

# Joint Reconstruction and Segmentation of Real 3D Data in Computed Tomography thanks to a Gauss-Markov-Potts Prior Model

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**Summary:** Computed Tomography (CT) is a powerful tool to reconstruct a volume in 3D and has a wide field of applications in industry for non-destructive testing. In these applications, the reconstruction process has a key importance to retrieve volumes that can be easily analyzed during the control. In this paper, in order to improve the reconstruction quality, we present a Gauss-Markov-Potts prior model for the object to reconstruct in a Bayesian framework. This model leads to a joint reconstruction and segmentation algorithm which is briefly described. The core of the paper is the application of the algorithm on real 3D data. We show that our method obtains better results than other state-of-art methods. We also propose reconstruction quality indicators without reference which uses both reconstruction and segmentation returned by the algorithm.

## Model for the projections in 3D X-ray CT

- Linear model based on Radon transform discretization and accounting for errors

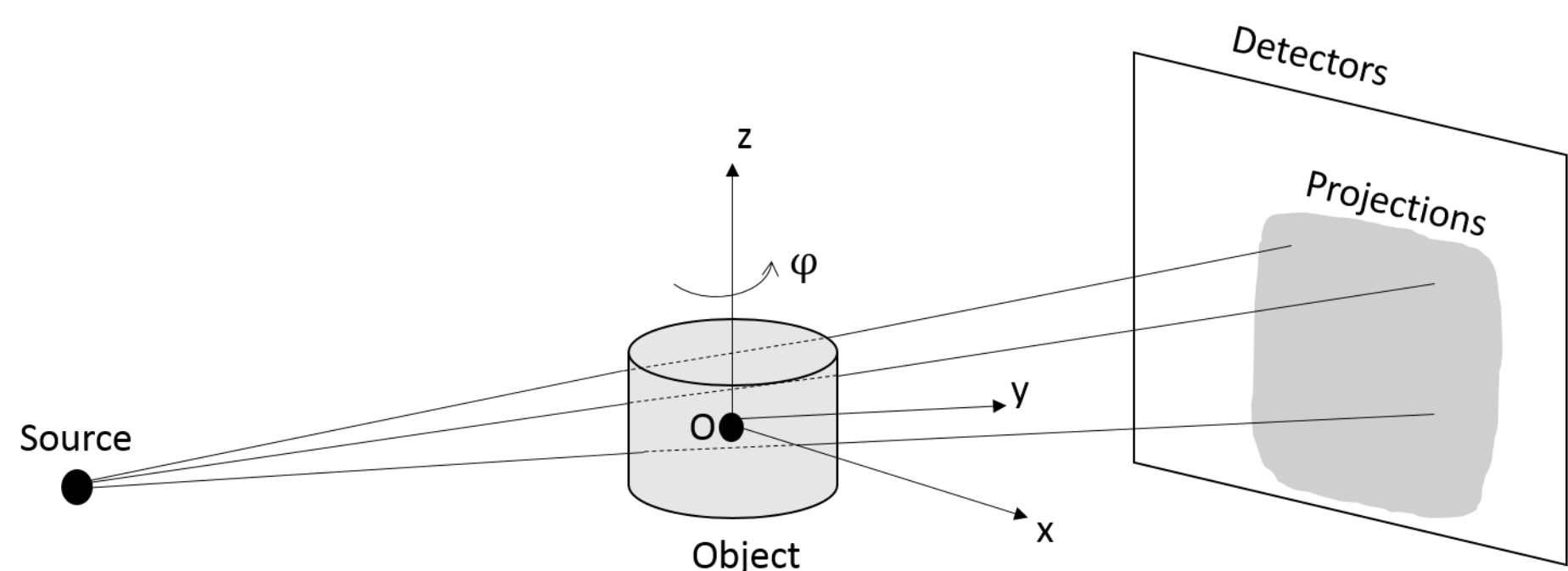
$$\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon}$$

with

$$p(\epsilon_i | \mathbf{v}_{\epsilon_i}) = \mathcal{N}(\epsilon_i | 0, \mathbf{v}_{\epsilon_i}), \forall i$$

- Conjugate prior on  $\mathbf{v}_{\epsilon_i}$ :

$$\mathbf{v}_{\epsilon_i} \sim \mathcal{IG}(\mathbf{v}_{\epsilon_i} | \alpha_{\epsilon_0}, \beta_{\epsilon_0}), \forall i$$



## Prior on the volume : Gauss-Markov-Potts model

- Label each voxel  $j$  depending on its material :  $\mathbf{z}_j = k$  if voxel  $j$  composes material  $k$ ,  $k \in \{1, \dots, K\}$
- $K$  : number of materials in the volume
- The distribution of  $\mathbf{f}_j$  depends on the label of voxel  $j$  :

$$\mathbf{f}_j \sim \mathcal{N}(\mathbf{m}_k, \mathbf{v}_k) \text{ if } \mathbf{z}_j = k$$

- Priors for the means  $\mathbf{m}$  and the variances  $\mathbf{v}$  :

$$\mathbf{m}_k \sim \mathcal{N}(\mathbf{m}_0, \mathbf{v}_0), \forall k \in \{1, \dots, K\}$$

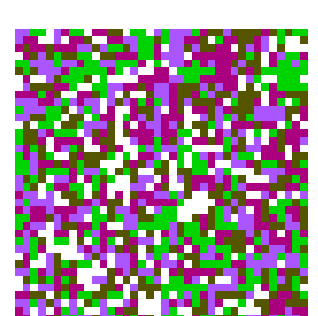
$$\mathbf{v}_k \sim \mathcal{IG}(\alpha_0, \beta_0), \forall k \in \{1, \dots, K\}$$

- The hidden field  $\mathbf{z}$  is modeled as a Potts field

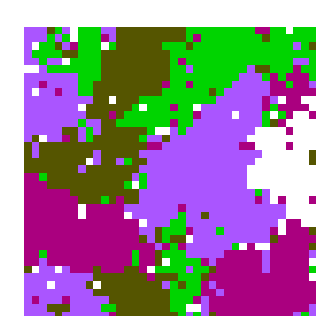
$$p(\mathbf{z}_j | \mathbf{z}_i, i \in \mathcal{V}(j); \alpha, \gamma_0) \propto \exp \left[ \sum_{k=1}^K \alpha_k \delta(\mathbf{z}_j - k) + \gamma_0 \sum_{i \in \mathcal{V}(j)} \delta(\mathbf{z}_j - \mathbf{z}_i) \right]$$



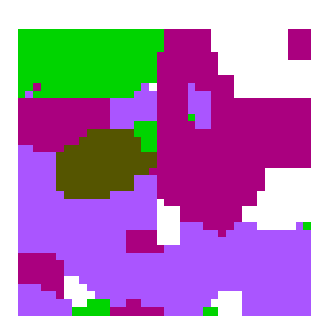
$\gamma_0 = 0.5$



$\gamma_0 = 0.7$



$\gamma_0 = 0.8$



$\gamma_0 = 1.6$

## Joint Maximization A Posteriori (JMAP)

- Alternate maximization based on Bayes rule

$$p(\mathbf{f}, \mathbf{z}, \mathbf{v}_{\epsilon}, \mathbf{m}, \mathbf{v} | \mathbf{g}; \mathcal{M}) \propto p(\mathbf{g} | \mathbf{f}, \mathbf{v}_{\epsilon}) p(\mathbf{f} | \mathbf{z}, \mathbf{m}, \mathbf{v}) p(\mathbf{v}_{\epsilon} | \alpha_{\epsilon_0}, \beta_{\epsilon_0}) p(\mathbf{z} | \alpha; \gamma_0) p(\mathbf{m} | \mathbf{m}_0, \mathbf{v}_0) p(\mathbf{v} | \alpha_0, \beta_0)$$

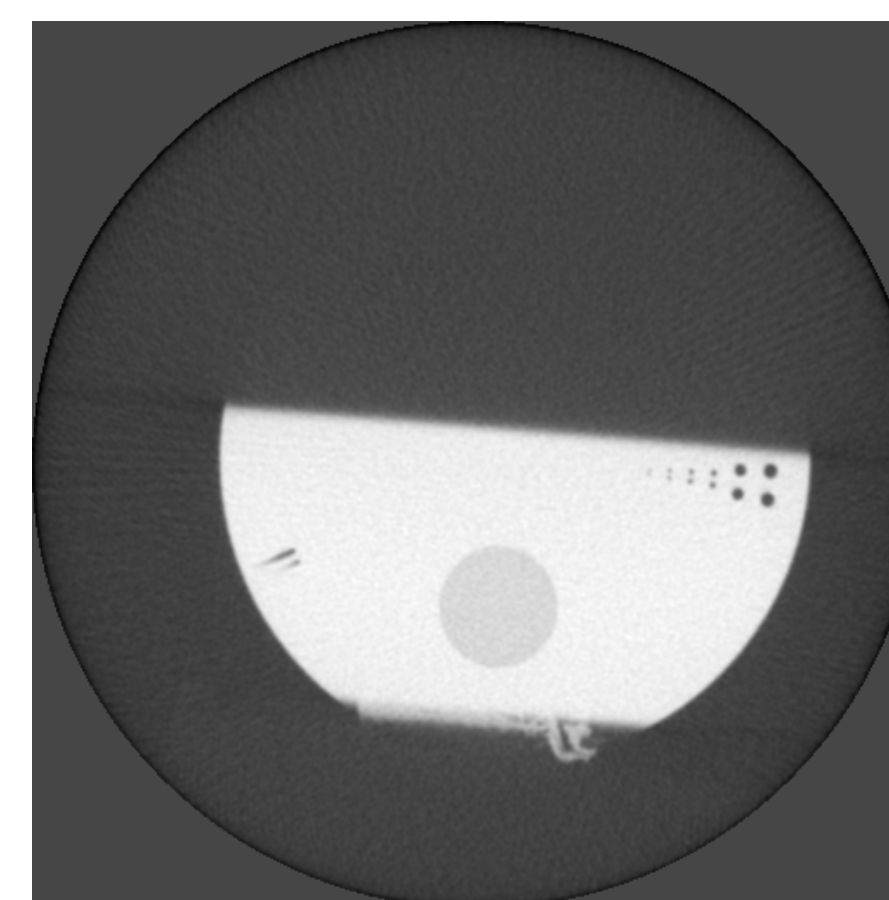
- Estimation of  $\mathbf{f}$  : minimize

$$J(\mathbf{f}) = \|\mathbf{g} - \mathbf{H}\mathbf{f}\|_{\mathbf{v}_{\epsilon}}^2 + \|\mathbf{f} - \mathbf{m}\mathbf{z}\|_{\mathbf{v}_{\mathbf{z}}}^2$$

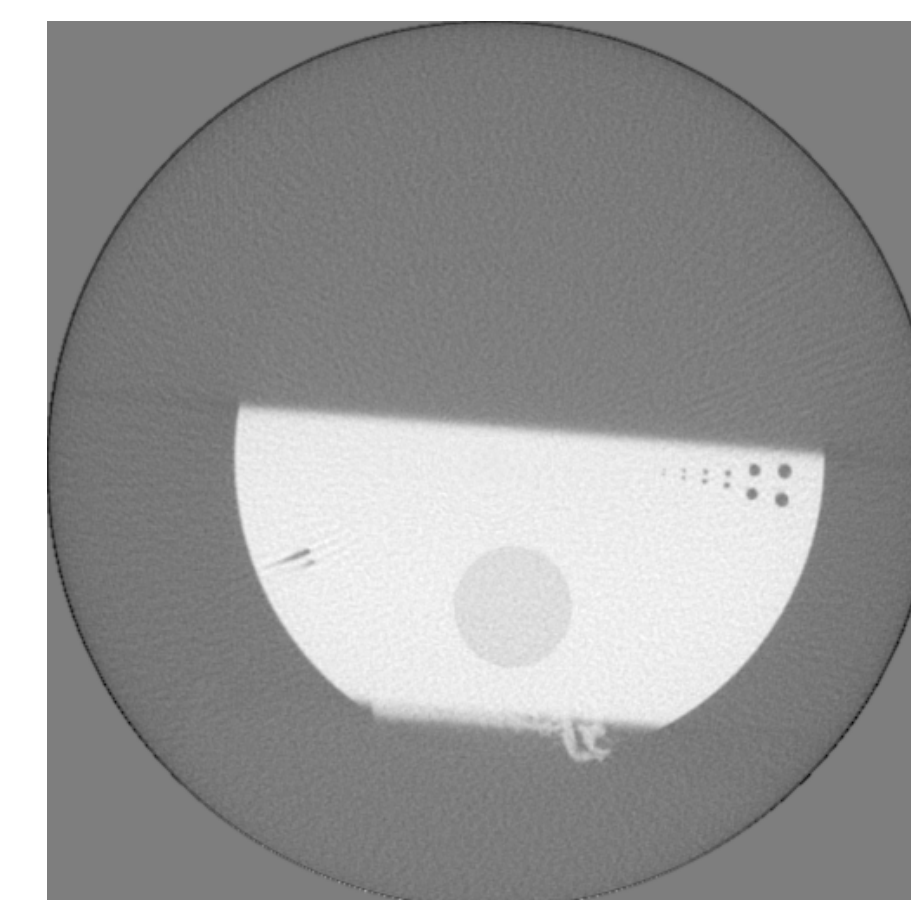
- $\mathbf{H}$  and  $\mathbf{H}^T$  implemented on GPU : ray-driven/voxel-driven pair
- Estimation of  $\mathbf{z}$  : Iterated Conditional Mode (ICM) (highly-parallelizable)
- Analytical formulas for the estimation of the other unknowns

## Results on Real IQI Volume

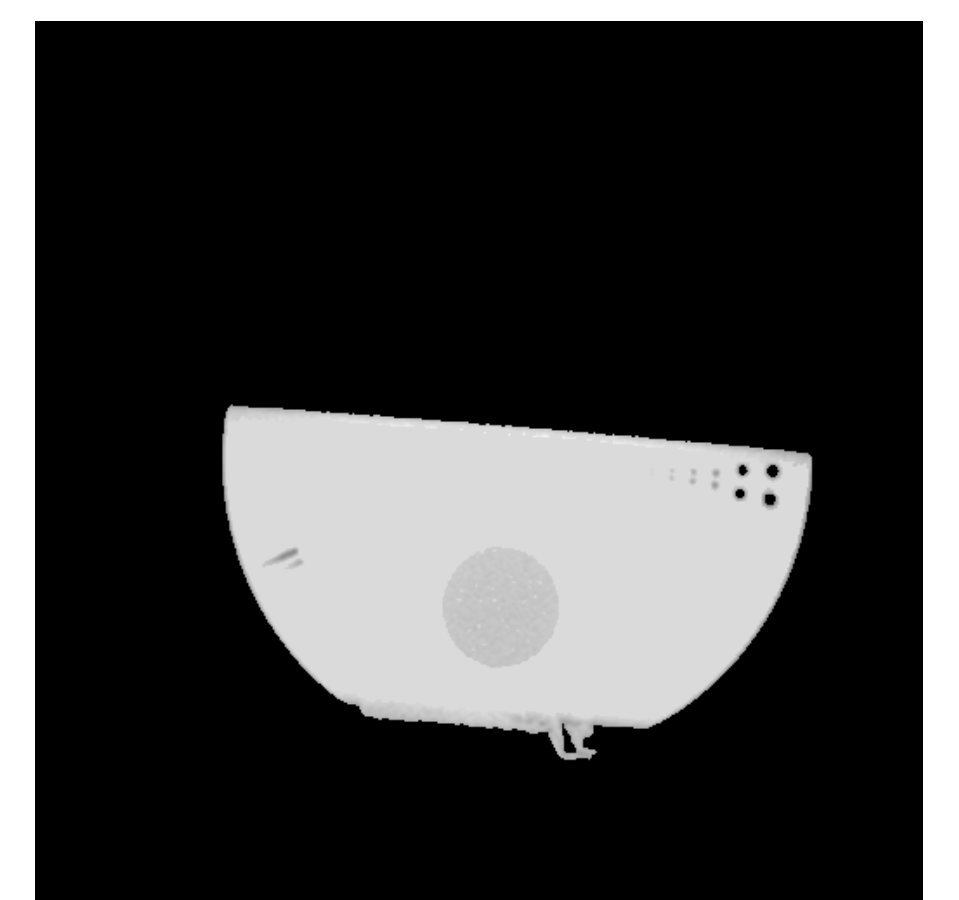
- Comparison with FDK and TV regularization



FDK



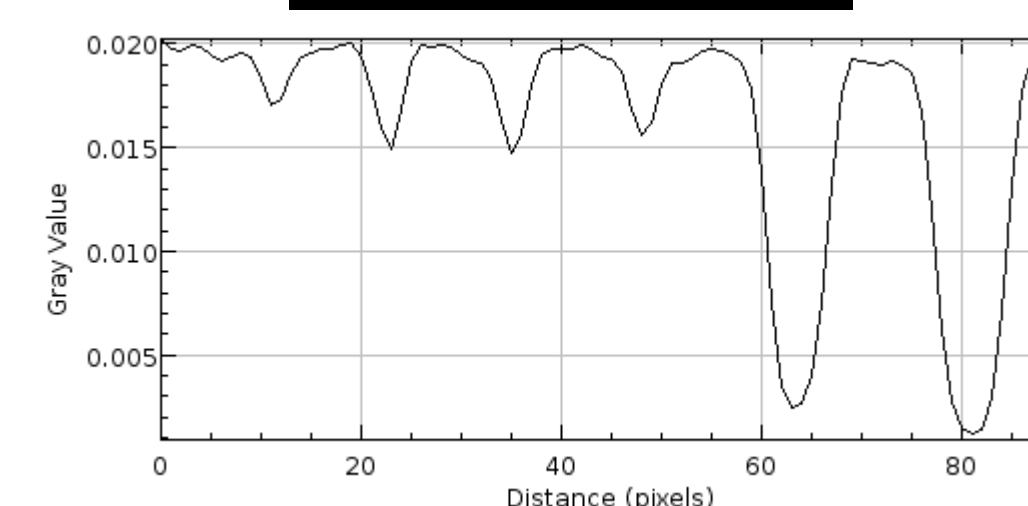
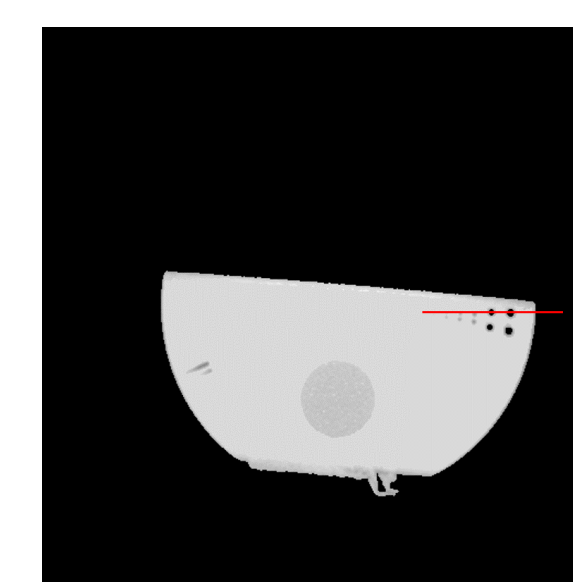
TV



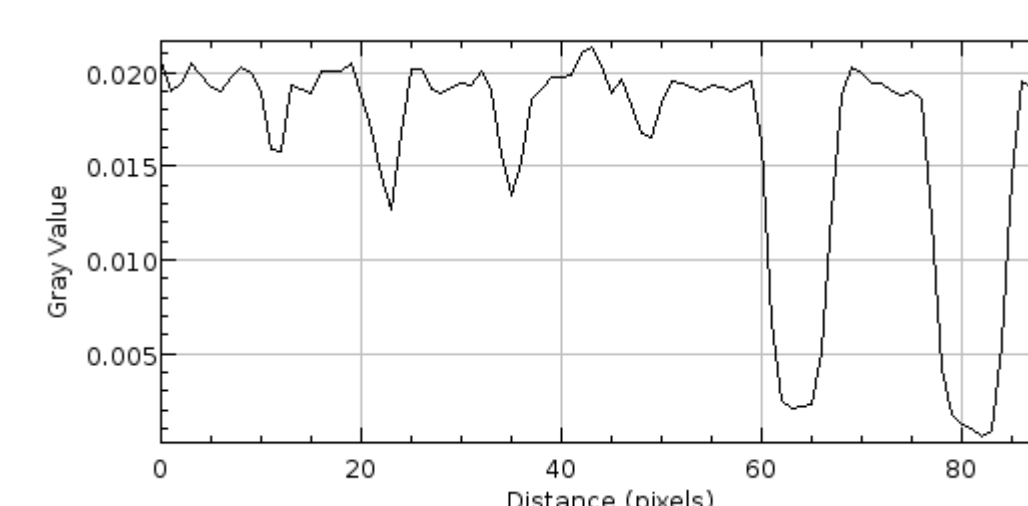
JMAP

## Profiles of the holes

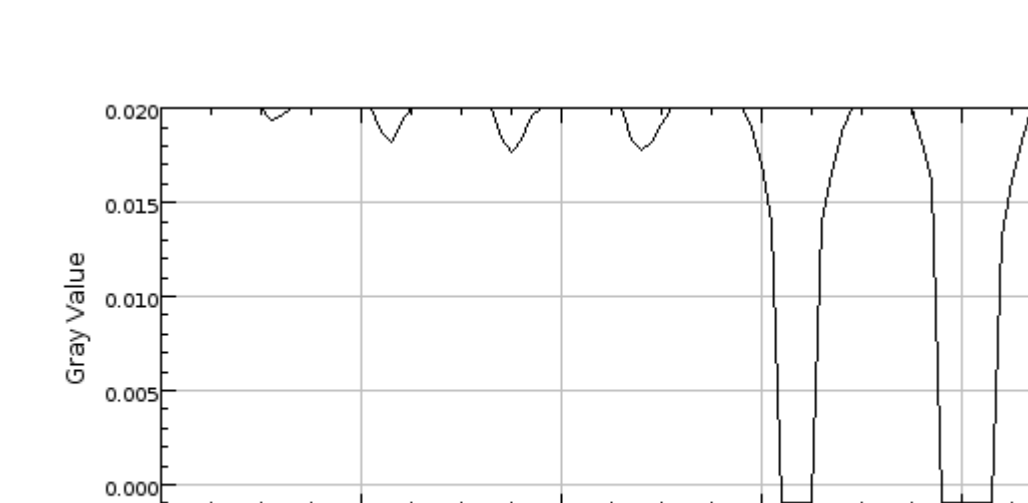
## and Peak-Height-Ratio



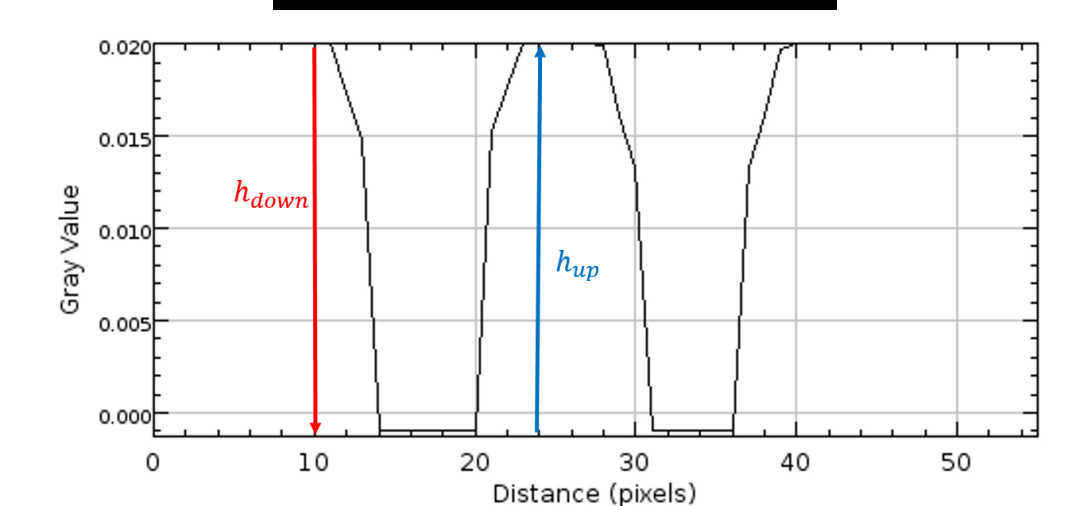
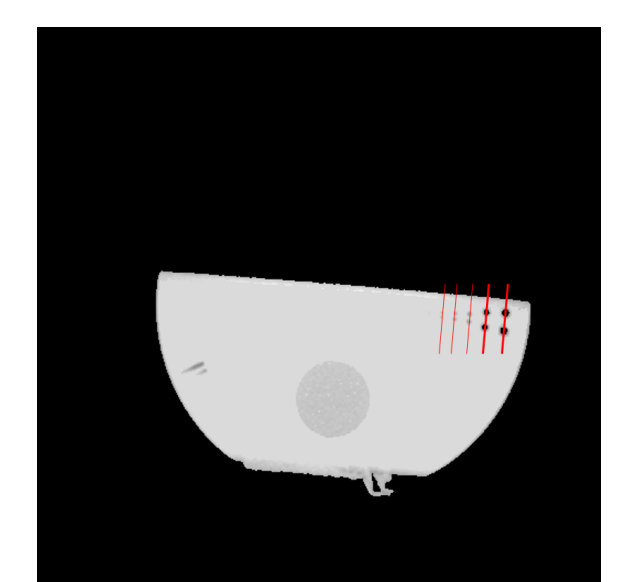
FDK



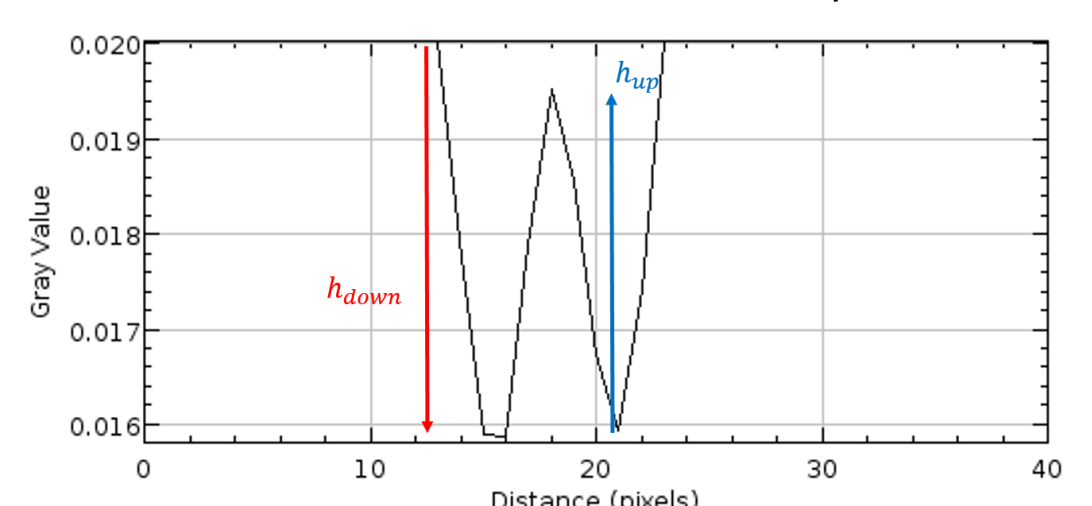
TV



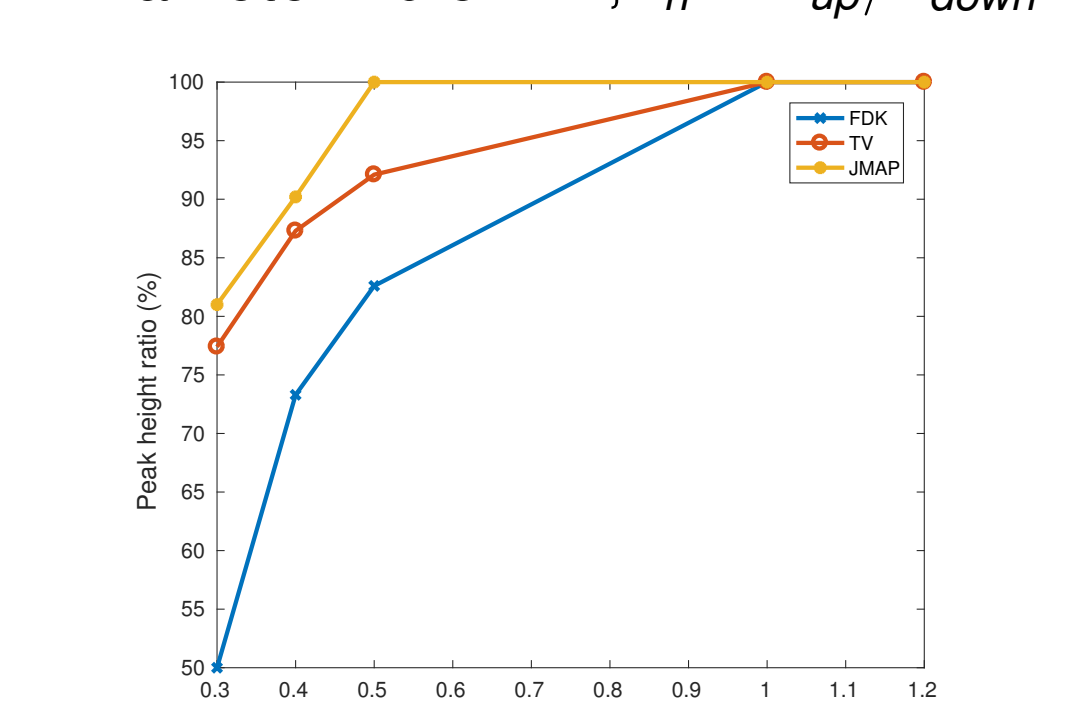
JMAP



Diameter : 1.2 mm,  $r_h = h_{up}/h_{down}$



Diameter : 0.5 mm,  $r_h = h_{up}/h_{down}$



## Conclusion and prospects

- Good results for our method in reconstructing industrial parts
- To do : GPU implementation of segmentation step