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► **To cite this version:**

Samir Ghouali, Mohammed Feham, Yassine Zakarya Ghouali. The Granger Causality Effect between Cardiorespiratory Hemodynamic Signals. Forging Connections between Computational Mathematics and Computational Geometry. Springer Proceedings in Mathematics & Statistics, 124, 2016, 978-3-319-16139-6. 10.5176/2251-1911_CMCGS14.50_23 . hal-01573108

HAL Id: hal-01573108

<https://hal.science/hal-01573108v1>

Submitted on 9 Aug 2017

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The Granger Causality Effect between Cardiorespiratory Hemodynamic Signals

Samir Ghouali, Mohammed Feham, and Yassine Zakarya Ghouali

Abstract Granger causality (GC) is one of the most popular measures to reveal causality influence of time series based on the estimated linear regression model and has been widely applied in economics and neuroscience due to its reliability, clarity, and robustness.

Granger causality has recently received increasing attention to study causal interactions of neurophysiological data; in this chapter we have developed a model of causality between the respiratory, hemodynamic, and cardiac signals, more specifically, a study based on the Granger causality between three ECG leads, blood pressure, central venous pressure, pulmonary arterial pressure, respiratory impedance, and airway CO₂. We selected 187 patients of 250 for our study, taken from Montreal General Hospital/MF (Massachusetts General Hospital/Marquette Foundation) databases. These signals are ideal for understanding causality and coupling (unidirectional or bidirectional).

In this approach we aim to analyze and understand the interactions between the signals mentioned above, and identify the significance of this interaction. The originality of this chapter is the number of variables selected for the study. Unlike the majority of studies that are conducted only with two variables, our study is multidimensional. The main advantage of a multidimensional and multivariable model is to solve a myriad of problems which is not the case in the two-dimensional studies.

Keywords Direction of information • Cardiorespiratory hemodynamic signals • Granger causality • Multivariate study • (MGH/MF) waveform database

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Introduction

Numerous studies in recent years have been devoted to the evaluation of causality; several applications of the latter are omnipresent in areas ranging from the economy [1, 2], climatology [3–5], directed information theory in networks [6], psychiatry [7], brain imaging field [8] and especially the analysis of biological systems, with a very special emphasis on the neural field [9–20], and the study of cardiac signals [21–29].

Although this one is not the universal definition of causality [30], it is commonly accepted that the notion of causality of two events describes why one event is caused by the other. According to this very general definition, in this chapter, we will look at the cardiovascular field. The importance of causality in this case appears in the spontaneous cardiovascular variability and complexity of cardiovascular regulation.

The detection and modeling of this causality depends strictly on all signals exploited to describe the observed interactions [31]. Characterization of the interdependence between the sensed signals is one of the most critical problems in cardiovascular pathophysiology [32]. From this idea, we can independently evaluate by what is called the strength of the relationship between two signals to a minimum.

The causality analysis has the ability to provide an original framework to identify the responsible mechanism for the spontaneous variations without the intervention of an artificial stimulus such as pharmacological intervention, an experiment on the patient or more severely surgery to obtain a specific causal relationship.

Causality is usually tested in time [33], frequency [34], and information domains [30]. In this chapter, methods assessing causality in time domain were chosen because they do not need to assume that the cardiovascular control mechanisms occur along specific temporal scales and the distribution of the statistic assessing causality under the null hypothesis of absence of a causal relationship between the two series is well known, thus allowing to easily keep under control the percentage of false causality detections.

In our case, we will look at the Granger causality because it perfectly studies multivariable models (several variables at once). The mathematical formulation of causality in measurable terms of predictability was given by Wiener [35]. Granger [1] introduced a specific notion of causality into time series analysis by evaluation of predictability autoregressive models.

The cardiovascular system is regulated by numerous control mechanisms acting to guarantee that the necessities of each physiological area are satisfied and that cardiovascular variables do not assume values incompatible with life. In the latter application, a large body of work has been developed to assess causality in both cardiovascular [24, 25] and cardiorespiratory [27] interactions.

Unfortunately, the world of physiological signals describing the behavior of a given system is not affected; it means that there's a restriction in the use of necessary

physiological signals, which implies a certain limitation in reliability of causality. To achieve this, we can give a small example of a study of causality between heart period (HP) and systolic arterial pressure (SAP), it would be preferable to include breathing (R) in the test [36]. According to several studies, neglecting (R), this can lead us to erroneous results in the causal interpretation mentioned above, this observation suggests that (R) should be included in this causality, the validation of this hypothesis has been approved by work [40] and they were able to uncover the importance of (R) in the causation (HP) and (SAP).

This demonstration gave us the reflex to study a large number of physiological signals from the outset, to eliminate any gaps, ignorance, and negligence of other signals, multivariate system identification approaches permit the dynamic characterization of the causal interactions among cardiovascular regulatory mechanisms responsible for coupling the variability between signals (e.g., heart rate, arterial pressure, and respiratory signal) [38]. Multivariate characterization is not solely helpful both to derive information about the gain and phase of the relationship linking any signal pair but also to estimate causality (i.e., who drives whom) in multivariate recordings.

Due to cardiopulmonary anatomy, there are strong mechanical interactions between the mechanical activity of the heart and respiratory movements which implies a change in atrial and pulmonary receptors [39]. The approach was applied to three ECG leads, ART (arterial pressure), PAP (pulmonary arterial pressure), CVP (central venous pressure), respiratory impedance, and airways CO₂, taken from the MGH/MF (Massachusetts General Hospital/Foundation Marquette) database. These signals are ideal for understanding causality and coupling (unidirectional or bidirectional).

The goal of the chapter is to propose to study the direction of causality between the signals mentioned previously; our contribution in this chapter is based on the following points:

- 3D Analysis of cardiovascular signals.
- Study bivariate/multivariate between the cardiovascular, respiratory, and hemodynamic signals.
- Have the necessary information and details for our next work to develop telemedicine applications on smartphones and especially on the part intended to signal analysis.

The remainder of this chapter is organized as follows: In “Definitions of Some Variables Studied,” we give some definition of the variables. In “Data and Methodology,” we will establish the data used and the methodology to follow. Then, in “The Method and Findings,” we present the model used and the result obtained. And finally in “Discussions,” we lead an analysis, scientific discussion, and a projection of perspectives.

Definitions of Some Variables Studied

ECG leads: Lead systems allow you to look at the heart from different angles. Each different angle is called a lead. The different leads can be compared to radiographs taken from different angles.

ART: Blood pressure is the pressure of blood in the arteries, also referred to as blood pressure because this pressure is the force exerted by the blood against the walls of arteries, tends the wall of the artery.

CVP: Central venous pressure (CVP) also known as right atrial pressure (RAP) describes the pressure of blood in the thoracic vena cava near the right atrium of the heart; it reflects the amount of blood returning to heart and the heart's ability to pump blood into the arterial system.

PAP: Pulmonary arterial pressure measures the pressure in the pulmonary arteries, the latter carries blood from the right side of the heart to the lungs.

Data and Methodology

Data Analysis

The Massachusetts General Hospital/Marquette Foundation (MGH/MF) Waveform database is a comprehensive collection of electronic records of hemodynamic and electrocardiographic signals of stable and unstable patients in intensive care units, operating rooms, and cath labs heart. It is the result of collaboration between physicians, biomedical engineers, and nurses of the Massachusetts General Hospital [40], which includes three ECG leads, arterial pressure, pulmonary arterial pressure, central venous pressure, respiratory impedance, and airway CO₂. This multidimensional cardiac data collected from various parts of body can effectively imitate the signals from various body sensor nodes.

The original dataset contains total 250 sets of cardiac signals, each containing 12–86 min in most cases are about an hour of recording. We selected 187 patients were selected on 250 for our simulation and contain all the signals mentioned above unlike the rest that does not have the typical data to our studies, these signals include cardiac events such as extrasystole, premature supraventricular tachycardia, bradycardia, extrasystole and ventricular stimulation, which are manually annotated by clinical professionals.

Methodology

In the analysis of the causality relationship, the choice of the appropriate technique is an important theoretical and empirical question. Granger causality is the most appropriate technique to study the relationship between hemodynamic,

cardiorespiratory, and electrocardiographic signals. The empirical strategy used in this chapter can be divided into three main stages. First, unit root tests in series are undertaken to determine the stationarity of the series. Second, the AIC (Ackaik) and SIC (Schwarz) criteria are used to determine the optimal lag used in the method of causation. Third and finally, we will test for multivariate causality proposed by Granger.

The Method and Findings

The Model Specification

The first causality was proposed and introduced by Wiener and Granger (Nobel 2003) and became a fundamental theory for the analysis of dynamic relationships between time series. Sims presented a slightly different specification of test by considering that future values help explain the present values.

In the remainder of this chapter, we will look at the multivariate Grange causality. Before beginning the multivariate Granger causality, it is necessary to move to the bivariate causality to see the difference and the limitation of the latter.

Bivariate Granger Causality Test

In this part, we will try to present the definitions of linear causality and discuss the following tests to identify the causal relationship between these two variables.

Granger causality test is designed to detect causal direction between two time series by examining a correlation between the current value of one variable and the past values of another variable. Based on Granger’s definition of causality Y is strictly Granger causing X if the conditional distribution of X_t , given the past observation X_{t-1}, X_{t-2}, \dots and Y_{t-1}, Y_{t-2}, \dots differs from the conditional distribution of X_t , given the past observation X_{t-1}, X_{t-2}, \dots only.

Intuitively, Y is a Granger cause of X if adding past observations of Y to the information set increases the knowledge on the distribution of current values of X. More precisely, the linear Granger causality is conducted based on the following two-equation model:

$$x_t = a_1 + \sum_{i=1}^k \alpha_i x_{t-i} + \sum_{i=1}^k \beta_i y_{t-i} + e_{1t} \tag{1}$$

and

$$y_t = a_2 + \sum_{i=1}^k \mu_i x_{t-i} + \sum_{i=1}^k \Omega_i y_{t-i} + e_{2t} \tag{2}$$

Where all $\{x_t\}$ and $\{y_t\}$ are stationary variables, e_{1t} and e_{2t} are the disturbances satisfying the regularity assumptions of the classical linear regression model and k is the optimal lag.

We say that the variable $\{y_t\}$ is not to Granger causality $\{x_t\}$ if and only if $\beta_i = 0$ in (1), for any $i = 1, \dots, k$. To better explain this, the past values of $\{y_t\}$ do not provide any additional information on the performance of $\{x_t\}$. In the same manner, and vice versa, $\{x_t\}$ does not Granger causality $\{y_t\}$ if and only if $\mu_i = 0$ in (2), for any $i = 1, \dots, k$.

We can test the causal relationships between two variables $\{x_t\}$ and $\{y_t\}$ by checking the null hypothesis separately:

$$H_0^1 : \beta_1 = \dots = \beta_k = 0 \text{ and } H_0^2 : \mu_1 = \dots = \mu_k = 0$$

1. If both Hypotheses H_0^1 and H_0^2 are accepted, there is no linear causal relationship between $\{x_t\}$ and $\{y_t\}$.
2. If hypothesis H_0^1 is accepted but hypothesis H_0^2 is rejected, then there exists linear causality running unidirectionally from $\{x_t\}$ to $\{y_t\}$.
3. If hypothesis H_0^1 is rejected but hypothesis H_0^2 is accepted, then there exists linear causality running unidirectionally from $\{y_t\}$ to $\{x_t\}$.
4. If both Hypotheses H_0^1 and H_0^2 are rejected, then there is feedback linear causal relationship between $\{x_t\}$ and $\{y_t\}$.

The purpose of the multidimensional study is that it allows us to quantitatively analyze and respond to a myriad of problems and suggestions which is not the case in bivariate studies (multivariate Granger causality: an estimation framework based on factorization of the spectral density matrix).

Multivariate Granger Causality

Consider a p dimensional multivariate stochastic process, $X(t) = [x_t, y_t, \dots, z_t]^T$, where the rank of the original matrix $X(t)$ is $[p, 1]$, we can estimate the model as MVAR (multivariate autoregressive) result:

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ \vdots \\ x_p(t) \end{pmatrix} = \sum_{K=1}^{\infty} \begin{bmatrix} A_{11}(K) & A_{12}(K) & A_{13}(K) & \dots & A_{1p}(K) \\ A_{21}(K) & A_{22}(K) & A_{23}(K) & \dots & A_{2p}(K) \\ \vdots & \vdots & & \dots & \vdots \\ \vdots & \vdots & & \dots & \vdots \\ A_{p1}(K) & A_{p2}(K) & A_{p3}(K) & \dots & A_{pp}(K) \end{bmatrix} \begin{pmatrix} x_1(t-K) \\ x_2(t-K) \\ \vdots \\ \vdots \\ x_p(t-K) \end{pmatrix} + \begin{pmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \vdots \\ \vdots \\ \varepsilon_p(t) \end{pmatrix} \tag{3}$$

where $A_{ij}(K)$ is the coefficient at K Th lag and $\varepsilon_i(t)$ is a corresponding error terms.

F-Test

Several statistics could be used to test the above hypotheses; one of the most commonly used statistics is the standard F-test. It is a statistical hypothesis test for testing the equality of two variances by taking the ratio of the two variances, it can be represented by:

$$F - test = \frac{\sigma_x^2}{\sigma_y^2} \tag{4}$$

Our Equations

First equation:

$$\begin{aligned}
 & \left(\begin{array}{c} ECG\ 1\ (t) \\ ART\ (t) \\ CVP\ (t) \\ PAP\ (t) \\ RESP\ (t) \\ CO2\ (t) \end{array} \right) = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \\
 & + \begin{bmatrix} A_{11,1} & A_{12,1} & A_{13,1} & A_{14,1} & A_{15,1} & A_{16,1} \\ A_{21,1} & A_{22,1} & A_{23,1} & A_{24,1} & A_{25,1} & A_{26,1} \\ A_{31,1} & A_{32,1} & A_{33,1} & A_{34,1} & A_{35,1} & A_{36,1} \\ A_{41,1} & A_{42,1} & A_{43,1} & A_{44,1} & A_{45,1} & A_{46,1} \\ A_{51,1} & A_{52,1} & A_{53,1} & A_{54,1} & A_{55,1} & A_{56,1} \\ A_{61,1} & A_{62,1} & A_{63,1} & A_{64,1} & A_{65,1} & A_{66,1} \end{bmatrix} \begin{bmatrix} ECG\ 1\ (t - 1) \\ ART\ (t - 1) \\ CVP\ (t - 1) \\ PAP\ (t - 1) \\ RESP\ (t - 1) \\ CO2\ (t - 1) \end{bmatrix} + \dots \\
 & + \begin{bmatrix} A_{11,k} & A_{12,k} & A_{13,k} & A_{14,k} & A_{15,k} & A_{16,k} \\ A_{21,k} & A_{22,k} & A_{23,k} & A_{24,k} & A_{25,k} & A_{26,k} \\ A_{31,k} & A_{32,k} & A_{33,k} & A_{34,k} & A_{35,k} & A_{36,k} \\ A_{41,k} & A_{42,k} & A_{43,k} & A_{44,k} & A_{45,k} & A_{46,k} \\ A_{51,k} & A_{52,k} & A_{53,k} & A_{54,k} & A_{55,k} & A_{56,k} \\ A_{61,k} & A_{62,k} & A_{63,k} & A_{64,k} & A_{65,k} & A_{66,k} \end{bmatrix} \begin{bmatrix} ECG\ 1\ (t - k) \\ ART\ (t - k) \\ CVP\ (t - k) \\ PAP\ (t - k) \\ RESP\ (t - k) \\ CO2\ (t - k) \end{bmatrix} \\
 & + \begin{bmatrix} A_{11,p} & A_{12,p} & A_{13,p} & A_{14,p} & A_{15,p} & A_{16,p} \\ A_{21,p} & A_{22,p} & A_{23,p} & A_{24,p} & A_{25,p} & A_{26,p} \\ A_{31,p} & A_{32,p} & A_{33,p} & A_{34,p} & A_{35,p} & A_{36,p} \\ A_{41,p} & A_{42,p} & A_{43,p} & A_{44,p} & A_{45,p} & A_{46,p} \\ A_{51,p} & A_{52,p} & A_{53,p} & A_{54,p} & A_{55,p} & A_{56,p} \\ A_{61,p} & A_{62,p} & A_{63,p} & A_{64,p} & A_{65,p} & A_{66,p} \end{bmatrix} \begin{bmatrix} ECG\ 1\ (t - p) \\ ART\ (t - p) \\ CVP\ (t - p) \\ PAP\ (t - p) \\ RESP\ (t - p) \\ CO2\ (t - p) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \tag{5}
 \end{aligned}$$

Second equation:

$$\begin{pmatrix} ECG\ 2(t) \\ ART(t) \\ CVP(t) \\ PAP(t) \\ RESP(t) \\ CO_2(t) \end{pmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}$$

$$+ \begin{bmatrix} A_{11,1} & A_{12,1} & A_{13,1} & A_{14,1} & A_{15,1} & A_{16,1} \\ A_{21,1} & A_{22,1} & A_{23,1} & A_{24,1} & A_{25,1} & A_{26,1} \\ A_{31,1} & A_{32,1} & A_{33,1} & A_{34,1} & A_{35,1} & A_{36,1} \\ A_{41,1} & A_{42,1} & A_{43,1} & A_{44,1} & A_{45,1} & A_{46,1} \\ A_{51,1} & A_{52,1} & A_{53,1} & A_{54,1} & A_{55,1} & A_{56,1} \\ A_{61,1} & A_{62,1} & A_{63,1} & A_{64,1} & A_{65,1} & A_{66,1} \end{bmatrix} \begin{bmatrix} ECG\ 2(t-1) \\ ART(t-1) \\ CVP(t-1) \\ PAP(t-1) \\ RESP(t-1) \\ CO_2(t-1) \end{bmatrix} + \dots$$

$$+ \begin{bmatrix} A_{11,k} & A_{12,k} & A_{13,k} & A_{14,k} & A_{15,k} & A_{16,k} \\ A_{21,k} & A_{22,k} & A_{23,k} & A_{24,k} & A_{25,k} & A_{26,k} \\ A_{31,k} & A_{32,k} & A_{33,k} & A_{34,k} & A_{35,k} & A_{36,k} \\ A_{41,k} & A_{42,k} & A_{43,k} & A_{44,k} & A_{45,k} & A_{46,k} \\ A_{51,k} & A_{52,k} & A_{53,k} & A_{54,k} & A_{55,k} & A_{56,k} \\ A_{61,k} & A_{62,k} & A_{63,k} & A_{64,k} & A_{65,k} & A_{66,k} \end{bmatrix} \begin{bmatrix} ECG\ 2(t-k) \\ ART(t-k) \\ CVP(t-k) \\ PAP(t-k) \\ RESP(t-k) \\ CO_2(t-k) \end{bmatrix}$$

$$+ \begin{bmatrix} A_{11,p} & A_{12,p} & A_{13,p} & A_{14,p} & A_{15,p} & A_{16,p} \\ A_{21,p} & A_{22,p} & A_{23,p} & A_{24,p} & A_{25,p} & A_{26,p} \\ A_{31,p} & A_{32,p} & A_{33,p} & A_{34,p} & A_{35,p} & A_{36,p} \\ A_{41,p} & A_{42,p} & A_{43,p} & A_{44,p} & A_{45,p} & A_{46,p} \\ A_{51,p} & A_{52,p} & A_{53,p} & A_{54,p} & A_{55,p} & A_{56,p} \\ A_{61,p} & A_{62,p} & A_{63,p} & A_{64,p} & A_{65,p} & A_{66,p} \end{bmatrix} \begin{bmatrix} ECG\ 2(t-p) \\ ART(t-p) \\ CVP(t-p) \\ PAP(t-p) \\ RESP(t-p) \\ CO_2(t-p) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

(6)

Third equation:

$$\begin{pmatrix} ECG\ 3(t) \\ ART(t) \\ CVP(t) \\ PAP(t) \\ RESP(t) \\ CO2(t) \end{pmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} + \begin{matrix} + \\ + \\ + \end{matrix} \begin{bmatrix} A_{11,1} & A_{12,1} & A_{13,1} & A_{14,1} & A_{15,1} & A_{16,1} \\ A_{21,1} & A_{22,1} & A_{23,1} & A_{24,1} & A_{25,1} & A_{26,1} \\ A_{31,1} & A_{32,1} & A_{33,1} & A_{34,1} & A_{35,1} & A_{36,1} \\ A_{41,1} & A_{42,1} & A_{43,1} & A_{44,1} & A_{45,1} & A_{46,1} \\ A_{51,1} & A_{52,1} & A_{53,1} & A_{54,1} & A_{55,1} & A_{56,1} \\ A_{61,1} & A_{62,1} & A_{63,1} & A_{64,1} & A_{65,1} & A_{66,1} \end{bmatrix} \begin{bmatrix} ECG\ 3(t-1) \\ ART(t-1) \\ CVP(t-1) \\ PAP(t-1) \\ RESP(t-1) \\ CO2(t-1) \end{bmatrix} + \dots \\
 + \begin{matrix} + \\ + \\ + \end{matrix} \begin{bmatrix} A_{11,k} & A_{12,k} & A_{13,k} & A_{14,k} & A_{15,k} & A_{16,k} \\ A_{21,k} & A_{22,k} & A_{23,k} & A_{24,k} & A_{25,k} & A_{26,k} \\ A_{31,k} & A_{32,k} & A_{33,k} & A_{34,k} & A_{35,k} & A_{36,k} \\ A_{41,k} & A_{42,k} & A_{43,k} & A_{44,k} & A_{45,k} & A_{46,k} \\ A_{51,k} & A_{52,k} & A_{53,k} & A_{54,k} & A_{55,k} & A_{56,k} \\ A_{61,k} & A_{62,k} & A_{63,k} & A_{64,k} & A_{65,k} & A_{66,k} \end{bmatrix} \begin{bmatrix} ECG\ 3(t-k) \\ ART(t-k) \\ CVP(t-k) \\ PAP(t-k) \\ RESP(t-k) \\ CO2(t-k) \end{bmatrix} \\
 + \begin{matrix} + \\ + \\ + \end{matrix} \begin{bmatrix} A_{11,p} & A_{12,p} & A_{13,p} & A_{14,p} & A_{15,p} & A_{16,p} \\ A_{21,p} & A_{22,p} & A_{23,p} & A_{24,p} & A_{25,p} & A_{26,p} \\ A_{31,p} & A_{32,p} & A_{33,p} & A_{34,p} & A_{35,p} & A_{36,p} \\ A_{41,p} & A_{42,p} & A_{43,p} & A_{44,p} & A_{45,p} & A_{46,p} \\ A_{51,p} & A_{52,p} & A_{53,p} & A_{54,p} & A_{55,p} & A_{56,p} \\ A_{61,p} & A_{62,p} & A_{63,p} & A_{64,p} & A_{65,p} & A_{66,p} \end{bmatrix} \begin{bmatrix} ECG\ 3(t-p) \\ ART(t-p) \\ CVP(t-p) \\ PAP(t-p) \\ RESP(t-p) \\ CO2(t-p) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \tag{7}$$

We discussed earlier, we will try to make a causality test in three dimensions, after extensive research of the model, and it was found that could write our mathematical equations

Empirical Result

Before beginning our results, we must clarify some details and signs:

- Corresponds to the causal direction between three ECG leads to (ART, CVP, PAP, RESP, and CO₂).
- Corresponds to the causal direction between (ART, CVP, PAP, RESP, and CO₂) to three ECG leads.
- *: Indicates statistical significance at 1 %.
- A value above the sign, is the value of F-statistic, which is considered a measure of the correlation between the variables studied.
- A value above the sign, which is in brackets, corresponds to the value of the probability of causation.
- (x. E x): (x exponential x).
- *MGH Number*: corresponds to a given patient.

The displayed results are partial; we only look at a sample of 30 patients randomly chosen, all this in aim of showing the methodology followed for causality. In our chapter, the following tables summarize the results obtained and the rate of causality in each direction represented by the F-statistic seen previously and the corresponding probability.

<i>MGH002</i>					
Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	13.4646*	5.1146*	11.8814*	1.7138*	1.6459*
	→	→	→	→	→
	(8.E-100)	(5.E-27)	(1.E-85)	(0.0019)	(0.0038)
ECG2	12.3759*	4.8100*	12.040*	1.3941	1.3370
	→	→	→		
	(5.E-90)	(1.E-24)	(6.E-87)	(0.0399)	(0.0631)
ECG3	5.80308*	3.13526*	7.27528*	0.83496	0.83210
	→	→	→		
	(1.E-32)	(5.E-12)	(5.E-45)	(0.7789)	0.7835

MGH005

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	3.3758* → (1.E-13) ←	5.1716* → (2.E-27) ←	8.2914* → (8.E-54) ←	6.3103* → (7.E-37) ←	5.0512* → (2.E-26) ←
ECG2	3.5804* → (3.E-15) ←	3.5811* → (3.E-15) ←	3.8269* → (5.E-17) ←	5.2412* → (5.E-28) ←	3.6362* → (1.E-15) ←
ECG3	4.8860* → (3.E-25) ←	4.2643* → (2.E-20) ←	8.1746* → (9.E-53) ←	3.1918* → (2.E-12) ←	3.1083* → (8.E-12) ←

MGH010

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	1.2906 → (0.0897)	1.4608 → (0.0225)	1.3528 → (0.0557)	1.1898 → (0.1772)	1.3872 → (0.0422)
ECG2	1.2241 → (0.1421)	1.4560 → (0.0235)	2.0112* → (6.E-05) ←	1.6922* → (0.0024) ←	1.8152* → (0.0006) ←
ECG3	1.2631 → (0.1088)	1.3919 → (0.0406)	1.5093 → (0.0145)	1.6529* → (0.0035) ←	1.8584* → (0.0004) ←

MGH016

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	5.81434* → (9.E-33) ←	9.07583* → (1.E-60) ←	8.58176* → (2.E-56) ←	8.94331* → (2.E-59) ←	6.14600* → (2.E-35) ←
ECG2	5.51416* → (3.E-30) ←	4.90756* → (2.E-25) ←	6.58831* → (3.E-39) ←	4.93960* → (1.E-25) ←	4.42447* → (1.E-21) ←
ECG3	3.86895* → (3.E-17) ←	4.25168* → (3.E-20) ←	4.58885* → (7.E-23) ←	4.40791* → (2.E-21) ←	3.79522* → (9.E-17) ←

MGH019

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	13.1779*	63.8833*	37.2080*	15.3122*	4.91764*
	(5 E-99)	(0.0000)	(0.0000)	(2.E-116)	(2.E-25)
	←	←	←	←	←
ECG2	30.9990*	17.460*	32.044*	21.700*	19.270*
	(2.E-259)	(5.E-136)	(7.E-269)	(1.E-174)	(2.E-152)
	←	←	←	←	←
ECG3	36.4516*	37.4346*	31.4419*	18.0569*	17.6442*
	(0.0000)	(0.0000)	(2.E-263)	(2.E-141)	(1.E-137)
	←	←	←	←	←

MGH025

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	18.9841*	21.9002*	20.9682*	14.2303*	14.2677*
	(6.E-150)	(2.E-176)	(5.E-168)	(1.E-106)	(5.E-107)
	←	←	←	←	←
ECG2	16.8669*	18.2168*	17.6601*	14.084*	12.5292*
	(1.E-130)	(6.E-143)	(7.E-138)	(8.E-105)	(2.E-91)
	←	←	←	←	←
ECG3	21.9698*	37.8166*	37.6249*	34.7434*	30.9402*
	(4.E-177)	(0.0000)	(0.0000)	(2.E-293)	(7.E-259)
	←	←	←	←	←

MGH030

Lags 45	ART	CVP	PAP	RESP	CO ₂
ECG1	6.80357*	6.69719*	16.6775*	10.6880*	11.7569*
	(3.E-40)	(3.E-39)	(3.E-126)	(2.E-73)	(1.E-82)
	←	←	←	←	←
ECG2	9.29697*	9.53410*	8.45248*	8.85198*	19.4575*
	(5.E-60)	(5.E-62)	(6 E-53)	(3 E-56)	(9.E-148)
	←	←	←	←	←
ECG3	4.39468*	3.27079*	17.1573*	12.3559*	12.5787*
	(6.E-21)	(1.E-12)	(2.E-130)	(6.E-88)	(7.E-90)
	←	←	←	←	←

MGH038

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	34.1336*	42.0905*	45.3102*	29.5296*	31.7334*
	→	→	→	→	→
	(8.E-288)	(0.0000)	(0.0000)	(5.E-246)	(5.E-266)
	←	←	←	←	←
ECG2	18.9646*	21.7894*	24.6819*	12.1942*	13.3511*
	→	→	→	→	→
	(1.E-149)	(2.E-175)	(7.E-202)	(2.E-88)	(9.E-99)
	←	←	←	←	←
ECG3	23.5344*	29.4159*	30.9951*	19.4842*	23.2000*
	→	→	→	→	→
	(2.E-191)	(5.E-245)	(2.E-259)	(2.E-154)	(2.E-188)
	←	←	←	←	←

MGH040

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	8.36020*	37.9324*	8.91593*	274.813*	297.264*
	→	→	→	→	→
	(2.E-54)	(0.0000)	(3.E-59)	(0.0000)	(0.0000)
	←	←	←	←	←
ECG2	0.89056	1.19442	1.16756	1.71417*	1.44555
			→	→	
	(0.6825)	(0.1723)	(0.2032)	(0.0019)	(0.0258)
			←		
ECG3	1.23840	0.61661	1.88425*	1.16677	1.14884
			→		
	(0.1292)	(0.9809)	(0.0003)	(0.2042)	(0.2270)
		←			

MGH051

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	2.24987*	2.95403*	3.94570*	1.22965	1.77085*
	→		→		
	(3.E-06)	(9.E-11)	(7.E-18)	(0.1371)	(0.0010)
	←	←	←		←
ECG2	1.69168*	0.47920	1.72074*	1.00992	1.30522
	→		→		
	(0.0024)	(0.9989)	(0.0017)	(0.4536)	(0.0803)
	←		←		
ECG3	2.16370*	0.41638	2.28666*	0.78672	1.57048*
	→		→		
	(8.E-06)	(0.9998)	(2.E-06)	(0.8496)	(0.0076)
	←		←		←

MGH053

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	10.3098* (2.E-71) ←	5.50973* (3.E-30) ←	6.75286* → (1.E-40) ←	10.1978* (1.E-70) ←	10.8550* (2.E-76) ←
ECG2	4.73162* (6.E-24) ←	4.03205* (1.E-18) ←	5.24456* → (4.E-28) ←	1.77028* (0.0010) ←	4.92457* (2.E-25) ←
ECG3	21.9699* (4.E-77) ←	7.94890* → (8.E-51) ←	15.4340* → (1.E-117) ←	24.9747* → (1.E-204) ←	24.6144* (3.E-201) ←

MGH057

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	4.67572* (2.E-23) ←	1.21919 (0.1469)	3.39351* (8.E-14) ←	7.15210* (5.E-44) ←	3.57831* → (4.E-15) ←
ECG2	4.67572* (2.E-23) ←	2.18725* (6.E-06) ←	3.6724* (7.E-16) ←	3.73050* → (3.E-16) ←	1.46234 (0.0222)
ECG3	4.99836* (4.E-26) ←	1.43633 (0.0279)	5.08211* (9.E-27) ←	6.47808* → (3.E-38) ←	4.33791* (7.E-21) ←

MGH063

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	18.4866* → (2.E-145) ←	11.2375* → (8.E-80) ←	6.15746* → (1.E-35) ←	2.87877* → (3.E-10) ←	1.32860 (0.0673)
ECG2	1.05992 (0.3630)	1.06988 (0.3460)	1.74440* (0.0013) ←	1.58709* → (0.0069) ←	2.75945* (2.E-09) ←
ECG3	0.97387 (0.5226)	1.30668 (0.0795)	1.20308 (0.1631)	1.00263 → (0.4674)	0.44818 (0.9996)

MGH083

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	12.7866*	22.5566*	20.9038*	17.7196*	14.8111*
	→	→	→	→	→
	(1.E-93)	(2.E-182)	(2.E-67)	(2.E-138)	(6.E-112)
ECG2	7.61496*	9.96653*	9.84395*	13.3686*	9.15010*
	→	→	→	→	→
	(6.E-48)	(2.E-168)	(2.E-67)	(6.E-99)	(3.E-61)
ECG3	7.68454*	11.0423*	9.93059*	12.8800*	8.41489*
	→	→	→	→	→
	(1.E-48)	(5.E-78)	(3.E-68)	(2.E-94)	(7.E-55)

MGH089

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	73.1543*	95.5537*	31.9302*	31.0302*	23.8196*
	→	→	→	→	→
	(0.000)	(0.000)	(8.E-268)	(1.E-259)	(5.E-194)
ECG2	11.3719*	10.9364*	2.96698*	5.51151*	2.26551*
	→	→	→	→	→
	(5.E-81)	(4.E-77)	(7.E-11)	(3.E-30)	(2.E-06)
ECG3	24.1756*	33.800*	25.4177*	25.3225*	18.7283*
	→	→	→	→	→
	(3.E-197)	(9.E-285)	(1.E-208)	(1.E-207)	(1.E-147)

MGH092

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	2.08356*	4.54662*	6.58337*	4.53687*	5.11720*
		→	→	→	
	(2.E-05)	(2.E-22)	(4.E-39)	(2.E-22)	(5.E-27)
ECG2	1.80886*	6.06665*	7.47223*	5.26159*	3.28409*
				→	
	(0.0007)	(8.E-35)	(1.E-46)	(3.E-28)	(5.E-13)
ECG3	1.81199*	10.0576*	21.9470*	11.5843*	2.75473*
		→	→	→	→
	(0.0006)	(3.E-69)	(6.E-177)	(7.E-83)	(2.E-09)

MGH100

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	4.91054*	16.0426*	3.79936*	20.7232*	3.24029*
	(2.E-25)	(4.E-123)	(8.E-17)	(9.E-166)	(1.E-12)
	←	←	←	←	←
ECG2	0.79440	7.89769*	1.16490	8.52316*	1.21279
	(0.8393)	(2.E-50)	(0.2065)	(8.E-56)	(0.1532)
		←		←	
ECG3	8.75542*	15.9131*	8.65074*	17.7816*	9.32829*
	(7.E-58)	(6.E-122)	(6.E-57)	(6.E-139)	(7.E-63)
	←	←	←	←	←

MGH112

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	4.15145*	3.49901*	11.9535*	12.3790*	3.28641*
	(2.E-19)	(1.E-14)	(3.E-86)	(5.E-50)	(4.E-13)
	←	←	←	←	←
ECG2	5.43397*	3.44163*	10.0673*	8.05470*	18.3041*
	(1.E-29)	(3.E-14)	(2.E-69)	(9.E-52)	(1.E-143)
	←	←	←	←	←
ECG3	34.8159*	23.0383*	29.4004*	23.7304*	55.1399*
	(6.E-294)	(7.E-187)	(9.E-245)	(3.E-193)	(0.0000)
	←	←	←	←	←

MGH120

Lags 41	ART	CVP	PAP	RESP	CO ₂
ECG1	3.10504*	18.4799*	2.90835*	1.37478	2.97306*
	(1.E-10)	(4.E-130)	(2.E-09)	(0.0558)	(6.E-10)
	←	←	←		←
ECG2	1.91610*	8.35619*	1.03304	1.13946	1.95470*
	(0.0004)	(9.E-49)	(0.4125)	(0.2494)	(0.0003)
	←	←			←
ECG3	5.81538*	20.848*	5.41456*	2.19320*	4.85186*
	(2.E-29)	(2.E-149)	(2.E-26)	(2.E-05)	(2.E-22)
	←	←	←	←	←

MGH125

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	37.2982*	59.1995*	56.2916*	41.9906*	46.1347*
	→ (0.0000)	→ (0.0000)	→ (0.0000)	→ (0.0000)	→ (0.0000)
	←	←	←	←	←
ECG2	23.9073*	32.6219*	34.3082*	27.3919*	26.0281*
	→ (8.E-195)	→ (4.E-274)	→ (2.E-289)	→ (1.E-226)	→ (4.E-214)
	←	←	←	←	←
ECG3	30.9119*	43.8797*	44.4954*	31.8713*	36.9114*
	→ (1.E-258)	→ (0.0000)	→ (0.0000)	→ (3.E-267)	→ (0.0000)
	←	←	←	←	←

MGH133

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	36.6303*	7.08807*	20.1787*	7.89099*	33.9250*
	→ (0.0000)	→ (2.E-43)	→ (8.E-161)	→ (2.E-50)	→ (7.E-286)
	←	←	←	←	←
ECG2	26.6880*	4.67989*	18.5284*	5.10445*	26.7680*
	→ (4.E-220)	→ (1.E-23)	→ (9.E-146)	→ (6.E-27)	→ (7.E-221)
	←	←	←	←	←
ECG3	11.8091*	5.58807*	11.1692*	6.56718*	10.2525*
	→ (7.E-85)	→ (1.E-30)	→ (3.E-79)	→ (5.E-39)	→ (5.E-71)
	←	←	←	←	←

MGH138

Lags 45	ART	CVP	PAP	RESP	CO ₂
ECG1	3.12395*	23.4754*	11.4000*	22.2300*	3.46775*
	→ (1.E-11)	→ (7.E-187)	→ (1.E-79)	→ (9.E176)	→ (4.E-14)
	←	←	←	←	←
ECG2	14.7510*	7.75526*	24.6622*	7.27460*	17.9733*
	→ (4.E-109)	→ (4.E-48)	→ (2.E-197)	→ (4.E-44)	→ (9.E-138)
	←	←	←	←	←
ECG3	2.65184*	30.2701*	8.61800*	22.8755*	2.22336*
	→ (1.E-08)	→ (2.E-247)	→ (2.E-55)	→ (2.E-181)	→ (5.E-06)
	←	←	←	←	←

MGH140

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	10.6366* → (7.E-73) ←	16.0300* → (2.E-120) ←	2.64371* → (1.E-08) ←	5.33928* → (3.E-28) ←	1.78006* → (0.0010) ←
ECG2	2.74211* → (3.E-09) ←	4.28183* → (5.E-120) ←	0.59752 → (0.9852) ←	1.26172 → (0.1202) ←	17.6785* → (4.E-135) ←
ECG3	1.10066 → (0.2976) ←	1.47381 → (0.0211) ←	0.55523 → (0.9932) ←	1.40337 → (0.0384) ←	53.3474* → (0.0000) ←

MGH145

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	23.3972* → (4.E-190) ←	37.4371* → (0.0000) ←	6.08542* → (5.E-35) ←	48.1748* → (0.0000) ←	33.5794* → (9.E-283) ←
ECG2	8.96339* → (1.E-59) ←	29.8548* → (5.E-249) ←	1.67581* → (0.0028) ←	17.5093* → (2.E-136) ←	10.5899* → (5.E-74) ←
ECG3	4.27008* → (2.E-20) ←	32.6098* → (5.E-274) ←	1.29463 → (0.0869) ←	28.2415* → (3.E-234) ←	11.3783* → (5.E-81) ←

MGH149

Lags 42	ART	CVP	PAP	RESP	CO ₂
ECG1	11.4165* → (1.E-74) ←	60.0942* → (0.000) ←	50.2848* → (0.000) ←	25.8305* → (1.E-194) ←	0.85858 → (0.7282) ←
ECG2	4.17901* → (4.E-18) ←	17.564* → (2.E-125) ←	12.394* → (1.E-82) ←	19.6183* → (1.E-142) ←	2.0663* → (6.E-05) ←
ECG3	5.30225* → (3.E-26) ←	17.5014* → (5.E-125) ←	14.5339* → (2.E-100) ←	18.8263* → (5.E-136) ←	2.81741* → (4.E-09) ←

MGH157

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	6.39863*	44.7048*	42.9332*	70.5331*	35.5973*
	→	→	→	→	→
	(1.E-37)	(0.0000)	(0.0000)	(0.0000)	(5.E-301)
ECG2	20.2817*	41.8912*	80.1427*	97.6992*	33.1709*
	→	→	→	→	→
	(1.E-161)	(0.0000)	(0.0000)	(0.0000)	(4.E-279)
ECG3	12.7252*	23.8653*	38.6965*	42.1619*	19.8184*
	→	→	→	→	→
	(4.E-93)	(2.E-194)	(0.0000)	(0.0000)	(2.E-157)

MGH164

Lags 45	ART	CVP	PAP	RESP	CO ₂
ECG1	10.6244*	38.4463*	39.8908*	3.08118*	0.46746
	→	→	→	→	→
	(9.E-73)	(0.000)	(0.000)	(2.E-11)	(0.9199)
ECG2	14.8957*	30.153*	25.192*	24.0169*	4.5827*
	→	→	→	→	→
	(2.E-110)	(2.E-246)	(3 E-202)	(9.E-192)	(2.E-22)
ECG3	41.7183*	37.8544*	30.4166*	62.3338*	32.2493*
	→	→	→	→	→
	(0.0000)	(0.0000)	(8.E-249)	(0.0000)	(4.E-265)

MGH172

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	29.6336*	103.428*	98.9313*	14.0733*	15.5381*
	→	→	→	→	→
	(5.E-247)	(0.0000)	(0.0000)	(3.E-105)	(1.E-118)
ECG2	5.59478*	17.9636*	16.0905*	9.22566*	5.18213*
	→	→	→	→	→
	(6.E-31)	(1.E-140)	(1.E-123)	(6.E-62)	(1.E-27)
ECG3	7.74507*	5.19981*	2.82488*	9.66148*	12.2956*
	→	→	→	→	→
	(4.E-49)	(1.E-27)	(7.E-10)	(8.E-66)	(3.E-89)

MGH177

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	1.63370*	1.95421*	1.63423*	1.62246*	5.68801*
	→				→
	(0.0043)	(0.0001)	(0.0043)	(0.0048)	(1.E-31)
	←	←	←	←	←
ECG2	14.7421*	4.41430*	11.2246*	9.16150*	7.20844*
	→		→		→
	(2.E-111)	(2.E-21)	(7.E-80)	(2.E-61)	(2.E-44)
	←	←	←	←	←
ECG3	0.96802	1.44906	4.72939*	1.05768	1.80505*
	→				
	(0.5340)	(0.0250)	(6.E-24)	(0.3668)	(0.0007)
		←		←	

MGH182

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	7.39968*	9.08936*	9.27866*	9.78691*	6.90695*
	→	→	→	→	→
	(4 E-46)	(9 E-61)	(2.E-62)	(6 E-67)	(7 E-42)
	←	←	←	←	←
ECG2	7.62345*	7.9467*	10.627*	9.12096*	8.97988*
	→	→	→	→	→
	(5 E-48)	(8 E-51)	(2 E-74)	(5 E-61)	(8 E-60)
	←	←	←	←	←
ECG3	5.74363*	7.42181*	8.01550*	11.576*	9.77919*
	→	→	→	→	→
	(4.E-32)	(3.E-46)	(2.E-51)	(8.E-83)	(8.E-67)
	←	←	←	←	←

MGH186

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	27.0971*	40.3905*	41.2741*	34.0191*	26.6640*
	→	→	→	→	→
	(7.E-224)	(0.0000)	(0.0000)	(9.E-287)	(6.E-220)
	←	←	←	←	←
ECG2	4.67641*	5.41086*	6.47160*	5.78547*	4.84396*
	→	→	→	→	→
	(2.E-23)	(2.E-29)	(3.E-38)	(2.E-32)	(7.E-25)
	←	←	←	←	←
ECG3	21.0929*	28.8383*	29.2925*	26.9987*	22.4236*
	→	→	→	→	→
	(4.E-169)	(9.E-240)	(7.E-244)	(5.E-223)	(3.E-181)
	←	←	←	←	←

MGH191

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	12.5219*	1.99626*	31.7273*	24.0795*	4.71440*
	→		→	→	
	(3.E-91)	(7.E-05)	(5.E-226)	(2.E-196)	(8.E-24)
	←	←	←	←	←
ECG2	4.41125*	1.80146*	10.9193*	7.43139*	1.38848
			→		
	(2.E-21)	(0.0007)	(6.E-77)	(2.E-46)	(0.0418)
	←	←	←	←	←
ECG3	8.60126*	1.30267	30.7626*	22.7601*	1.02445
	→		→	→	
	(2.E-56)	(0.0819)	(3.E-257)	(2.E-184)	(0.4265)
	←	←	←	←	←

MGH195

Lags 45	ART	CVP	PAP	RESP	CO ₂
ECG1	18.5186*	15.4394*	13.7019*	21.6494*	16.2000*
		→	→	→	→
	(1.E-145)	(1.E-117)	(6.E-102)	(3.E-174)	(1.E-124)
	←	←	←	←	←
ECG2	41.2513*	10.1139*	30.5860*	9.51562*	35.1716*
	→	→	→	→	→
	(0.0000)	(8.E-70)	(1.E-255)	(2.E-64)	(3.E-297)
	←	←	←	←	←
ECG3	8.53683*	10.1623*	8.12275*	24.5725*	9.66749*
	→	→	→	→	→
	(6.E-56)	(3.E-70)	(2.E-52)	(7.E-202)	(7.E-66)
	←	←	←	←	←

MGH198

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	11.4239*	60.0530*	46.7532*	28.9611*	7.84743*
	→	→	→	→	→
	(2.E-81)	(0.0000)	(0.0000)	(7.E-241)	(6.E-50)
	←	←	←	←	←
ECG2	7.34840*	34.0190*	28.3777*	21.4622*	4.70172*
	→	→	→	→	→
	(1.E-45)	(9.E-287)	(1.E-235)	(2.E-172)	(1.E-23)
	←	←	←	←	←
ECG3	8.90013*	43.8040*	38.3062*	23.9416*	3.79965*
	→	→	→	→	→
	(4.E-59)	(0.0000)	(0.0000)	(4.E-195)	(8.E-17)
	←	←	←	←	←

MGH202

Lags 45	ART	CVP	PAP	RESP	CO ₂
ECG1	17.3587*	26.6135*	6.24928*	9.14276*	0.76490
	(3.E-132)	(7.E-215)	(1.E-35)	(6.E-60)	(0.8737)
	←	←	←	←	
ECG2	10.5459*	15.500*	7.10172*	9.24585*	13.1002*
	(4.E-72)	(1.E-115)	(1.E-42)	(7.E-61)	(9.E-95)
	←	←	←	←	←
ECG3	85.9955*	67.0712*	71.9574*	3.19624*	13.3631*
	(0.0000)	(0.0000)	(0.0000)	(3.E-12)	(8.E-97)
	←	←	←	←	←

MGH227

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	17.3587*	26.6135*	6.24928*	9.14276*	0.76490
	(3.E-132)	(7.E-215)	(1.E-35)	(6.E-60)	(0.8737)
	←	←	←	←	
ECG2	7.11487*	27.5168*	26.1840*	25.4145*	6.04228*
	(1.E-43)	(1.E-227)	(1.E-215)	(1.E-208)	(1.E-34)
	←	←	←	←	←
ECG3	2.68736*	32.9532*	36.3138*	33.3783*	2.52247*
	(5.E-09)	(4.E-277)	(2.E-307)	(6.E-281)	(6.E-08)
	←	←	←	←	←

MGH229

Lags 46	ART	CVP	PAP	RESP	CO ₂
ECG1	8.36020*	37.9324*	8.91593*	274.813*	297.264*
	(2.E-54)	(0.000)	(3.E-59)	(0.000)	(0.0000)
	←	←	←	←	←
ECG2	0.89056	1.1944	1.16756	1.71417*	1.44555
	(0.6825)	(0.1723)	(0.2032)	(0.0019)	(0.0258)
				←	
ECG3	1.23840	0.61661	1.88425*	1.16677	1.14884
	(0.1292)	(0.9809)	(0.0003)	(0.2042)	(0.2270)
			←		

Discussions

In order to establish Granger causality on the multivariate analysis dealing with cardiac, respiratory, and hemodynamic signals, in contrast to existing work on the heart by taking generally only two signals (two variables) to maximum, our work focuses on 187 patients, but we displayed the results for 13, because the required sizes of the item force us to do that.

Our study was fulfilling on a case-by-case basis, we will accomplish that for two patients randomly just to see correspondence (Granger causality/Current status of patients).

MGH002: We evaluate after the measures that all blood pressures have a bilateral relationship with the cardiac signals and the rate of CO₂ and breathing as they only have a bilateral relationship only with ECG1 and this may be explained by a vicious circle generated by the heart ectopic that causes disorders of blood tensions which in it turn acted on the heart highlighting the disorder.

MGH019: Atrial fibrillation is a serious phenomenon, which is characterized by rapid acute contractions and disordering of the auricle of the heart, which can cause cardiac arrest. Due to this, there is a dangerous increase in blood pressure, the resulting physical stress causes hyperventilation, this pressure increase complicates the task of the heart that has been already weakened and disturbs the phenomenon of oxygen uptake and expulsion of CO₂ from pulmonary alveoli, which aggravates the situation of the heart that has a workload coupled with a deficiency of these drivers.

We will present a table that accounts the results in proportion to better understand the influence of signals studied. The table below contains all the results (186 patients).

ALL patients	ART (%)	CVP (%)	PAP (%)	RESP (%)	CO ₂ (%)
ECG1	85.71	73.46	91.83	83.67	75.51
	→	→	→	→	→
	97.91	81.63	93.87	93.87	93.87
	←	←	←	←	←
ECG2	87.75	75.51	87.75	85.71	81.63
	→	→	→	→	→
	93.87	79.59	89.79	97.95	95.91
	←	←	←	←	←
ECG3	79.59	69.38	85.71	83.67	71.42
	→	→	→	→	→
	87.75	77.55	91.83	89.79	91.83
	←	←	←	←	←

Conclusion

Our methods are based and validated by the Granger causality. The mathematical search result obtained by this method could confirm the cardiorespiratory hemodynamic anatomy. The knowledge and the quantitative understanding of these interactions are critical in monitoring people at risk situations (awakening from anesthesia, age-related pathologies that followed pregnant women, etc.). So for our future telemedicine applications it is a real progress towards the complete analysis of signals received. Based on these results, and with the inclusion of all the interdependencies with these specific degrees protocols; that enable an excellent intervention.

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