, w_1, w_2 are three new particles, which we call w -weaktons. These are all massless particles with spin $J = \frac{1}{2}$. Each of them carries a weak charge, and only w^* and \bar{w}^* carry a strong charge. Also, the neutrinos do not carry electric charge, w^* carries $2/3$ electric charge, w_1 carries $-1/3$ electric charge and w_2 carries $-2/3$ electric charge.

1) *Weakton constituents of particles*

The weakton constituents of charged leptons and quarks are given by

$$\begin{aligned}
 (3.2) \quad & e = \nu_e w_1, w_2, \quad \mu = \nu_\mu w_1 w_2, \quad \tau = \nu_\tau w_1 w_2, \\
 & u = w^* w_1 \bar{w}_1, \quad c = w^* w_2 \bar{w}_2, \quad t = w^* w_2 \bar{w}_2, \\
 & d = w^* w_1 w_2, \quad s = w^* w_1 w_2, \quad b = w^* w_1 w_2,
 \end{aligned}$$

where c, t and d, s, b are distinguished by the spin arrangements.

The weakton constituents of the mediators and their dual mediators are given by

$$\begin{aligned}
 (3.3) \quad & \gamma = \cos \theta_w w_1 \bar{w}_1 - \sin \theta_w w_2 \bar{w}_2 \quad (\uparrow\uparrow, \downarrow\downarrow) \quad \text{vector photon,} \\
 & \gamma_0 = \cos \theta_w w_1 \bar{w}_1 - \sin \theta_w w_2 \bar{w}_2 \quad (\uparrow\downarrow, \downarrow\uparrow) \quad \text{scalar photon,} \\
 & g^k = w^* \bar{w}^* (\uparrow\uparrow, \downarrow\downarrow), \quad \text{vector gluons,} \\
 & g_0^k = w^* \bar{w}^* (\uparrow\downarrow, \downarrow\uparrow) \quad \text{scalar gluons}
 \end{aligned}$$

The ν -mediator ν has spin-0 with the following weakton constituents:

$$(3.4) \quad \nu = \alpha_1 \nu_e \bar{\nu}_e + \alpha_2 \nu_\mu \bar{\nu}_\mu + \alpha_3 \nu_\tau \bar{\nu}_\tau \quad (\downarrow\uparrow), \quad \sum_{l=1}^3 \alpha_l^2 = 1.$$

Each gluon carries two strong charges and two weak charges, and participates both the weak and strong interactions. Both photon and the ν mediator only carry respectively two weak charges, and participate the weak interaction, but not the strong interaction. All three mediators carry no electric charge.

2) Mass generation mechanism

For a particle moving with velocity v , its mass m and energy E obey the Einstein relation

$$(3.5) \quad E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Usually, we regard m as a static mass which is fixed, and energy is a function of velocity v .

Now, taking an opposite viewpoint, we regard energy E as fixed, mass m as a function of velocity v , and the relation (3.5) is rewritten as

$$(3.6) \quad m = \sqrt{1 - \frac{v^2}{c^2}} \frac{E}{c^2}.$$

Thus, (3.6) means that a particle with an intrinsic energy E has zero mass $m = 0$ if it moves at the speed of light $v = c$, and will possess nonzero mass if it moves with a velocity $v < c$.

All particles including photons can only travel at the speed sufficiently close to the speed of light. Based on this viewpoint, we can think that if a particle moving at the speed of light (approximately) is decelerated by an interaction force \vec{F} , obeying

$$\frac{d\vec{P}}{dt} = \sqrt{1 - \frac{v^2}{c^2}} \vec{F},$$

then this massless particle will generate mass at the instant. In particular, by this mass generation mechanism, several massless particles can yield a massive particle if they are bound in a small ball, and rotate at velocities less than the speed of light.

For the mass problem, we know that the mediators:

$$(3.7) \quad \gamma, g^k, \nu \quad \text{and their dual particles,}$$

have no masses. To explain this, we note that these particles in (3.3) and (3.4) consist of pairs as

$$(3.8) \quad w_1 \bar{w}_1, \quad w_2 \bar{w}_2, \quad w^* \bar{w}^*, \quad \nu_l \bar{\nu}_l.$$

The weakton pairs in (3.8) are bound in a circle with radius R_0 as shown in Figure 3.1. Since the interacting force on each weakton pair is in the direction of their connecting line, they rotate around the center O without resistance. As $\vec{F} = 0$ in the moving direction, by the relativistic motion law:

$$(3.9) \quad \frac{d}{dt} \vec{P} = \sqrt{1 - \frac{v^2}{c^2}} \vec{F},$$

the massless weaktons rotate at the speed of light. Hence, the composite particles formed by the weakton pairs in (3.8) have no rest mass.

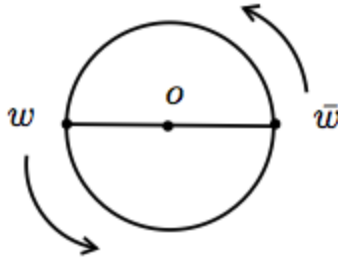


FIGURE 3.1

For the massive particles

$$(3.10) \quad e, \mu, \tau, u, d, s, c, t, b,$$

by (3.2), they are made up of weakton triplets with different electric charges. Hence the weakton triplets are arranged in an irregular triangle as shown in Figure 3.2. Consequently, the weakton triplets rotate with nonzero interacting forces $\vec{F} \neq 0$ from the weak and electromagnetic interactions. By (3.9), the weaktons in the triplets at a speed less than the speed of light due to the resistance force. Thus, by the mass generating mechanism above, the weaktons become massive. Hence, the particles in (3.10) are massive.

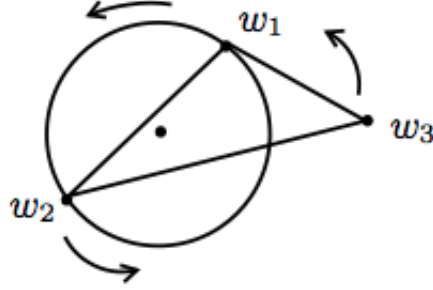


FIGURE 3.2

3.2. Photon cloud model of electrons.

3.2.1. *Photon clouds of electrons.* The weakton constituents of an electron are $\nu_e w_1 w_2$. Therefore, an electron carries three weak charges, which are the source of the weak interaction. A photon has the weakton constituents as given by (3.3), and carries two weak charges.

The weak force formula between the naked electron and a photon γ is given by; see [13]:

$$(3.11) \quad \begin{aligned} F &= -g_w(\rho_\gamma)g_w(\rho_e)\frac{d}{dr}\left[\frac{1}{r}e^{-kr} - \frac{B}{\rho}(1+2kr)e^{-2kr}\right], \\ &= g_w(\rho_m)g_w(\rho_e)e^{-kr}\left[\frac{1}{r^2} + \frac{1}{rr_0} - \frac{4B}{\rho}\frac{r}{r_0^2}e^{-kr}\right], \end{aligned}$$

where $k = 1/r_0 = 10^{16} \text{ cm}^{-1}$, $g_w(\rho_m)$ and $g_w(\rho_e)$ are the weak charges of mediators and the naked electron, expressed as

$$g_w(\rho_\gamma) = 2\left(\frac{\rho_w}{\rho_\gamma}\right)^3 g_w, \quad g_w(\rho_e) = 3\left(\frac{\rho_w}{\rho_e}\right)^3 g_w,$$

and B/ρ is a parameter determined by the naked electron and the photon.

By the weak force formula (3.11), there is an attracting shell region of weak interaction between naked electron and the photon

$$(3.12) \quad F < 0 \quad \text{for } \rho_1 < r < \rho_2,$$

as shown in Figure 1.1, which is reproduced in Figure 3.3 for convenience. Since photons carry weak charges, they are attached to the

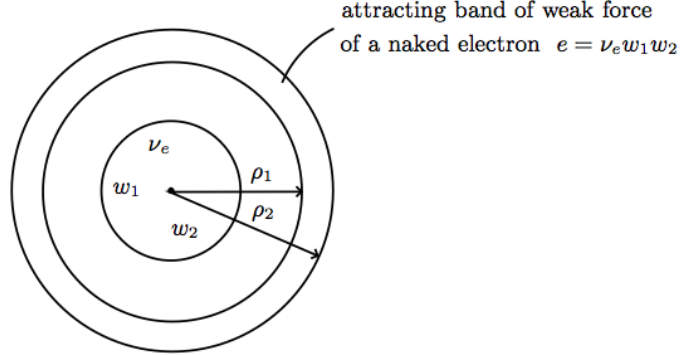


FIGURE 3.3. Electron structure.

electron in the attracting shell region (3.12), forming a cloud of photons. The irregular triangle distribution of the weaktons ν_e, w_1, w_2 generate a small moment of force on the mediators. Meanwhile there also exist weak forces between them. Therefore the bosons will rotate at a speed less than the speed of light, and generate a small mass attached to the naked electron $\nu_e w_1 w_2$.

3.2.2. Angular momentum rule. The Angular Momentum Rule 3.1 below was first discovered in [14, 13, 15]. It ensures that the photons in the clouds of electrons can only be scalar photons $J = 0$, and consequently the photon cloud of an electron does not change the spin of the electron $J = 1/2$.

Angular Momentum Rule 3.1. *Only the fermions with spin $J = \frac{1}{2}$ and the bosons with $J = 0$ can rotate around a center with zero moment of force. The particles with $J \neq 0, \frac{1}{2}$ will move on a straight line unless there is a nonzero moment of force present.*

Also we remark that based on the mechanism of decay and scattering of particles, weakton exchanges may occur during the following γ - γ scattering process, leading to the transformation between scalar photons and vector photons:

$$\gamma + \gamma \longrightarrow \gamma + \gamma,$$

and the corresponding weakton constituent exchange is given by

$$w_1\bar{w}_1(\uparrow\uparrow) + w_1\bar{w}_1(\downarrow\downarrow) \rightleftharpoons w_1\bar{w}_1(\uparrow\downarrow) + w_1\bar{w}_1(\downarrow\uparrow).$$

This observation shows that although the photons in the photon cloud of an electron can only scalar photons, both scalar and vector photons are abundant in Nature.

3.3. Photon absorption and radiation mechanism of electrons.

The attracting shell region in Figure 3.3 of an electron results in the ability for the electron to attract and emit photons. A macroscopic system is immersed in a sea of photons (mediators). When a photon enters the attracting shell region of an electron, it will be absorbed by the electron. An electron emits photons as its velocity changes, which is called the bremsstrahlung. Also, when the orbiting electron jumps from higher energy level to a lower energy level, it radiates photons. Hence electrons in the system are constantly in a state of absorbing and emitting photons, resulting changes on their energy levels. As we shall see in the next section, at the equilibrium of absorption and radiation, the average energy level of the system maintains unchanged, and represents the temperature of the system; at the same time, the number (density) of photons in the sea of photons represents the entropy (entropy density) of the system.

The reasons why bremsstrahlung can occur is unknown in classical theories. Based on the electron structure theory in Section 3.2, this phenomenon can be easily explained.

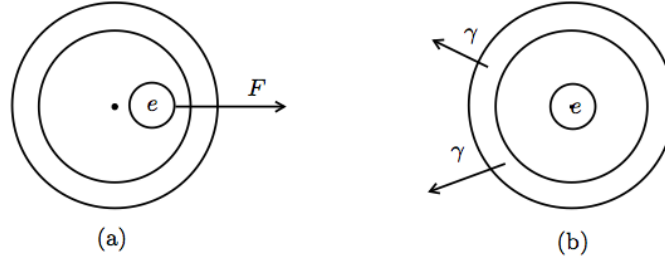


FIGURE 3.4. (a) The naked electron is accelerated or decelerated in an electromagnetic field; and (b) the mediators (photons) fly away from the attracting shell region under a perturbation of moment of force.

In fact, if an electron is situated in an electromagnetic field, then the electromagnetic field exerts a Coulomb force on the naked electron $\nu_e w_1 w_2$, but not on the attached neutral mediators. Thus, the naked electron changes its velocity, which draws the mediator cloud to move

as well, causing a perturbation to moment of force on the mediators. As the attracting weak force in the shell region (3.6) is small, under the perturbation, the centrifugal force makes some mediators in the cloud, such as photons, flying away from the attracting shell region, and further accelerated by the weak repelling force outside this shell region to the speed of light, as shown in Figure 3.4.

3.4. Energy levels of particles. This section is based on [16, 13]. The mass m , energy E and the momentum \vec{p} of a particle obey the Einstein energy-momentum relation; see among many others [10, 13]:

$$(3.13) \quad E^2 = m^2 c^4 + c^2 p^2.$$

There are different energy levels, which can undergo changes by 1) absorbing and/or emitting photons, and 2) exchanging the interior constituents. The weaktons are elementary particles, and all other particles are composite. For composite particles, the energy levels determined by their constituents are called intrinsic energy levels, which can change through exchanging constituents.

Energy levels of particles play an important role in statistical physics. There are many particles in Nature, and we are interested in energy levels of the following particles, which play crucial role in statistical physics:

$$(3.14) \quad \textit{photons, electrons, and atoms.}$$

Hereafter we focus on the energy levels of these particles.

3.4.1. Energy levels of photons. The weakton constituents of a photon are the following two weaktons, symmetrically bounded together by the weak force:

$$\gamma = w_i \bar{w}_i \quad i = 1, 2.$$

It suffices for us to consider the bounded states of one weakton. As the weakton $w_i (i = 1, 2)$ are massless, the wave function describing them is the two-component Weyl spinor:

$$(3.15) \quad \psi = (\psi_1, \psi_2),$$

and the corresponding wave equations are

$$(3.16) \quad (\vec{\sigma} \cdot \vec{D}) \frac{\partial \psi}{\partial t} = c(\vec{\sigma} \cdot \vec{D})^2 \psi - \frac{ig_w}{2\hbar} \{(\vec{\sigma} \cdot \vec{D}), A_0\} \psi,$$

where $\{A, B\} = AB + BA$ is the anti-commutator, $\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ is the Pauli matrix operator, the operator \vec{D} is defined by

$$(3.17) \quad \vec{D} = \nabla + i \frac{g_w}{\hbar c} \vec{W},$$

with $W_\mu = (W_0, \vec{W})$ being the weak interaction potential of weaktons.

The spectral equation of photons can be derived by setting

$$\psi = e^{-i\lambda t/\hbar} \varphi, \quad \varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix},$$

where λ is the bounding energy. We infer from (3.16) that

$$(3.18) \quad -\hbar c(\vec{\sigma} \cdot \vec{D})^2 \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} + \frac{ig_w}{2\hbar} \{(\vec{\sigma} \cdot \vec{D}), W_0\} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = i\lambda(\vec{\sigma} \cdot \vec{D}) \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}.$$

Since the weaktons are confined in the photon, we can set $\psi = 0$ outside of the photon. Consequently, we have the following boundary condition:

$$(3.19) \quad \varphi = 0 \quad \text{for } |x| = \rho_\gamma,$$

where ρ_γ is the radius of photons.

We can then derive the following conclusions for energy levels of photons using the above linear eigenvalue problem (3.18) and (3.19).

(1) There are finite number of negative eigenvalues for (3.18) and (3.19), representing the bounding energies of the weaktons:

$$(3.20) \quad \infty < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N < 0;$$

(2) There are finite number of energy levels for photons, given by

$$(3.21) \quad E_k = E_0 + \lambda_k \quad \text{for } 1 \leq k \leq N,$$

where E_0 is the intrinsic energy of the two weakton constituents of the photon. Hence the energy levels of a photon are finite:

$$(3.22) \quad 0 < E_1 \leq E_2 \leq \cdots \leq E_N;$$

(3) The frequencies of a photon are discrete:

$$(3.23) \quad \omega_k = E_k/\hbar, \quad \Delta\omega_k = \omega_{k+1} - \omega_k = (\lambda_{k+1} - \lambda_k)/\hbar;$$

(4) The number of energy levels of photons can be estimated as follows:

$$(3.24) \quad N = \left(\frac{B_w \rho_\gamma g_w^2}{\beta_1 \rho_w \hbar c} \right)^3 \simeq 10^{90},$$

and the energy differences can be estimated as

$$(3.25) \quad \Delta E \simeq \frac{E_{\max} - E_{\min}}{N} = \frac{\lambda_N - \lambda_1}{N} \simeq 10^{-45} \text{eV},$$

which is small and unobservable.

3.4.2. *Energy levels of electrons.* The electrons are massive with three weakton constituents: $\nu_e w_1 w_2$. As mentioned earlier, these three weaktons possess different electric charges, and are arranged in an irregular triangle as shown in Figure 3.2, becoming the massive. Hence they are governed by three Dirac spinors:

$$\psi^j = (\psi_1^j, \dots, \psi_4^j), \quad j = 1, 2, 3.$$

It is then easy to derive the energy level equation for an electron as follows:

$$(3.26) \quad -\frac{\hbar^2}{2m_j}(\nabla + i\frac{2g_w}{\hbar c}\mathbf{W})^2\varphi^j + 2(2g_w W_0 + \vec{\mu}_j \cdot \text{curl}\mathbf{W})\varphi^j = \lambda\varphi^j, \\ \varphi^j = 0 \quad \text{for } j = 1, 2, 3, \quad |x| = \rho_e,$$

where ρ_e is the radius of an electron, $W_\mu = (W_0, \mathbf{W})$ is the weak interaction potential, $\varphi^j = (\varphi_1^j, \varphi_2^j)$ are the eigenstates of the j-th weakton, and

$$\vec{\mu}_j = \frac{\hbar g_w}{2m_j} \vec{\sigma}$$

is the weak magnetic moment of the j-th weakton.

We derive from (3.26) the following conclusions:

- 1) The intrinsic energy levels of electrons are finite and discrete.
- 2) The number N of intrinsic energy levels of an electron can be approximately estimated as

$$N = \left[\frac{4}{\lambda_1} \frac{B_w \rho_e^2}{\rho_w} \frac{m_w c}{\hbar} \frac{g_w^2}{\hbar c} \right]^{3/2} \sim 10^{45},$$

where ρ_e is the radius of the electron, ρ_w is the radius of the weakton, B_w is the weak interaction parameter in (3.11), m_w is the mass of the constituent weaktons of the electron caused by nonzero interacting force from the weak and the electromagnetic interactions, and λ_1 is the first eigenvalue of $-\Delta$.

- 3) In view of the photon cloud structure that an electron consists of the naked electron and the shell-layer of its photons cloud, the total number N of energy levels of electrons is about

$$N = \text{number of intrinsic energy levels} \\ \times \text{number of energy levels of photon} \simeq 10^{135}.$$

3.4.3. *Energy levels of atoms.* Classical energy levels was developed based on the Bohr atomic models and the Schrödinger equations. The is made up of nucleus and the orbiting electrons, the nucleus is made up of protons p and neutrons n , which are made up of three quarks:

$p = uud$, $n = udd$. In addition, the weakton constituents of upper and down quarks are $u = w^*w_1\bar{w}_1$ and $d = w^*w_1w_2$. Therefore, the energy levels of an atom is the sum of energy levels of the nucleus and the energy levels of the orbiting electrons, and the energy levels of the nucleus are determined by

$$\text{energy levels of a nucleus} = E_k^1 + \lambda_j^1,$$

where E_k^1 are the energy levels of nucleons, λ_j^1 are the negative eigenvalues of the spectral equation for the atom, representing the bounding energies bounding the nucleons. Therefore,

$$E_k^1 = E_l^2 + \lambda_j^2 + \text{energy levels of absorbed mediators of the nucleons.}$$

Here E_l^2 are the energy levels of quarks, λ_j^2 are the negative eigenvalues of the spectral equation of the nucleons, representing the bounding energy between quarks. Finally,

$$E_l^2 = E_0^3 + \lambda_j^3 + \text{energy levels of the mediators absorbed by quarks,}$$

where E_0^3 is the intrinsic energy of the weaktons in the quark, and λ_j^3 is the negative eigenvalues of the spectral equation for the quark, representing the bounding energies bounding the weaktons inside the quarks.

3.4.4. Physical conclusions of energy levels of particles. In summary, we have the following physical conclusions for energy levels of particles, which provide the particle physics foundation of statistical physics.

- (1) The energies of micro-particles are on their energy levels, and there are finite number of energy levels, which are discrete;
- (2) Particles can jump to different different energy levels by a) absorbing or emitting photons, and b) exchanging their constituent particles.
- (3) The energy of a particle obeys the Einstein energy-momentum relation (3.13). When the energy level of a particle changes, its mass and momentum will undergo changes as well. For a fixed energy level, the mass and the momentum can undergo transformations between each other.
- (4) The number N of energy levels is large, and the gap between adjacent energy levels are small; they can be estimated roughly estimated as

$$\text{number of energy levels} \simeq \begin{cases} 10^{90} & \text{for photons,} \\ 10^{135} & \text{for electrons,} \\ 10^{300} & \text{for atoms,} \end{cases}$$

$$\Delta E_k = E_{k+1} - E_k \simeq 10^{-45} \text{ eV} \quad \text{for photons.}$$

4. ENERGY LEVEL FORMULA OF TEMPERATURE

We have introduced the photon cloud structure of subatomic particles in the previous section. In this section, we use such structure of sub-atomic particles to reveal the nature of temperature and entropy.

Basically, among basic constituents of matter, electrons, protons and neutrons are fundamentally important. Both protons and neutrons are confined in the nucleons, and electrons are the only charged particles abundant inside the matter. The essence of thermal radiation is the radiation and absorption of photons. With the photon cloud structure of electrons, electrons and photons form a pair of conjugate physical carriers for absorption and emission associated with thermal radiation. On the other hand, thermal energy is the conjugate relation between temperature and entropy. Hence a correct statistical theory of heat must make a precise connection of the following correspondence:

$$(4.1) \quad \begin{array}{c} \text{conjugation between electrons and photons} \\ \Updownarrow \\ \text{conjugation between temperature and entropy.} \end{array}$$

4.1. Derivation of temperature formula. The main objective of this section is to derive the following temperature formula:

$$(4.2) \quad kT = \begin{cases} \sum_n \left(1 - \frac{a_n}{N}\right) \frac{a_n \varepsilon_n}{N(1 + \beta_n \ln \varepsilon_n)} & \text{for classical systems,} \\ \sum_n \left(1 + \frac{a_n}{g_n}\right) \frac{a_n \varepsilon_n}{N(1 + \beta_n \ln \varepsilon_n)} & \text{for Bose systems,} \\ \sum_n \left(1 - \frac{a_n}{g_n}\right) \frac{a_n \varepsilon_n}{N(1 + \beta_n \ln \varepsilon_n)} & \text{for Fermi systems.} \end{cases}$$

If we view

$$\bar{\varepsilon} = \frac{1}{N} \sum_n a_n \varepsilon_n$$

as the average energy level for the thermodynamical system, then the above formulas shows that temperature is simply the weighted average energy level of the system.

Hereafter we derive these formulas using the basic distributions.

Classical systems. Consider a classical equilibrium thermodynamic system with energy levels of the particles given by

$$(4.3) \quad \varepsilon_1, \quad \varepsilon_2, \dots, \quad \varepsilon_{N_E}.$$

By the MB distribution (2.12), the total energy of the system is

$$(4.4) \quad E = \sum_n a_n \varepsilon_n = \frac{N}{Z} \sum_n g_n \varepsilon_n e^{-\varepsilon_n/kT},$$

where N is the total number of particles, and $Z = \sum_n g_n e^{-\frac{\varepsilon_n}{kT}}$ is the partition function.

When we find the total energy E , we can view (4.4) as an equation defining an implicit function of the temperature T in terms of the energy levels in (4.3):

$$T = T(\varepsilon_1, \dots, \varepsilon_{N_E}).$$

Physically, it means that under the invariance of the total energy E , the distribution $\{a_n\}$ changes as $\{\varepsilon_n\}$ vary, leading to the change of the temperature T . Hence we can assume the following expression of T :

$$(4.5) \quad T = \sum_n \alpha_n T(\varepsilon_n),$$

where the coefficients α_n are to be determined.

Physically, it is natural to assume that

$$(4.6) \quad \begin{aligned} & \text{fluctuations on a specific energy level } \varepsilon_n \text{ will only lead to fluctuations} \\ & \text{on the energy } a_n \varepsilon_n \text{ on the level } \varepsilon_n \text{ in the total energy} \\ & E = \sum_m a_m \varepsilon_m. \end{aligned}$$

Mathematically, by (4.6), the implicit function relation can be determined using the following variation:

$$(4.7) \quad \begin{aligned} 0 = \frac{\partial E}{\partial \varepsilon_n} &= N g_n \frac{\partial}{\partial \varepsilon_n} \left[\frac{\varepsilon_n}{Z} e^{-\varepsilon_n/kT} \right] \\ &= N \left[g_n \left(\frac{1}{Z} - \frac{\varepsilon_n}{Z^2} \frac{\partial Z}{\partial \varepsilon_n} \right) e^{-\varepsilon_n/kT} \right. \\ &\quad \left. + \frac{g_n \varepsilon_n}{Z} e^{-\varepsilon_n/kT} \left(-\frac{1}{kT} + \frac{\varepsilon_n}{kT^2} \frac{\partial T}{\partial \varepsilon_n} \right) \right], \end{aligned}$$

$$(4.8) \quad \begin{aligned} \frac{\partial Z}{\partial \varepsilon_n} &= \left(-\frac{g_n}{kT} + \frac{g_n \varepsilon_n}{kT^2} \frac{\partial T}{\partial \varepsilon_n} \right) e^{-\varepsilon_n/kT} \\ &= -\frac{Z}{N} \frac{a_n}{kT} + \frac{a_n \varepsilon_n}{kT^2} \frac{Z}{N} \frac{\partial T}{\partial \varepsilon_n}. \end{aligned}$$

Consequently, we have

$$0 = a_n - \frac{a_n \varepsilon_n}{Z} \frac{\partial Z}{\partial \varepsilon_n} - \frac{a_n \varepsilon_n}{kT} + \frac{a_n \varepsilon_n^2}{kT^2} \frac{\partial T}{\partial \varepsilon_n}$$

$$= a_n + \frac{a_n^2 \varepsilon_n}{N} \frac{1}{kT} - \frac{a_n^2 \varepsilon_n^2}{N k T^2} \frac{\partial T}{\partial \varepsilon_n} - \frac{a_n \varepsilon_n}{kT} + \frac{a_n \varepsilon_n^2}{k T^2} \frac{\partial T}{\partial \varepsilon_n},$$

which implies the following differential equation for $T(\varepsilon_n)$:

$$(4.9) \quad \frac{\partial T}{\partial \varepsilon_n} = \frac{T}{\varepsilon_n} - k \left(1 - \frac{a_n}{N} \right)^{-1} \frac{T^2}{\varepsilon_n^2}.$$

Let $x = \varepsilon$, then we infer from (4.9) that

$$(4.10) \quad T' = \frac{T}{x} - k \left(1 - \frac{a_n}{N} \right)^{-1} \frac{T^2}{x^2}.$$

Now let

$$y \stackrel{\text{def}}{=} \frac{T}{x}, \quad \text{then } T' = xy' + y.$$

Equation (4.10) becomes

$$(4.11) \quad xy' = -k \left(1 - \frac{a_n}{N} \right)^{-1} y^2.$$

Physically, we may assume that

$$(4.12) \quad \frac{a_n}{N} \simeq \text{constant}.$$

Then (4.11) is equivalent to

$$(4.13) \quad - \left(1 - \frac{a_n}{N} \right) \int \frac{dy}{ky^2} = \int \frac{dx}{x}.$$

Since $x = \varepsilon_n$ carries the dimension of energy, we write the solution of (4.13) as follows:

$$\frac{(1 - a_n/N)}{ky} = \ln \frac{\varepsilon_n}{\varepsilon_0} + C_n,$$

where ε_0 is the unit of energy. Hence we obtain that

$$kT(\varepsilon) = \left(1 - \frac{a_n}{N} \right) \frac{\varepsilon_n}{C_n + \ln \varepsilon / \varepsilon_0}.$$

As discussed earlier, all $T(\varepsilon_m)$ should take the same form, we obtain that

$$kT = \sum \left(1 - \frac{a_n}{N} \right) \frac{\alpha_n \varepsilon_n}{C_n + \ln \varepsilon_n / \varepsilon_0}.$$

Let $\beta_n = 1/C_n$ and $\theta_n = \alpha_n/C_n$, then we have

$$(4.14) \quad kT = \sum_n \left(1 - \frac{a_n}{N} \right) \frac{\theta_n \varepsilon_n}{1 + \beta_n \ln \varepsilon / \varepsilon_0}.$$

We now try to determine the coefficients θ_n . For this purpose, we first define the translational derivative with respect to all energy levels:

$$(4.15) \quad T' = \lim_{\Delta\varepsilon \rightarrow 0} \frac{1}{\Delta\varepsilon} \left[T(\varepsilon_1 + \Delta\varepsilon, \varepsilon_2 + \Delta\varepsilon, \dots) - T(\varepsilon_1, \varepsilon_2, \dots) \right] \\ = \sum_n \theta_n \left(1 - \frac{a_n}{N} \right) \left(1 - \beta_n - \beta_n \ln \frac{\varepsilon_n}{\varepsilon_0} \right).$$

Then we take the translation derivative on both sides of (4.4). By the physical assumption (4.6), we obtain that $\delta\varepsilon_n = \delta\varepsilon$ for all n and

$$(4.16) \quad 0 = \delta E = \sum_n g_n \frac{\partial}{\partial \varepsilon_n} \left[\frac{\varepsilon_n}{Z} e^{-\varepsilon_n/kT} \right] \delta \varepsilon_n.$$

Also by (4.6), we have

$$(4.17) \quad g_n \frac{\partial}{\partial \varepsilon_n} \left[\frac{\varepsilon_n}{Z} e^{-\varepsilon_n/kT} \right] = g_n \left(\frac{1}{Z} - \frac{\varepsilon_n}{Z^2} \frac{\partial Z}{\partial \varepsilon} \right) e^{-\varepsilon_n/kT} \\ + \frac{g_n \varepsilon_n}{Z} \left(-\frac{1}{kT} + \frac{\varepsilon_n}{kT^2} T' \right) e^{-\varepsilon_n/kT},$$

where T' is as in (4.15), and by (4.6), we use the following approximation for the contribution of $\partial Z / \partial \varepsilon$ to the n -th energy level:

$$(4.18) \quad \frac{\partial Z}{\partial \varepsilon} = g_n \left(-\frac{1}{kT} + \frac{\varepsilon_n}{kT^2} T' \right) e^{-\varepsilon_n/kT}.$$

Hence by (4.16)-(4.18), we obtain that

$$(4.19) \quad kT = \sum_n \frac{a_n}{N} \left(1 - \frac{a_n}{N} \right) \varepsilon_n - \sum_n \frac{a_n}{N} \left(1 - \frac{a_n}{N} \right) \frac{T'}{T} \varepsilon_n^2.$$

On the other hand,

$$(4.20) \quad kT = \sum_n \theta_n \left[1 - \frac{a_n}{N} \right] \varepsilon_n \times \frac{1}{1 + \beta_n \ln \varepsilon_n / \varepsilon_0} \\ = \sum_n \left\{ \theta_n \left[1 - \frac{a_n}{N} \right] \varepsilon_n - \theta_n \left[1 - \frac{a_n}{N} \right] \beta_n \varepsilon_n \ln \frac{\varepsilon_n}{\varepsilon_0} \right\}.$$

We deduce then from (4.19) and (4.20) that

$$(4.21) \quad \theta_n = \frac{a_n}{N}, \quad \beta_n = \frac{T' \varepsilon_n}{T \ln(\varepsilon_n / \varepsilon_0)},$$

and consequently (4.2) for classical systems follows.

Quantum systems. For a quantum system, we first recall the Bose-Einstein statistics (2.16) or the Fermi-Dirac statistics (2.20):

$$(4.22) \quad a_n = \frac{g_n}{e^{(\varepsilon_n - \mu)/kT} \pm 1} \quad \left(\begin{array}{l} + \text{ for FD} \\ - \text{ for BE} \end{array} \right).$$

The total energy is written as

$$(4.23) \quad E = \sum_n \frac{g_n \varepsilon_n}{e^{(\varepsilon_n - \mu)/kT} \pm 1}.$$

As in the classical particle system case, with the assumption (4.6), by differentiating E with respect to ε_n , we obtain that

$$(4.24) \quad \frac{a_n}{g_n} \frac{\varepsilon_n (\varepsilon_n - \mu)}{kT^2} e^{\frac{\varepsilon_n - \mu}{kT}} T' = \frac{a_n}{g_n} \frac{\varepsilon_n}{kT} e^{\frac{\varepsilon_n - \mu}{kT}} - 1.$$

By (4.22), we have

$$(4.25) \quad e^{\frac{\varepsilon_n - \mu}{kT}} = \frac{g_n \pm a_n}{a_n} \quad \left(\begin{array}{l} + \text{ for FD} \\ - \text{ for BE} \end{array} \right).$$

We infer then from (4.24) and (4.25) that

$$(4.26) \quad T' = \frac{T}{\varepsilon - \mu} - \frac{g_n}{g_n \pm a_n} \frac{kT^2}{\varepsilon (\varepsilon - \mu)}.$$

Here $T' = \frac{\partial T}{\partial \varepsilon_n}$ and $\varepsilon = \varepsilon_n$. The solution of (4.26) is

$$(4.27) \quad kT(\varepsilon_n) = \left(1 \pm \frac{a_n}{g_n} \right) \frac{\varepsilon}{C_n + \ln \varepsilon} \quad \left(\begin{array}{l} + \text{ for FD} \\ - \text{ for BE} \end{array} \right).$$

Then as in the case for classical particle systems, we derive the following temperature formula:

$$(4.28) \quad kT = \sum_n \left(1 \pm \frac{a_n}{g_n} \right) \frac{a_n \varepsilon_n}{N(1 + \beta_n \ln \varepsilon_n)} \quad \left(\begin{array}{l} + \text{ for FD} \\ - \text{ for BE} \end{array} \right).$$

4.2. Physical meaning of the temperature formula. Equation (4.2) enables us to have a better understanding on the essence of temperature. In short,

- 1) *the essence of temperature T is (weighted) average energy level,*
- 2) *the temperature T is a function of distributions $\{a_n\}$ and the energy levels $\{\varepsilon_n\}$, and*
- 3) *the parameters $\{\beta_n\}$ in the temperature formula reflects the property of the material.*

We now discuss some further physical implications of the temperature formula.

1) *Absolute zero for Fermi particle systems.* For a Fermi particle thermodynamic system,

$$(4.29) \quad T = 0 \text{ K} \quad \Longleftrightarrow \quad \text{either } a_n = g_n \text{ or } a_n = 0.$$

Basic quantum mechanics shows that if the lower energy level is not fully occupied, then particles on the higher energy level are not stable, and will spontaneously jump to lower energy levels, unless there are always photons that excite the particles on the higher energy level. Consequently, (4.29) can be rewritten as

$$(4.30) \quad T = 0 \text{ K} \quad \Longleftrightarrow \quad \begin{cases} a_n = g_n & \text{for } n = 1, \dots, m, \\ a_n = 0 & \text{for } n > m. \end{cases}$$

This is an exact solution of the temperature formula for Fermi particle systems.

Solid state systems at $T = 0$ Kelvin are usually Fermi systems, since in a solid state system, atoms and molecules are fixed at lattice points, and the corresponding energy levels are determined by the orbiting electrons. Therefore such systems can be regarded as Fermi systems consisting of orbiting and free electrons.

2) *Absolute zero for Bose particle systems.* For a Bose particle system, we have

$$(4.31) \quad T = 0 \text{ K} \quad \Longleftrightarrow \quad \varepsilon_1 = 0, \ a_1 = N \text{ and } a_n = 0 \ \forall n > 1.$$

This corresponds exactly to the Bose-Einstein condensation. With temperature at absolute zero, states in a Bose particle system can only be in two forms: a gaseous state or a condensed state of a subsystem in an object.

3) *Classical systems.* For a classical particle system with temperature at absolute zero, we have

$$(4.32) \quad T = 0 \text{ K} \quad \Longleftrightarrow \quad a_1 = N \text{ and } a_n = 0 \ \forall n > 1.$$

Here we do not need to assume $\varepsilon_1 = 0$, which corresponds to superconductivity or condensation states of superfluids.

The above results derived from the temperature formula are in agreement with physical facts on $T = 0\text{K}$.

4) *Existence of highest temperature.* Based on the theory of energy levels in Section 3.4, the number of energy levels of all particles are finite. Consequently, we infer from the temperature formula (4.2) the

upper limit of T :

$$(4.33) \quad kT_{\max} < \varepsilon_{\max}, \quad \varepsilon_{\max} \stackrel{\text{def}}{=} \max_n \frac{\varepsilon_n}{1 + \beta_n \ln \varepsilon_n}.$$

We used the temperature formula for classical particle systems to derive (4.33), since at high temperature, the system can be regarded as a classical particle system.

5. THEORY OF ENTROPY

5.1. Physical meaning of entropy. As the electrons in the system represent in a natural way all the particles in the system, the energy level theory of temperature amounts to saying that

$$(5.1) \quad \text{temperature } T = \text{average energy level of electrons.}$$

We now develop the theory of entropy based on the dual relation (4.1). In view of both (4.1) and (5.1), we deduce the following new description of entropy:

$$(5.2) \quad \text{entropy} = \text{certain sense of number of photons in the system.}$$

This equivalence (5.2) provides a starting point for the new theory of entropy, which we shall explore in this section.

5.1.1. Physical supports of entropy as number of photons. 1) The first law of thermodynamics amounts to saying that for a given thermodynamical system, the internal energy consists of thermal energy, mechanical energy, interaction energy, etc, which can transform among each other and from one system to another, maintaining the total internal energy invariant. In particular we have

$$(5.3) \quad SdT + TdS = 0,$$

which shows that in an isolated system, thermal fluctuation follows the rule that temperature increasing or decreasing corresponds to entropy decreasing or increasing.

At the same time, it is clear that

$$(5.4) \quad \begin{aligned} &\bullet \text{ a particle absorbs photons if and only if its energy level} \\ &\quad \text{increases and the number of photons between particles} \\ &\quad \text{in the system decreases, and} \\ &\bullet \text{ a particle emits photons if and only if its energy level} \\ &\quad \text{decreases and the number of photons between particles} \\ &\quad \text{in the system increases.} \end{aligned}$$

It is clear that (5.3) and (5.4) are consistent. This shows clearly that the first law of thermodynamics offers a strong support for entropy being the number of photons in the system depicted in (5.2).

2) Long-range transfer is one important characteristic of thermal energy $Q = TS$. The temperature T is the average energy levels of particles, does not possess the long-range transfer feature, and can only be transferred through kinetic energies of particles. Therefore, the long-range transfer can only be achieved through the entropy S . On the other hand, it is clear that photons radiation is the only possible candidate. Hence (5.2) should be valid, and in other words, the characteristic of long-range transfer of thermal energy provides a physical support for (5.2).

3) First we call have the following law of entropy transfer.

Law of Entropy Transfer 5.1. *Assume the transfer of thermal and other forms energies is negligible. When two thermodynamic systems undergo thermal exchange, the entropy increasing for one system always leads to the entropy decreasing for the other system. Also, the entropy increases for the heat input system, and decreases for the heat output system.*

This law supports (5.2). Without particle exchange, thermal energy can only be transferred through either thermal radiation or transfer of kinetic energy of the system particles. With thermal radiation, energy levels of particles in heat *output* system decreases. It is clear then that with photon numbers and energy levels in equilibrium, decreasing of energy levels leads to the absorption of more photons, reducing the number of photons. For heat input system, the kinetic energy and energy levels of particles increase. This increase of energy levels causes emission of more photons, for photon numbers and energy levels to returning to their original equilibrium.

This verifies the agreement between the law of entropy transfer and the entropy theory (5.2).

5.2. Photon number formula of entropy. For a given thermodynamic system, in view of (5.2), we characterize entropy as the number of photons in the photon clouds between system particles, or the photon density of the photon gas in the system. As discussed earlier, thermal radiation is simply photon radiation (γ radiation). Also, photons are Bosons and obey the Bose-Einstein distribution. In this case, since the total number of photons is not fixed, the chemical potential $\mu = 0$. Then the BE distribution is written as

$$(5.5) \quad a_n = \frac{g_n}{e^{(\varepsilon_n - \mu)/kT} - 1}.$$

Hence the total energy of the photon gas in the system is given by

$$(5.6) \quad E = \sum_n a_n \varepsilon_n.$$

The corresponding partition function Z is given by

$$(5.7) \quad Z_B = \prod_n [1 - e^{-\varepsilon_n/kT}]^{-g_n}, \quad \ln Z_B = \sum_n g_n \ln [1 - e^{-\varepsilon_n/kT}]^{-1}.$$

Consequently, by the entropy formula (2.18):

$$S = k \left[\ln Z_B - \beta \frac{\partial}{\partial \beta} \ln Z_B \right],$$

we obtain that

$$(5.8) \quad S = k \sum_n \left[g_n \ln \frac{e^{\varepsilon_n/kT}}{e^{\varepsilon_n/kT} - 1} + \frac{g_n}{e^{\varepsilon_n/kT} - 1} \cdot \frac{\varepsilon_n}{kT} \right].$$

By the distribution (5.5), we have

$$\frac{e^{\varepsilon_n/kT}}{e^{\varepsilon_n/kT} - 1} = 1 + \frac{a_n}{g_n},$$

by which we infer from (5.8) that

$$(5.9) \quad S = k \sum_n \left[g_n \ln \left(1 + \frac{a_n}{g_n} \right) + \frac{a_n \varepsilon_n}{kT} \right].$$

Since for any photon gaseous system, we always have $a_n \ll g_n$, which implies that

$$\ln \left(1 + \frac{a_n}{g_n} \right) \simeq \frac{a_n}{g_n}.$$

Therefore, we derive from (5.9) the following photon number formula of entropy:

$$(5.10) \quad S = kN_0 \left[1 + \frac{1}{kT} \sum_n \frac{\varepsilon_n a_n}{N_0} \right],$$

where $N_0 = \sum_n a_n$ is the total number of photons, and $\sum_n \frac{\varepsilon_n}{kT} a_n$ represents the number of photons in the sense of average energy level. Notice that N_0 accounts only the photons between systems particles, not those in the clouds of electrons. Basically, thanks to the mechanism of photon radiation and absorption mechanism, at the equilibrium of absorption and radiation, the average energy level of the system maintains unchanged, and represents the temperature of the system; at the same time, the number (density) of photons in the sea of photons represents the entropy (entropy density) of the system.

5.3. Law of Temperature. By the temperature formula and the entropy formula, we arrive immediately the following law of temperature.

Theorem 5.2 (Law of temperature). *The following physical assertions hold true for temperature:*

- (1) *There are minimum and maximum values of temperature with $T_{\min} = 0$ and T_{\max} being given by (4.33);*
- (2) *When the number of photons in the system is zero, the temperature is at absolute zero; namely, the absence of photons in the system is the physical reason causing absolute zero temperature;*
- (3) *(Nernst Theorem) With temperature at absolute zero, the entropy of the system is zero;*
- (4) *With temperature at absolute zero, all particles fills all lowest energy levels.*

6. NATURE OF HEAT

The theory of temperature and entropy developed in the previous sections provide a theoretical foundation for the theory of heat. We further explore in this section the consequence of the theory to reveal the nature of heat.

6.1. Thermal energy. First, in classical thermodynamics, thermal energy is defined as

$$(6.1) \quad \Delta Q = \Delta U - \Delta W,$$

where ΔQ represents the thermal energy absorbed by the system, ΔU is the change of internal energy, and ΔW is the work done by the system. At a thermal equilibrium, we have

$$(6.2) \quad dU = TdS - pdV.$$

Physically, this differential equation can be understood as follows. First, for a given thermodynamical system, the absorbed (released) heat dQ is given by

$$dQ = dU.$$

On the other hand,

$$dQ = TdS + SdT,$$

Therefore

$$(6.3) \quad dU = TdS + SdT.$$

Since the volume of the system can change, the change of system temperature corresponds to the work done by the system:

$$(6.4) \quad SdT = \text{work} (= -pdV).$$

Then substituting SdT in (6.3) by (6.4), we arrive at (6.2).

Now we use the statistical theory of heat presented in the previous two sections to explain the above thermodynamic process. By the entropy formula (5.10), the thermal energy can be written as

$$(6.5) \quad Q_0 = ST = E_0 + kN_0T,$$

where E_0 is the total energy of photons as given by (5.6), and N_0 is the number of photons. By (6.5), the absorbing thermal energy is given by

$$dQ = dQ_0 = dE_0 + kTdN_0 + kN_0dT,$$

by which we infer from (6.3) that

$$(6.6) \quad dU = dE_0 + kTdN_0 + kN_0dT.$$

With the same reason as (6.4), we obtain then that

$$(6.7) \quad dU = dE_0 + kTdN_0 - pdV.$$

Eq. (6.7) is the differential equation for the new theory of heat expressed in (6.5), which is the equivalent form of the classical equation (6.2). Its physical meaning is clear: When the system absorbs thermal energy dQ (photon energy), the increased internal energy consists of two parts: one is to do the work $-pdV$, and the other is to change the total energy E_0 and the number of photons N_0 in the system. By the entropy formula (5.10), the change of E_0 and N_0 amounts to the change of entropy with constant temperature; namely,

$$dE_0 + kTdN_0 = TdS.$$

Hence the new theory of heat we established in this article is consistent with the classical theory, and importantly, offers the new physical meaning.

6.2. Balance between temperature and entropy. The statistical theory of heat presented in the previous sections tells us that thermal energy is the product of temperature and entropy, representing the energy of the photons in the system. Entropy represents the number of photons, and the temperature is the average energy level of the matter particles in the system.

Also, temperature and entropy can transfer between each other. When the system matter particles absorb photons, the temperature increases and the entropy decreases; while the particles radiate photons, the temperature decreases and the entropy increases. We need now to examine conditions for radiation and absorption of photons.

1) *Absorption condition.* Based on the photon cloud structure of electrons, each electron possesses a layer which absorbs photons. The

allowable photons in each such layer is finite, and if the layer is saturated with photons, no more photons can be absorbed. Based on the Bohr atomic theory, the orbiting energy level of an atom is given by

$$(6.8) \quad E_1 < E_2 < \cdots < E_k < E_{\max},$$

where E_k is the highest energy level, and E_{\max} is the escaping energy. For an electron at the energy level, it can absorb photons with energy level $E = E_{i+j} - E_i$ such that $E < E_{\max} - E_i$.

2) *Radiation condition.* There is no photon radiation for particles with uniform motion. Also, for an electron at energy level E_i with all lower energy levels in (6.8) filled, it will not radiate photons. The typical particle radiations include: atomic radiation, bremsstrahlung, Cherenkov radiation, and the radiation of electromagnetic polaritons. Basic electromagnetism shows that for bremsstrahlung, the emitting energy per unit time is given by

$$(6.9) \quad W = \frac{1}{6\pi^2\varepsilon_0} \frac{e^2 a^2}{c^3},$$

where ε_0 is the electric permittivity, e is the electric charge, a is the acceleration, and c the speed of light.

3) *Vibration mechanism of photon absorption and radiation.* We can see then that a particle can only absorb and radiate photons while experiencing vibratory motion. By (6.9), the higher the frequency of the vibration of the particle, the larger the absorbing and radiating energy. Also, only the vibratory kinetic energy of the particle can be transferred to the energy levels of the particles, and the average kinetic energy of the macroscopic system does not effect much of the energy levels of the micro-particles in the system. The vibration of the particles in the system is caused by collisions between system particles, by collisions between the system particles and the photons (mediators), and by absorbing and radiating photons.

4) *Transformation and balance between temperature and entropy.* For particles in high speed vibration and collision, the rate of photon emission and absorption increases, causing the energy and number density of photons to increase, and consequently leading to the increase of entropy density. Conversely, absorbing more photons by the system leads to the temperature increase. Hence it is clear that there is a natural connection between temperature and entropy.

6.3. Zeroth law of thermodynamics. The zeroth law of thermodynamics states as follows

Zeroth Law of Thermodynamics 6.1. *If two thermodynamic systems are each in thermal equilibrium with a third, then they are in thermal equilibrium with each other.*

This law is considered as the basis for temperature, and it is a common view that this law would lead to the following conclusion:

$$(6.10) \quad \textit{the temperatures are the same for all thermodynamic systems that are in equilibrium with each other.}$$

We examine this conclusion with the new theory of heat. First we recall the energy level temperature equation (4.2), restated here for convenience:

$$(6.11) \quad kT = \sum_n \left(1 - \frac{a_n}{N}\right) \frac{a_n \varepsilon_n}{N(1 + \beta_n \ln \varepsilon_n)},$$

which shows clearly that the temperature depends on three ingredients: the distribution $\{a_n\}$, the energy levels $\{\varepsilon_n\}$ and the parameters $\{\beta_n\}$. The energy levels $\{\varepsilon_n\}$ and the parameters $\{\beta_n\}$ are system (object)-dependent. The distribution $\{a_n\}$ depends also on the total energy E_0 of photons and the total number N_0 of photons in the system. Hence with the same total photon energy and the same photon density, different systems (objects) induce different temperatures. Therefore the conclusion (6.10) is meaningless for different objects.

With the new theory of heat, what the zeroth law truly implies is that

$$(6.12) \quad \textit{the energies and number densities of photons are the same for all thermodynamic systems that are in equilibrium with each other.}$$

Therefore, with the new theory the photon density in the object can be viewed as a measure for its temperature.

6.4. Caloric theory of heat. The current accepted theory of heat is also called the mechanical theory of heat, which related the heat with mechanical work. The theory was first introduced in 1798 by Benjamin Thompson, and further developed by such great scientists as Sadi Carnot, Rudolf Clausius, and James Clerk Maxwell.

The caloric theory of heat is an obsolete theory that heat is made up of a fluid called caloric that is massless and flows from hotter bodies to colder bodies. It was considered that caloric was a massless gas that exists in all matter, and is conserved. However the transfer between heat and mechanical work makes the caloric theory obsolete.

The statistical theory of heat presented in this article is developed based on physical theories on fundamental interactions, the photon

cloud model of electrons, the first law of thermodynamics, statistical theory of thermodynamics, radiation mechanism of photons, and energy level theory of micro-particles. The theory utilizes rigorous mathematics to reveal the physical essence of temperature, entropy and heat.

The statistical theory of heat established in this paper revives the old caloric theory in the sense that

$$(6.13) \quad \textit{photons are the caloric of heat.}$$

Photons possess all the required characteristics of caloric of heat: massless, ability to penetrate to matter, conserved in a certain sense, association of temperature with the quantity of photons in the system (object), and so on. One most important characteristic of the new theory is that it gives a natural explanation of the long-range heat transfer.

For example, for the heat phenomena associated with friction, the new theory indicates that the kinetic energy is transferred to system particles through friction, increasing the vibration kinetic energy of the particles. Then based on the radiation and vibration mechanism of photons, the high speed vibration of particles increases the energy levels of the particles and the absorbing and radiating frequencies of the surrounding photons; this leads to the cumulation of high density and high energy level photons. Hence the friction will increase the temperature of the object.

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(TM) DEPARTMENT OF MATHEMATICS, SICHUAN UNIVERSITY, CHENGDU, P. R. CHINA

E-mail address: matian56@sina.com

(SW) DEPARTMENT OF MATHEMATICS, INDIANA UNIVERSITY, BLOOMINGTON, IN 47405

E-mail address: showang@indiana.edu