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Aurélien Hazan. Stock-flow consistent macroeconomic model with nonuniform distributional constraint. 2017. hal-01571202

**HAL Id: hal-01571202**

**<https://hal.science/hal-01571202>**

Preprint submitted on 1 Aug 2017

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# Stock-flow consistent macroeconomic model with nonuniform distributional constraint

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## Abstract

We report on results concerning a partially aggregated Stock Flow Consistent (SFC) macro-economic model in the stationary state where the sectors of banks and firms are aggregated, the sector of households is disaggregated, and the probability density function (pdf) of the wealth of households is exogenous, constrained by econometric data. It is shown that the equality part of the constraint can be reduced to a single constant-sum equation, which relates this problem to the study of continuous mass transport problems, and to the sum of iid random variables. Existing results can thus be applied, and provide marginal probabilities, and the location of the critical point before condensation occurs. Various numerical experiments are performed using Monte Carlo sampling of the hit-and-run type, using wealth and income data for France.

*Keywords:* economics, physics and society, constraint satisfaction, monte carlo, random network, finance, mass transport, condensation

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While the neoclassical standpoint on macroeconomics has adapted in response to growing criticism in the aftermath of the subprime crisis [1], it still relies on intensely debated hypotheses, for example rational expectations, utility functions, or representative agents, that form the building blocks of Dynamic Stochastic General Equilibrium (DSGE).

Other modelling approaches include “the Agent Based (...) approach, which conceives the economy as a complex adaptive system populated by heterogeneous locally interacting agents, and the Stock Flow Consistent framework (...), which provides a comprehensive and fully integrated representation of the real and financial sides of the economy through the adoption of rigorous accounting rules based on the quadruple entry principle developed by Copeland” [2].

SFC macroeconomic models, that date back to the 1950’s, enforce local conservation of money in a holistic perspective. They usually stand at the aggregate level, and cover a large scope of economic phenomena [3]. Thanks to quadruple-entry bookkeeping, they are able to guarantee that real and financial transactions, involving many agents belonging to different sectors, stay balanced [4, 2.6].

Agent Based Models (ABM), on the other hand, adopt a bottom-up perspective, and are able to recover stylized facts, starting from low-level discrete-time dynamical description. Most of the time, closed form solutions are not available for ABM (though recent works anchored in statistical physics started to fill that gap [5, 6]).

The link between SFC and microeconomic models started to be explored during the 1970’s [7] and is now an active research topic. Agent Based Models that enforce SFC rules (SFC-AB) have been studied over the last years in the post-keynesian community. They rely on economic models, multi-agent programming, and a critical phase of calibration [2]. Little theoretical results exist to explain the behavior of such systems.

In this article, we look for a tradeoff between the complexity of the model, and the availability of theoretical results. Our interest goes to increasing the number of agents, with linear behaviors, in a stationary state. What can be said at the population level, that is concerning the distribution of the different variables?

To answer this question, we first look at theoretical properties available for SFC models: mathematically, SFC can be formalized as a system of difference equations. Depending on the nature of transactions flows,

an analytic expression can be found for the stationary state (if any) and for time-dependent transients [4] of aggregated models. In [8], the stationary state of a particular, fully-disaggregated SFC model with random connectivity, was seen as a Constraint Satisfaction Problem (CSP), with solutions lying inside a random convex polytope. Marginal pdf of money stocks and flows were numerically estimated, and the effect of various flow knockouts was evaluated in the case of a BMW model [4, 7] which describes an economy where private banks create money through loans.

In the present article another stand is taken: while the accounting constraints are maintained, the marginal pdf of several variables is considered *exogenous*, that is constrained so as to respect empirically observed data. The marginal pdf of the other variables still is an unknown that we seek to approximate. This approach differs from recent SFC-AB works e.g. [2] where an initial uniform distribution is supposed.

We thus report on the following original results: in the case of a partially aggregated SFC model of the BMW type where banks and firms are represented by one agent respectively, the set of accounting identities that define the SFC model can be reduced to a constant sum over individual wealth  $M_i$ , the distribution of which is constrained by a weight function  $f(m)$ . The properties of the sum  $\sum M_i$  are discussed, using previous investigations by [9, 10, 11] that concluded to the existence of a phase transition leading to condensation. The marginal single site pdf of all variables are deduced, for various usual weight functions. Furthermore a simple Monte Carlo sampling algorithm is available, and allows to simulate the behavior of a simple economy.

Section 1 depicts the SFC model, the distributional constraints imposed, and the simplified constant sum problem obtained after gaussian elimination ; section 2 discusses how the latter is related to continuous mass transport studies in the statistical physics litterature. Numerical simulations and results are examined in section 3. Section 4 and 5 conclude.

## 1. Partially aggregated SFC model with constrained pdf

In this section, the elements of the BMW model are briefly exposed: the balance sheet, the transaction matrix that define the flows of money occurring between an origin and a destination, and the behavioral equations that specify the flows. Notations follow Godley and Lavoie [4] and are explained in Tab. 2, more detail concerning accounting conventions can be found in Appendix. Tab. 1 represents a balance sheet, that is the summary of assets, liabilities, and capital of all agents, at a fixed time such as the end of the year. The particular model is a partially aggregated BMW model, with private money, no state nor central bank, where agents are grouped by sector (firms, banks, households). More detail can be found in [4, 7], and a fully disaggregated version is discussed in [8]. It follows from this simplified setup that the stock of capital  $K$ , the stock of loans  $L$ , and the sum of deposits  $\sum M_i$  are equal.

	Households			Firms	Banks	$\Sigma$
	1	2	3	1	1	
Money deposits	$M_1$				$-M_1$	0
		$M_2$			$-M_2$	0
			$M_3$		$-M_3$	0
Loans				$-L_1$	$L_1$	0
Fixed capital				$K_1$		$K_1$
Balance (net worth)	$-V_{h1}$	$-V_{h2}$	$-V_{h3}$			$-\sum_i V_{hi}$
$\Sigma$	0	0	0	0	0	0

Table 1: Example of balance sheet of the BMW model with many households, one bank and one firm:  $nw = 3$ ,  $nf = 1$ ,  $nb = 1$ .  $M_i, L_j, K_k$  are the individual money deposits, loans, and tangible capital.  $V_{hi}$  is the net worth of individual households.

During the period (say a year) that separates two balance sheets, many transactions occur such as consumption, investment, wage, depreciation, interest (on loans and deposits). The transactions corresponding to the partially aggregated BMW model are summarized in Tab. A.4, and explained in a detailed way in

the Appendix. The balance sheet at time  $t + 1$  is the result of applying these transactions to the balance sheet at time  $t$ .

Finally, the behavioral equations are :

$$AF = \delta K_{t-1} \quad (1)$$

$$C_{d,i} = \alpha_{0,i} + \alpha_1 Y D_i + \alpha_2 M_{t-1,i} \quad (2)$$

$$I_d = \gamma(\kappa Y_{t-1} - K_{t-1}) + AF \quad (3)$$

Eq.(1) states that the depreciation of tangible capital is proportional to its past stock  $K_{-1}$ , eq.(2) is the consumption demand  $C_{d,i}$  of household  $i \in [1, nw]$  which is a mixture of an autonomous term, of the disposable income  $Y D_i$  and of the wealth accumulated by households  $M_{t-1,i}$  at the previous time step. Lastly, eq.(3) sets a target investment level, that depends both on amortization and on some target capital  $\kappa Y_{t-1}$ , where  $Y_t$  is the total production at time  $t$ , defined as the sum of consumption supply  $C_s$  and investment supply  $I_s$ .

In the steady-state regime considered in this article, the term  $\kappa Y_{t-1} - K_{t-1}$  vanishes as well as changes in deposits and loans :

$$\Delta M_i = 0 \quad (4)$$

$$\Delta L = 0 \quad (5)$$

$$I_d = AF = \delta K_{-1} \quad (6)$$

Notations are gathered in Table 2. The subscripts  $d$  and  $s$  stand for demand and supply,  $i$  represents quantities related to individual households,  $t$  stands for the time instant.

Variable	Label
money deposit	$M$
capital	$K$
loans to firms	$L$
investment	$I$
interest on loans	$IL$
wage bill	$WB$
depreciation allowance	$AF$
interest on workers deposits	$ID$
consumption of workers	$C$

Table 2: Labels associated with the different monetary variables, after [4]. The subscripts  $d$  and  $s$  stand for demand and supply,  $i$  represents quantities related to individual households,  $t$  stands for the time instant.

### 1.1. Properties of the partially aggregated model

In the steady-state unidimensional case, Godley and Lavoie find that  $\sum M = kY$  and  $Y = \frac{\alpha_0}{(1-\alpha_1)(1-\delta k) - k\alpha_2}$ .



### 1.2. Modelling the distribution of wealth and income

The distribution of wealth and income display several regularities, as many authors have shown since the 19th century. Economists such as V. Pareto have studied the distribution of wealth, and found a good fit to a power law among the richest. Statisticians, and more recently, physicists became interested in the topic. It is now well accepted that the bulk of the distribution of wealth and income fits a Gamma or log-normal law. The findings of Pareto concerning the tail of the distribution, were confirmed and refined by many subsequent studies. For incomes, various empirical estimations found a paretian tail  $x^{-\gamma}$  with an exponent  $\gamma \in [1.5, 4]$  across time and countries [12, p.25].

In the case of France, income and wealth data are not available publicly at the level of individuals or households. Recent empirical works by economists and statisticians take advantage of tax tabulations to estimate the percentiles of the distribution of wealth among the population [13].

Fig. 1(a) shows the empirical cumulative distribution function (cdf) of french income in 2010 provided by the World Inequality Database (see Appendix B), as well as a gamma distribution with adequate parameters. Fig. 1(c) shows the cdf of french wealth the same year, and a lognormal fit. Fig. 1(b,d) show the corresponding probability density functions (pdf).

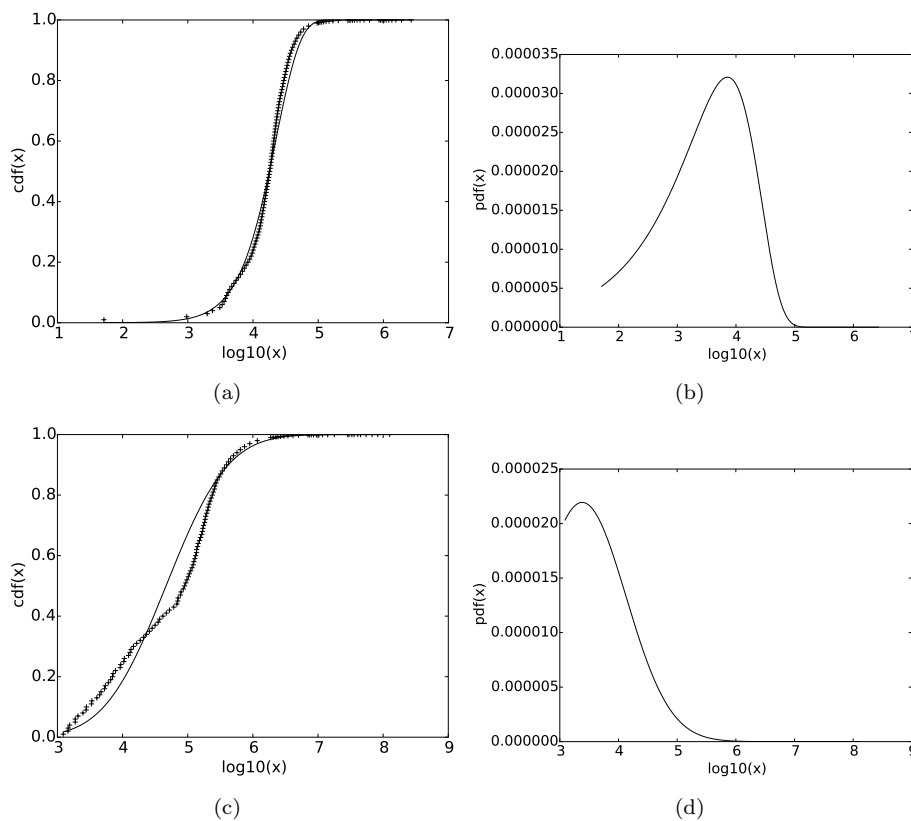


Figure 1: (a) empirical cdf of fiscal annual income in France in the year 2010 represented by +, and in plain line the cdf of a Gamma variable with shape parameter  $a = 1.46$  and scale parameter  $1/\lambda = 1.55 \cdot 10^4$ ; (b) pdf of the Gamma distribution; (c) empirical cdf of wealth in France in the year 2010 represented by +, and in plain line the cdf of a lognormal variable with shape parameter  $a = 1.72$  and scale parameter  $4.64 \cdot 10^4$ ; (d) pdf of the lognormal distribution.

Many nonparametric and parametric models of income and wealth have been discussed in the literature [12]. We compared several among them: a piecewise constant bulk with a Pareto tail [14], a nonparametric method [13], a monotone polynomial interpolation in the log domain [15], and parametric models as shown in Fig 1. For simplicity, the distribution of income is modelled by a single Gamma distribution on the full

range covered by empirical data, and wealth by a lognormal distribution. The fit of the income model to the empirical cdf is rather good, and sufficient for our needs in this article in the case of wealth.

In sec. 3, the topic of drawing random samples from a multivariate distribution, with identically distributed components, given additional linear constraints, is discussed.

## 2. Analogy with continuous mass transport and sum or random variables models

Since some of the results obtained for mass transport models may be of interest to the problem in sec. 1.1, let us summarize a few of them. The total mass, replaced by money, is constant and the number of sites is replaced by the number of economic agents. As pointed out in [9, 10] the problems of mass transport and sum of random iid variables in the large deviation regime can be mapped.

Dynamical models of mass transport on a lattice have been the subject of many works: it was observed that when the mass density increases above some critical threshold, a condensed steady-state could appear under some distributional conditions. Condensation means that a finite fraction of the total mass can concentrate on a single site. The analogy between mass transport and macroeconomic modelling was noticed already in [11].

To explain this phenomenon, it was first remarked that for some dynamical models (such as the ZRP), a factorized steady-state exists and can be computed. In that case, examined in [16, 9], the steady-state probability and the partition function write:

$$P(\{m_l\}) = Z(M, L)^{-1} \prod_{l=1}^L f(m_l) \delta\left(\sum_{l=1}^L m_l - M\right) \quad (11)$$

$$Z(M, L) = \prod_{l=1}^L \left[ \int_0^\infty dm_l f(m_l) \right] \delta\left(\sum_{l=1}^L m_l - M\right) \quad (12)$$

where  $f(m_l)$  is the single-site weight,  $M$  is the total mass,  $L$  the number of sites, and  $\delta\left(\sum_{l=1}^L m_l - M\right)$  embodies the constant sum constraint. When  $f(m_l)$  is normalised,  $Z(M, L)$  in eq.(12) is the probability that a sum of  $L$  random variables with pdf  $f(m_l)$  sum to  $M$ . The single-site weight  $f(m_l)$  can be related to the distribution of wealth or income in sec. 1.2, and to  $p(m)$ , the single-site marginal probability.  $p(m)$  is an unknown, that can differ from  $f(m)$ , because it takes into account the fixed total amount of money:

$$p(m) = \int dm_2 \dots dm_L P(m, m_2, \dots, m_L) \delta\left(\sum_{l=2}^L m_l + m - M\right) \quad (13)$$

The properties of the partition function eq.(12), the value of the condensation threshold, the size of the condensate and the value of  $p(m)$  were examined in great detail in [9] and further extended to the presence of two constraints [17, 18].

In the thermodynamic limit  $L, M \rightarrow +\infty$  with  $M/L$  finite, three cases are examined in [9]:  $f(m)$  decreases faster than the exponential, slower than  $m^{-2}$ , or slower than the exponential but faster than  $m^{-2}$ .

For example, when  $f(m)$  is a paretian distributions  $1/m^{-\gamma}$  with  $\gamma \in [2, 3]$ , the typical outcome is not condensed, but a phase transition exists, governed by the mean value  $\langle f \rangle$  of  $f(m)$ . If  $\langle f \rangle$  is smaller than the average density  $M/L$ , condensation occurs: a single site will contain the excess mass. If  $\langle f \rangle > M/L$ , the system is in the fluid phase. In the limit case  $\langle f \rangle = M/L$ , the system is in the critical state. Condensation does not correspond to an observed phenomenon in available data, and will not be considered here, thus we suppose that  $\langle f \rangle \geq M/L$ . Nevertheless we must keep in mind that at the critical value  $\langle f \rangle = M/L$  or in its neighborhood, sampling algorithms can fail because of the phase transition.

If  $\gamma$  belongs to the interval  $[1, 2]$ ,  $f(m)$  has a very broad tail. The variables  $M_i$  experience large fluctuations, and the sum  $\sum_{i=1}^N M_i$  has condensed typical outcomes [17]. The theoretical analysis of  $Z(M, L)$  in [9, 7.2] shows that no phase transition occur, however a pseudocondensate -i.e. a bump in the tail of  $p(m)$ - emerges.

The expression of the marginal probability  $p(m)$  is another interesting result. While  $f(m)$  is known,  $p(m)$  is unknown a priori, because it results from incorporating the constant-sum constraint  $\delta\left(\sum_{i=1}^L m_i - M\right)$  in the sampling process. In [9] several expressions of  $p(m)$  are reviewed. In the grand-canonical ensemble approximation that neglects the global interaction between the particles,  $p(m) = f(m)e^{-\mu m}$ , where  $\mu$  is the negative of the chemical potential. A similar form is found in [17, §3.1] with the Density Functional Method.  $\mu$  must be found so that:

$$\rho = M/L = \int_0^{+\infty} m f(m) e^{-\mu m} dm / \int_0^{+\infty} f(m) e^{-\mu m} dm \quad (14)$$

Such a value doesn't exist for all densities  $M/L$  when  $f(m)$  decreases slower than the exponential but faster than  $m^{-2}$ , which gives rise to the aforementioned phase transition. Let us examine the consequences in the cases covered in this article. If the weight function  $f$  is a gamma distribution, that has an exponential tail, then eq.(14) always has a solution. For example if  $\langle f \rangle = M/L$ , then  $\mu = 0$  is a solution, and  $f(m) = p(m)$ .

If the weight function  $f$  is a lognormal distribution -i.e. with a subexponential tail- such that  $\langle f \rangle > M/L$ , then eq.(14) still has a solution.

In the canonical ensemble, more detailed closed form expressions were reported in [9].

### 3. Numerical experiments

In this section the methods employed and results observed in several experimental settings are discussed: in sec. 3.1, the different flavors of the hit-and-run sampler are compared. In sec. 3.2 the theoretical marginal values of wealth and income for the partially aggregated BMW model are examined and compared to sampled values.

#### 3.1. Sampling methods

In this section we present briefly the algorithms used to simulate the terms of the sum  $\sum X_i = cst$ , where  $(X_i)$  are iid random variables, the distribution of which are constrained by the weight functions  $f(m)$ . The parameters of functions  $f(m)$  are fit to empirical data, whether income or wealth, as seen in sec. 1.2.

In order to compute all the variables in the economic model in sec.1 knowing these samples, one can simply use the different relations in the echelon form of eq.(8).

The Hit-and-run (HR) sampler is a standard Monte-Carlo algorithm that can be used in this case. If  $f$  is uniform, it can be proved to converge to a uniform distribution, with a zero rejection rate [19]. The HR sampler was extended to the non-uniform case, with the drawback of losing this last property. The issue of decreasing the rejection rate to accelerate sampling is still an open problem [20]. Convergence is guaranteed in  $\mathcal{O}^*(d^4)$  for a large class of target distributions in dimension  $d$ , provided that the HR implementation uses the Hypersphere Direction (HD) scheme to update the direction during the random walk inside the polytope given by linear constraints. Since HR is a Monte-Carlo algorithm, successive samples are correlated. One solution to mitigate this problem is to keep only a fraction of the generated samples, thanks to a thinning factor [21].

Coordinate Direction (CD) is another way to update the direction, simpler to implement because it doesn't require any change of basis. It can be modified to converge quicker than HD, as shown in [22]. However, on the opposite of HD, its convergence is not guaranteed [19, p.724].

Several works in the field of metabolic networks research use HR, to sample feasible metabolic flows in a uniform [23, 24] or possibly non-uniform way [25]. SFC macro models were also studied with this algorithm in [8].

#### 3.2. Numerical results

With the equation eq.(10) that links wealth to income for each agent, one can compute the cdf of  $WB_{s,i}$  from the distribution of  $M_i$  that was modelled in sec. 1.2 and fit to wealth data. The obtained cdf can be compared with the direct fit of  $WB_{s,i}$  to WID income data. If the parameters of the BMW model depicted



in section 1 are unknown, then can then be estimated, by minimizing the distance between the two cdf. If they are known from previous studies, the procedure serves as a consistency check. An illustration is shown in Fig.2(a), and provides the set of parameters presented in Tab. 3.  $r$ ,  $\delta$ ,  $k$  are provided by the litterature.  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_2$  are the free parameters that need to be estimated. Let us emphasize the necessity to rescale all the variables because of the full determination of  $\sum M$  given the set of parameters, as seen in eq.(9).

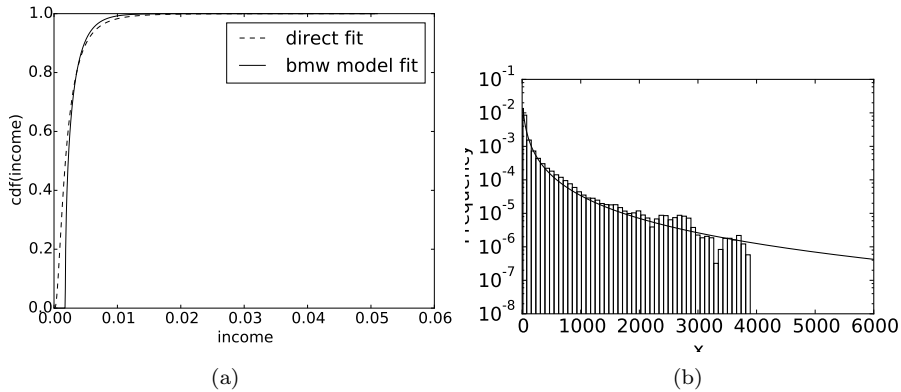


Figure 2: (a) Comparison between the direct fit of  $WB_s$  to WID income data, and the cdf of  $WB_s$  derived from the direct fit of  $M_i$  to WID wealth data (b) histogram of HR-HD wealth samples in dimension  $d = 100$ . Plain line is the weight function  $f$  corresponding to a lognormal distribution with shape  $\sigma = 1.72$ . Scaling is such that  $\langle f \rangle = M/L$ . Thinning factor is 1000.

Parameter	Value
$\alpha_0$	0.001
$\alpha_1$	0.75
$\alpha_2$	0.02
$r$	0.03
$\delta$	0.1
$k$	5.5

Table 3: Values of parameters used in Fig. 2(a).

Fig.2(b) represents the histogram of wealth samples generated by a HR-HD sampler, that approximates  $p(m)$ . One can notice that  $p(m)$  is close to the weight distribution  $f(m)$ , which is expected as mentioned in sec. 3.1, except in the region of the tail, because of the finiteness of the sample.

As was recalled in sec. 3.1, there is no general proof of convergence for HR-CD, that would be available for a large class of distributions. We found some examples where HR-CD failed to sample correctly the distribution of weights  $f(m)$  with very broad tails, even though we stayed in the typical fluctuation regime  $\langle f \rangle \geq M/L$ : first a monotone polynomial fit  $m \rightarrow \frac{1}{m} \sum_{i=0}^N \beta_i \log(m)^i$ , used to model the wealth; then a lognormal distribution with shape parameter  $\sigma = 1.72$ . This failure is not mitigated using the acceleration method in [22]. It calls for some improvement on the algorithms currently used (HR-HD), because of the high rejection rate due to sampling a high-dimensional distribution.

#### 4. Discussion

In this paper a partially aggregated SFC model was studied. This is a restriction to more general problems, such as disaggregated models, and also random topology models, where the connectivity is not fixed. The partially aggregated model is useful as a limit case, because some additive results can be obtained, as was seen above. However it adds restriction on the type of distribution considered. For example, one

may want to decouple the distributions of wealth and income. This can still be achieved with the partially aggregated model, for example considering  $\alpha_{0,i}$  as random variables, rather than fixed constants.

Furthermore, one may question the usefulness of sampling as a tool to study the marginal distributions of variables in the model, since closed form expressions of the cdf can be obtained. Once again, the partially aggregated model is a limit case. In the case of fully disaggregated models, it is not likely that a simple equation will relate  $WB_{s,i}$  and  $M_i$ . Some constant quantities might appear though, as in the case of metabolic networks [26].

## 5. Conclusion

In this article the problem of finding the marginal probabilities of the variables in a steady-state partially aggregated SFC macroeconomic model is addressed, with distributional constraints imposed on individual wealth variables. This last feature can be thought of as expressing the fact that the distribution of wealth or income are exogenous, as the result of an evolving balance of power between capital and workforce. This improves over [8] where an SFC model was seen as a Constraint Satisfaction Problem, but without *a priori* constraint on distributions.

The following results are reported:

- using standard algebra, we find that the linear system of equations that forms part of the problem is underdetermined and amounts to a constant sum equation.
- we recall relevant results from the theory of mass transport, that give an approximation of the marginal probabilities, given the weight distributions. Since condensation is not a admissible outcome for the system state, we deduce some constraint on the parameters of the weight distribution.
- we fit empirical wealth data to a classical distribution, and compute the corresponding cdf for income. The latter is compared to an direct empirical fit of income data, which allows us to estimate some free parameters.
- finally, sampled solutions to the initial problem are shown, with various hit-and-run algorithms.

This last result can be used by practitioners in the SFC community that are interested in distributional phenomena, and can be compared with the results obtained using time-averaged SFC/ABM models.

In future work we will compare the limit results obtained here in the case of a partially aggregate model to more general settings. First an extension to disaggregated models will be examined. Thanks to recent developments in the analysis of metabolic networks we hope to overcome the curse of dimensionality; which will be compared to accelerations of hit-and-run algorithms such as [20]. An extension to random network, following [27, 28, 29], will also be addressed.

Furthermore in this article, we limited our scope to BMW models, that form just a part of SFC models. More models, that fall in the category of linear dynamical systems, will be analyzed, and more empirical data will be included.

Lastly, the dynamical behavior may be of interest, for example steadily growing economies, or stability properties [30], that are related to the occurrence of crises.

## Appendix A. Transaction matrix for the BMW model

Tab. A.4 sums up the different transactions in the modelled economy. Firms and banks are each represented by a single agent. The number of households is set to  $nw = 3$  in this example but can take any strictly positive value.

	Households			Production Firms		Banks		$\Sigma$
	1	2	3	Current 1	Capital 1	Current 1	Capital 1	
Consumption	$-C_{d1}$			$C_{s1}$				0
		$-C_{d2}$		$C_{s2}$				0
			$-C_{d3}$	$C_{s3}$				0
Investment				$I_{s1}$	$-I_{d1}$			0
				$I_{s2}$	$-I_{d2}$			0
Wage	$WB_{s1}$			$-WB_{d1}$				0
		$WB_{s2}$		$-WB_{d2}$				0
			$WB_{s3}$	$-WB_{d3}$				0
Depreciation				$-AF_1$	$AF_1$			0
Interest on loans				$-IL_1$		$IL_1$		0
Interest on deposits	$ID_1$					$-ID_1$		0
		$ID_2$				$-ID_2$		0
			$ID_3$			$-ID_3$		0
Change in loans				$\Delta L_1$			$-\Delta L_1$	0
Change in deposits	$-\Delta M_1$						$\Delta M_1$	0
		$-\Delta M_2$					$\Delta M_2$	0
			$-\Delta M_3$				$\Delta M_3$	0
$\Sigma$	0	0	0	0	0	0	0	0

Table A.4: Transaction matrix of the BMW model with many households, on firm and one bank: agents  $nw = 3$ ,  $nf = 1$ ,  $nb = 1$ . Households must buy goods from the only available firm. The firm is buying capital goods from itself. In the stationary case,  $\Delta L = \Delta M = 0$ .

## Appendix B. Econometric data

Econometric data in sec. 1.2 are taken from the World Inequality Database <http://wid.world>. Tab. B.5 extracts some corresponding information among the documentation supplied by the authors. For both variables “the base unit is the individual (rather than the household) but resources are split equally within couples. The population is comprised of individuals over age 20”.

Variable Name	Year	Description	Unit	Ref.	Code
Net personal wealth	2012	Net personal wealth threshold value at a given percentile. Net personal wealth is the total value of non-financial and financial assets (housing, land, deposits, bonds, equities, etc.) held by households, minus their debts.	EUR constant 2015	[31]	thweal992j
Fiscal income	2012	Fiscal income threshold value at a given percentile. Fiscal income is defined as the sum of all income items reported on income tax returns, before any deduction. It includes labour income, capital income and mixed income.	EUR constant 2015	[31]	tfinc992j

Table B.5: Extract of the WID documentation.

## Appendix C. Acknowledgements

Open-source software were used to perform this research: Python, Cython, R, Sage, L<sup>A</sup>T<sub>E</sub>X.

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