



HAL
open science

A bio-energetic model of cyclist for enhancing pedelec systems

Nadia Rosero, John Jairo Martinez Molina, Henry Leon

► **To cite this version:**

Nadia Rosero, John Jairo Martinez Molina, Henry Leon. A bio-energetic model of cyclist for enhancing pedelec systems. IFAC WC 2017 - 20th IFAC World Congress, Jul 2017, Toulouse, France. 10.1016/j.ifacol.2017.08.918 . hal-01575847

HAL Id: hal-01575847

<https://hal.science/hal-01575847>

Submitted on 21 Aug 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A bio-energetic model of cyclist for enhancing pedelec systems

Nadia Rosero, John J. Martinez* Henry Leon**

* Univ. Grenoble Alpes, CNRS, GIPSA-lab, F-38000 Grenoble, France
(e-mail: nadia.rosero, john.martinez@gipsa-lab.fr).

** Univ. de la Sabana, Faculty of Medicine, Chia, Colombia
(e-mail: henrylear@clinicaunisabana.edu.co)

Abstract: The paper presents a whole-body bio-energetic model of a cyclist which includes the mechanical dynamics of the bike. This model could be used to solve control-design problems for pedelec systems. The behavior of some physiological variables during cycling is reproduced by keeping an energy aware transfer flow. The modeling approach considers three main levels: i) physiological, ii) bio-mechanical and iii) pure-mechanical. Physical laws of energy/mass conservation were applied to simulate the ways in which energy is stored, transferred and dissipated at each level. A simulation example shows a scenario of a physiological test.

Keywords: Physiological model, modeling of human performance, control in system biology, bio-energetics.

1. INTRODUCTION

Cycling is an activity in which the human body is combined with a very friendly machine, the bike. Several authors agree that this human-machine system is one of the most efficient means of transport because it requires less energy per unit distance and per unit mass than any other form of land transportation like is described in Jeukendrup et al. (2000). Nowadays, several ways to improve the cycling performance have been explored, for instance, new advances in aerodynamics, body position, new wear material, chain-ring shape, among others, have dramatically increased the success in cycling sports.

The use of electrical motors is intended to extend the use of bikes in towns, but electrical assistance has been conceived without regarding the physiological state of the cyclists in most commercial e-bikes. However, some remarkable advances in research have been developed, for example the works by Meyer et al. (2015), Giani et al. (2014), Le et al. (2008) and Corno et al. (2015a). These works use cyclist models mostly conceived in an intuitive way, trying to reproduce some well-known behavior of the metabolic system. The models by Ma et al. (2009), Fayazi et al. (2013) and Corno et al. (2015b), fit most of the observed performances, but they do not describe the complete energetic interaction between the physiological, bio-mechanical and pure-mechanical components of the cyclist-bike system.

In addition, keeping the importance of the energy concept in physics; a system can be viewed as a set of subsystems that exchange energy among themselves and the environment. An interesting point concerns the fact that energies from different domains can be combined simply by adding up the individual energy contributions. Lastly, the role of energy and the interconnections between subsystems can provide the basis for various control strategies.

While cycling an electrical bike, the human being is one of the sources of power to generate movement in conjunction with the battery-motor pack. The power required to produce motion passes through a chain containing power conversion, storage and dissipation at different levels.

In order to design a more intelligent control system for pedelecs, it could be necessary to consider a model which is able to reproduce the behavior of the bio-energetic system of the cyclist. Such a model has to be simple enough and energy-aware to be combined with previous electro-mechanical models that describe the electrical bike dynamics. In this way, we could incorporate the physiological state of the cyclist and/or new therapeutic objectives in future control solutions.

In this paper, we present an energetic model of the cyclist-bike system, which is intended to solve future control problems related to the optimisation of pedelec systems. It is based on physical laws that describe hydraulic and electro-mechanical systems. Hence, the behavior of the system state respects the energy/mass conservation laws and allows the association of the storage, transfer and dissipation of power in a more intuitive way. According to the classification and descriptions given by Abbiss and Laursen (2005) and Noakes (2000), it could be considered like an energy supply/energy depletion model.

The model allows the description of the dynamical behavior at three different levels: metabolic, bio-mechanical and pure-mechanical level. The latter corresponds to the interaction of the cyclist with the bike dynamics which facilitates the simulation of practically any scenario. This paper is organized as follows: Firstly, the problem statement is presented in Section 2. Second, the description of the proposed model is presented and discussed in Section 3. Finally, Section 4 depicts a simulated example including a ramp of work-test scenario.

2. PROBLEM STATEMENT

2.1 Some physiological aspects to be considered

The proposed model is intended to describe the behavior of certain physiological variables accompanied by bio-mechanical and pure-mechanical dynamical interactions. Two metabolic pathways have been considered i) an aerobic pathway and ii) an anaerobic pathway. The following additional issues have been addressed during the modeling process:

- 1) The model has to describe the power contributions of every bio-energetic pathway, in a dynamical way.
- 2) The model has to reproduce the power conversion efficiency observed in every metabolic pathway.
- 3) The model has to be able to describe the energy consumption of muscles for both i) during isometric action (i.e. without muscular motion) and ii) during concentric/eccentric action (e.g. during pedaling).
- 4) The model has to reproduce the dependency of the Maximal Voluntary Contraction (MVC) with respect to the speed at which a muscle changes its length.
- 5) The model has to be simple, only retaining a few components to describe sources, storage, dissipation and conversion of energy at each stage of the model (i.e. for physiological and bio-mechanical stages).
- 6) Finally, the model has to allow the computation of a fatigue index that is compatible with the physiological energy storage process.

2.2 Expected use of the proposed model

The use of a bike as a therapeutic object can be more suitable to help patients with particular pathologies. However, this requires the adaptation of the electrical assistance for every patient and a possible exchange of information, in real-time, about the physiological, bio-mechanical and pure mechanical states.

Even if the proposed model can be a parameter varying or a very uncertain model, its structure maintains the main dynamical relationships between physiological, bio-mechanical and mechanical variables. In this way, it can be suitable for control design of novel electrical assistance systems. In addition, it could be used for simulation, test and validation of existing and future control strategies for pedelecs.

3. MODEL DESCRIPTION

In this paper we propose an electro-hydraulic based model of the bio-energetic system of a cyclist, which allows to describe three different interconnected sub-systems: i) a physiological stage represented as an hydraulic system, where mass balance is applied for every metabolic pathway to obtain the system equations, ii) a bio-mechanical stage like an electric engine in which electro-mechanical laws of motion were applied to obtain the dynamical equations for the transformation of physiological energy in mechanical one, and iii) a pure mechanical stage which includes the dynamics of a bike by applying Newton's equations of motion.

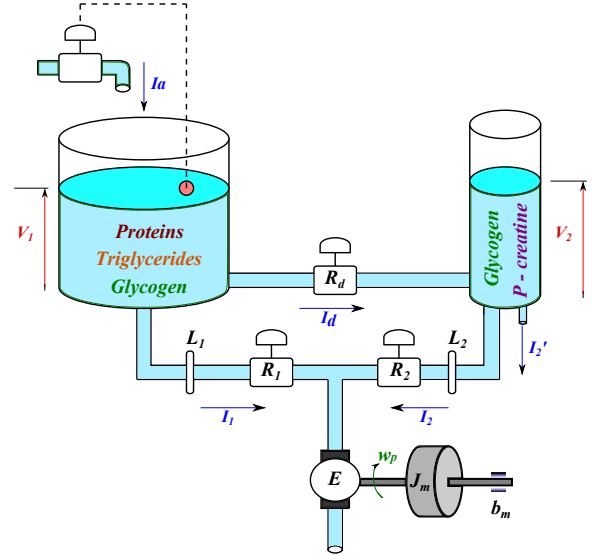


Fig. 1. Electro-hydraulic based model of the bio-energetic system of a cyclist.

Table 1. Nomenclature cyclist-bike model

Sym	Type	Meaning
I_a	Input	Amount of oxygen and nutrients transported by the cardiovascular system to the muscles
V_1	State	Vessel 1 level. Linked to the aerobic energy.
V_2	State	Vessel 2 level. Linked to the anaerobic energy.
I_1	State	Vessel 1 outflow. ATP from the aerobic pathway.
I_2	State	Vessel 2 outflow. ATP from the anaerobic pathway.
ω_p	State	Pedaling frequency. Measured at pedal level.
I_d	Variable	Flow between vessels 1 and 2.
I_2'	Variable	Losses when anaerobic pathway is used.
C_1	Parameter	Capacitance of vessel 1.
C_2	Parameter	Capacitance of vessel 2.
R_d	Parameter	Resistance of I_d flow. Regulates the recovery dynamics.
R_1	Parameter	Resistance of I_1 flow. Represents the voluntary desire to apply a force.
R_2	Parameter	Resistance of I_2 flow. Represents the involuntary and complementary action of the anaerobic pathway.
L_1	Parameter	Inductance at vessel 1 output. Models the dynamics of aerobic ATP synthesis.
L_2	Parameter	Inductance at vessel 2 output. Models the dynamics of anaerobic ATP synthesis.
K_e	Parameter	Counter-electromotive force. $E = K_e \cdot \omega_p$.
J_m	Parameter	Muscular inertia.
J_{eq}	Parameter	Equivalent inertia. It includes the cyclist, bike and wheels.
b_m	Parameter	Muscle viscous friction coefficient. It is useful to model isometric action.

3.1 The physiological stage

Consider the Fig.1 and the nomenclature summarized in Table 1. In a body, the cardiovascular system permits the blood to circulate and transport nutrients from and to the

cells, for instance the Fick Principle relates the oxygen consumption with the cardiac output (heart rate HR by stroke volume SV) and the tissular oxygen extraction, this aspect is modeled as the input flow I_a . Here, I_a is controlled to regulate the oxygen supply and other necessary nutrients to produce energy in muscles. The stored energy takes the form of molecules of glycogen, triglycerides and proteins that are used for the aerobic system, and molecules of glycogen and phosphocreatine that are used for the anaerobic system. These stored molecules are modeled as the volume of the vessels, proportional to the levels V_1 and V_2 . Hydraulic models have been used previously for understanding the behavior of energy supply by different pathways during exercise, for example the cited ones by Morton (2006).

Mass balance equations for the vessel 1 (used for the aerobic system) and the vessel 2 (used for the anaerobic system) model the storage process of nutrients and in particular the required time to fill and/or to empty these vessels. For the aerobic system we have:

$$C_1 \frac{dV_1}{dt} = I_a - I_1 - I_d \quad (1)$$

and for the anaerobic system:

$$C_2 \frac{dV_2}{dt} = I_d - I_2 - I'_2 \quad (2)$$

The constants C_1 and C_2 represent the capacitance of vessel 1 and 2 respectively (i.e. the cross-sections of each vessel). The value of C_1 is linked to the endurance of the cyclist and/or the availability of muscle fibers type I, while C_2 is an image of the anaerobic capacity of the cyclist and/or the availability of muscle fibers type II according to Coyle et al. (1992) and Jeukendrup et al. (2000).

The flow I_a is a function of the aerobic vessel level V_1 and a given metabolic reference value. It is assumed that this term regulates the level V_1 and it is proportional to the HR and the oxygen uptake $\dot{V}O_2$. For simplicity, this action is modeled by a saturated proportional control loop, as follows:

$$I_a = \begin{cases} -k_p(V_1 - V_1^{ref}) & \text{if } I_a \leq I_{max} \\ I_{max} & \text{otherwise} \end{cases} \quad (3)$$

where the term V_1^{ref} could be obtained from the volume of available energy (in form of proteins, lipids and glycogen) at the beginning of the workout and it will decrease during the exercise. Here the proposed variation of V_1^{ref} takes into account observations in endurance time by Morton (2006) and his previous works. Then, we have:

$$\dot{V}_1^{ref} = -\frac{1}{C_a} I_a \quad (4)$$

where the initial condition $C_a V_1^{ref}(0)$ corresponds at the average amount of oxygen and nutrients that can be consumed above basal level in an endurance test. C_a is a constant parameter.

The maximum value of I_a will be I_{amax} which models the cardiovascular limit to provide oxygen and other nutrients to the aerobic system. This value is in fact an image of the maximum oxygen uptake $\dot{V}O_{2max}$. The flow I_d between the two vessels is described by the following equation:

$$I_d = \frac{V_1 - \alpha V_2}{R_d} \quad (5)$$

with α a constant parameter satisfying $0 \leq \alpha \leq 1$ and R_d a resistance which limits the flow between vessels. It mainly models the molecular transfer between both the storage systems. The parameter α can be used to finely model this exchange.

When an anaerobic effort is being carried out, the human body synthesizes Adenosine Triphosphate (ATP) from Phosphocreatine and Adenosine Diphosphate (ADP) or Anaerobic glycolysis; while this, it contracts an oxygen debt that will be paid off once the exercise is over. This phenomenon is known as Excess Post Exercise Oxygen Consumption (EPOC). The flows I_a and I_d reproduce this phenomenon. In particular, I_a will increase during the effort to assure that the level V_1 is high as much as possible and to recover the initial value after the workout.

In addition, the use of the anaerobic pathway produce an augmentation of lactate and acidosis caused by the hydrogen-ions H^+ product of anaerobic glycolysis, which saturate the clearance mechanisms as is described in Binzoni (2005) and McArdle et al. (2006). In Fig. 1, the flow I'_2 represents the fact, that this pathway is less efficient than aerobic one, generating losses in energy use, even if it is fast and synthesise big amounts of ATP.

The human body cannot easily store ATP and it needs to be continuously created. Here, the synthesis of ATP molecules is modeled by two differential equations which relate the energy storage in body as the source of power (proportional to V_1 and V_2), the dissipated energy required to synthesize ATP (I_1 and I_2) and the stored one in muscles (proportional to L_1 and L_2). Thus, the dynamics of the flows I_1 and I_2 , describing the ATP synthesis dynamics, will be governed by the following equations:

$$L_1 \frac{dI_1}{dt} = V_1 - R_1 I_1 - E \quad (6)$$

$$L_2 \frac{dI_2}{dt} = V_2 - R_2 I_2 - E \quad (7)$$

where the constants L_1 and L_2 allows the characterization of the necessary time-response for generating ATP from the aerobic and anaerobic pathways, respectively. The synthesis of ATP molecules is related to the production of muscle force. Hence, the flows (or currents) I_1 and I_2 are images of the produced force from the aerobic and anaerobic pathways, respectively.

These dynamics are inspired from an electrical motor. As in electrical motors, the counter-motive force, denoted here with the symbol E , is proportional to the pedal angular speed ω_p (assuming that this speed is proportional to the speed of changing the muscle length).

The effort E pushes against the current $I_1 + I_2$ which induces it. It will be given by

$$E = K_e \omega_p \quad (8)$$

where K_e is a positive constant, which describes the power conversion from physiological energy to bio-mechanical one. Since the produced torque T_e is proportional to the sum of currents, i.e.

$$T_e = K_e (I_1 + I_2) \quad (9)$$

Thus, the bio-mechanical power (without considering mechanical power losses and storage) can be calculated as

$$P_{bmech} = E (I_1 + I_2) = T_e \omega_p \quad (10)$$

and the physiological power can be calculated as the sum of two contributions:

$$P_{phy} = (V_1 I_1) + (V_2 I_2) \quad (11)$$

In the case of an isometric exercise, when muscle group moment equals to resistance moment; ω_p and bio-mechanical power P_{bmech} will be zero even if the torque T_e is not zero. However, the physiological power P_{phy} will take a different value depending on the resistances R_1 and R_2 which, in turn depend on desired torques or forces.

If we consider the case of a constant applied force, that is

$$I_1 + I_2 = \left(\frac{V_1 - K_e \omega_p}{R_1} \right) + \left(\frac{V_2 - K_e \omega_p}{R_2} \right) \quad (12)$$

Then, the muscle fatigue appears in (12) as a consequence of the reduction of the levels V_1 and V_2 during an effort. The word “fatigue” is interpreted in this paper as the inability to produce required/desired power, and exhaustion as a condition acquired by accumulated fatigue.

According to Kenney et al. (2015) during long term exercises, fatigue coincides among other factors with a decreased concentration on muscle glycogen, no matter its rate of depletion. Here, the glycogen concentration is directly related with levels V_1 and V_2 . Then, we propose two indices of fatigue as:

$$SoF_1 = \frac{V_1(0) - V_1(t)}{V_1(0)} \quad (13)$$

$$SoF_2 = \frac{V_2(0) - V_2(t)}{V_2(0)} \quad (14)$$

Since both vessels levels represent available muscular substrates, the proposed indices are related with peripheral fatigue. The index SoF_2 could be the responsible of the sensation of fatigue due to metabolic acidosis and increase of lactate, while SoF_1 is more linked to glycogen depletion.

In addition to the previous exposed properties of the model, the chosen structure for modeling the human force generation reproduces the force-velocity relationship accepted for the physiology community. From (12), the variations of the torque or force (proportional to $I_1 + I_2$) with respect to the angular speed ω_p will be

$$\frac{\partial(I_1 + I_2)}{\partial \omega_p} = - \left(\frac{1}{R_1} + \frac{1}{R_2} \right) K_e \quad (15)$$

which means that the force decreases as long as the speed increases, because derivative is always negative. In other words, it will be harder to produce muscular force at high speeds. Hence, the proposed model seems to be well adapted to the observations presented by the physiology community.

3.2 The bio-mechanical stage

The role of resistances: The resistance R_1 allows the inclusion of the “voluntary” desire of applying force through

I_1 , and R_2 represents the “involuntary” but complementary role of the anaerobic pathway to apply I_2 . For instance, big values of resistances imply zero forces, and the smallest value of resistances provides the maximal forces that muscles can produce.

The maximum isometric force (i.e. a static force without movement of the muscles) that an individual can develop is called Maximum Voluntary Contraction (MVC). Here it can be computed using (12) for $\omega_p = 0$, given that currents are proportional to the cyclist force.

Suppose an individual desires to apply a given force which imposes a reference on I_1 , denoted I_1^{ref} . It is assumed that the desire to perform a force produces electrical signals coming from motor neurons. These electrical signals produce muscular actions. This voluntary control action is modeled as a proportional-integral control loop, that is

$$R_1 = K_p e_1 + K_i \int_0^t e_1 dt \quad (16)$$

where e_1 stands for the force error signal $e_1 := I_1 - I_1^{ref}$. The constants K_p and K_i will depend on the capability of an individual to reproduce a desired force.

Concerning the resistance R_2 , it has to follow an internally generated reference which allows to perform more or less anaerobic flow according to the state of the aerobic pathway. Here this control action is modeled as follows:

$$R_2 = K_p e_2 + K_i \int_0^t e_2 dt \quad (17)$$

where e_2 stands for the force error signal $e_2 := I_2 - I_2^{ref}$, with

$$I_2^{ref} = I_1^{ref} - I_1 = -e_1 \quad (18)$$

That reference models the natural mechanism which intends to guarantee that the total desired force is assured by the sum of the aerobic and anaerobic contributions, i.e. $I_1 + I_2 = I_1^{ref}$ as much as possible. However, in practice the variables I_1 and I_2 do not perfectly track their references. The tracking error will depend on the state of the aerobic and the anaerobic pathway. In particular, the state of the vessels levels V_1 and V_2 and the effort E .

The terms $-R_1 I_1$ and $-R_2 I_2$ in (6) and (7), respectively, have to be negative in order to model dissipative control actions (these actions do not generate power, they can only dissipate).

We consider the same control gains K_p and K_i to simplify the model. Thus, the time-response for each metabolic pathway is mainly assured by the choice of inductances L_1 and L_2 (for instance, the condition $L_1 > L_2$ models a slower aerobic dynamics than an anaerobic one).

In Fig.1, the electro-mechanical stage described by equations (6)-(7) can be used to model the intersection between the physiological domain and the bio-mechanical one.

Recalling that the counter-motion force is $E = K_e \cdot \omega_p$, while the torque T_e is proportional to the sum of currents $I_1 + I_2$, that is $T_e = K_e (I_1 + I_2)$. The second Newton law for rotational motion can be used for obtaining the dynamics of the pedaling angular speed ω_p . That is,

Table 2. Parameters of simulation example

Parameter	Value	Units
C_1	1.23	m^2
C_2	0.41	m^2
R_d	2	$s \cdot m^{-2}$
L_1	0.92	H
L_2	0.0092	H
K_e	0.4612	$V \cdot s \cdot rad^{-1}$
b_m	0.1646	$N \cdot m \cdot s \cdot rad^{-1}$
$J_m + J_{eq}$	11.11	$kg \cdot m^2$
α	1	-
K_p	5000	-
K_i	50	-

$$J_m \frac{d\omega_p}{dt} = T_e - b_m \omega_p - T_p \quad (19)$$

where T_p is the torque at the pedal level, which is often measured in practice. The constants J_m and b_m model the muscular inertia and muscle viscous friction coefficient, respectively. Thus, during pedaling, the torque provided by the physiological stage, T_e , is converted in torque at the pedal level, T_p , but a fraction of this power is stored in the inertia and a second one is dissipated by friction.

3.3 The pure mechanical stage

A very simplified motion equation concerning the mechanical cyclist-bike dynamics can be established as follows:

$$J_{eq} \frac{d\omega_p}{dt} = T_p - T_{load} \quad (20)$$

where the inertia J_{eq} represents the equivalent inertia observed at the pedal level, it includes the bike, wheels and cyclist equivalent inertia. The load torque T_{load} includes all the dissipation terms which appears during bike motion. For instance, T_{load} includes mainly the aerodynamic losses, rolling resistance and a component of the gravity force due to slope.

Combining equations (19) and (20) and rearranging the torque produced by the cyclist at the physiological stage, we obtain

$$(J_{eq} + J_m) \frac{d\omega_p}{dt} = T_e - b_m \omega_p - T_{load} \quad (21)$$

which is more suitable for simulating the whole-body bio-energetic behavior because the torque produced at the physiological level T_e , explicitly appears into the motion equation.

4. A SIMULATION EXAMPLE

4.1 Simulation data

The simulation has been performed to illustrate the dynamical behavior of the proposed model. The parameters have been chosen to fit available static and dynamical data from physiological tests. Even if the used parameters do not correspond to a particular individual, the results allows to evaluate the pertinence of the proposed model. Used parameters are summarized in Table 2.

4.2 Description of the presented scenario

A simulated scenario is proposed in order to illustrate the ability of the model to reproduce several physiological

behaviors and phenomena during energy depletion and during the recovery process.

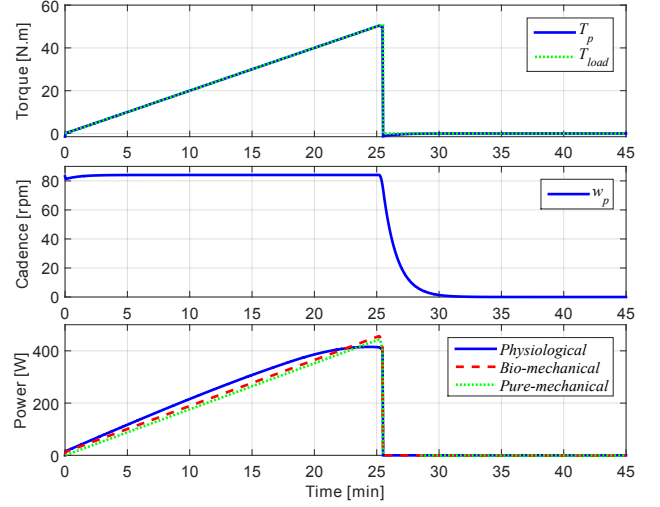


Fig. 2. Torque, cadence and power

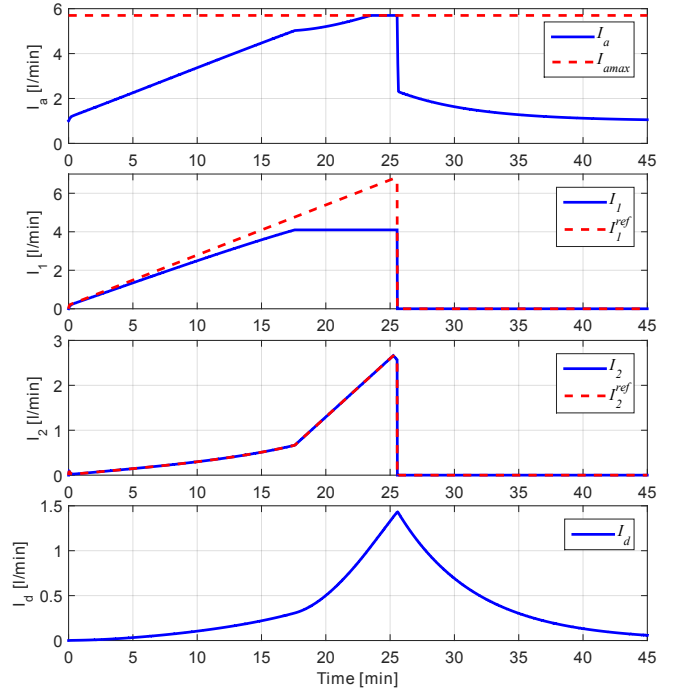


Fig. 3. Flows during simulation scenario.

Fig. 2 shows an scenario related to the incremental cycling test with resistance protocol, i.e. the resistance torque T_{load} is linearly increased during the time, while cyclist is required to maintain constant the pedaling frequency ω_p (freely chosen by him previously). The test is terminated when the cadence fell to more than 10 rpm below the chosen one for more than 10 s.

Fig. 3 shows the main flows of the model. The cyclist starts from a basal level $V_1(0)$ and $I_a(0) = 1l/min$ at $t = 0$, which correspond to basal values. The load torque T_{load} increases as a ramp from $0N \cdot m$ to $50N \cdot m$, as it is depicted in Fig. 2. This produces a cyclist torque T_p , mostly provided by the aerobic pathway until around

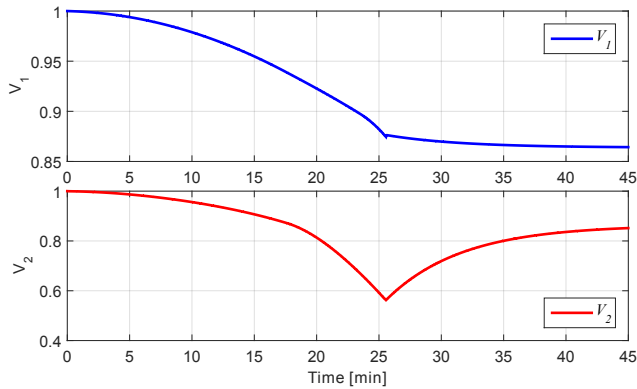


Fig. 4. Levels in vessels 1 and 2

$t = 18min$. At this time could be considered the anaerobic threshold and the beginning of lactate accumulation. The contribution of aerobic pathway is saturated from this point until the end of the workout.

Since the beginning of the exercise, the flow I_a increases to provide the required oxygen and nutrients, a change in the slope is evident after the anaerobic threshold.

Fig. 4 shows how the levels V_1 and V_2 decrease during the increments in efforts. An important change in the anaerobic vessel level V_2 is observed specially in the last five minutes of the workout, producing an important level of fatigue. Remark also, that both metabolic pathways provide energy, noted by the decrements in vessels levels. However, the main source of energy comes from the aerobic pathway. The anaerobic pathway only intervenes when the aerobic system is not able to provide the desired effort. After an important effort, the cyclist feels fatigue sensation given by the low level in V_2 and he is not able to sustain the cadence, then the T_{load} is set to zero (see Fig. 2) at $t = 27min$. During the recovery time, the body is able to replenish the aerobic vessel as it is seen in level V_1 , but the replenishment of anaerobic vessel is very slow.

The provided powers are also seen in Fig. 2. The power rises in a proportional way to the load until its maximum value around $t = 26min$, time at which the anaerobic system produces important quantities of ATP. Physiological power P_{phy} , bio-mechanical P_{bmec} and the mechanical one P_{mec} , which is equal to $T_p \cdot \omega_p$ were calculated. The losses of energy during the transfer can be observed by the difference between the curves.

5. CONCLUSIONS AND FUTURE WORK

This work presents a dynamical model of physiological and bio-mechanical variables within the human while cycling. This bio-energetic approach allows to obtain information about energy diminution in the cyclist in the framework of conceiving an adapted assistance to his requirements.

The structure of the model incorporates the comprehension about internal control loops in the body, the losses in muscles depending of the load or speed variation, and relationships between some measurable variables like HR, applied torque, and angular speed in a simple way. This, make it feasible for implementation in an embedded and light system for commercial bikes.

Future work includes optimal calibration of parameters given static or dynamic data of a specific cyclist. Furthermore, the insertion of the model in a control strategy could improve the use of energy originated in both sources, battery pack and human being for a self-sustaining strategy in pedelecs.

REFERENCES

- Abbiss, C.R. and Laursen, P.B. (2005). Models to explain fatigue during prolonged endurance cycling. *Sports Medicine*, 35(10), 865–898.
- Binzoni, T. (2005). Saturation of the lactate clearance mechanisms different from the lactate shuttle determines the anaerobic threshold: prediction from the bioenergetic model. *Journal of physiological anthropology and applied human science*, 24(2), 175–182.
- Corno, M., Berretta, D., Spagnol, P., and Savaresi, S.M. (2015a). Design, control, and validation of a charge-sustaining parallel hybrid bicycle.
- Corno, M., Giani, P., Tanelli, M., and Savaresi, S.M. (2015b). Human-in-the-loop bicycle control via active heart rate regulation. *Control Systems Technology, IEEE Transactions on*, 23(3), 1029–1040.
- Coyle, E., Sidossis, L., Horowitz, J., and Beltz, J. (1992). Cycling efficiency is related to the percentage of type I muscle fibers. *Medicine and science in sports and exercise*, 24(7), 782.
- Fayazi, S.A., Wan, N., Lucich, S., Vahidi, A., and Mocko, G. (2013). Optimal pacing in a cycling time-trial considering cyclist’s fatigue dynamics. In *American Control Conference (ACC), 2013*, 6442–6447. IEEE.
- Giani, P., Corno, M., Tanelli, M., and Savaresi, S.M. (2014). Cyclist heart rate control via a continuously varying transmission. *Proc. 19th IFAC World Congr*, 912–917.
- Jeukendrup, A.E., Craig, N.P., and Hawley, J.A. (2000). The bioenergetics of world class cycling. *Journal of Science and Medicine in Sport*, 3(4), 414–433.
- Kenney, W.L., Wilmore, J., and Costill, D. (2015). *Physiology of Sport and Exercise 6th Edition*. Human kinetics.
- Le, A., Jaitner, T., Tobias, F., and Litz, L. (2008). A dynamic heart rate prediction model for training optimization in cycling (p83). In *The Engineering of Sport 7*, 425–433. Springer.
- Ma, L., Chablat, D., Bennis, F., and Zhang, W. (2009). Dynamic muscle fatigue evaluation in virtual working environment. *arXiv preprint arXiv:0901.0222*.
- McArdle, W.D., Katch, F.I., and Katch, V.L. (2006). *Essentials of Exercise Physiology*. Lippincott Williams & Wilkins.
- Meyer, D., Zhang, W., and Tomizuka, M. (2015). Sliding mode control for heart rate regulation of electric bicycle riders. In *ASME 2015 Dynamic Systems and Control Conference*. American Society of Mechanical Engineers.
- Morton, R.H. (2006). The critical power and related whole-body bioenergetic models. *European Journal of Applied Physiology*, 96(4), 339–354.
- Noakes, T. (2000). Physiological models to understand exercise fatigue and the adaptations that predict or enhance athletic performance. *Scandinavian Journal of Medicine & Science in Sports*, 10(3), 123–145.