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Zoé Mesnil

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# NEW OBJECTIVES FOR THE NOTIONS OF LOGIC TEACHING IN HIGH SCHOOL IN FRANCE: A COMPLEX REQUEST FOR TEACHERS

Zoé MESNIL

Université Paris Diderot, Laboratoire de Didactique André Revuz

*If everyone agrees that logic is needed to do mathematics, there are divergences concerning the role of mathematical logic in acquiring the necessary and sufficient knowledge in this area. In France today there are new syllabuses in which notions of logic are explicitly mentioned. How do teachers integrate them into their teaching? In this paper, through a study of the French syllabuses and textbooks, I show the strong constraints and ill-defined conditions for this teaching of logic notions. I also describe the contents of an in-service training course for teachers, which aims to give them tools for this teaching.*

*Key words: logic-language-reasoning-textbooks-teacher's training*

## INTRODUCTION

In the introduction of the proceedings of the 19<sup>th</sup> ICMI Study Conference: Proof and Proving in Mathematics Education (2009), the authors note that some research should be pursued to understand the role of logic in the teaching of proof. The experience of teaching formal logic in high school during the time of "modern mathematics" in the 1970s in different countries has shown that this approach does not directly provide students with effective tools to improve their abilities in expression and reasoning. In most of these countries, logic then disappeared from high school syllabuses, but students' difficulties in expression and reasoning persist and the debate on the role of logic in the treatment of these problems remains open.

In France, logic was briefly re-introduced in the 2001 syllabus. The 2009 syllabus, which is still in application, goes even further: it includes objectives for "mathematical notations and reasoning" which are linked to notions such as connectives AND/OR, negation, conditional propositions, equivalence, different types of reasoning. Behind these notions there are objects which are defined and studied by mathematical logic. But the aim stated by the syllabus is not to teach mathematical logic, but to convey the knowledge in logic necessary to mathematical activity. Then, two main questions emerge: How can teachers identify the logic they have to teach? What kind of (pre-service and in-service) training should they have to efficiently organize their teaching? The French context, where logic has been re-introduced in high school syllabus, is suitable for providing elements of answers to these questions.

These questions brought me to study the system of conditions and constraints in which the teacher makes his teaching choices. A more detailed study of syllabuses and textbooks from 1960 highlights the features of this system. First of all, the history

of teaching logic in mathematics in France is a tormented history, and some teachers never got courses about notions of logic during their studies. Furthermore, since this teaching has been absent from syllabuses over the past years, teachers did not really have to think about it during their preservice training. Finally, a first glance at the activities proposed in the textbooks and at the content of the pages dedicated to the notions of logic mentioned in the syllabus shows diversity in the interpretation of this syllabus, but also a certain lack of knowledge of the notions at work. But prior to this study, more general questions should be raised about the links between logic and mathematics and between logic and mathematics teaching. One of the aims of this study is to contribute to the reflection on the necessary training in logic for teachers, so they could teach the underlying logic in mathematical activity.

In this paper, I will mainly present this study. It shows the difficulties encountered by French high school teachers in integrating notions of logic in their mathematics teaching. One axis of this work is to show the importance of language<sup>1</sup> issues in the contribution of logic to the mathematical activity. This dimension is not taken into consideration in a precise way in the documents used as resources by teachers. Logic is often seen as linked to reasoning, in general and more specifically in mathematics. But it is also linked to the setting-up and the functioning of a language which allows to describe the structure of propositions and of reasoning, and the links between both. I see mathematical logic as a reference theory dealing with objects which may be tools to analyze our mathematical discourse and to understand the ambiguities inevitably linked to the use of certain formulations which are informal or implicit in the mathematical language.

First of all, I will present some quick thoughts on logic and language, and some didactical studies that have shown difficulties for some students, probably linked to language. Later on, I will present the study of the system of conditions and constraints I have previously stated. Finally, I will briefly describe the content choices made for a training called "Initiation to logic" for teachers in activity.

## **LOGIC AND LANGUAGE IN THE MATHEMATICAL ACTIVITY**

The study of different moments in the history of logic shows that the constitution and description of a logic is accompanied by a necessary formalization of language, in the sense of a codified formatting, whose codes vary of course depending on authors and eras. For example, in his logical work, *The Organon*, Aristotle (Greek philosopher, IV<sup>e</sup> century BC) began to explain what he called a proposition (enunciative sentence, in which there is truth or error), and then classified these propositions according to two criteria, quality (affirmative or negative) and quantity (universal or particular). He obtained four types of propositions that are the building blocks of syllogisms, which could be treated in a formal way, because the propositions can be replaced by variables. Nevertheless, Aristotle did not want to formalize the relationship between

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<sup>1</sup> The use of the term "language" does not refer here particularly to common language, or to the mathematical one, but to the use by someone of signs he organizes with the intention of communicating something.

the parts forming these propositions and did not use logical constants like connectives and quantifiers.

Since the beginning, some mathematicians have sought the necessary and sufficient formalization of their language that ensures the infallibility of reasoning. For some of these mathematicians, language had to be totally formalized, to reach the univocity of the meaning of expressions and the possibility of a formal treatment of these expressions, independently of their meaning. For another part of these mathematicians, too much formalization does not allow the intuitive progress of reasoning. Mathematical logic can be seen as the culmination of this research. An important step has been taken by modeling mathematical language and reasoning, allowing a further exploration of the properties of these logical systems using mathematical tools.

These epistemological considerations lead me to the assumption that mathematical logic can provide tools, probably first to the mathematician and mathematics teacher, to analyze the language they use in their mathematical activity, and to detect its ambiguities. For the teacher, an additional challenge is to provide tools for the analysis of students' reasoning, allowing to highlight another possible understanding. This can occur, for example with the propositions "if ... then ..." which are implicitly universally quantified, but quantification is not always perceived by the students. That may lead them to give a response which is not expected, and is yet the result of a correct reasoning. For example, some pupils could say that the sentence "if  $n$  is prime, then  $n$  is odd" is neither false, nor true, while almost all mathematicians say it is false (you can find a more detailed example with the "maze" task described in (Durand-Guerrier, 2004)).

In a more general way, various studies based on experiments with university students show the difficulties they have encounter in understanding and proving quantified statements (Dubinsky, Yiparaki, 2000, Arsac, Durand-Guerrier, 2003, Chellougui, 2009, Roh, 2010). For most students engaged in proving if a statement is true or false, the relationship between the quantified formulation of a statement and the framework of its proof is not clear. Thus, while recognizing the role of informal statements in memorizing mathematical results, J. Selden and A. Selden make the assumption that the ability to unpack the logic of an utterance by formally writing it is related to the ability to ensure the validity of a proof of this statement (Selden & Selden, 1995).

Unpacking the logic can be seen as writing the statements and making explicit some conventions, for example about quantification. The predicate language did not become the universal language in which the mathematicians express themselves, but a reference language, and, depending on the nature of their activities (research, drafting a communication, course ...), they use formulations whose logical structure is more or less exhibited. For example, calling a predicate  $P$  on a variable  $n$  which is a natural number, mathematicians commonly say " $P(n)$  when  $n$  is big enough." But to explain to students how to show this property, they may reformulate this as " $P(n)$  for

all  $n$  beyond a certain value". And in a course, they could write "there exists  $N_0$  such that for all  $n \geq N_0$ ,  $P(n)$ ". This interplay between different formulations, which is easy for a mathematician, does not always seem so simple for students! Though, I believe that a rewording work is important because it contributes to the construction of what J. Selden and A. Selden call "statement images" (Selden & Selden, 1995, p 133):

These are meant to include all of the alternative statements, examples, nonexamples, visualizations, properties, concepts, consequences, etc., that are associated with a statement.

This invites us to think about activities for students to develop the ability to rewording, which is rarely an explicit goal of education and rarely proposed as an explicit task, and to think about the knowledge that teachers need in order to engage in such activities. S. Epp mentions the need for solid knowledge concerning language and logic (Epp, 1999, p3):

When given by teachers with a solid command of mathematical language and logic, such feedback can be of enormous benefit to students' intellectual development.

## **HOW CAN A FRENCH HIGH SCHOOL MATHEMATICS TEACHER TEACH LOGIC TODAY?**

### **Research questions, study materials and methodology**

There are some goals concerning "mathematical notations and reasoning" in the first high school year<sup>2</sup> mathematical syllabus, launched in September 2010. In these instructions, some objects of mathematical logic are explicitly mentioned. Therefore, teachers have to build up a teaching allowing to reach these aims. While doing so, they have choices to make, for which they are submitted to a system of institutional conditions and constraints. I think it is interesting to study this system in order to understand the teachers' practices concerning logic, and in order to think about the training they need for that. I have conducted this study with the three following questions: what is the scholarly knowledge of reference for this teaching? ("*savoir savant*" (Chevallard, 1985))? What are the notions to teach (the knowledge to be taught, "*savoir à enseigner*")? Where and why do we find logic in the high school mathematics teaching in France (ecological approach (Artaud, 1997))? I have looked for the answers by analyzing different documents from 1960<sup>3</sup> until now. I think the epistemological approach is essential to understand the current choices.

The study of syllabuses and their joined instructions contributes to answer these three questions. I have noticed the notions of logic present in these documents, as well as the terms used to speak about logic. These terms give information about the specific function attributed to logic, in relation with language and reasoning. I have completed this study by searching in the APMEP<sup>4</sup> periodicals for information on the reactions

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<sup>2</sup> 15-16 years old students.

<sup>3</sup> 1960 is the first year of the introduction of logic in the high school mathematics syllabuses in France.

<sup>4</sup> Association des Professeurs de Mathématiques de l'Enseignement Public, the most important association

and expectations of the teachers who are on the field of teaching. I have searched for the presence or not of articles on logic, and of debates concerning its teaching. Finally, I have studied what was said about logic in the textbooks. These textbooks are seen both as a possible interpretation of syllabus, proposing a "dressed knowledge" ("*savoir apprêté*") [Ravel, 2003] and as a resource for teachers. I have tried to determine if and how the notions of variable, proposition, connectives and quantifiers were introduced, more specifically if and how their syntactic and semantic aspects<sup>5</sup> were present and linked. I have also searched what kinds of tasks were designed.

The analysis shows the complexity of the current demand: the conditions are ill-defined and the constraints are strong.

### **What is the scholarly knowledge of reference for the teaching of logic of mathematics?**

A part of this complexity lies in the logic itself. The question that interests us here is not the teaching of mathematical logic as a branch of mathematics. The question rather has to do with the teaching of the logic of mathematics, which I defined as "the art of organizing one's speech in that discipline, seen under the double aspect of syntactic correction and semantic validity". One of the difficulties with this logic of mathematics is that there is no reference content, no consensus on the knowledge in this area that is needed to do mathematics and on the words which should be used to phrase this knowledge. Here is an example of the potential problems with the lack of common knowledge reference. In the objectives of the new syllabus, students must "be trained on examples to properly use the logical connectives "and", "or" and to distinguish their meanings from the common meanings of "and", "or" in the usual language." But neither in the syllabus, nor in the accompanying document on this subject, entitled "Resources for high school first year, Notations and mathematical reasoning" can we find a definition of the logical connectives "and", "or". However, the distinction between the use of these words in the language of mathematics and in everyday language is not limited to the inclusive or exclusive character of the connective "or". Yet it is the only thing mentioned in the textbooks or in the accompanying document.

Another essential difference lies in the fact that in mathematical language, the connectives "and", "or" link two propositions, which is not always the case in the everyday language, even if it is spoken about mathematics. For instance, if we try to show the logical structure of the proposition "Sets A and B are not empty and disjoint", there will be three "and" that will correspond to connectives between propositions (in capital letters), and one that will not match: "Set A is not empty AND set B is not empty AND sets A and B are disjoint". Not only is this distinction

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of mathematics teachers in France.

<sup>5</sup> In its syntactic aspect, a connective is an operator on propositions and its semantic aspect is given by the truth tables.

not mentioned, but in addition to that, the study of certain exercises proposed in textbooks show confused conceptions about notions that yet seem as simple and usual in mathematics as the connectives "and", "or". This shows that even if someone has studied mathematics and therefore had used logic for this activity, it is still not enough for him to constitute a knowledge that could sustain a teaching of certain notions of logic to students. I make the hypothesis that the part of the mathematical logic which is generally called "propositional calculus and predicate calculus" is a corpus of knowledge that can be a reference to teachers in their teaching of mathematics logic. The one condition for that is for it to be linked with the study of language and reasoning practices in mathematics. No such reference is explicitly proposed nowadays, as opposed to what happened at the time of modern mathematics: in 1970, the instructions along with the new syllabus of the first high school year gave definitions and first properties of connectives and quantifiers.

### **What is the knowledge to be taught ?**

Concepts of logic are mentioned for the first time in the mathematics syllabus for students in their first year of high school in 1960. During the middle of the twentieth century, the French mathematicians were strongly influenced by the Bourbaki group of mathematicians, whose axiomatic style spread into teaching. The reform called "modern mathematics" came into force in high school with the syllabus for first-year high school students in 1969. This syllabus was based on the idea of a unified mathematic, that could be used in experimental sciences as well as in human ones. Mastering the language of mathematics was then essential, and it was the essential function of logic. All textbooks from that time start with a first chapter on set theory and logic, which are the foundations of this language. The syntactic and semantic aspects of the notions of logic were present. The instructions of 1970 specifically say that "the chapter *Language of sets* should become more of a practical introduction at any time in the course than of a dogmatic preamble", but they do not give examples of this practical introduction. The APMEP periodicals have published several articles between 1960 and 1975, linked to logic (theoretical presentations or stories of didactical experiences), and were spokesmen of animated debates such as the one concerning the use of quantifiers symbols. This introduction of logic in the high school mathematics syllabi then seemed like an ambitious project, linked to the acquisition of mathematical language, and was sustained by a certain wish to train teachers, not only to mathematical logic but also to modern mathematics in general. But if we look at textbooks, we can see that most of the time this logic has been reduced to simply a formal presentation of notions of logic, without linking it to mathematical activity. This presentation was not adapted to the initiation to mathematics logic for a mass of student reaching high school.

In 1981 came a radically different new syllabus. It was the time of the "counter-reform", in which logic was explicitly excluded from mathematics teaching. Modern mathematics have been vehemently accused of being too formal, elitist and not linked to mathematics applications. Logic taught by then and some representative elements

such as the tables of truth are almost symbols of this excessive formalism. This might be an explanation to the fact that none of the first high school year textbooks give tables of truth, even if some of them describe their contents, saying for instance that "the proposition **P and Q** is true only in the case where P and Q are both of them true".

This lasted until the implementation of the 2001 syllabus, which states that "training in logic is part of the requirements of high school classes". This text is included in the 2009 syllabus and is supplemented by a table setting targets for "notations and mathematical reasoning". These objectives relate to certain objects of mathematical logic, but are rather vague. It is also specified there that "the concepts and methods relevant of the mathematical logic should not be the object of specific courses but should naturally take their place in all the syllabus chapters". I will give as examples two of these objectives: "Students are trained, based on examples, to correctly use the logical connectives "and", "or" and to distinguish their meanings from the common meanings of "and", "or" in the everyday language. They are also trained to wisely use universal and existential quantifiers (the symbols  $\forall$ ,  $\exists$  are not due) and to spot implicit quantifications in certain propositions, particularly in the conditional ones." We have already seen that these objectives are not clear, concerning the logical connectives "and", "or", and that the Resources document which come with the syllabus does not take in charge all the important aspects of these notions. The second example concerns quantifications, mainly the implicit ones. We have evoked p3 certain didactical studies that have shown how these quantifications created difficulty for the students, for instance in the understanding of the propositions "if...when... ". Still, this does not appear in the Resources document. The syllabus instructions put logic both on the side of language and the reasoning one (presentation of the different reasoning types). But the links between logic and mathematics are not the same as in the time of modern mathematics. Here, it is essentially about specifying the mathematical language's particularities related to the everyday language. Furthermore, the proposition, a base element of the mathematical language, is absent from syllabuses and the Resources document.

This tormented history of logic in teaching mathematics in high school mainly has two consequences that contribute to the complexity of what is actually asked from teachers. On the one hand, all of them do not have the same training for these notions. On the other hand, there has not been a continuous thinking, particularly a didactical one, about that teaching of logic notions in high school.

Furthermore, we have seen that logic should be taught along with other notions, which represents a strong constraint as far as time is concerned. It should also be caught were it lies, which implies that teachers should be lucid and serene enough to be able to spot it.



## Logic in the textbooks

I will only talk here about the current textbooks. Textbooks authors should follow the directions, even though imprecise, given in the syllabus, to provide teachers with tasks for their students. Thus, in most textbooks published in September 2010, we can find pages offering a brief overview of the concepts of logic mentioned in the syllabus. These pages are not a separate chapter. They are a sort of glossary which students and teachers can refer to during certain tasks concerning logic. An analysis of ten mathematics textbooks published in 2010 shows a diversity in the presentations. Some books have one approach that can be called "propositional", which means they constitute a kind of "grammar of mathematical propositions". Other books have an approach that can be called "natural", which means that they take common language as the starting point to speak about mathematical language, while specifying the requirements for this discipline, in particular the requirements of univocal meaning for each word. Because the syllabus demands that logic does not constitute a course on its own, very few textbooks use the terms "definitions", "properties" in their pages, even though it is exactly what they give. Exercises associated to logic are essentially "True or False" ones, and do not allow to work on the language. Finally, we can find mistakes, such as the confusion between "if...then..." and "therefore", in the pages talking about logic or in the exercises of certain textbooks. The lack of knowledge of some teachers concerning logic makes it difficult for them to spot and analyze these mistakes.

We have seen through the study of the documents contributing to define the *knowledge to be taught* that this knowledge is not clear for the mathematics high school teacher today in France. It appears to us that it is therefore necessary to offer them trainings that will give them the tools needed for this teaching. Because logic is as essential to address language issues as it is to address the reasoning issues in the mathematical activity, I think that mathematics teachers should have tools allowing them to think about the way they speak, and about the ambiguities and implicits that lie under some usual formulations in our practice of mathematics. Mathematical logic seems a possible reference for this reflection.

## AN INITIATION TO LOGIC IN THE FRAMEWORK OF A CONTINUOUS TRAINING FOR TEACHERS.

The Institut de Recherche pour l'Enseignement des Mathématiques (IREM)<sup>6</sup> at the Paris Diderot University proposed in 2011 a training course called "Introduction to logic" as part of the continuous training for teachers (this training course had already been organized in 2010 and renewed in 2012). This course was led in collaboration with René Cori, professor in the logic team of the Paris Diderot University. Fifty teachers (the number of places was limited) enrolled in this course, forty of them

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<sup>6</sup> Research Institut for Mathematics Teaching

were effectively attending. The training took place during three days of 6 hours each (two consecutive days in January, then one separated day a month later). One of the training goals is to give the trainees knowledge in mathematical logic. This does not mean lecturing about mathematical logic. It is all about teaching logic for teachers, a logic in context, at the service of mathematical activity. What is proposed is an analysis and a critical look at mathematical language with which teachers are already familiar. An important place is given to the notion of variable, that we will present as being characteristic of mathematical language in relation to the common language, and the multiple ways mathematics use to implicitly quantify their statements. The logical connectives are then presented as operators on propositions, which means they allow, starting from one or two propositions, to "create" a third one. This syntactic aspect is separated from the semantic aspect broached by giving the truth tables of these connectives. The notions of tautology and propositions logically equivalent are defined and put in relation with the practices of reasoning. An important moment is dedicated to implication: establishing its truth table creates reactions. Then, it is essential to note that the negative of a conditional proposition is not a conditional proposition. We also discuss at length about the implicit universal quantification associated to the formulation "if... then...". We finally suggest few developments on the study of theories.

Another important part of the training is a more practical aspect, based on the study of the school textbooks. Basing themselves on selected parts, the trainees do a critical analysis in small groups. At the end, we share our work. This practical exercise allows to show the misunderstandings there can be about some notions, and was based on the taught theoretical components (this work was done after having spoken about variables, connectives and quantifiers). We also devote time during the third day for the trainees who wish to present activities they have done in class, so that we can discuss about it.

This training is a field for experimentation for my research. I am trying to implement data collection and analysis tools.

## **CONCLUSION**

With this contribution, I wanted to participate to the reflection on teaching notions of logic, particularly in showing the importance of not neglecting the contribution of the logic in problematics related to language and not just to reasoning. Anyway, language and reasoning are not two isolated poles, there is an interaction between the structure of mathematical propositions and the structure of their proof. Moreover, rewording is a common activity in the proving process in mathematics.

In France today the new syllabuses for high school give goals concerning mathematical notations and reasoning, and mathematics teachers then have to explicitly teach some notions of logic. But through a study of documents defining the *knowledge to be taught*, we have shown that their choices for this teaching were made in ill-defined conditions, and were subject to strong constraints. It therefor seems

essential to me that the mathematics teachers have tools allowing them to think about the way they speak, and about the ambiguities and implicits that lie under some usual formulations in our practice of mathematics. Mathematical logic seems a possible reference for this reflection.

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