



**HAL**  
open science

# A simple expression for the map of Asplund's distances with the multiplicative Logarithmic Image Processing (LIP) law

Guillaume Noyel, Michel Jourlin

## ► To cite this version:

Guillaume Noyel, Michel Jourlin. A simple expression for the map of Asplund's distances with the multiplicative Logarithmic Image Processing (LIP) law. 12th European Congress for Stereology and Image Analysis 2017 (ECSIA 2017), University of Kaiserslautern; Fraunhofer-Institut für Techno- und Wirtschaftsmathematik (ITWM), Sep 2017, Kaiserslautern, Germany. hal-01576183v2

**HAL Id: hal-01576183**

**<https://hal.science/hal-01576183v2>**

Submitted on 24 Jan 2018

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Copyright

# A simple expression for the map of Asplund's distances with the multiplicative Logarithmic Image Processing (LIP) law

Guillaume Noyel\*, and Michel Jourlin

\*International Prevention Research Institute, 95 cours Lafayette, 69006 Lyon, France,  
www.i-pri.org

Noyel and Jourlin (2017) have shown that the map of Asplund's distances, using the multiplicative LIP law, between an image,  $f \in \overline{\mathcal{I}} = [0, M]^D$ ,  $D \subset \mathbb{R}^N$ ,  $M \in \mathbb{R}$ ,  $M > 0$ , and a structuring function  $B \in [0, M]^{D_B}$ ,  $D_B \subset \mathbb{R}^N$ , namely the probe, is the logarithm of the ratio between a general morphological dilation  $\lambda_B f$  and an erosion  $\mu_B f$ , in the lattice  $(\overline{\mathcal{I}}, \leq)$ :

$$As_B^{\Delta} f = \ln \left( \frac{\lambda_B f}{\mu_B f} \right) = \ln \left( \frac{\bigvee_{h \in D_B} \{\tilde{f}(x+h)/\tilde{B}(h)\}}{\bigwedge_{h \in D_B} \{\tilde{f}(x+h)/\tilde{B}(h)\}} \right), \text{ with } f > 0 \text{ and } \tilde{f} = \ln(1 - f/M). \quad (1)$$

The morphological dilation and erosion with a translation invariance (in space and in grey-level) are defined, for additive structuring functions as (Serra, 1988; Heijmans and Ronse, 1990):

$$\begin{aligned} (\delta_B(f))(x) &= \bigvee_{h \in D_B} \{f(x-h) + B(h)\} = (f \oplus B)(x), \text{ (dilation)} \\ (\varepsilon_B(f))(x) &= \bigwedge_{h \in D_B} \{f(x+h) - B(h)\} = (f \ominus B)(x), \text{ (erosion)} \end{aligned} \quad (2)$$

and for multiplicative structuring functions as (Heijmans and Ronse, 1990):

$$\begin{aligned} \bigvee_{h \in D_B} \{f(x-h) \cdot B(h)\} &= (f \dot{\oplus} B)(x), \text{ (dilation)} \\ \bigwedge_{h \in D_B} \{f(x+h)/B(h)\} &= (f \dot{\ominus} B)(x), \text{ (erosion)}. \end{aligned} \quad (3)$$

If  $f > 0$  and  $B > 0$ , we have  $(f \dot{\oplus} B) = \exp(\bigvee_{h \in D_B} \{\ln f(x-h) + \ln B(h)\}) = \exp(\ln f \oplus \ln B)$ . Let us define, the reflected structuring function by  $\tilde{B}(x) = B(-x)$  and  $\tilde{f} = \ln(-\hat{f})$ . We obtain:

$$\begin{aligned} \lambda_B f &= \bigvee_{h \in D_B} \{\tilde{f}(x+h)/\tilde{B}(h)\} = \bigvee_{-h \in D_B} \{\tilde{f}(x-h)/\tilde{B}(-h)\} = (-\tilde{f}) \dot{\oplus} (-1/\tilde{B}) \\ &= \exp \left( \bigvee_{-h \in D_B} \{\ln(-\tilde{f}(x-h)) + \ln(-1/\tilde{B}(h))\} \right), \text{ with } \tilde{f} < 0 \text{ and } \tilde{B} < 0 \\ &= \exp \left( \ln(-\tilde{f}) \oplus (-\ln(-\tilde{B})) \right) = \exp \left( \hat{f} \oplus (-\hat{B}) \right). \end{aligned} \quad (4)$$

Similarly, we have  $\mu_B f = (-\tilde{f}) \dot{\ominus} (-\tilde{B}) = \exp(\hat{f} \ominus \hat{B})$ . Using the previous expressions of  $\lambda_B f$  and  $\mu_B f$  into equation 1, we obtain:

$$As_B^{\Delta} f = \ln \left( \frac{\lambda_B f}{\mu_B f} \right) = \ln \left( \frac{\exp(\hat{f} \oplus (-\hat{B}))}{\exp(\hat{f} \ominus \hat{B})} \right) = [\hat{f} \oplus (-\hat{B})] - [\hat{f} \ominus \hat{B}] = \delta_{-\hat{B}} \hat{f} - \varepsilon_{\hat{B}} \hat{f}. \quad (5)$$

Therefore, the map of Asplund's distances with the LIP multiplication is the difference between a dilation and an erosion with an additive structuring function (i.e. a morphological gradient).

## References

- Heijmans H, Ronse C (1990). The algebraic basis of mathematical morphology i. dilations and erosions. *Computer Vision Graphics and Image Processing* 50:245 – 295.
- Noyel G, Jourlin M (2017). Double-sided probing by map of Asplund's distances using Logarithmic Image Processing in the framework of Mathematical Morphology. Accepted to the 13th International Symposium on Mathematical Morphology, Fontainebleau, France, May 15 - 17, 2017, <http://cmm.ensmp.fr/ismm2017/>.
- Serra J (1988). *Image analysis and mathematical morphology: Theoretical advances*, vol. 2. Academic Press.