



**HAL**  
open science

## Spin-squeezed atomic crystal

Dariusz Kajtoch, Emilia Witkowska, Alice Sinatra

► **To cite this version:**

Dariusz Kajtoch, Emilia Witkowska, Alice Sinatra. Spin-squeezed atomic crystal. 2017. hal-01570313v1

**HAL Id: hal-01570313**

**<https://hal.science/hal-01570313v1>**

Preprint submitted on 28 Jul 2017 (v1), last revised 30 Aug 2018 (v2)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Spin-squeezed atomic crystal

Dariusz Kajtoch,<sup>1,2</sup> Emilia Witkowska<sup>2</sup> and Alice Sinatra<sup>1</sup>

<sup>1</sup>Laboratoire Kastler Brossel, ENS-PSL, CNRS, UPMC-Sorbonne Universites and Collège de France, Paris, France

<sup>2</sup>Institute of Physics, PAS, Aleja Lotnikow 32/46, PL-02668 Warsaw, Poland

(Dated: July 28, 2017)

We propose a method to obtain a regular arrangement of two-level atoms in a three-dimensional optical lattice with unit filling, where all the atoms share internal state coherence and metrologically useful quantum correlations. Such a spin-squeezed atomic crystal is obtained by adiabatically raising an optical lattice in an interacting two-component Bose-Einstein condensate. We study numerically and analytically the spin-squeezing dynamics, the adiabaticity condition and the limits imposed by particle losses.

PACS numbers: 03.75.Gg, 03.75.Kk, 42.50.Dv, 06.30.Ft

*Introduction* - One of the most successful applications of cold atoms is atomic clocks that provide the best time standards. The intrinsic uncertainty in the measured clock transition frequency, so called standard quantum limit  $\Delta\omega \propto (\mathcal{T}\sqrt{N})^{-1}$  where  $\mathcal{T}$  is the interrogation time and  $N$  the number of atoms, has been already reached in atomic fountains using a microwave transition [1] and it will become a limiting factor even for the new generation of optical lattice clocks [2]. Quantum correlations and spin squeezing in an atomic ensemble [3] allow to increase the clock stability beyond this limit [4, 5]. This will push even further the fantastic precision of the best atomic clocks in the long term, besides being an immediate practical advantage in experiments where the atom number cannot be indefinitely increased as in trapped atoms atom-chip [5, 6] or optical lattice [2] clocks.

In this theoretical work we propose a method to prepare a spin-squeezed state of ultra-cold atoms in an optical lattice potential dedicated for atomic clocks. The idea sketched in Fig. 1 brings together two quantum state preparation protocols which have been successfully realized separately: the superfluid to Mott insulator transition in an optical lattice [7–9] and spin squeezing in an atomic ensemble [10–13]. The resulting spatial configuration with one atom per site, already recognized as very promising for atomic clocks [14, 15], shows in addition quantum correlations allowing for metrological gain with respect to independent atoms. Taking a different point of view, our scheme allows to “extract” the entanglement among Bose-condensed atoms that are initially in a common spatial mode [16], to obtain a state where each atom is in a separate mode, and can be individually addressed and manipulated with a quantum gas microscope [17, 18]. The entanglement can then be used any quantum information or quantum metrology task. A full characterization of the entangled state by performing measurements on specific partitions is also possible [19].

In the following, we formalize the idea sketched in Fig. 1. Using an adiabatic approximation we provide analytical and numerical results for the squeezing, the relevant time scales and their scaling with the atom number.

We consider a two component condensate with repulsive interactions in the non-demixing regime [22] with

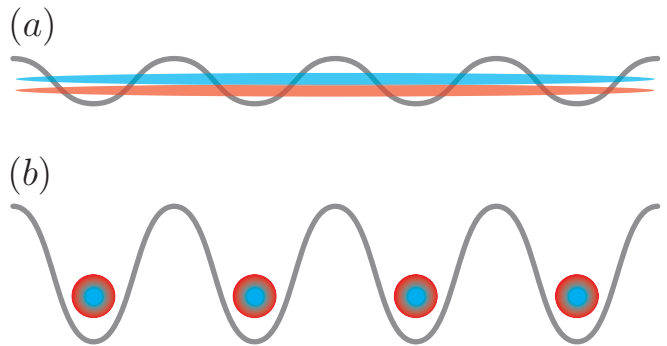


FIG. 1. (a) Initially, a Bose-Einstein condensate of atoms in an internal state  $a$  is prepared in a shallow 3D optical lattice. At time  $t = 0$  an electromagnetic  $\pi/2$ -pulse puts each atom in a coherent superposition two internal states  $a$  and  $b$ , and the binary atomic interactions between cold atoms start the generation of squeezing in the system [20, 21]. (b) Simultaneously, the lattice height is gradually increased, in such a way that the system enters the Mott insulator phase at “best squeezing time”  $t_{\text{best}}$  for which squeezing is the largest, thus freezing the “best squeezing”  $\xi_{\text{best}}^2 = \xi^2(t_{\text{best}})$  (see equation (2)) in the Mott insulator phase.

symmetric coupling constants describing  $s$ -wave interactions between atoms in the two states,  $g_{aa} = g_{bb}$ , and an adjustable interspecies coupling  $g_{ab} < g_{aa}$  [11, 23];  $g_{ij} = 4\pi\hbar^2 a_{ij}/m$  where  $a_{ij}$  is the  $s$ -wave scattering length for one atom in state  $i$  and one in state  $j$ ,  $m$  is the atomic mass and  $\hbar$  is the Planck constant. The system, at zero temperature, is confined in a 3D uniform optical lattice with periodic boundary conditions. The total atom number  $N$  is equal to the number of lattice sites  $M$  [24]. The lattice height is increased with a linear ramp

$$V_0(t) = V_{\text{init}} + (V_c - V_{\text{init}})t/t_{\text{best}} \quad (1)$$

from the initial height  $V_{\text{init}}$  to the critical value  $V_c$  on a time scale given by the best squeezing time  $t_{\text{best}}$ . We assume that the system dynamics is confined to the lowest Bloch band and it is described by the two-component Bose-Hubbard model [25], with time-dependent hopping  $J(V_0)$  and interaction terms  $U_\sigma(V_0)$ ,  $\sigma = a, b, ab$  [26]. We

quantify the spin-squeezing using the parameter [3, 4]

$$\xi^2 = \frac{N \langle \Delta \hat{S}_\perp^2 \rangle_{\min}}{\langle S \rangle^2}, \quad (2)$$

where  $\langle S \rangle$  is the length of the mean collective spin and  $\langle \Delta \hat{S}_\perp^2 \rangle_{\min}$  is the minimal variance of the spin orthogonally to the mean spin direction. The collective spin operators,  $\hat{S}_\pm = \hat{S}_x \pm i\hat{S}_y = \int d^3r \hat{\Psi}_a^\dagger(\mathbf{r}) \hat{\Psi}_b(\mathbf{r})$ ,  $\hat{S}_z = (\hat{N}_a - \hat{N}_b)/2$  where  $\hat{\Psi}_\sigma$  is the bosonic field operator and  $\hat{N}_\sigma$  the atom number operator in the internal state  $\sigma$ , give access to spin squeezing as a function of time.

*Initial state and evolution* - Starting from a condensate in internal state  $a$ , the mixing  $\pi/2$  pulse puts the system in a superposition of Fock states with  $N_a$  atoms in the internal state  $a$  and  $N_b = N - N_a$  atoms in state  $b$ . Concerning the spatial wave function, in our theoretical analysis we assume that after the pulse each Fock state is in the spatial ground state for the corresponding atom number  $|\Psi_{N_a, N_b}^0(0^+) : N_a, N_b\rangle$  in the shallow lattice. We thus neglect the thermal excitations initially present in the system and the excitations created by the mixing pulse. A clean way to avoid any excitation in the experiment would be to perform the pulse for an ideal gas at zero temperature in a completely flat potential, and subsequently ramp up adiabatically the interactions [27] and the lattice. The very stringent condition of having no excitations at all is however not necessary in practice for our proposal. Based on our studies of spin squeezing at finite temperature [28, 29], we expect that the final squeezing will not be affected as long as the initial non-condensed fraction before the pulse is much smaller than the targeted squeezing, which can be achieved in the weakly interacting limit at sufficiently low temperature.

In the adiabatic approximation, during the evolution following the mixing pulse, each Fock state of the prepared superposition remains in an instantaneous ground state and picks up a time-dependent phase factor determined by the ground state energy  $E_0(N_a, N_b, t)$

$$|\psi(t)\rangle = 2^{-N/2} \sum_{N_a=0}^N \sqrt{\binom{N}{N_a}} e^{-\frac{i}{\hbar} \int_0^t dt' E_0(N_a, N - N_a, t')} \times |\Psi_{N_a, N - N_a}^0(t) : N_a, N - N_a\rangle. \quad (3)$$

We assume that the squeezing is measured after raising the lattice to reach the deep Mott phase with one atom per site in the limit of vanishing tunnel coupling, which allows to disentangle the internal and external degrees of freedom in state (3). A consequence of this fact is that the action of the collective spin operators on the Fock states then obeys simple rules, e.g.  $\hat{S}_+ |\Psi_{N_a, N_b}^0 : N_a, N_b\rangle \approx \sqrt{(N_a + 1)N_b} |\Psi_{N_a+1, N_b-1}^0 : N_a + 1, N_b - 1\rangle$ , as for the two-mode Fock state where only internal degrees of freedom are considered. If the atom number is large, one can expand the ground state energy  $E_0(N_a, N_b, t)$  in the phase factor of each Fock state in (3) around  $\langle \hat{N}_a \rangle =$

$\langle \hat{N}_b \rangle = N/2$  up to the second order [30]

$$\frac{1}{\hbar} \int_0^t dt' E_0(N_a, N_b, t') \simeq \Phi_0(N, t) + T(t) \frac{(N_a - N_b)^2}{4}. \quad (4)$$

There is no linear term in (4) because of the  $a$ - $b$  symmetric situation we consider, and the phase factor  $\Phi_0(N, t)$  depending on the total number of atoms does not play any role in the spin dynamics and can be neglected. Introducing chemical potentials  $\mu_\sigma(t) = \partial_{N_\sigma} E_0(N_a, N_b, t)$  and the parameter  $\chi(t) = \frac{1}{2}(\partial_{N_a} - \partial_{N_b})[\mu_a(t) - \mu_b(t)]/\hbar$

$$T(t) = \int_0^t dt' \chi(t'). \quad (5)$$

By changing the time variable from  $t$  to the dimensionless  $T(t)$ , one then recovers the one-axis twisting (OAT) model [20] with an Hamiltonian proportional to  $\hat{S}_z^2$ . In particular, in the large  $N$  limit, the squeezing optimized over time  $\xi_{\text{best}}^2 = \xi^2(t_{\text{best}})$  is  $\xi_{\text{best}}^2 \simeq 3^{1/6}/N^{2/3}$  and the best squeezing time is

$$t_{\text{best}} \simeq \frac{3^{1/6}}{N^{2/3}} \left[ \frac{1}{V_c - V_{\text{init}}} \int_{V_{\text{init}}}^{V_c} dV_0 \chi(V_0) \right]^{-1} \quad (6)$$

where we used  $T(t_{\text{best}}) \simeq 3^{1/6}/N^{2/3}$  and the linear ramp (1). We introduce the term ‘‘dynamic OAT model’’ for the quadratic approximation (4) in the phase factors, as opposite to ‘‘static OAT model’’ for which we directly take a time independent Hamiltonian  $\hat{\mathcal{H}} = \hbar\chi(0)\hat{S}_z^2$  corresponding to the initial conditions after the pulse. In order to show the squeezing dynamics in our system and to test the validity of the dynamic OAT model, we evaluate numerically the energy phase factors in (3) using the Gutzwiller method [25, 31–33]. Precisely, the total mean-field energy is minimized in the subspace of fixed mean number of atoms in each component, i.e.  $\langle \hat{N}_a \rangle = N_a$  and  $\langle \hat{N}_b \rangle = N_b$ , by a projection method similarly to Ref.[34]. The results are shown in Fig. 2(a-b). The squeezing dynamics slows down while approaching the phase transition, and finally freezes in the Mott insulating phase. Note that the numerical results from equation (3) including the whole non-linearity (green solid line) and the quadratised dynamic OAT model (red circles) are in agreement up to the phase transition already for  $N = 125$ . In Fig. 2(b) we show the time dependence of  $T(t)$ . As expected, the dynamic OAT model predicts a larger value of  $t_{\text{best}}$  than the static one. We show the scaling of the best squeezing time with the total number of atoms  $N$  in Fig. 2(c). A fit gives  $t_{\text{best}} \propto N^{0.35}$ , a slightly less favorable scaling than for the static and homogeneous OAT model, for which  $\chi(0)^{\text{hom}} = 8(a_{aa} + a_{bb} - 2a_{ab})E_R/\pi\lambda\hbar N$  and  $t_{\text{best}} \propto N^{1/3}$ . The same figure confirms that dependence of  $t_{\text{best}}$  on the scattering lengths suggested by the static and homogeneous OAT model, approximately holds.

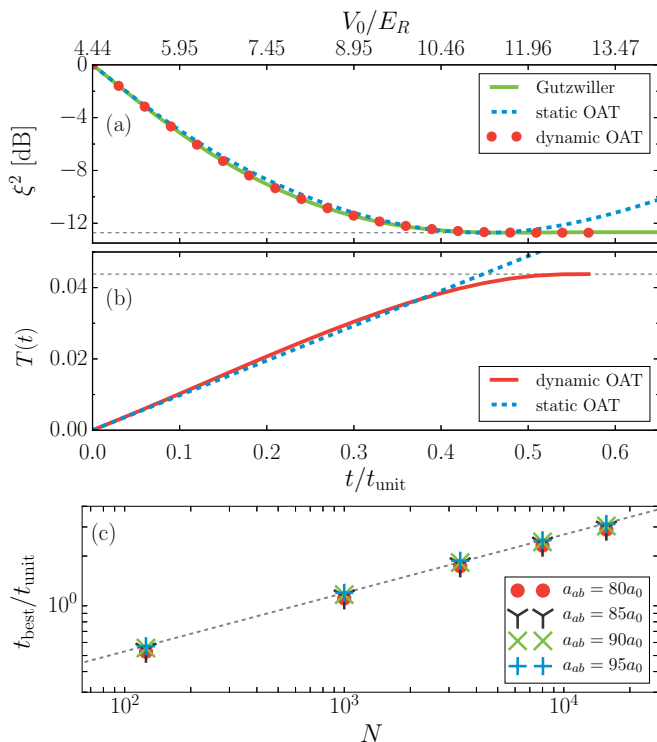


FIG. 2. (Color online) Numerical results: (a) Spin-squeezing parameter  $\xi^2$  as a function of  $t/t_{\text{unit}}$  (bottom x-axis) and  $V_0/E_R$  (top x-axis) for the linear ramp;  $E_R = 2\pi^2\hbar^2/(m\lambda^2)$  is the recoil energy and  $\lambda/2$  the lattice period.  $t_{\text{unit}}^{-1} = (a_{aa} + a_{bb} - 2a_{ab})E_R/\hbar\lambda$ . The green solid line shows numerical results with  $E_0$  in (3) calculated with the Gutzwiller method, while the red dots result from the dynamic OAT model (4). Predictions of the static OAT model (see text) are shown for comparison (blue dashed lines). (b)  $T(t)$  for the dynamic OAT model (red solid line) and the static OAT model (blue dashed line). Parameters are  $N = 125$ ,  $a_{aa} = a_{bb} = 100.4a_0$  and  $a_{ab} = 95a_0$ , where  $a_0$  is the Bohr radius. The dashed horizontal lines represent  $\xi_{\text{best}}^2$  and  $T(t_{\text{best}})$  of the OAT model for  $N = 125$ . By definition, the dynamic curves touch the dashed horizontal lines at the best squeezing time  $t_{\text{best}}$ . (c) Scaling of the best squeezing time  $t_{\text{best}}$  with  $N$  for  $a_{aa} = a_{bb} = 100.4a_0$  and different values of  $a_{ab}$ . The dashed line is a fit  $t_{\text{best}} \propto N^\alpha$  with  $\alpha = 0.353 \pm 0.004$ .

*Adiabaticity* - In the superfluid phase, where delocalization of atoms is energetically more favorable, a large majority of particles occupy the zero quasi-momentum Bloch state of the lowest band giving rise to condensation in momentum space. When the depletion of the condensate is small, the Bogoliubov method can capture the physical properties of the Bose-Hubbard model [35], although it cannot be pushed too far towards the phase transition boundary [35, 36]. Taking advantage of the fact the squeezing in our scheme develops entirely in the superfluid phase, we use the number-conserving Bogoliubov theory [37, 38] to derive the validity conditions of the adiabatic approximation at heart of our treatment.

The starting point is the two-component Bose-

Hubbard Hamiltonian [25] in quasi-momentum representation. This Hamiltonian has the same form as the Hamiltonian of a two-component Bose-Einstein condensate [39] with the field operators  $\hat{\Psi}_\sigma(\mathbf{r})$  expanded in a plane wave basis, provided the kinetic energy  $\epsilon_{\mathbf{q}} = \hbar^2\mathbf{q}^2/2m$  and the coupling constants  $g_\sigma$ ,  $g_{ab}$  are replaced by  $\epsilon_{\mathbf{q}}(t) = -2J(V_0)\sum_\gamma \cos(\mathbf{q} \cdot \mathbf{e}_\gamma)$  and  $U_\sigma(V_0)$ ,  $U_{ab}(V_0)$  respectively, where  $\mathbf{e}_\gamma$  are primitive lattice vectors. By introducing the number-conserving operators,  $\hat{\Lambda}_{\mathbf{q},a} = \hat{a}_0^\dagger \hat{a}_{\mathbf{q}}/\sqrt{N_a}$  and  $\hat{\Lambda}_{\mathbf{q},b} = \hat{b}_0^\dagger \hat{b}_{\mathbf{q}}/\sqrt{N_b}$ , one diagonalizes the quadratic Hamiltonian using the generalized Bogoliubov transformation

$$\hat{\Lambda}_{\mathbf{q},\sigma}(t) = u_{\mathbf{q},+}^\sigma(t)\hat{\beta}_{\mathbf{q},+}(t) + v_{\mathbf{q},+}^\sigma(t)\hat{\beta}_{-\mathbf{q},+}^\dagger(t) + u_{\mathbf{q},-}^\sigma(t)\hat{\beta}_{\mathbf{q},-}(t) + v_{\mathbf{q},-}^\sigma(t)\hat{\beta}_{-\mathbf{q},-}^\dagger(t), \quad (7)$$

where  $\hat{\beta}_{\mathbf{q},\pm}(t)$  are Bogoliubov quasi-particle operators satisfying bosonic commutation relations and  $u_{\mathbf{q},\pm}^\sigma(t)$ ,  $v_{\mathbf{q},\pm}^\sigma(t)$  are known functions [22, 39]. The Bogoliubov transformation in Eq. (7) is the same as in [39] with the same notations, provided that equations are expressed in terms of  $\Delta E_{\mathbf{q}}(t) = \epsilon_{\mathbf{q}}(t) - \epsilon_0(t)$ .

In order to calculate the number of excitations created by the lattice rump starting at zero temperature with no excitation, we write the Heisenberg equation of motion for  $\hat{\beta}_{\mathbf{q},\pm}(t)$ , and proceed similarly to appendix C of Ref.[27]. One can show that Bogoliubov modes  $\hat{\beta}_{\mathbf{q},+}(t)$  and  $\hat{\beta}_{\mathbf{q},-}(t)$  evolve independently, and are coupled to modes  $\hat{\beta}_{-\mathbf{q},+}(t)$  and  $\hat{\beta}_{-\mathbf{q},-}(t)$  respectively

$$i\hbar \frac{d}{dt} \begin{pmatrix} \hat{\beta}_{\mathbf{q},\pm}(t) \\ \hat{\beta}_{-\mathbf{q},\pm}^\dagger(t) \end{pmatrix} = \begin{pmatrix} \hbar\omega_{\mathbf{q},\pm}(t) & -i\hbar\Omega_{\mathbf{q},\pm}(t) \\ -i\hbar\Omega_{\mathbf{q},\pm}(t) & -\hbar\omega_{\mathbf{q},\pm}(t) \end{pmatrix} \times \begin{pmatrix} \hat{\beta}_{\mathbf{q},\pm}(t) \\ \hat{\beta}_{-\mathbf{q},\pm}^\dagger(t) \end{pmatrix}, \quad (8)$$

where the Bogoliubov spectrum is defined by  $(\hbar\omega_{\mathbf{q},\pm}/\Delta E_{\mathbf{q}})^2 = 1 + \tilde{u}_a + \tilde{u}_b \pm \sqrt{(\tilde{u}_a - \tilde{u}_b)^2 + 4\tilde{u}_{ab}}$  with  $\tilde{u}_\sigma = U_\sigma n_\sigma/\Delta E_{\mathbf{q}}$  and  $n_\sigma = N_\sigma/M$  for  $\sigma = a, b$ ,  $\tilde{u}_{ab} = U_{ab}\sqrt{n_a n_b}/\Delta E_{\mathbf{q}}$  and  $\Omega_{\mathbf{q},\pm}(t) = \frac{1}{2} \frac{d}{dt} \log \left( \frac{\Delta E_{\mathbf{q}}(t)}{\hbar\omega_{\mathbf{q},\pm}(t)} \right)$ . The total fraction of excitations  $\frac{1}{N} \sum_{\mathbf{q} \neq 0} n_{\mathbf{q}}^{\text{ex}}(t)$  with  $n_{\mathbf{q}}^{\text{ex}}(t) = \sum_{\epsilon=\pm} \langle \Psi_{\text{bog}}(0) | \hat{\beta}_{\mathbf{q},\epsilon}^\dagger(t) \hat{\beta}_{\mathbf{q},\epsilon}(t) | \Psi_{\text{bog}}(0) \rangle$  stays small, as long as the adiabaticity condition

$$|\Omega_{\mathbf{q},\pm}(t)| \ll |2\omega_{\mathbf{q},\pm}(t)| \quad (9)$$

is verified for each quasi-momentum  $\mathbf{q} \neq \mathbf{0}$ . From the expression of  $\Omega_{\mathbf{q},\pm}(t)$  and for the linear ramp (1) one gets the expression of the adiabatic time for each mode

$$t_{\text{adiab},\pm}^{\mathbf{q}} = \frac{(V_c - V_{\text{init}})\hbar E_R}{4\Delta E_{\mathbf{q}}(V_0)} \left| \frac{d}{dV_0} \left( \frac{\Delta E_{\mathbf{q}}(V_0)}{\hbar\omega_{\mathbf{q},\pm}(V_0)} \right) \right| \quad (10)$$

which should be evaluated at  $\mathbf{q} = (2\pi/N^{1/3}l, 0, 0)$  with the lattice spacing  $l = \lambda/2$ , where the condition (9) is

most stringent, and maximized over the ramp duration  $V_0 \in [V_{\text{init}}, V_c]$ . In the large atom number limit one gets

$$t_{\text{adiab},\pm} \underset{N \rightarrow \infty}{\simeq} \frac{N^{1/3}}{2\pi} \frac{(V_c - V_{\text{init}})l}{4J(V_0)} \left| \frac{d}{dV_0} \left( \frac{J(V_0)}{c_{\pm}(V_0)} \right) \right|, \quad (11)$$

where  $c_{\pm}^2 = (l/\hbar)^2 J(V_0) ((\hbar\omega_{\mathbf{q},\pm}/\Delta E_{\mathbf{q}})^2 - 1) \Delta E_{\mathbf{q}}$  is the sound velocity of the phonon-like Bogoliubov excitation branches. We see from Eq. (11) that the adiabatic time shares almost the same scaling with  $N$  as the best squeezing time. Furthermore, the negative branch  $t_{\text{adiab},-}$  is always larger than  $t_{\text{adiab},+}$  as long as one of the components is not completely depleted. This implies that  $t_{\text{adiab},-}$  alone sets the adiabatic time scale. We checked numerically that the condition  $t_{\text{best}} \gg t_{\text{adiab},-}$  holds for all parameters that we consider. For example, for  $N = 10^4$  and parameters in Fig. 2,  $t_{\text{adiab}}/t_{\text{best}} \approx 0.032$ .

*Decoherence* - While two and three-body losses are suppressed in the Mott insulator phase, they cannot be neglected in the superfluid phase, and play an important role as soon as the lost fraction of atoms at the squeezing time becomes comparable to the squeezing  $\xi_{\text{best}}^2$  that one would have in the absence of decoherence [40]. For two particular configurations in Fig. 3 we show the lost fraction and the expected squeezing  $\xi_{\text{best}}^2$  in the absence of decoherence as a function of the initial atom number. As the lost fraction increases with  $N$ , while  $\xi_{\text{best}}^2$  decreases, the crossing of the two curves gives the maximum atom number  $N_{\text{max}}$  for which the decoherence due to atom losses can be neglected. The lost fraction is obtained by solving rate equations for the mean atom number

$$\begin{aligned} \frac{d}{dt} \langle \hat{N}_{\sigma} \rangle(t) &= -\gamma_{\sigma}^{(2)}(t) g_{\sigma}^{(2)}(t) \langle \hat{N}_{\sigma} \rangle^2(t) \\ &\quad -\gamma_{ab}^{(2)}(t) g_{ab}^{(2)}(t) \langle \hat{N}_a \rangle(t) \langle \hat{N}_b \rangle(t) \end{aligned} \quad (12)$$

for two-body losses, and

$$\frac{d}{dt} \langle \hat{N}_{\sigma} \rangle = -\gamma_{\sigma}^{(3)}(t) g_{\sigma}^{(3)}(t) \langle \hat{N}_{\sigma} \rangle^3 \quad (13)$$

for three-body, where  $\gamma_{\sigma}^{(m)}(t) = \frac{K_{\sigma}^{(m)}}{M^{m-1}} \int d^3r w^{2m}(\mathbf{r}, t)$  and  $\gamma_{ab}^{(2)}(t) = \frac{K_{ab}^{(2)}}{2M} \int d^3r w^4(\mathbf{r}, t)$ , with  $K_{\sigma}^{(m)}$  the  $m$ -body loss rate constants for component  $\sigma$ ,  $w(r)$  is the Wannier function identical for the  $M$  sites, and  $g_{\sigma}^{(m)}$  is the on-site normalized  $m$ -body correlation function in our time-dependent lattice, for example  $g_a^{(2)} = \langle a_i^{\dagger} a_i^{\dagger} a_i a_i \rangle / \langle a_i^{\dagger} a_i \rangle^2$ , calculated numerically in the adiabatic approximation using the Gutzwiller method. In the case of the transition  $|F = 1, m_F = 1\rangle \leftrightarrow |F = 2, m_F = 1\rangle$  for  $^{87}\text{Rb}$  atoms in Fig. 3(a), where the interspecies scattering length  $a_{ab}$  can be tuned by Feshbach resonance [11], decoherence due to two-body losses is never negligible above  $N_{\text{max}} \approx 40$

atoms, and limits the squeezing to 10dB already for a hundred atoms. On the other hand, for the transition  $|F = 1, m_F = 1\rangle \leftrightarrow |F = 2, m_F = -2\rangle$  in Fig. 3(b), where the interspecies interaction can be tuned by slightly shifting the optical lattices for the two components [23] and

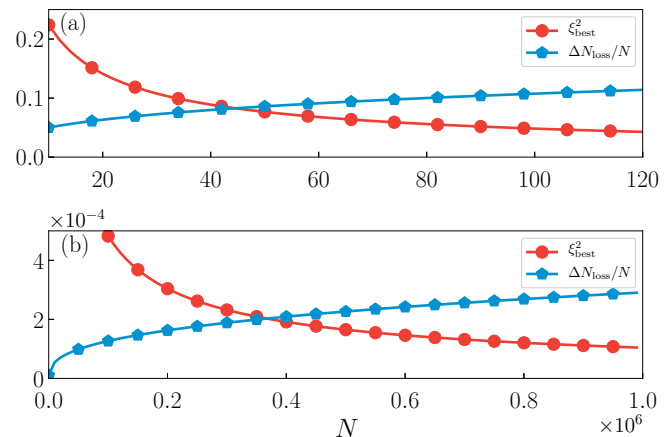


FIG. 3. Best squeezing  $\xi_{\text{best}}^2 = \xi^2(t_{\text{best}})$  in the absence of decoherence and lost fraction due to 2-body and 3-body losses at the time  $t_{\text{best}}$  as function of the atom number in  $^{87}\text{Rb}$  for the transitions (a)  $|1, 1\rangle \leftrightarrow |2, -1\rangle$  with Feshbach tuned  $a_{ab} = 95a_0$  and dominant two body losses  $K_b^{(2)} = 8.1 \times 10^{-20} m^3/s$  [41] and  $K_a^{(2)} = 1.708 \times 10^{-19} m^3/s$  [42] and (b)  $|1, -1\rangle \leftrightarrow |2, -2\rangle$  with three body losses only  $K_a^{(3)} = 5.4 \times 10^{-42} m^6/s$  [43] and  $K_b^{(3)} = 1.8 \times 10^{-41} m^6/s$  [44].

two body losses are absent [45], the limit that is now imposed by three body losses is much less constraining and it does not influence the results up to large atom numbers  $N_{\text{max}} \approx 4 \times 10^5$ .

*Conclusions* - We propose a method to prepare a regular arrangement of spin-entangled atoms, which we call a spin-squeezed atomic crystal, by bringing an interacting two-component Bose-Einstein condensate across the superfluid-to-Mott transition in a time-dependent optical lattice. We theoretically study the formation of such a crystal in the adiabatic approximation, which we justify by a separate study in the superfluid regime where the spin correlations build up. We finally calculate the maximal system size for which the effect of decoherence due to two- and three- body losses can be neglected for parameters of current experimental relevance. If the method turns out to be efficient, it could provide a unique system to a variety of quantum metrology tasks and applications.

*Acknowledgments* - This work was supported by the Polish National Science Center Grants DEC-2015/18/E/ST2/00760, by the CNRS PICS-7403, and partially by the PL-Grid Infrastructure. D.K. acknowledges support from the French Government (BGF).

[1] G. Santarelli, P. Laurent, P. Lemonde, A. Clairon, A. G. Mann, S. Chang, A. N. Luiten, and C. Salomon,

Phys. Rev. Lett. **82**, 4619 (1999).



- [2] B. J. Bloom, T. L. Nicholson, J. R. Williams, S. L. Campbell, M. Bishof, X. Zhang, W. Zhang, S. L. Bromley, and J. Ye, *Nature* **506**, 71 (2014).
- [3] D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, *Phys. Rev. A* **50**, 67 (1994).
- [4] D. J. Wineland, J. J. Bollinger, W. M. Itano, F. L. Moore, and D. J. Heinzen, *Phys. Rev. A* **46**, R6797 (1992).
- [5] C. Deutsch, F. Ramirez-Martinez, C. Lacroûte, F. Reinhard, T. Schneider, J. N. Fuchs, F. Piéchon, F. Laloë, J. Reichel, and P. Rosenbusch, *Phys. Rev. Lett.* **105**, 020401 (2010).
- [6] R. Szmuk, V. Dugrain, W. Maineult, J. Reichel, and P. Rosenbusch, *Phys. Rev. A* **92**, 012106 (2015).
- [7] M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch, and I. Bloch, *Nature* **415**, 39 (2002).
- [8] T. Fukuhara, S. Sugawa, M. Sugimoto, S. Taie, and Y. Takahashi, *Phys. Rev. A* **79**, 041604 (2009).
- [9] J. Catani, L. De Sarlo, G. Barontini, F. Minardi, and M. Inguscio, *Phys. Rev. A* **77**, 011603 (2008).
- [10] M. F. Riedel, P. Böhi, Y. Li, T. W. Hänsch, A. Sinatra, and P. Treutlein, *Nature* **464**, 1170 (2010).
- [11] C. Gross, T. Zibold, E. Nicklas, J. Estève, and M. K. Oberthaler, *Nature* **464**, 1165 (2010).
- [12] O. Hosten, N. J. Engelsen, R. Krishnakumar, and M. A. Kasevich, *Nature* **529**, 505 (2016), letter.
- [13] K. C. Cox, G. P. Greve, J. M. Weiner, and J. K. Thompson, *Phys. Rev. Lett.* **116**, 093602 (2016).
- [14] T. Akatsuka, M. Takamoto, and H. Katori, *Phys. Rev. A* **81**, 023402 (2010).
- [15] T. Akatsuka, M. Takamoto, and H. Katori, *Nat. Phys.* **4**, 954 (2008).
- [16] N. Killoran, M. Cramer, and M. B. Plenio, *Phys. Rev. Lett.* **112**, 150501 (2014).
- [17] W. S. Bakr, J. I. Gillen, A. Peng, S. Folling, and M. Greiner, *Nature* **462**, 74 (2009).
- [18] J. F. Sherson, C. Weitenberg, M. Endres, M. Cheneau, I. Bloch, and S. Kuhr, *Nature* **467**, 68 (2010).
- [19] P. van Loock and A. Furusawa, *Phys. Rev. A* **67**, 052315 (2003).
- [20] M. Kitagawa and M. Ueda, *Phys. Rev. A* **47**, 5138 (1993).
- [21] A. Sorensen, L.-M. Duan, J. I. Cirac, and P. Zoller, *Nature* **409**, 63 (2001).
- [22] E. Timmermans, *Phys. Rev. Lett.* **81**, 5718 (1998).
- [23] D. Jaksch, H.-J. Briegel, J. I. Cirac, C. W. Gardiner, and P. Zoller, *Phys. Rev. Lett.* **82**, 1975 (1999).
- [24] In an experimental realization this would be an optical lattice in a flat bottom potential [46] which releases the constraint of having exactly a filling factor of one to reach the Mott transition [47].
- [25] J. Wernsdorfer, M. Snoek, and W. Hofstetter, *Phys. Rev. A* **81**, 043620 (2010).
- [26] We used simplified forms for the hopping  $J(V_0) = 4E_R(V_0/E_R)^{3/4}\exp(-2\sqrt{V_0/E_R})/\sqrt{\pi}$ , and interactions terms  $U_\sigma(V_0) = g_\sigma(V_0/E_R)^{3/4}k^3/(\pi\sqrt{8\pi})$  [48].
- [27] K. Pawłowski, M. Fadel, P. Treutlein, Y. Castin, and A. Sinatra, *Phys. Rev. A* **95**, 063609 (2017).
- [28] A. Sinatra, E. Witkowska, J.-C. Dornstetter, Y. Li, and Y. Castin, *Phys. Rev. Lett.* **107**, 060404 (2011).
- [29] A. Sinatra, E. Witkowska, and Y. Castin, *The European Physical Journal Special Topics* **203**, 87 (2012).
- [30] Sinatra, A. and Castin, Y., *Eur. Phys. J. D* **4**, 247 (1998).
- [31] D. S. Rokhsar and B. G. Kotliar, *Phys. Rev. B* **44**, 10328 (1991).
- [32] J. Zakrzewski, *Phys. Rev. A* **71**, 043601 (2005).
- [33] D. Jaksch, V. Venturi, J. I. Cirac, C. J. Williams, and P. Zoller, *Phys. Rev. Lett.* **89**, 040402 (2002).
- [34] F. Y. Lim and W. Bao, *Phys. Rev. E* **78**, 066704 (2008).
- [35] D. van Oosten, P. van der Straten, and H. T. C. Stoof, *Phys. Rev. A* **63**, 053601 (2001).
- [36] T. A. Zaleski and T. K. Kopeć, *Phys. Rev. A* **84**, 053613 (2011).
- [37] Y. Castin and R. Dum, *Phys. Rev. A* **57**, 3008 (1998).
- [38] C. W. Gardiner, *Phys. Rev. A* **56**, 1414 (1997).
- [39] B. Oleś and K. Sacha, *Journal of Physics A: Mathematical and Theoretical* **41**, 145005 (2008).
- [40] Y. Li, Y. Castin, and A. Sinatra, *Phys. Rev. Lett.* **100**, 210401 (2008).
- [41] M. Egorov, R. P. Anderson, V. Ivannikov, B. Opanchuk, P. Drummond, B. V. Hall, and A. I. Sidorov, *Phys. Rev. A* **84**, 021605 (2011).
- [42] B. Opanchuk, M. Egorov, S. Hoffmann, A. I. Sidorov, and P. D. Drummond, *EPL* **97** (2012), 10.1209/0295-5075/97/50003.
- [43] K. M. Mertes, J. W. Merrill, R. Carretero-González, D. J. Frantzeskakis, P. G. Kevrekidis, and D. S. Hall, *Phys. Rev. Lett.* **99**, 190402 (2007).
- [44] J. Söding, D. Gury-Odelin, P. Desbiolles, F. Chevy, H. Inamori, and J. Dalibard, *Appl. Phys. B* **69** (1999), 10.1007/s003400050805.
- [45] S. Tojo, T. Hayashi, T. Tanabe, T. Hirano, Y. Kawaguchi, H. Saito, and M. Ueda, *Phys. Rev. A* **80**, 042704 (2009).
- [46] A. L. Gaunt, T. F. Schmidutz, I. Gotlibovych, R. P. Smith, and Z. Hadzibabic, *Phys. Rev. Lett.* **110**, 200406 (2013).
- [47] E. Lundh, *The European Physical Journal D* **46**, 517 (2008).
- [48] I. Bloch, J. Dalibard, and W. Zwerger, *Rev. Mod. Phys.* **80**, 885 (2008).