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A simple expression for the map of Asplund's distances with the multiplicative Logarithmic Image Processing (LIP) law

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Noyel and Jourlin (2017) have shown that the map of Asplund's distances, using the multiplicative LIP law, between an image, $f \in \overline{\mathcal{I}} = [0, M]^D$, $D \subset \mathbb{R}^N$, $M \in \mathbb{R}$, $M > 0$, and a structuring function $B \in [O, M]^{D_B}$, $D_B \subset \mathbb{R}^N$, namely the probe, is the logarithm of the ratio between a general morphological dilation $\lambda_B f$ and an erosion $\mu_B f$, in the lattice $(\overline{\mathcal{I}}, \leq)$:

$$As_B^{\triangle} f = \ln\left(\frac{\lambda_B f}{\mu_B f}\right) = \ln\left(\frac{\vee_{h\in D_B}\{\widetilde{f}(x+h)/\widetilde{B}(h)\}}{\wedge_{h\in D_B}\{\widetilde{f}(x+h)/\widetilde{B}(h)\}}\right), \text{ with } f > 0 \text{ and } \widetilde{f} = \ln\left(1 - f/M\right). \quad (1)$$

The morphological dilation and erosion with a translation invariance (in space and in grey-level) are defined, for additive structuring functions as (Serra, 1988; Heijmans and Ronse, 1990):

$$\begin{array}{rcl} (\delta_B(f))(x) & = & \vee_{h\in D_B}\{f(x-h) + B(h)\} = (f \oplus B)(x) \text{ , (dilation)} \\ (\varepsilon_B(f))(x) & = & \wedge_{h\in D_B}\{f(x+h) - B(h)\} = (f \ominus B)(x) \text{ , (erosion)} \end{array} \quad (2)$$

and for multiplicative structuring functions as (Heijmans and Ronse, 1990):

$$\begin{array}{rcl} \vee_{h\in D_B}\{f(x-h).B(h)\} & = & (f \dot{\oplus} B)(x) \text{ , (dilation)} \\ \wedge_{h\in D_B}\{f(x+h)/B(h)\} & = & (f \dot{\ominus} B)(x) \text{ , (erosion).} \end{array} \quad (3)$$

If $f > 0$ and $B > 0$, we have $(f \dot{\oplus} B) = \exp\left(\vee_{h\in D_B}\{\ln f(x-h) + \ln B(h))\}\right) = \exp\left(\ln f \oplus \ln B\right)$. Let us define, the reflected structuring function by $\overline{B}(x) = B(-x)$ and $\widehat{f} = \ln\left(-\widetilde{f}\right)$. We obtain:

$$\begin{array}{rcl} \lambda_B f & = & \vee_{h\in D_B}\left\{\widetilde{f}(x+h)/\widetilde{B}(h)\right\} = \vee_{-h\in D_B}\left\{\widetilde{f}(x-h)/\widetilde{B}(-h)\right\} = (-\widetilde{f})\dot{\oplus}(-1/\widetilde{\overline{B}}) \\ & = & \exp\left(\vee_{-h\in D_B}\left\{\ln\left(-\widetilde{f}(x-h)\right) + \ln\left(-1/\widetilde{\overline{B}}(h)\right)\right\}\right) \text{ , with } \widetilde{f} < 0 \text{ and } \widetilde{\overline{B}} < 0 \\ & = & \exp\left(\ln\left(-\widetilde{f}\right) \oplus \left(-\ln\left(-\widetilde{\overline{B}}\right)\right)\right) = \exp\left(\widehat{f} \oplus \left(-\widehat{\overline{B}}\right)\right). \end{array} \quad (4)$$

Similarly, we have $\mu_B f = (-\widetilde{f})\dot{\ominus}(-\widetilde{B}) = \exp\left(\widehat{f} \ominus \widehat{B}\right)$. Using the previous expressions of $\lambda_B f$ and $\mu_B f$ into equation 1, we obtain:

$$As_B^{\triangle} f = \ln\left(\frac{\lambda_B f}{\mu_B f}\right) = \ln\left(\frac{\exp\left(\widehat{f} \oplus \left(-\widehat{\overline{B}}\right)\right)}{\exp\left(\widehat{f} \ominus \widehat{B}\right)}\right) = \left[\widehat{f} \oplus \left(-\widehat{\overline{B}}\right)\right] - \left[\widehat{f} \ominus \widehat{B}\right] = \delta_{-\widehat{\overline{B}}}\widehat{f} - \varepsilon_{\widehat{B}}\widehat{f}. \quad (5)$$

Therefore, the map of Asplund's distances with the LIP multiplication is the difference between a dilation and an erosion with an additive structuring function (i.e. a morphological gradient).

References

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