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Marx, maths, and MEGA 2

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The authors offer special thanks to Irina Antonova, Pradip Baksi, Bertram Schefold, and Annette Vogt, and to all members of 'marxmath'.

This paper examines two areas where the MEGA 2 project can advance the understanding of Marx's thought; although the contribution of MEGA 2 to each is very different, the areas are linked by their subject matter — Marx's engagement with the mathematical sciences — and by other personalities involved.

In the first part of the paper we deal with the question of Marx's mathematical manuscripts, the majority of which are apparently already available and translated into several languages, but awaiting their definitive publication in their place in the MEGA. In the second part we deal with Marx's contact with nineteenth-century thought in probability and statistics, a story whose full elucidation cannot be undertaken until more volumes of his Notebooks are available, and especially that which is known to contain his notes on Quetelet.

The connection between the two parts lies in the following letter from Marx to Engels (31/05/1873):

> I have been telling Moore about a problem with which I have been racking my brains for some time now. However, he thinks it is insoluble, at least pro tempore, because of the many factors involved, factors which for the most part have yet to be discovered. The problem is this: you know about those graphs in which the movements of prices, discount rates, etc. etc., over the year, etc., are shown in rising and falling zigzags. I have variously attempted to analyze crises by calculating these ups and downs as irregular curves and I believed (and still believe it would be possible if the material were sufficiently studied) that I might be able to determine mathematically the principal laws governing crises. As I said, Moore thinks it cannot be done at present and I have resolved to give it up for the time being.

This has drawn the response that Moore was badly informed, and should have drawn Marx's attention to the methods of Fourier analysis, first published in 1822 (Kolman 1968). Given that one knows of Marx's extensive work on the calculus it is natural to think first that he had in mind a law in the form of polynomials describing the various time series. On the other hand, Moore's objection about the 'many factors involved' suggests that what he and Marx had in mind was instead something on the lines of modern multiple regression.

In our second section we will suggest a possible reading of Marx which would suggest further possibilities for the kind of statistical apparatus he might have been groping for

Editing Marx's Mathematical Manuscripts

Marx's mathematical formation

Marx received his early training in mathematics at the Gymnasium of Trier (Treves), the Rhineland city where he was born. In a report to Berlin on the staff, 'many were said to exercise a bad influence on the boys', while one master, Steininger, who taught

Marx natural science and mathematics, had 'an innate propensity to opposition'.[2] At his graduation, in 1835, his knowledge of mathematics was considered 'adequate'.[3] This means that he had some knowledge of elementary arithmetic, algebra to the quadratic equations, and plane and solid geometry. He also may have had trigonometry, and a little higher algebra, analytical geometry and calculus. There is no indication that he showed any interest in mathematics during the two following decades before he settled in London and was working on *Capital*.

In a letter to Engels of January 11, 1858, he wrote:

> 'In elaborating the *principles* of economics I have been so damnably held up by errors in calculation that in despair I have applied myself to a rapid revision of algebra. I have never felt at home with arithmetic. But by making a detour via algebra, I shall quickly get back into the way of things.' A few years later on July 6, 1863, he explained that mathematics was a hobby for him: 'Since I now spend 10 hours a day working *ex officio* at economics, I can hardly be expected to waste my leisure hours on reading these schoolboy essays. So, for the present the thing's been filed away. My spare time is now devoted to differential and integral calculus.'

It is interesting to note that Marx does not establish a relationship other than marginal between his interest in mathematics and his work in economics. It is worthwhile to mention a letter of Engels to Friedrich Albert Lange, on 29 March 1865, which on the contrary establishes a connection between Marx's mathematics and Hegel's. Engels writes: 'There is a remark about old Hegel which I cannot let pass without comment: you deny him any deeper knowledge of the mathematical sciences. Hegel knew so much mathematics that none of his disciples was capable of editing the numerous mathematical manuscripts he left behind. The only man who, to my

[2] Quoted in Nicolaievsky, Boris.; Maenchen Helfen Otto.1936, pp. 12-13, Karl Marx, man and fighter, Methuen And Company Limited.

[3] Struik, 1948 – (other reference to come)

knowledge, has enough understanding of mathematics and philosophy to be able to do so is Marx.'

His interest in calculus became stronger in the 1870s and he not only criticised several mathematicians but also tried to develop his own methods. From their correspondence, Engels seems to have been for a while rather sceptical; nevertheless in his speech at Marx's funeral, he celebrated his many achievements asserting proudly: 'But in every single field which Marx investigated — and he investigated very many fields, none of them superficially — in every field, even in that of mathematics, he made independent discoveries'. (*Der Sozialdemokrat*, March 22, 1883).

Two years later Engels mentioned his intention to publish Marx's writing along with his own studies in *Anti-Dühring* (1885 Preface: London, September 23, 1885): 'Since Karl Marx's death, however, my time has been requisitioned for more urgent duties, and I have therefore been compelled to lay aside my work. For the present I … must wait to find some later opportunity to put together and publish the results which I have arrived at, perhaps in conjunction with the extremely important mathematical manuscripts left by Marx.' Thereafter we know of no attempt by Engels to publish the MMM and they only re-appear after World War I, as their story joins that of E. J. Gumbel (Gumbel, [1927], Vogt 1995).

Ryazanov works on Marx's collected papers

We have to introduce briefly D. Ryazanov (1870–1938) and the first attempt at producing Marx Engels Collected Papers. Born in Odessa, Ryazanov joined the Narodnik revolutionaries at the age of 15, for which he was arrested and passed five years in prison. He adhered to socialism in 1887, then to Marxism. Before World War I, he spent most of his life in prison or in exile, during which he started writing a history of the First International. He returned to Russia following the February Revolution in 1917, and joined the Bolsheviks. Ryazanov participated in the October Revolution and later founded the Marx-Engels Institute, which became one of the main institutions of Soviet philosophy, and was named its director in 1920. Ryazanov

also created 'MEGA' (Marx-Engels-Gesamtausgabe), which was to publish the complete works (Gesamtausgasbe) of Marx and Engels. During this period, 'he unceasingly participated in the life of the party and trade unions as a conscious marxist, a democratic communist, in other words, opposed to any dictatorship over the proletariat' (Boris Souvarine, David D. Ryazanov, 1931). Defending the trade unions' autonomy against the will of the party, he was excluded from any political responsibility in 1921, at the end of the 'war communism' period and the beginning of the NEP ('New Economic Policy'). Souvarine praised Ryazanov's conduct: 'He then devoted himself entirely to historical work and marxist culture, outside of the factions and groupings, maintaining his critical spirit and his faculties of judgement intact, preserving the Institute from passing fads, and maintaining the best traditions of scientific, qualified, honest and conscientious work there, a happy contrast with the proceedings of institutions entrusted to servile functionaries'. Eventually he was arrested and jailed without trial in February 1931.

According to Anderson (1997), Ryazanov had established a far-reaching plan for MEGA 1, part of which was actually published during the years 1928-35. He divided MEGA 1 into three series, each of which was to contain writings in the original language in which Marx or Engels had written them, usually German, English, or French as well as a rigorous scholarly apparatus of footnotes and prefaces. The distribution of texts was planned as follows: Series I. Philosophical, Economic, Historical, and Political Works; Series II. Capital and other economic manuscripts; Series III. Letters from and to Marx and Engels. In a 1923 report on his plans for MEGA 1 to Moscow's Socialist Academy, Ryazanov referred to a fourth or 'final group' of Marx's writings, 'the notebooks,' which he indicated would be of use mainly to Marx biographers. Ryazanov, otherwise an enthusiastic editor, was reluctant to publish these excerpts he considered as a 'waste of time' by Marx or even 'inexcusable pedantry'. It seems that he has for second advice for 'some mathematical notebooks' making an exception for them, which were slated for publication. That is when Gumbel entered the picture.

E. J. Gumbel

Emil Julius Gumbel was born in 1891, the son of a Jewish banker. He studied economics in Munich and became an assistant in the seminar devoted to statistics and insurance where he got his PhD in 1913. After his experiences during World War I he turned pacifist and organised some agitation to end the war. After the war he began to study physics with Albert Einstein who was also a determined pacifist and was to recommend him to Ryazanov. Gumbel remained distant from political ideologies, large political parties and closer to trade-unions, and small dissident organisations.[4]

In many aspects, Gumbel's political model was Bertrand Russell.[5] 'Gumbel's admiration was nourished by the breadth and brilliance of Russell's work, which ranged from mathematics to philosophy and included contemporary moral and political issues' (Arthur D. Brenner, *Emil J. Gumbel: Weimar German Pacifist and Professor*, Boston: Brill Academic Publishers. 2001). After the war, Gumbel translated into German not only Russell's most recent scholarly book, *Introduction to Mathematical Philosophy* (London, 1919), but also selected essays by Russell on pacifism, socialism, and social organization, which Gumbel put together for a book entitled *Politische Ideale*. To the latter compendium, Gumbel added three essays of his own: one explaining to the general reader the significance of Russell's scientific work; an assessment of his place in the British antiwar movement; and a summary of Russell's (hopeful if critical) views on Bolshevism and Soviet Russia. Russell expressed 'his ardent desire to see the realization of socialism and his pleasure than it was being attempted in Russia, but was sharply critical of the Bolsheviks for failing to live up to communist ideology.'[6]

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[4] Christian Jansen, Leben und Maximen des politisch engagierten Mathematikers Emil Julius Gumbel, 213-226, in E. Eichhorn, E. J. Thiele (Hrsg.) *Vorlesungen zum Gedenken an Felix Hausdorf*, Heldermann Verlag Berlin 1994.

[5] In 1962. when Gumbel attempted to resume a long-broken contact with Russell, he wrote. 'I shared and I share your views.' Gumbel to Russell, 3 June 1962, McMaster University, Bertrand Russell Archive (BRA) (Brenner 2001, p.).

[6] Gumbel to Russell, 31 October 1921, BRA quoted by Brenner, p.

Brenner reports that Einstein 'helped promote Gumbel's academic career' and wrote 'numerous letters praising Gumbel's scholarly potential and achievements'.[7] Taking into account their respective backgrounds, it is not surprising that Ryazanov invited Gumbel to become the editor of the MMM. Both were independent thinkers and not prone to view matters through an ideologically biased lens.

Gumbel's own 1926 report stated that before the beginning of World War I, D. Ryazonov 'discovered amid Marx's papers in the SPD archives a great number of exercise and note books devoted to mathematics'. He then began to think about their publication, but he did not succeed at this time. He proposed to F. Adler to publish only what was entirely completed and ready to print. Gumbel indicated that the Institute K. Marx and F. Engels had taken photographs of all mathematical manuscripts which were stored there when Gumbel arrived in Moscow (winter 1925). Ryazonov asked him to prepare them for publication. He gave then a first description of the MMM which represent 865 sheets A4 (viertelbogen) in very small writing. By 1927, Gumbel considered that the MMM have been deciphered and were ready to print'. He had them classified into 4 categories: A. Calculations; B. Extracts; C. Drafts; D. Independent works. For the Calculations (A) there was no text to be found so that 'they are deprived of any philosophical thought which could belong to Marx'. Sometimes he was able to ascertain the sources Marx used. Gumbel tells that Marx could have just copied the original texts as they stood in the sources but sometimes Marx did not follow the development to be found in the source, and eventually many extracts are compound of short remarks the sources of which are impossible to discover.

Sheynin (2003) quoted from a letter to Einstein his general evaluation of the MMM; on 30 April 1926, Gumbel informed Einstein:

> 'Ich war jetzt sechs Monate in Moskau und habe im Marx – Engels Institut die sogen.

[7] Gumbel became professor of statistics in Heidelberg thanks to a letter of recommendation from Bortkiewitch who rarely wrote such letters

> Mathematischen Manuskripte von Marx druckreif gemacht. Es handelt sich dabei um Notizen zur Differenzialrechnung, die ein gewisses philosophisches Interesse besitzen und zeigen, dass Marx die Anfangsgründe des Differenzierens wohl beherrscht hat. Meine Arbeitsbedingungen waren außerordentlich günstig. Allerdings lebt die Mehrzahl der dortigen Gelehrten in großer Notlage.'

Actually Gumbel considered that only the drafts and independent works were suitable for publication. From his report it is not very clear where he put the border between drafts, excerpts and independent works (for example, he lists five drafts: three of them considered 'ready to print', one only 'suitable to edit', the last one is not qualified, perhaps because it includes 'calculations').

Later on Gumbel asserts that 'only the manuscripts which don't include calculations or excerpts should be edited' (p. 40). These independent works are said to represent 203 sheets (out of 865) whereas drafts account for 60 pages.

Gumbel's edited manuscripts were intended to form volume XVI in the first Collected Papers (MEGA 1). Gumbel announced that they were ready to print in 1926, but the volume XVI was never to appear. At least Gumbel hoped that some comments and remarks he has prepared could be published.

The first partial disclosure of Marx's mathematical manuscripts

Unfortunately publication was repeatedly delayed until Ryazanov was arrested in 1931, and eventually executed in 1938. As Kolman (1892-1972) writes, 'work on the manuscripts was given a new direction' (Kolman 1968, p. 225). Actually Vladimir Viktorovich Adoratskii (1878–1945) was given the direction of the Institute, whereas responsibility for the publication of the MMM was allocated to Ernest Kolman who presented them to a conference of mathematicians in Zurich in 1932 and later published a paper telling 'his' story of the 1968 edition. His story is important to understand the reception of the MMM, but it is obviously flawed so that it is necessary to have a second look at him. Kolman has an unusual story (Marko, tba).

Kolman, a Czech, was conscripted into the Austro-Hungarian army in 1914 and eventually became a prisoner of the Russians. In 1917 he met Lenin and took an active part in the October Revolution. During the 20s and 30s Kolman occupied a great number of significant functions in the Party and in the Soviet state, as well as in many scientific and social organizations. He took an active part in philosophic discussions in the Soviet Union, which consisted mainly of severe criticism of the so-called 'mechanists and Mensheviking idealists' with the consequence that 'many representatives of these philosophic tendencies disappeared not only from active philosophic life but also into labor camps'.

In the early 30s the Central Committee of the Communist Party of the U.S.S.R. appointed Kolman to the Board of Editors of the journal *Pod znamenem marksizma* (*Under the Banner of Marxism*) in which the first disclosure of the MMM was made in 1933. In 1934 Kolman received the title of 'Doctor of Philosophic Sciences', the highest possible degree in scientific life in the U.S.S.R., and in 1939 he became a full professor of mathematics. After the end of World War II, Kolman returned to Czechoslovakia, becoming the Head of the Department of Propaganda at the Central Committee of the Communist Party. A few months after the Communist Party take-over of political power in February 1948, he was arrested in Prague after urging the regime to speed up the construction of socialism in Czechoslovakia. He was transported to Moscow and jailed till Stalin's death. After his rehabilitation, he remained a few years in Moscow and returned to Prague in 1959.

If his experience turned him into an anti-Stalinist, nonetheless 'for him it is Marxist-Leninist philosophy that is a correct and truthful philosophy and 'the only scientific philosophy' of our time' (Kovaly). He stressed 'the division of all philosophy into two opposite camps: idealism and materialism' using 'the principle not only in philosophy but also in the natural and social sciences' (Kovaly, 343). During the early 60s in a series of discussions on dialectics with Jean-Paul Sartre, he 'defended the objective dialectic both in nature and society, and the subjective dialectic of our thinking as a reflection of the objective dialectic' (ibidem).

In 1976 Kolman migrated to Sweden, reportedly because he was sick of the Brezhnev regime. In 1979 he was to publish an autobiography the title of which implies eventual repentance.[8]

According to Seneta (2004) there is documentary evidence that Kolman had some solid mathematical training before his conscription into the Austro-Hungarian army. Nevertheless there was a 11- or 12-year break before he resumed 'academic' activity, which may explain the largely polemical and philosophical, rather than technical, nature of his subsequent mathematical writings; probably his career owed much more to his political views than to his scientific credentials. Alongside with his 'misled' route, one can but take cautiously his statements about the MMM.

Kolman (1968) states that after 'he became acquainted with the transcribed portion of the manuscripts and with the preparatory work toward their publication', '[he] was convinced that E. Gumbel was unable to appreciate completely either the importance of their publication or their philosophical and historical-mathematical significance'. Kolman adds that 'At [his] suggestion the board of directors of the Institute enlisted for the work on the manuscripts S. A. Yanovskaya' (ibidem Kolman p. 226). Sheynin and Seneta quote a similar sentence from Kolman's autobiography which proves how limited was his repentance. Gumbel is described as a 'mediocre German mathematician'. How much weight should be placed on Kolman's evaluation of Gumbel will be seen in due course.

In 1931 Kolman attended in London the second international congress of the history of science and technology, and presented a paper on Marx's notebooks, including some remarks on the mathematical manuscripts. A few months later, in an international congress of mathematicians held in Zurich he presented a report 'A new foundation of the differential calculus by Karl Marx' 'which discussed one of the works contained in Marx's manuscripts' (227). A year later some portions of the

[8] (The misled generation. We should not have lived this way) (*Die verirrte Generation. So hätten wir nicht leben sollen. Eine Biographie,* Frankfurt a/M, 1979.

MMM were published in the journal *Pod znamenem marksizma* (N° 1, pp, 14-115.) accompanied by the article 'On the Mathematical Manuscripts of K. Marx' by the team leader, S. A. Yanovskaya.

The 1968 edition

S.A. Yanovskaya (1896–1966) was to be recognized as the editor for the 1968 publication of the MMM. Through her biography by Bazhanov, we learn that Yanovskaya studied mathematics at Novorossiisky University (Odessa). Her teachers were skilful and original mathematicians such as S.O. Shatunovsky (who in 1901–2 proclaimed that the law of excluded middle is not valid for infinite sets) and I.Yu. Timchenko (history of mathematics, especially history of the theory of analytical functions). She then worked as an instructor at the Odessa Region Communist Party headquarters – largely ignoring mathematics. In 1924 Yanovskaya entered the Institute of Red Professors (established for persons from the lower classes) and two years after graduating (1931) she became a Full Professor without defending her doctorate (her degree was defended in 1935). Yanovskaya matured as a Party functionary working with academics, and she closely followed the purity of her colleagues' thoughts as well as their ideological devotion.

Her papers (with Kolman or alone) give an idea of her approach to history of mathematics. According to her Frege, Russell, Couturat, and Cantor are very close in their views to true idealism and mysticism ('the example of which is Platonism'). More generally speaking, asserts Yanovskaya, 'bourgeois science in the imperialist era does not evolve from Hegel to Marx, Engels, and Lenin, but regresses' [Kol'man, Yanovskaya 1931, p. 118–119]. In the 1930's Yanovskaya strongly criticized idealism in mathematics, and unmasked 'the bourgeois philosophy of mathematics'.

Bazhanov presents her biography as an 'epiphany', considering particularly her shift from hard-line communist orthodoxy and hostility towards 'bourgeois minded' Soviet-Russian mathematicians to the vigorous support of mathematical logic.

After the 1933 publication the complete edition was expected to take place quickly but the Second World War intervened (Baksi 1994). The archives and the library of the Institute were shifted to the Far East. After the War the pace of work picked up slowly. Yanovskaya had a heavy load dealing with mathematical logic in the Moscow University; although she was given a new assistant, Konstantin Alekscievich Rybnikov, in the 1950s one can note only the publication of a note on Marx entitled 'On The Concept of Function' in the journal *Voprossy Filosofii* (No. 11, 1958).

Yanovskaya died in 1966 and it was only two years later that a more complete edition was released. It includes a publisher's note but there is no mention of Gumbel or Kolman and the tribute paid to Yanovskaya is minimal.

From 1968 to a new edition in the MEGA 2

The 1968 edition is much more complete. We suspect that most of the material selected by Gumbel has been published with a description of some excerpts. While Marx's text is in its original language, in this edition the whole apparatus is Russian, so that the reception was rather slow. Bu six years later an version was published in West Germany by Wolfgang Endemann and draw attention from reviewers and translators. During the last three decades translations have appeared in many western languages (Italian, English, French, Spanish, Portuguese), and J. Dauben mentions also translation into Chinese and Japanese. In 1994, Pradip Baksi undertook an extensive translation of the 1968 edition into Bengali and then into English.

But meanwhile the MEGA 2 project has been launched. Shortcomings were noticed in the 1968 edition and it was decided to include the MMM in the new MEGA. Irina Antonova (1999, 2009) was in charge of the project and in 1988 she told Pradip Baksi that it would be completed in 1990. But in 1989 the Wall fell and the MEGA 2 project was rescued by the IMES; however, we are still a long way to a definitive scholarly edited publication of the MMM.

Marx on probability and statistics

In this part of the paper we document and assess Marx's interest in statistical and probabilistic ideas, discuss difficult points in relation to his possible sources which the MEGA 2 may help to clear up, and consider the reception of this aspect of his thought.

Marx cites Quetelet

As we will show below, key texts of Marx are suffused by probabilistic and statistical ideas. Despite this, it is at present difficult to say much that is definite about Marx's sources for the statistical notions he employs. It is evident from both specific references and the substantive content of many of his texts that he drew on the work of the Belgian statistician Quetelet; however, the exact sites of Marx's inspiration in the latter's work remain as yet largely unidentified.

These questions may be elucidated by two aspects of the MEGA project. The first is the publication of further volumes of Marx's notebooks, and the second is the reconstruction of his library and that of Engels.

To our knowledge, Marx cites Quetelet in support of his argument in four places: in Chapter 13 of *Capital* Vol. 1; in Chapter 50 of *Capital* Vol. III; in an article published in the *New-York Daily Tribune* for 17-18 February 1853; and in an article published in the *Neue Oder-Zeitung* of 8 February 1855. There is also a discussion of Quetelet's work in one of Marx's letters to Kugelmann (3 March 1869).

Only in the *Tribune* article does he suggest which parts of Quetelet's numerous works he is appealing to. Unfortunately for the scholar wishing to trace the provenance of Marx's statistical ideas – which, in the Vol. I case especially, involve some technically sophisticated concepts – this sole approach to specific citation raises more questions than it answers.

In the *Tribune* article Marx criticises the death penalty on the grounds that the regularity of the annual statistics of crime demonstrates that the murder rate, as that of other crimes, is a function of social conditions, an idea for which he invokes the support of Quetelet:

> Mr. A. Quételet, in his excellent and learned work, *l'Homme et ses Facultés*, says:
>
> 'There is a *budget* which we pay with frightful regularity — it is that of prisons, dungeons and scaffolds.... We might even predict how many individuals will stain their hands with the blood of their fellow men, how many will be forgers, how many will deal in poison, pretty nearly the same way as we may foretell the annual births and deaths.'
>
> And Mr. Quételet, in a calculation of the probabilities of crime published in 1829, actually predicted with astonishing certainty, not only the amount but all the different kinds of crimes committed in France in 1830. That it is not so much the particular political institutions of a country as the fundamental conditions of modern *bourgeois* society in general, which produce an average amount of crime in a given national fraction of society, may be seen from the following table, communicated by Quételet, for the years 1822-24. We find in a number[9] of one hundred condemned criminals in America and France:

[9] Actually, a percentage, as suggested not only by the 'Total' row but at the point where Quetelet gives this date in the *Treatise*.

Age	Philadelphia	France
Under twenty-one years	19	19
Twenty-one to thirty	44	35
Thirty to forty	23	23
Above forty	14	23
Total	100	100

> Now, if crimes observed on a great scale thus show, in their amount and their classification, the regularity of physical phenomena — if as Mr. Quételet remarks, 'it would be difficult to decide in respect to which of the two' (the physical world and the social system) 'the acting causes produce their effect with the utmost regularity' — is there not a necessity for deeply reflecting upon an alteration of the system that breeds these crimes, instead of glorifying the hangman who executes a lot of criminals to make room only for the supply of new ones?

Here Marx cites at least two works: (1) Quetelet's ground-breaking book on human and social statistics of which the first edition, *Sur l'homme et le développement de ses facultés, ou Essai de physique sociale* was published in 1835 (the English translation *A Treatise on Man and the Development of His Faculties* came out in 1842), (2) the un-named work of 1829, and (perhaps) some third work containing the statistics tabulated by Marx (since he does not specify whether they are taken from the *Treatise* or the 1829 work, or some other).

There are several puzzles here. First, Marx's use of the French title of the *Treatise* suggests that it was that which he used. But the only known record of Marx reading Quetelet is an unpublished notebook of autumn 1851 held at the IISS in Amsterdam which appears, to go by the summary on the IISS web-site, to contain notes from the *English* edition of 1842.[10] Since the composition of his *Tribune* article (on January 28, 1853) came only a little more than 15 months after a close reading of this English

[10] Henceforth these two works will be referred to as the *Essai* and the *Treatise* respectively.

edition, one wonders why Marx quoted the *French* title. One explanation might be that Marx thought that giving an English title for a work by a francophone author would confuse his American readers. Another might be that those readers would find a French title more satisfyingly exotic.

A third possible explanation might note that Marx had lived in Brussels in the mid-1840s when the Belgian statistician's *Essai* was enjoying its first fame, and suggest that Marx had read that version at that time and that the pressure of turning out material for the *Tribune* caused him to rely on his memory of the earlier reading rather than his notes of the more recent.

However, the first quotation given by Marx is almost identical to a passage on page 6 of the 1842 translation; also Marx must have had at least some notes by him from which to quote the table of figures. Since this table appears in both the French and English editions (it is at page 95 of the latter; the table is also reproduced in the much-expanded second French edition of 1869) it seems most likely that he did indeed reproduce it from either the *Essai* or the *Treatise* (or at any rate his notes of the latter), rather than some third source.

The second puzzle is Marx's citation of an 1829 work by Quetelet. The only work published by the latter in that year seems to be *Recherches statistiques sur le royaume des Pays-Bas*; that work, as its name suggests, deals primarily with the statistics of the Low Countries, but much space is devoted to comparing the state of society there with in France and Britain. However, it is difficult to find in it a definite quantitative prediction as described by Marx. Quetelet is undoubtedly convinced that the numbers of different types of crimes in the various countries will remain pretty constant in the short run, but the nearest he comes to claiming powers of prediction is the following:

> Ainsi l'on passe d'une annee a l'autre, avec la triste perspective de voir les memes crimes se reproduire dans le meme ordre et attirer les memes peines dans les memes proportions. Triste condition de l'espece humaine! La part des prisons, des fers et de

> l'echafaud, semble fixee pour elle avec autant probabilite que les revenus de l'etat. Nous pouvons enumerer d'avance combien des individus souilleront leurs mains du sang de leurs semblables, combien seront faussaires, combien empoisenneurs, a peu pres comme on peut enumerer d'avance les naissances e les deces qui doivent avoir lieu.

Hence one would like information about Marx's precise source on this point. Note also that the last sentence in the quotation is virtually identical with the passage in the *Essai* (and its equivalent in the *Treatise*) which Marx quotes in full. It should also be noted that the second direct quotation offered by Marx is, although it has a ring of Quetelet about it, one which we have not yet been able to trace.

This brings us to a further puzzle: in the letter to Kugelmann already mentioned Marx remarks:

> Quételet is now *too old* for one still to make any sort of experiment with him. He rendered great services in the past by demonstrating that even the apparently casual incidents of social life possess an inner necessity through their periodic recurrence and their periodic average incidence. But he was *never* successful in interpreting this necessity. And he made no progress, but simply extended the material for his observations and calculations. He is today no further on than he was *before* 1830. [emphases in original|

The lack of preamble to Marx's comment *to* Kugelmann suggests that this part of his letter was responding to some earlier comment or question *from* Kugelmann: if so, it would be interesting to know what it was. The likeliest candidate would seem to be that Kugelmann noted the contemporary publication of a substantially enlarged second edition of Quetelet's *Essai* in 1869 edition (as *Physique Sociale etc*, thus making the former sub-title the main title, an interesting change of emphasis), and that Marx is here giving his impressions of this edition (assuming that it in fact appeared early enough in the year for Marx to have seen it; Kugelmann's original letter might have been inspired by advance news of Quetelet's publication). This hypothesis is

strengthened by the fact that the extra matter in the *Physique Sociale* mainly consists of masses of further data.

Whether or not this is correct, Marx's comments suggest familiarity with Quetelet's writings over four decades from 1829 at the latest to 1869. But Quetelet had suffered a stroke in 1855 and seems to have published little on social statistics after 1848 apart from extending the *Essai* as noted. Be that as it may, Marx's comments suggest that he thought that even the *Essai* added nothing essentially new to Quetelet's pre-1835 work. As we have seen, Quetelet's claim about the possibility of quantitative predictions of crime is made in identical terms in both his 1829 *Recherches* and in the *Essai* of 1835 (and hence in the *Treatise* of 1842). If Marx had indeed seen the *Recherches* one can see the basis for his assessment of Quetelet's failure to make progress after 1830, even if he had not in fact seen the *Physique Sociale* in 1869.

Once again, one would very much like to see records of Marx's reading, and details of the books owned by him and Engels, in order to substantiate or refute our conjectures.

Probabilistic and statistical ideas in Marx's work

The foregoing would be matter for little more than footnotes at the appropriate places in the *MEGA*, were it not for the fact that it is easy to show that chance and necessity (or probabilistic notions, in today's terminology) were key to Marx's thinking from his earliest days. Moreover, at least two key arguments in *Capital* depend on statistical thinking that is (perhaps) intuitive and, despite its informal expression, of some sophistication by the professional standards of the time. Furthermore, the totality of Marx's thought in this area shows him to be a precursor of recent developments in the conception of market economies as complex adaptive systems.

Thus access to Marx's 1851 notes on Quetelet's *Treatise*, whatever other notebooks may come to light, and information about his library, will be key to understanding the development of Marx's ideas on probability and statistics.

The following section of the paper provides documentation to demonstrate the early and continued interest of Marx in this area; it is preliminary and indicative, rather than exhaustive.

Early writing

The first mention of 'chance' in Marx's writing comes right at the beginning; in his *Reflections of a Young Man on The Choice of a Profession* (1835) he begins:

> Nature herself has determined the sphere of activity in which the animal should move, and it peacefully moves within that sphere, without attempting to go beyond it, without even an inkling of any other. To man, too, the Deity gave a general aim, that of ennobling mankind and himself, but he left it to man to seek the means by which this aim can be achieved …
>
> … Serious consideration of this choice, therefore, is certainly the first duty of a young man who is beginning his career and does not want to leave his most important affairs to chance.

These are conventional enough sentiments from a young man brought up in a *milieu* where the ideals of the Enlightenment and public service were taken for granted. What is striking in the light of later writing is the final phrase quoted here. The standard Enlightenment view of the notion of chance was summed up by Hume: ' 'tis commonly allowed by philosophers that what the vulgar call chance is nothing but a secret and conceal'd cause' (cited from Hacking 1991). De Moivre, one of the pioneers of modern probabilistic analysis, objected to the idea of chance as inherently atheistical:

> [S]uch Laws, as well as the original Design and Purpose of the establishment, must All be *from without*; the Inertia of matter, and the nature of all created Beings, rendering it impossible that any thing should modify its own essence, or give to itself, or to any thing else, an original determination or propensity. And hence, if we blind not ourselves with metaphysical dust, we shall be led, by a short and obvious way, to the

acknowledgement of the great *Maker* and *Governour* of all: *Himself all-wise, all powerful and good.*

De Moivre, *The doctrine of chances* page 251-2; cited in Pearson (1978: 161)

We find a similar disdain for the intellectual status of chance expressed in another early text from Marx, his uncompleted tragedy *Oulanem* (1837):

Pertini. Chance! Such is the language of philosophers
When reason doesn't come to rescue them.

MECW pp.588–607

Deducing the sentiments of an author from the words he puts in his characters' mouths is always tricky, especially when the work is as fragmentary as *Oulanem*; however, Pertini seems an unsatisfactory character (he has prior to the point of our quotation deliberately concealed prior acquaintance with the eponymous hero of the drama, for what appear to be malevolent purposes), so we are not obliged to assume that he speaks for Marx.[11] Note also that, compared with Hume, it is now not the vulgar, but philosophers, who are said to abuse the notion of chance.

Furthermore, in the next scene of the drama we find the eponymous hero, Oulanem, explicitly attacking the contemporary scientific world-view:

Soon I shall clasp Eternity and howl
Humanity's giant curse into its ear.
Eternity! …
An evil artifice contrived to taunt us,
Who are but clockwork, blind machines wound up
To be the calendar-fools of Time; to be,

[11] Indeed, at one point Pertini comments on his resemblance to the character of Mephistopheles in Goethe's *Faust*; see Prawer, p.16. However, as Prawer notes elsewhere, 'Marx … often speaks with Mephistopheles' voice' (Prawer p.327)

> Only that something thus at least might happen;
>
> And to decay, that there might be decay!
>
> MECW pp.588–607

Whether or not Pertini speaks for Marx in the previous passage, we feel sure that Oulanem does so here, for reasons that will become apparent.

Commitment to determinist laws of nature, coupled nonetheless with a belief in the existence of human free will, led inevitably to a dualistic conception of mind and body.

Dualism and religion were precisely the targets aimed at in Marx's doctoral dissertation (1841), which explicitly praises Epicurus for designing his physics to allow human free will and for rejecting religion (McLellan 1980: 59; McCarthy; Gabaude). He compares the atomist physics of Democritus and Epicurus and defends the latter's doctrine of the spontaneous declination, or 'swerve', of the atom against other commentators, precisely because this theory is designed to permit free-will. He approvingly quotes Epicurus:

> Necessity … does not exist, but some things are accidental, others depend on our arbitrary will. … It would be better to follow the myth about the gods than to be a slave to the *heimarinene* [what has been decreed, destiny] of the physicists.

For Democritus, the natural motion of the atoms is straight downwards, but Marx agrees with Lucretius that, if so, it is hard to see how nature produces any further determinations, since none of the atoms will interact: '[I]f the atoms were not to decline, neither their repulsion nor their meeting would have taken place, and the world would never have been created' (Marx 1975: 52); see also Marx's fourth preparatory notebook on Epicurean philosophy (1839), where the atom's spontaneity is derived dialectically from its concept.

Notice that chance makes room for free will, for human self-determination; where the latter is lacking, chance is realised as determinism and *vice versa*. Thus in Marx's essays on freedom of the press (1842: MECW I: pp. 132–181) he says 'the sphere in which absence of freedom is dominant becomes a matter of pure chance'. Likewise, in his *Critique of Hegel's Philosophy of Right* (1843–1844) Marx complains that 'Hegel resolves all difficulties through a natural necessity which is the antithesis of freedom'.

First studies in political economy

With this background it is not surprising that, when Marx comes to study political economy, what strikes him is the way in which the atomism of an economy of generalised free exchange paradoxically produces both randomness and constraint. In the *Economic and Philosophical Manuscripts* (April – August 1844) we read that 'the worker is sure to lose and to lose most from the gravitation of the market price toward the natural price …[because] … [t]he sudden chance fluctuations in market price … hit profit less than wages'.

While composing the *Manuscripts* Marx was beginning his study of contemporary political economy with his 'Excerpts from James Mill's *Elements of Political Economy*'. After 84 quotations from Mill, Marx's *first* comment is:

> Both on the question of the relations of money to the value of metal and in his demonstration that the cost of production is the sole factor in the determination of value Mill succumbs to the error … of defining an *abstract law* without mentioning the fluctuations or the continual suspension through which it comes into being. If e.g. it is an *invariable* law that in the last analysis - or rather in the sporadic (accidental) coincidence of supply and demand – the cost of production determines price (value), then it is no less an *invariable law* that these relations do not obtain, i.e. that value and the cost of production do not stand in any necessary relation. Indeed, supply and demand only ever coincide momentarily thanks to a previous fluctuation in supply and demand, to the disparity between the cost of production and the exchange value. This is the *real* movement, then, and the above-mentioned law is no more than an abstract,

> contingent as one-sided movement in it. Yet recent economists dismiss it as accident, as inessential. Why? Because if the economists were to attempt to fix this movement in the sharp and precise terms to which they reduce the whole of economics this would produce the following basic formula: laws in economics are determined by their opposite, lawlessness. The true law of economics is *chance*, and we learned people arbitrarily seize on a few moments and establish them as laws.' (emphases in original)

(Marx used a translation of Mill's book, by J. T. Parisot, Paris, 1823.)

The ideas of this passage are repeated in Marx's pamphlet *Wage labour and capital* – originally written as a lecture to the German Workingmen's Club in Brussels in 1847 – where he summarises his thought thus: 'the total movement of this disorder is its order'; still clearer is the formula in *The poverty of philosophy* (chapter 1, section 2), written in the winter of 1846-47 against Proudhon:

> If M. Proudhon admits that the value of products is determined by labour time, he should equally admit that it is the fluctuating movement alone that in a society founded on individual exchanges makes labour the measure of value. *There is* no ready-made constituted 'proportional relation', but *only a constituting movement*. (emphases added)

The relation between value and labour is the heart of all controversy over Marx's political economy, of course.

So far we have illustrated Marx's interest in probabilistic ideas (translating the contemporary 'chance' into modern language). We have yet to show him making use of statistical concepts in the narrower sense, in other words that of numerical summaries of data, conceived of as observations of random variables. This seems to be first found in *The German Ideology* (Autumn 1845–mid-1846) where social formations up to the present are contrasted with communism:

> [T]he communal relationship into which the individuals of a class entered, and which

> was determined by their common interests over against a third party, was always a community to which these individuals belonged only as *average individuals*, only insofar as they lived within the conditions of existence of their class … . With the community of revolutionary proletarians, on the other hand, … it is just the reverse … . It is just this combination of individuals (assuming the advanced stage of modern productive forces, of course) which puts the conditions of the free development and movement of individuals under their control — conditions which were previously abandoned to *chance* and had won an independent existence [emphases added]
>
> MECW Vol. 5, p.80

Note how the idea of the 'average individual' is here linked to the recurring contrast between self-determination and chance.

Preparations for *Capital*

The uncertainties of a capitalist economy, and the need for probabilistic and statistical concepts to theorise such an economy, find expression in at least six places in the *Grundrisse*, and one each in *A Contribution to the Critique of Political Economy* and *Theories of Surplus Value*.

In the text which has come down to us as the 'Introduction' to the *Grundrisse* (Notebook M, August-September 1857) there is an enigmatic note from Marx to himself which reads

> (7) This conception appears as necessary development. But legitimation of chance. How? (Of freedom also, among other things) (Influence of means of communication. World history has not always existed; history as world history a result.) [Penguin p.109]

It is not clear from the context what conception 'appears as necessary development', but it seems that it is the final idea in point (6), how relations of production develop unevenly as legal relations.

There are two relevant passages in Notebook I of the *Grundrisse*, both related to his critique of Darimon as a representative of time-chit theorists. In the first, we find Marx once again making use of the average, in connection with convertibility between bank notes and gold; those who oppose legal regulation of this note that

> this … is achieved on the average by a bank of issue which lends against bills of exchange … and charge that their opponents despite everything never achieved better than this average measure of security. The latter is a fact. The average, by the way, is not to be despised, and calculations on the basis of averages have to form the basis for banks just as well as for all insurance companies etc. (p.131)

In the second, where Marx discusses the convertibility of time-chits we find him moving beyond the average to more sophisticated statistics in connection with 'mercantile speculation, whose calculus of probabilities depends both on the median price averages which figure as the centre of oscillation, and on the average peaks and average troughs of oscillation above or below this centre'. It is not clear quite what Marx means by a 'median price average', but he seems primarily to think of the average as a statistic of a time series.

> This average [that of the value of commodities as determined by labour time] appears as an external abstraction if it is calculated out as the average figure of an epoch, e.g. 1 lb. of coffee = 1s. if the average price of coffee is taken over 25 years; but it is very real if it is at the same time recognized as the driving force and the moving principle of the oscillations which commodity prices run through during a given epoch. (p.137)

Possibly Marx also has in mind that the distribution of prices should also be considered in cross-section as well as longitudinally, as we shall see shortly.

Be that as it may, what does seem clear here is that Marx not only conceives of the average price as a statistic of a random variable expressing some underlying generating process, but that he also thinks of the limits of the range over which it varies as statistics ('average peaks and average troughs of oscillation'). In this connection, it is a

fascinating coincidence that Gumbel, the first editor of Marx's mathematical manuscripts, achieved fame in his professional career as a pioneer in, precisely, the statistics of extreme values of samples from random variables; he is commemorated by the Gumbel distribution, Type I of the family of generalised extreme value distributions.

When Marx turns to discussing the operation of a hypothetical system of bank-issued time-chits he writes that:

> [if] all owners of commodities … want to sell at their exchange value, then they will not await the *chance arrival or non-arrival of a buyer*, but go immediately to the bank, unload their commodities on to it, and obtain their exchange value symbol, money, for them. (p.154, emphasis added)

The implication is that the random process by which buyers arrive determines whether a seller can indeed obtain the full exchange-value of their commodity, or instead has to settle for less; in other words that market prices are cross-sectional random variables as well as time-series ones. While we resist the temptation to portray Marx as a precursor of every possible statistical notion, it is nonetheless the case that he here anticipates the concerns of queueing theory (formally pioneered by Erlang in 1909).

The last passage we note from Notebook I is very significant in showing Marx's conception of the way in which the freedoms of the market place, and the chance processes to which these give rise, nonetheless come to dominate the participants and make them unfree. Furthermore, it will be important in considering Marx's reception. We therefore quote it at length:

> As much, then, as the whole of this movement appears as a social process, and *as much as the individual moments of this movement arise from the conscious will and particular purposes of individuals, so much does the totality of the process appear as an objective interrelation, which arises spontaneously from nature*; arising, it is true, from the mutual

> influence of conscious individuals on one another, but neither located in their consciousness, nor subsumed under them as a whole. Their own collisions with one another produce an *alien* social power standing above them, produce their mutual interaction as a process and power independent of them. Circulation, because a totality of the social process, is also the first form in which the social relation appears as something independent of the individuals, but not only as, say, in a coin or in exchange value, but extending to the whole of the social movement itself. *The social relation of individuals to one another as a power over the individuals which has become autonomous, whether conceived as a natural force, as chance or in whatever other form, is a necessary result* of the fact that the point of departure is not the free social individual. Circulation as the first totality among the economic categories is well suited to bring this to light. (pp.196–197; first and last emphases added; 'alien' is Marx's emphasis)

The sentence we shall return to in considering Marx's sources, and his later reception, is the one to which he himself added emphasis: 'Their own collisions with one another produce an *alien* social power standing above them … '. For the present we merely note how this recalls his engagement with Epicurean physics, in which the spontaneous swerve of the atom leads to the collisions which produce the world.

We do not here have space for an extended presentation of further relevant passages from the *Grundrisse* and elsewhere, but we note the following: Notebook II contains further reflections on the stochastic formation of exchange value, free will, and the 'chance influences of nature'; Notebook VI discusses the uncertainty to which the free labourer is subjected, and the role of chance in the metamorphoses of capital from exchange to production; Notebook VII revisits the ideas reported above in connection with the circulation of money, and the risky returns to wage labour.

Because of its relevance for an important passage in *Capital* we single out for special notice the section in Notebook VII where Marx records Robert Owen expressing the view that 'small masters with small capitals have only little chance of success'.

Capital

As noted above, two out of the four places known to us where Marx cites Quetelet in support of his points occur in *Capital*, one in Volume I and another in Volume III. However, the latter Volume is the site of a further set of passages that demonstrate how Marx's thought found expression in words that have clear counterparts in formal statistical terms. We discuss these three sites in order.

Vitally important, from the point of view of considering Marx's engagement with statistics as such, as opposed to probabilistic ideas in general, is the reference to Quetelet in *Capital* Volume I, Chapter 13, 'Co-operation'. In the first footnote to the chapter Marx quotes Edmund Burke, and cites Quetelet, in support of his explanation of his concept of average social labour:

> 'Unquestionably, there is a good deal of difference between the value of one man's labour and that of another from strength, dexterity, and honest application. But I am quite sure, from my best observation, that any given five men will, in their total, afford a proportion of labour equal to any other five within the periods of life I have stated; that is, that among such five men there will be one possessing all the qualifications of a good workman, one bad, and the other three middling, and approximating to the first, and the last. So that in so small a platoon as that of even five, you will find the full complement of all that five men can earn.' (E. Burke, 1. c., pp. 15, 16.) Compare Quételet on the average individual.

Note that Burke's attention is focussed on the *total* labour performed, *not* the average per worker. Despite frequent references to the average it seems clear from the main text that for Marx, and the capitalist, the product of the working day is the total labour of the workers employed; this is a random variable found by summing the random variables constituted by the individual labours of each worker, which are social labour only in so far as they are employed by capitalists employing other social labours.

Hence the total product is the sum of independent, identically-distributed random variables, in modern language. Moreover, the degree to which the total labour is equal to average social labour multiplied by the number of workers depends on the number of workers employed.

> These individual differences, or 'errors' as they are called in mathematics, compensate one another, and vanish, whenever a certain minimum number of workmen are employed together. … *From the point of view, however, of the capitalist who employs these 12 men, the working-day is that of the whole dozen*. … *But if the 12 men are employed in six pairs, by as many different small masters, it will be quite a matter of chance, whether each of these masters produces the same value*, and consequently whether he realises the general rate of surplus-value. … Thus the laws of the production of value are only fully realised for the individual producer, when he produces as a capitalist, and employs a number of workmen together, whose labour, by its collective nature, is at once stamped as average social labour. (emphases added)

This will be recognised as an appeal to a central limit theorem (CLT). Although the essence of this argument had been familiar currency among natural scientists for many decades by the time of Volume I's publication in 1867, it was not formulated as a theorem and proved rigorously until much later in the century. (It was first described by de Moivre in 1733, and rescued from obscurity by Laplace in 1812, but mathematical proof was left to Lyapunov in 1901).

Marx's reference to the 'error' terminology for individual variation suggests that he was at home with the contemporary literature and thus by implication with Q's importation of statistics from astronomy. However, neither Quetelet"s *Treatise*, nor his popular textbook on probability, appear to contain any statement relating to the law of large numbers, apart from a repeated insistence on the need to have as large a sample as possible in order to calculate a reliable average. More information about *what* Marx's read would be welcome; one would like also to know *when* he read it, because

it would be interesting to know whether he read Owen in the light of Quetelet, or *vbice versa*.

The classical version of the CLT is restricted to the case of random variables with finite variance, and the limit is the normal, or Gaussian, distribution. Relax the requirement of finite variance and finite means, in the generalised CLT, and the limiting distribution will be a member of the family of stable distributions, which includes the Gaussian as a boundary case; all other members are heavy-tailed, and may have skewed forms (and for some of the latter, with bounded support).[12] In the language of stable distributions, Marx's argument is that the random variable describing individual labour is in the domain of attraction of the random variable describing social labour; it can be shown that the only possible distributions for which this is true are members of the stable family.[13]

Since the (value) product of labour presumably has a lower bound of zero its distribution is likely to be right skewed and it is therefore the GCLT which is relevant to social labour. We now argue that Marx not only possessed a notion akin to that of a density distribution, but was aware of the possibility of distributions that are skewed, or leptokurtic, or both.

In Chapter 10 of *Capital* Volume III Marx discusses the equalization of the general rate of profit

> Between those spheres that approximate more or less to the social average, there is again a tendency to equalization, which seeks the 'ideal' mean position, i.e. a mean position which does not exist in reality. In other words, it tends to shape itself around this ideal as a norm.
>
> Marx (1981: 273)

[12] Other well-known cases are the Cauchy and Lévy distributions; at the opposite boundary to the normal is Dirac's delta.

[13] Strictly speaking, the variables are the *products* of the labours.

Note that Marx says that the tendency to profit rate equalisation is a tendency 'to s*hape itself around* this ideal [mean] as a norm' (our emphasis); in other words, profit rate equalisation *is* the 'shape around the ideal' towards which the tendency is directed – in other words, it is the formation of a profit-rate distribution ('shape itself around' the mean). This might be thought a fanciful interpretation, were it not that a few pages later (Marx 1981: 283-4) Marx not only describes a probability density, but also discusses how variations in its shape – symmetric or not, light- or heavy-tailed – will affect the relation of the mean to the whole:

> If only a relatively small proportion [of commodities] are produced in worse conditions [than the average ones], and another portion in better conditions, so that the individual value of the one part is greater than the mean value of the great bulk of the commodities, and that of the other part lower than this mean, then these two extremes will cancel one another out so that the average value of the commodities at the extremes is the same as the value of the mass of average commodities …

Marx's prefatory 'if' implies the possibility of contrary conditions, such as skewness, and heavy tails, possibilities which he indeed proceeds to discuss. He also envisages censoring of the data (by the market):

> We leave aside here the situation where the market is over-supplied, in which case it is always the proportion which is produced under the most favourable conditions that governs the market price; …

If we are right in interpreting Marx as having in mind something corresponding to the modern notion of the probability density function then he would seem to be some way ahead of Quetelet, who considered explicitly only ranges and averages[14] (Mosselmans 2005), and indeed of Galton. Not only is Galton's interest in deviations from the average clearly also present in Marx, but the latter's discussion of skewed and

[14] However, Quetelet's appeal to the correction of astronomical observations, in his justification of the 'average man', certainly *implies* the Gaussian distribution (Mosselmans 2005).

heavy-tailed distributions is ahead of Galton, who dubbed the Gaussian distribution 'normal' (Hacking 1990: 184).

It thus becomes especially interesting to know the source of Marx's discussion: since we know that he read Quetelet's *Treatise* we know that he had had the opportunity to see what is effectively a probability density illustration: Plate 4, which includes a 'curve showing, at different ages, the degrees of the Propensity to crime' (precisely the topic referenced by Marx in his *Tribune* article. Furthermore, the curve is notably skewed (and that the data is of course right-censored). Once again, we need whatever evidence the 1851 notebook has to offer.

The second citation of Quetelet in *Capital* comes in Volume III, Chapter 50, 'The Illusion created by Competition', where Marx discusses how prices of production are formed out of the fluctuations of market-prices: 'The same domination of the regulating averages will be found here that Quetelet pointed out in the case of social phenomena' (Marx 1981: 1000). As we have seen from his notes on Mill, the fluctuations of supply and demand, and hence of market prices occupied Marx from the beginning of his studies in political economy.

However, we should note that the emphasis has apparently been reversed since those early studies; here prices of production are 'regulating' averages of market prices, whereas in his polemic against Proudhon it is 'only [the] constituting movement' of individual exchanges that make labour the measure of value. But Volume III as published is the product of Engels' editing, and the extent to which the latter fulfilled Marx's intentions has been questioned by Heinrich (1996). As Heinrich concludes, '[e]very future discussion of Marx's economic theory will have to refer to Marx's original manuscript'.

Marx's reception

The belief that Marx proposed iron laws of history is so widespread a misconception that to document it would be both arduous and tedious. Instead we quote a

particularly apposite example: Ian Hacking makes no reservation when he speaks of Marx 'fabricating an iron necessity out of the very same numerals, the identical official statistics, that I have incorporated into an account of the taming of chance' (1990: 8) and adds that:

> Marx read the statistics of Engel or Quetelet or Farr with indifference, divining with their aid the underlying laws of society that bind it in a totally nonstatistical necessity.
>
> Hacking (1990: 132)

Even the sketch provided in this article should indicate that whatever one makes of Marx's views on necessity, they were certainly not 'nonstatistical'. Hacking is an erudite scholar of the history of statistics, and it would be unfair to criticise him for mistaking the position in an area where even specialists in Marx have done little work.

Such work might have been done in the former Soviet Union, given the presence of (a) strong institutional support for the study of Marx, including copies of his manuscripts, (b) a significant indigenous statistical tradition, and (c) the German émigré statistician Gumbel actually working on Marx's mathematical manuscripts. Unfortunately the practical politics of the Soviet Union, especially following the rise of Stalin, made practical work in economic and social statistics both controversial and hazardous, while the philosophical implications of probabilistic ideas were equally likely to attract the attention of the Stalinist ideologue Ernest Kolman (Seneta 2003).

The degree of Kolman's understanding may be judged from essay in the 1968 edition of Marx's mathematical manuscripts:

> Marx's statement on the statistical nature of economic mechanisms as mechanisms of large-scale processes has an exceptionally great methodological significance for mathematical statistics. These mechanisms express the interactions of individual processes in the laws of probability; they dominate over any variations from the mean. Marx returned repeatedly to this problem.

Kolman (1968: 222)

As we have seen, while Marx certainly 'returned repeatedly' to statistical issues, to summarise his thought by asserting that in it 'economic mechanisms … dominate over any variations from the mean' is simplistic at best. Moreover, as evidence of Marx's repeated return, Kolman offers the quotation from the *Grundrisse* about 'average peaks and average troughs', which we discussed above and connected with the professional achievement of Gumbel, about whose work on the manuscripts Kolman is so dismissive.

Thus it is not surprising to find that Western commentators, where they notice Marx's statistical concerns at all, stumble equally badly. To confine ourselves once more to a single apposite case, consider the translator's note to Marx's citation of Quetelet in *Capital* Volume III, Chapter 50:

> Marx's attitude towards Quételet, in so far as it can be inferred from a few brief references, is interesting and characteristic: the regularities Quételet demonstrates in social phenomena are ingenious, but not particularly significant. Cf. 'Parties and Cliques' [the article from *Neue Oder-Zeitung*]

Marx (1981: 1000)

An ironic consequence of this is that the work which has done most to interest present-day Marxist economists in probabilistic approaches (Farjoun and Machover 1983) itself fails to recognise the extent and significance of statistical thinking of Marx's presentation in *Capital*. Whereas they see their application of statistical mechanical ideas to the *dis*solution – as they put it – of the transformation problem posed in Volume III by returning to the analysis of Volume I, Marx's discussion is already statistically informed, as we have seen above (see also Wells 2007: 121 ff).

Conclusion

Marx's study of the calculus is moderately well-known, albeit incompletely documented. In relation to its exact motivation, in the context of his wider concerns, much remains to be elucidated. In particular, one would like to know how far his attention was motivated by the philosophical problems of the subject and its relations with dialectical thinking (Carchedi 2003) rather than possible applications to political economy.

Our second section has shown that much of Marx's thinking was couched in language that, when read with the right spectacles, appears clearly statistical in form. Given this, one may question the degree to which the mathematical manuscripts were motivated by potential application of the calculus. On the other hand a sceptical reader might ask why, if probabilistic thought was really so important to Marx, it has been so little noticed — or less politely, whether our own concerns have led us to see in Marx what is not really there. To this objection we reply, with Marx:

> OWING TO A CERTAIN JUDICIAL BLINDNESS, even the best minds fail to see, on principle, what lies in front of their noses. Later, when the time has come, we are surprised that there are traces everywhere of what we failed to see … and surprised to find what is newest in what is oldest.
>
> MECW 42:557

Great credit is due to Farjoun and Machover for their insight in applying the methods of statistical mechanics to Marx's political economy, not least for helping us to see 'what lies in front of our noses'.

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