

A simple expression for the map of Asplund's distances with the multiplicative Logarithmic Image Processing (LIP) law

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Noyel and Jourlin (2017) have shown that the map of Asplund's distances, using the multiplicative LIP law, between an image, $f \in \overline{\mathcal{I}} = [0, M]^D$, $D \subset \mathbb{R}^N$, $M \in \mathbb{R}$, $M > 0$, and a structuring function $B \in [0, M]^{D_B}$, $D_B \subset \mathbb{R}^N$, namely the probe, is the logarithm of the ratio between a general morphological dilation $\lambda_B f$ and an erosion $\mu_B f$, in the lattice $(\overline{\mathcal{I}}, \leq)$:

$$As_B^{\Delta} f = \ln \left(\frac{\lambda_B f}{\mu_B f} \right) = \ln \left(\frac{\bigvee_{h \in D_B} \{ \tilde{f}(x+h)/\tilde{B}(h) \}}{\bigwedge_{h \in D_B} \{ \tilde{f}(x+h)/\tilde{B}(h) \}} \right), \text{ with } f > 0 \text{ and } \tilde{f} = \ln(1 - f/M). \quad (1)$$

The morphological dilation and erosion with a translation invariance (in space and in grey-level) are defined, for additive structuring functions as (Serra, 1988; Heijmans and Ronse, 1990):

$$\begin{aligned} (\delta_B(f))(x) &= \bigvee_{h \in D_B} \{ f(x-h) + B(h) \} = (f \oplus B)(x), \text{ (dilation)} \\ (\varepsilon_B(f))(x) &= \bigwedge_{h \in D_B} \{ f(x+h) - B(h) \} = (f \ominus B)(x), \text{ (erosion)} \end{aligned} \quad (2)$$

and for multiplicative structuring functions as (Heijmans and Ronse, 1990):

$$\begin{aligned} \bigvee_{h \in D_B} \{ f(x-h) \cdot B(h) \} &= (f \dot{\oplus} B)(x), \text{ (dilation)} \\ \bigwedge_{h \in D_B} \{ f(x+h)/B(h) \} &= (f \dot{\ominus} B)(x), \text{ (erosion)}. \end{aligned} \quad (3)$$

If $f > 0$ and $B > 0$, we have $(f \dot{\oplus} B) = \exp(\bigvee_{h \in D_B} \{ \ln f(x-h) + \ln B(h) \}) = \exp(\ln f \oplus \ln B)$. Let us define, the reflected structuring function by $\tilde{B}(x) = B(-x)$ and $\tilde{f} = \ln(-\hat{f})$. We obtain:

$$\begin{aligned} \lambda_B f &= \bigvee_{h \in D_B} \{ \tilde{f}(x+h)/\tilde{B}(h) \} = \bigvee_{-h \in D_B} \{ \tilde{f}(x-h)/\tilde{B}(-h) \} = (-\tilde{f}) \dot{\oplus} (-1/\tilde{B}) \\ &= \exp \left(\bigvee_{-h \in D_B} \{ \ln(-\tilde{f}(x-h)) + \ln(-1/\tilde{B}(h)) \} \right), \text{ with } \tilde{f} < 0 \text{ and } \tilde{B} < 0 \quad (4) \\ &= \exp \left(\ln(-\tilde{f}) \oplus (-\ln(-\tilde{B})) \right) = \exp \left(\hat{f} \oplus (-\hat{B}) \right). \end{aligned}$$

Similarly, we have $\mu_B f = (-\tilde{f}) \dot{\ominus} (-\tilde{B}) = \exp \left(\hat{f} \ominus \hat{B} \right)$. Using the previous expressions of $\lambda_B f$ and $\mu_B f$ into equation 1, we obtain:

$$As_B^{\Delta} f = \ln \left(\frac{\lambda_B f}{\mu_B f} \right) = \ln \left(\frac{\exp \left(\hat{f} \oplus (-\hat{B}) \right)}{\exp \left(\hat{f} \ominus \hat{B} \right)} \right) = \left[\hat{f} \oplus (-\hat{B}) \right] - \left[\hat{f} \ominus \hat{B} \right] = \delta_{-\hat{B}} \hat{f} - \varepsilon_{\hat{B}} \hat{f}. \quad (5)$$

Therefore, the map of Asplund's distances with the LIP multiplication is the difference between a dilation and an erosion with an additive structuring function (i.e. a morphological gradient).

References

- Heijmans H, Ronse C (1990). The algebraic basis of mathematical morphology i. dilations and erosions. *Computer Vision Graphics and Image Processing* 50:245 - 295.
- Noyel G, Jourlin M (2017). Double-sided probing by map of Asplund's distances using Logarithmic Image Processing in the framework of Mathematical Morphology. Accepted to the 13th International Symposium on Mathematical Morphology, Fontainebleau, France, May 15 - 17, 2017, <http://cmm.ensmp.fr/ismm2017/>.
- Serra J (1988). *Image analysis and mathematical morphology: Theoretical advances*, vol. 2. Academic Press.