

1 **Supplement for: Computation and characterization of local sub-filter-scale**
2 **energy transfers in atmospheric flows**

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ABSTRACT

16 In the supplementary material, we present additional information on the
17 thermodynamic \mathcal{D}_ℓ^T and dynamical components $\mathcal{D}_\ell(\vec{u})$ of the DR indicator.
18 We also present the evolution of DR indicator and its components during
19 Jolina and Katrina hurricanes. We include two supplementary tables with the
20 values of statistical quantities of $\mathcal{D}_\ell(\vec{u}, b)$ distributions by arranged by height.

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TABLE 1. Statistics of $\mathcal{D}_\ell(\vec{u}, b)$ distributions as a function of the height for the ERA-Interim reanalysis.

height	mean($\mathcal{D}_\ell(\vec{u}, b)$)	std($\mathcal{D}_\ell(\vec{u}, b)$)	skew($\mathcal{D}_\ell(\vec{u}, b)$)	kurt($\mathcal{D}_\ell(\vec{u}, b)$)
1000 hPa	0.0000	0.0003	-12.5696	276.9933
950 hPa	0.0002	0.0003	1.8389	14.4944
850 hPa	0.0004	0.0006	2.6106	14.7987
700 hPa	0.0005	0.0008	2.5612	13.7117
600 hPa	0.0008	0.0012	2.7417	14.9112
500 hPa	0.0011	0.0017	2.8925	15.3062
400 hPa	0.0011	0.0018	2.8953	15.0074
300 hPa	0.0011	0.0018	2.9440	15.3355
250 hPa	0.0005	0.0011	3.0691	17.6553
200 hPa	-0.0003	0.0010	-3.2327	20.4761
150 hPa	-0.0003	0.0009	-2.2594	19.2577
100 hPa	-0.0004	0.0010	-3.4291	20.3697

TABLE 2. Statistics of $\mathcal{D}_\ell(\vec{u}, b)$ distributions as a function of the height for the NCAR reanalysis.

height	mean($\mathcal{D}_\ell(\vec{u}, b)$)	std($\mathcal{D}_\ell(\vec{u}, b)$)	skew($\mathcal{D}_\ell(\vec{u}, b)$)	kurt($\mathcal{D}_\ell(\vec{u}, b)$)
1000 hPa	-0.0000	0.0006	-7.6942	101.0839
950 hPa	0.0003	0.0006	2.4538	26.8792
850 hPa	0.0005	0.0008	2.3793	13.9458
700 hPa	0.0007	0.0010	1.9269	7.7386
600 hPa	0.0011	0.0015	1.9876	7.9285
500 hPa	0.0015	0.0021	2.1588	8.9876
400 hPa	0.0016	0.0024	2.3421	10.0828
300 hPa	0.0016	0.0026	2.5646	11.6123
250 hPa	0.0010	0.0019	2.7246	14.4601
200 hPa	-0.0000	0.0012	-1.9678	14.2280
150 hPa	-0.0003	0.0011	-2.5738	12.7600
100 hPa	-0.0004	0.0010	-2.7603	12.9706

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 29 height: 1000 hPa, 500 hPa and 100hPa. (a,d) yearly averages, (b,e) winter averages, (c,f)
 30 summer averages. 7

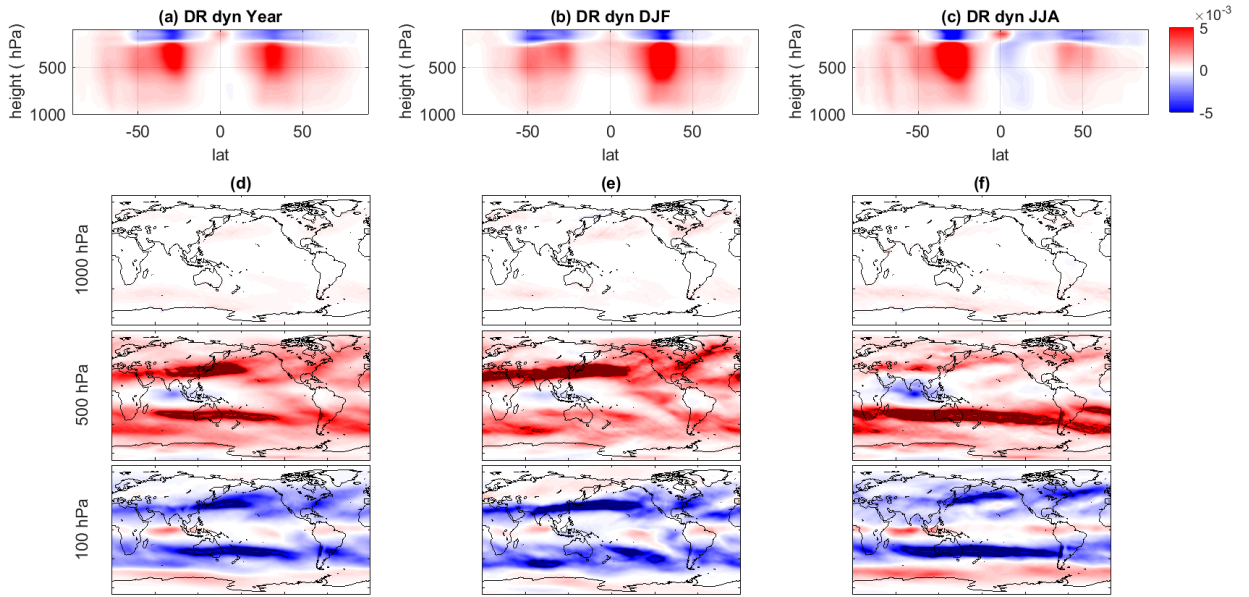
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 36 NCAR reanalysis . (a-c) longitudinal averages, (d-e-f) maps at three different fixed height:
 37 1000 hPa, 500 hPa and 100hPa. (a,d) yearly averages, (b,e) winter averages, (c,f) summer
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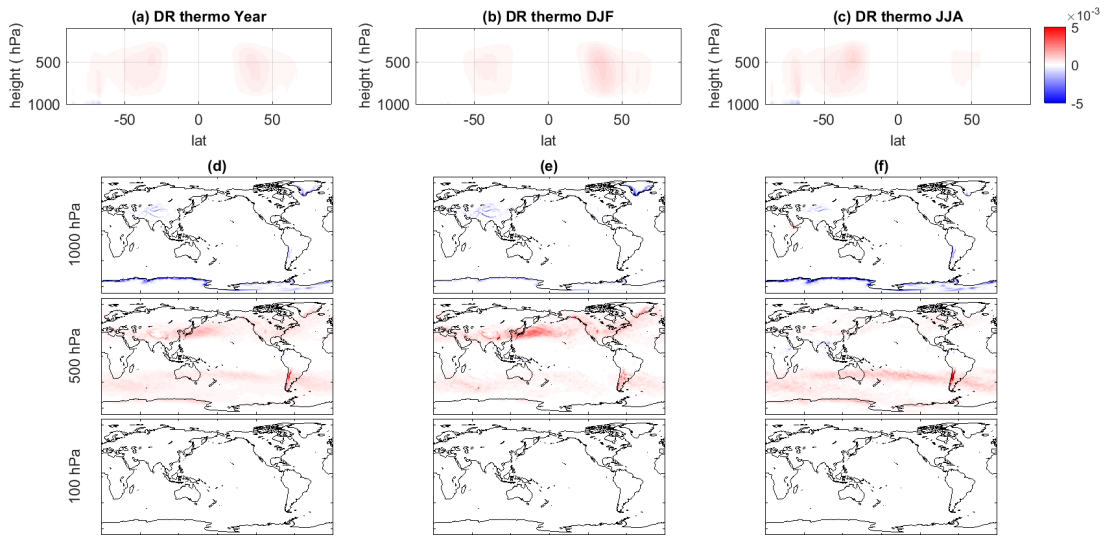
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 40 the NCAR reanalysis (Thermodynamic component only). (a-c) longitudinal averages, (d-e-
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43 **Fig. 5.** 3D structure of the wind field (green cones) and DR indicator for the Hurricane Jolina.
 44 (a,d,g,j) Dynamic component $\mathcal{D}_\ell(\vec{u})$ only. (b,e,h,k) Thermodynamic component \mathcal{D}_ℓ^T only.
 45 (c,f,i,l) Total $\mathcal{D}_\ell(\vec{u}, b)$ contribution. Results for ERA-Interim reanalysis, for $\ell = 220$ km.
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 47 colorscale indicates values $\mathcal{D}_\ell(\vec{u}, b) > 0.001$. Time from top to bottom. 11

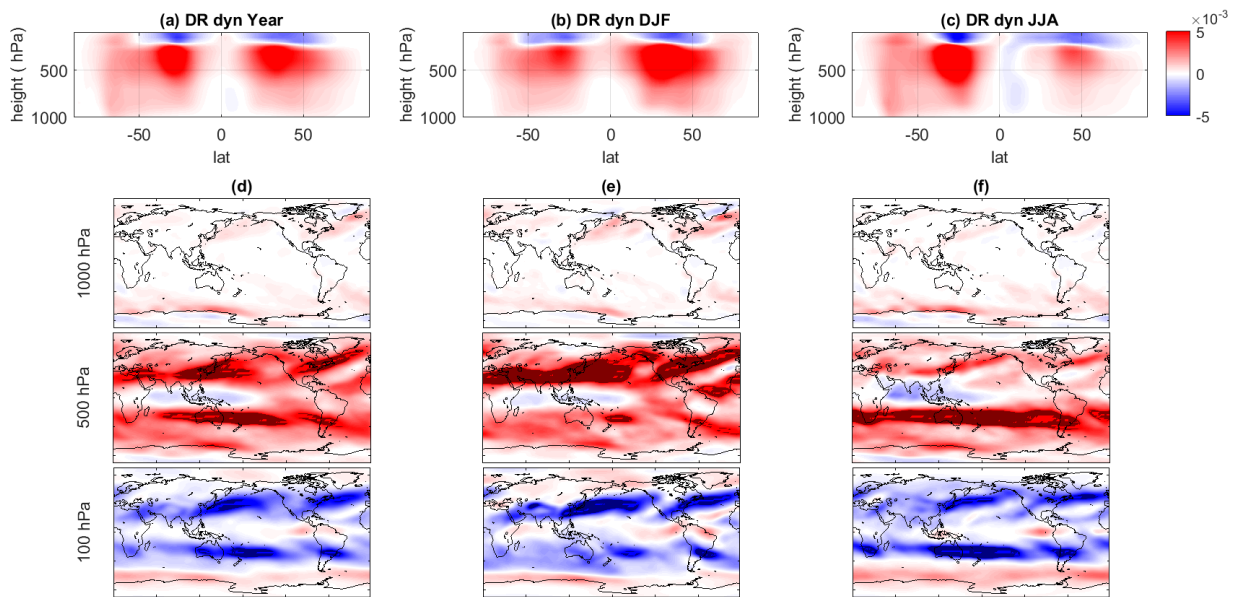
48 **Fig. 6.** 3D structure of the wind field (green cones) and DR indicator for the Hurricane Katrina.
 49 (a,d,g,j) Dynamic component $\mathcal{D}_\ell(\vec{u})$ only. (b,e,h,k) Thermodynamic component \mathcal{D}_ℓ^T only.
 50 (c,f,i,l) Total $\mathcal{D}_\ell(\vec{u}, b)$ contribution. Results for ERA-Interim reanalysis, for $\ell = 220$ km.
 51 Red: isosurfaces at $\mathcal{D}_\ell(\vec{u}, b) = 0.001$. Dark Blue: isosurfaces at $\mathcal{D}_\ell(\vec{u}, b) = -0.001$. The
 52 colorscale indicates values $\mathcal{D}_\ell(\vec{u}, b) > 0.001$. Time from top to bottom. 12



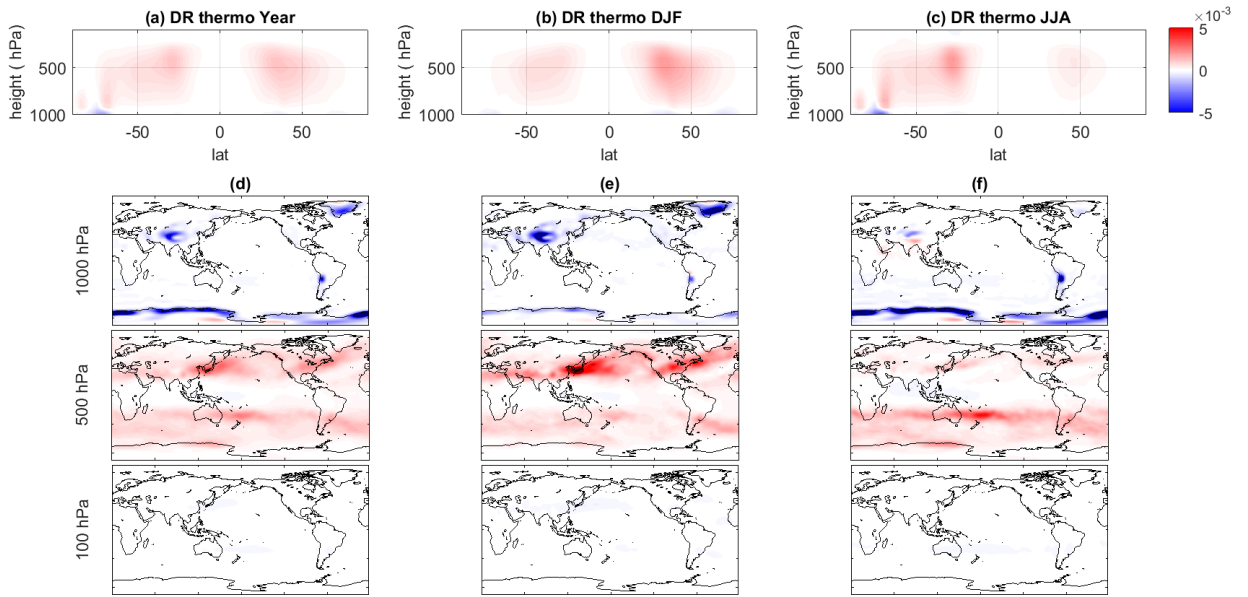
53 FIG. 1. Distribution of the dynamical component of the DR indicator $\mathcal{D}_\ell(\vec{u})$ for $\ell = 220$ Km and the ERA-
 54 Interim reanalysis. (a-c) longitudinal averages, (d-e-f) maps at three different fixed height: 1000 hPa, 500 hPa
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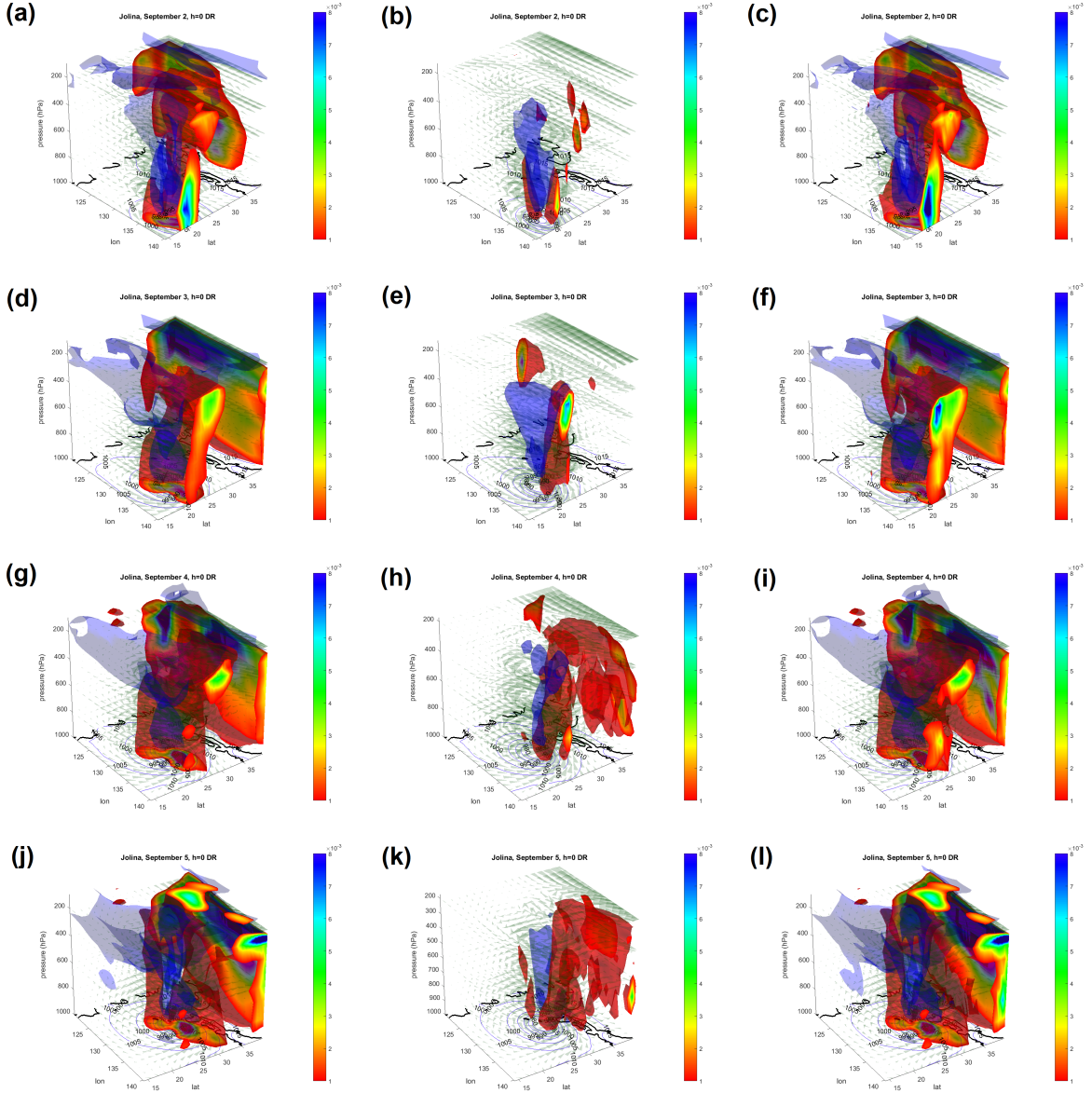
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 57 Interim reanalysis. (a-c) longitudinal averages, (d-e-f) maps at three different fixed height: 1000 hPa, 500 hPa
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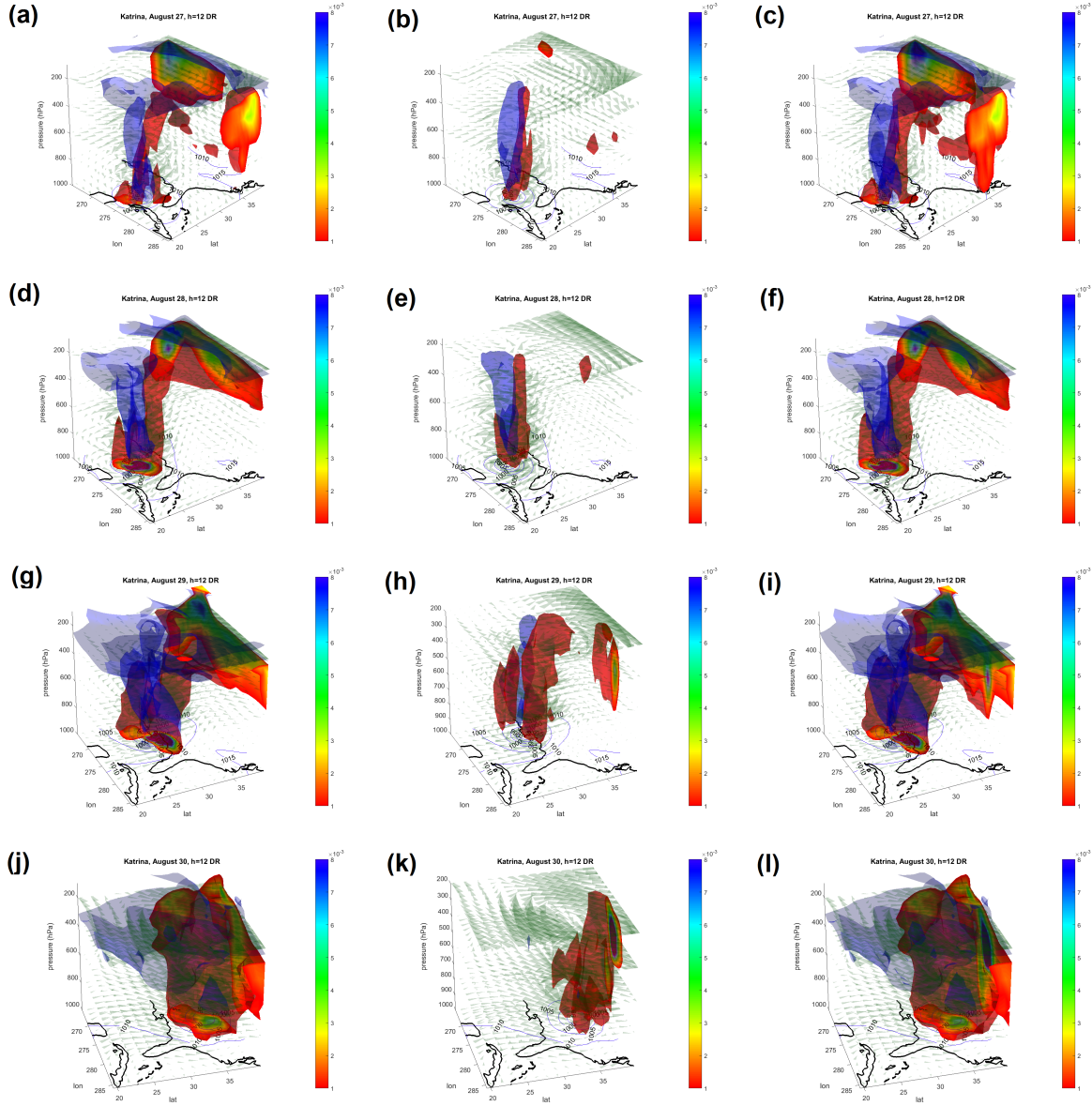
59 FIG. 3. Distribution of the dynamical component of the DR indicator $\mathcal{D}_\ell(\vec{u})$ for $\ell = 220$ Km and the NCAR
 60 reanalysis . (a-c) longitudinal averages, (d-e-f) maps at three different fixed height: 1000 hPa, 500 hPa and
 61 100hPa. (a,d) yearly averages, (b,e) winter averages, (c,f) summer averages.



62 FIG. 4. Distribution of the thermodynamic component of the DR indicator \mathcal{D}_ℓ^T for $\ell = 220$ Km and the NCAR
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 64 height: 1000 hPa, 500 hPa and 100hPa. (a,d) yearly averages, (b,e) winter averages, (c,f) summer averages.



65 FIG. 5. 3D structure of the wind field (green cones) and DR indicator for the Hurricane Jolina. (a,d,g,j)
66 Dynamic component $\mathcal{D}_\ell(\vec{u})$ only. (b,e,h,k) Thermodynamic component \mathcal{D}_ℓ^T only. (c,f,i,l) Total $\mathcal{D}_\ell(\vec{u}, b)$ contri-
67 bution. Results for ERA-Interim reanalysis, for $\ell = 220$ km. Red: isosurfaces at $\mathcal{D}_\ell(\vec{u}, b) = 0.001$. Dark Blue:
68 isosurfaces at $\mathcal{D}_\ell(\vec{u}, b) = -0.001$. The colorscale indicates values $\mathcal{D}_\ell(\vec{u}, b) > 0.001$. Time from top to bottom.



69 FIG. 6. 3D structure of the wind field (green cones) and DR indicator for the Hurricane Katrina. (a,d,g,j)
70 Dynamic component $\mathcal{D}_\ell(\vec{u})$ only. (b,e,h,k) Thermodynamic component \mathcal{D}_ℓ^T only. (c,f,i,l) Total $\mathcal{D}_\ell(\vec{u}, b)$ contri-
71 bution. Results for ERA-Interim reanalysis, for $\ell = 220$ km. Red: isosurfaces at $\mathcal{D}_\ell(\vec{u}, b) = 0.001$. Dark Blue:
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