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1 **Computation and characterization of local sub-filter-scale energy transfers**
2 **in atmospheric flows**

3 Davide Faranda^{1,2}, Valerio Lembo³, Manasa Iyer⁴, Denis Kuzay⁵,
4 Sergio Chibbaro⁴, Francois Daviaud⁶ & Berengere Dubrulle⁶

5 ¹*Laboratoire des Sciences du Climat et de l'Environnement, LSCE/IPSL, CEA-CNRS-UVSQ,*
6 *Université Paris-Saclay, F-91191 Gif-sur-Yvette, France*

7 ²*London Mathematical Laboratory, 14 Buckingham Street, London, WC2N 6DF, UK*
8 *davide.faranda@lsce.ipsl.fr*

9 ³*Meteorological Institute, University of Hamburg, Grindelberg 5, 20146 Hamburg, Germany*

10 ⁴*Sorbonne Université, UPMC Univ Paris 06, CNRS UMR 7190, Institut Jean le Rond*
11 *d'Alembert, Paris, France*

12 ⁵*Univ Lyon, Ens de Lyon, Univ Claude Bernard, CNRS, Laboratoire de Physique, F-69342 Lyon,*
13 *France*

14 ⁶*SPEC, CEA, CNRS, Université Paris-Saclay, CEA Saclay 91191 Gif sur Yvette cedex, France*

ABSTRACT

16 Atmospheric motions are governed by turbulent motions associated to non-
17 trivial energy transfers at small scales (direct cascade) and/or at large scales
18 (inverse cascade). Although it is known that the two cascades coexist, energy
19 fluxes have been previously investigated from the spectral point of view but
20 not on their instantaneous spatial and local structure. Here, we compute local
21 and instantaneous sub-filter scale energy transfers in two sets of reanalyses
22 (NCEP-NCAR and ERA-Interim) in the troposphere and the lower strato-
23 sphere for the year 2005. The fluxes are mostly positive (towards subgrid
24 scales) in the troposphere and negative in the stratosphere reflecting the baro-
25 clinic and barotropic nature of the motions respectively. The most intense
26 positive energy fluxes are found in the troposphere and are associated with
27 baroclinic eddies or tropical cyclones. The computation of such fluxes can
28 be used to characterize the amount of energy lost or missing at the smallest
29 scales in climate and weather models.

30 1. Introduction

31 Large scale atmospheric motions are modeled via a set of nonlinear equations known as primi-
32 tive equations (Smagorinsky 1963). They include an equation for mass conservation, the thermal
33 energy equation and the evolution equations of velocity fields. The latter are derived from the
34 Navier-Stokes equations on spherical geometry under the hydrostatic assumption, i.e. that vertical
35 motion is much smaller than horizontal motion. Such equations are the cornerstone of all weather
36 and climate models and their explicit implementation depends on the choice of coordinate system
37 and the resolution of the model (Cao and Titi 2007; Klein 2010; Stevens and Bony 2013; Cao et al.
38 2015). In the classical turbulence phenomenology, valid homogeneous flows, energy is injected
39 at large scales, transferred downscale at a constant averaged rate ε (Kolmogorov (1941) cascade)
40 and dissipated at small scales by viscous effects (Frisch 1995). This phenomenology is based on
41 the so-called Karman-Howarth-Monin (KHM equation), derived directly from the Navier-Stokes
42 equation, that reads:

$$\frac{1}{2}\partial_t E + \varepsilon = -\frac{1}{4}\vec{\nabla}_\ell \cdot \langle \delta \vec{u} (\delta u)^2 \rangle + \nu \nabla_\ell^2 E, \quad (1)$$

43 where u is the velocity field, ν the molecular viscosity, $\langle \rangle$ means statistical average, ε is the mean
44 non-dimensional energy injection rate, $\delta \vec{u} = \vec{u}(\vec{x} + \vec{\ell}) - \vec{u}(\vec{x})$ is the velocity increments over a dis-
45 tance ℓ and $E(\ell) = \langle (\delta u)^2 \rangle / 2$ is a measure of the kinetic energy at scale ℓ .

46 Turbulence in the atmosphere is strongly influenced by density stratification and rotation (Holton
47 and Hakim 2012). Turbulence in such condition is known to develop a complex dynamics, as re-
48 vealed by accurate numerical simulations and laboratory experiments (Levich and Tzvetkov 1985;
49 Schertzer et al. 1997; Falkovich 1992; Pouquet and Marino 2013). Depending on the scale of the
50 flow, energy transfers can be directed either towards smaller scales (direct cascade) or towards
51 larger scales (inverse cascade) (Bartello 1995). To this aim, Augier et al. (2012) recently consid-

52 ered a set of primitive equations for incompressible, non-diffusive and inviscid stably stratified
 53 fluid in the Boussinesq approximation, in order to account for both the kinetic energy (KE) and
 54 available potential energy (APE) in a modified version of the KHM equation. The primitive equa-
 55 tions for the stratified Boussinesq fluid were written as:

$$\begin{aligned}
 \vec{\nabla} \cdot \vec{u} &= 0, \\
 \partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla}) \vec{u} &= -\vec{\nabla} p + b \vec{e}_z + \nu \Delta \vec{u}, \\
 \partial_t b + (\vec{u} \cdot \vec{\nabla}) b &= -N^2 u_z + \kappa \Delta b,
 \end{aligned} \tag{2}$$

56 where u is the velocity field, u_z its vertical component, p the rescaled pressure, ν and κ the viscos-
 57 ity and diffusivity, $N = \sqrt{-(g/\rho_0)(d\bar{\rho}/dz)}$ the constant Brunt-Väisälä frequency, \mathbf{e}_z the vertical
 58 unit vector, and $b = -\rho'g/\rho_0$ the buoyancy perturbation, g the acceleration due to gravity, ρ_0 a
 59 reference density, $\bar{\rho}(z)$ the mean profile and ρ' a density perturbation. The resulting generalized
 60 KHM equation was written as:

$$\begin{aligned}
 \frac{1}{2} \partial_t E + \varepsilon &= -\frac{1}{4} \vec{\nabla}_\ell \cdot \langle \delta \vec{u} \left[(\delta u)^2 + \frac{(\delta b)^2}{N^2} \right] \rangle, \\
 &\equiv \vec{\nabla} \cdot \vec{J},
 \end{aligned} \tag{3}$$

61 where E is now

$$E = \frac{1}{2} \langle (\delta u)^2 \rangle + \langle ((\delta b)^2/N^2) \rangle, \tag{4}$$

62 and we have omitted contributions due to viscosity and diffusivity. This shows that the flux \vec{J} is
 63 made of a KE flux and an APE flux. This equation stems directly from the energy conservation law
 64 governing atmospheric dynamics at the dissipative scales. Energy conservation in the atmosphere
 65 is clearly described by the Lorenz Energy Cycle (LEC) formulation (Lorenz 1955), showing the
 66 conversion of APE into KE and then into dissipative heating. In a spectral sense, the KE and APE
 67 fluxes can have different direction, so that the resulting energy flux can be positive, or negative,

68 depending on the scale, isotropy or stratification, and the corresponding direction of the energy
69 cascade is hard to predict. In other words it is not clear to what extent is applicable a picture of
70 turbulence in a stratified atmosphere in which a clear separation of scales exists between 2-D and
71 3-D turbulence and whether the two pictures are associated with cascades in opposite directions
72 (Lovejoy and Schertzer 2010).

73 The lack of general consensus about the direction of cascades also reflects in the interpretation
74 of observational-based results and implementation of turbulence in numerical models. Observed
75 energy spectra in the troposphere and in the lower stratosphere (Nastrom and Gage 1985) exhibit
76 $k^{-5/3}$ law, generally connected to direct cascades, and/or k^{-3} power laws, associated to an inverse
77 energy cascade. The inverse cascade has been historically associated to the quasi-geostrophic two
78 dimensional dynamics induced by rotation (Charney 1971), and fed by baroclinic instability. Tung
79 and Orlando (2003) simulated the Nastrom-Gage energy spectrum of atmospheric turbulence as
80 a function of wavelength with a two-level quasi-geostrophic model, and were able to obtain both
81 spectral behaviours with this simple dynamics. Kitamura and Matsuda (2006) analysed the role of
82 stratification and rotation in the generation of the cascades, observing that in experiments without
83 planetary rotation, the obtained spectral slope was steeper and energy transfer to larger vertical
84 wave-numbers was increased. Some theories for a mesoscale inverse cascade for stratified (not
85 quasi-geostrophic) turbulence were proposed by Gage (1979) and Lilly (1983) but these are no
86 longer considered viable. According to Lindborg (2005), atmospheric mesoscale $-5/3$ energy
87 spectra can be explained by the existence of a direct cascade arising in the limit of strong stratifi-
88 cation while the role of planetary rotation is to inhibit the cascade process at large scales leading
89 to an accumulation of kinetic energy and steepening of the kinetic energy spectrum at small wave
90 numbers. Evidence on the existence of a direct energy cascade comes from high resolution direct
91 numerical simulations of stratified flows (Lindborg 2006). They also suggest that the direction of

92 the cascade may be crucially dependent on the ratio of the Brunt-Väisälä frequency to the rotation
93 frequency.

94 A way to clarify the situation is to compute the energy fluxes. In the classical picture of turbu-
95 lence, such energy transfers are related directly to the skewness velocity increments $\langle \delta \mathbf{u} (\delta u)^2 \rangle$,
96 and the direction of the cascade is provided by the sign of this quantity (positive for direct cas-
97 cade, negative for inverse cascade). This quantity which is global, since it relies on space-average,
98 has thus been used in the past to quantify the direction of the energy transfer. From the observed
99 stratospheric third-order structure function, Lindborg and Cho (2001) argued that there is a for-
100 ward energy cascade in the mesoscale range of atmospheric motions. In that study the authors
101 pointed out that for scale smaller than 100 km the statistical inhomogeneities can be neglected
102 while this assumption is not valid for larger scales. Similar conclusions also hold for the study of
103 intense phenomena such as in tropical cyclones (Tang and Chan 2015) and suggests that one must
104 be careful in using skewness based approaches to infer energy fluxes depending on the homogene-
105 ity assumptions.

106 Another approach to compute energy fluxes for atmospheric model is to rely on the spectral ki-
107 netic energy budgets (see e.g. Augier and Lindborg (2013) and Peng et al. (2015)). However, like
108 the skewness approach, these computations only provide a global in space estimate of the energy
109 transfers and not their local value, nor their instantaneous spatial distribution.

110 An important breakthrough was made when Duchon and Robert (2000) reformulated the energy
111 budget of the Navier-Stokes equations into a form allowing for the definition of energy transfers
112 local in space and time and valid for any geometry including when strong inhomogeneity and
113 anisotropy are present. Its ability to provide interesting information about energy transfers at a
114 given scale ℓ has been so far exploited in the experimental set-up of the Von Karman swirling flow
115 to measure the scale to scale energy transfers and non viscous energy dissipation (Kuzzay et al.

116 2015; Saw et al. 2016). The Duchon and Robert indicator requires only the 3D velocity fields and
117 provides, for each instant, 3D maps of the sub-filter energy transfers at a scale ℓ . The interest of
118 this formulation is that it is devoid of any adjustable parameters unlike, for exemple, local esti-
119 mates of energy budgets based on LES methods Kuzzay et al. (2015).

120 In this work we adapt the definition of such indicators to the atmospheric dynamics providing the
121 first local maps of sub-filter-scale energy transfers without any adjustable parameter. The goal of
122 this work is i) to identify and characterize the atmospheric motions responsible for large energy
123 transfers and ii) to compute global time and spatial average and assess whether the reanalyses
124 over(under)-represent energy fluxes. The paper is structured as follows. After presenting the indi-
125 cator, we will study these transfers in the NCEP-NCAR and ERA-Interim reanalyses - to ensure
126 that results are model independent - for the year 2005 and investigate: i) the vertical and horizontal
127 global averages, ii) the distribution of energy transfers at different scales, iii) two case studies of
128 intense cyclones that occurred in 2005 (Katrina and Jolina) and also correspond to extreme en-
129 ergy transfers according to our indicator. We finally discuss the implications of our results on a
130 theoretical and practical level.

131 2. Methods

132 For any solutions of the Navier-Stokes equations, Duchon and Robert (2000) defined energy
133 transfers in a fluid at an arbitrary scale ℓ using a local energy balance equation

$$\partial_t E^\ell + \partial_j \left(u_j E^\ell + \frac{1}{2} (u_j \hat{p} + \hat{u}_j p) + \frac{1}{4} (\widehat{u^2 u_j} - \widehat{u^2} u_j) - v \partial_j E^\ell \right) = -v \partial_j u_i \partial_j \hat{u}_i - \mathcal{D}_\ell, \quad (5)$$

134 where u_i are the components of the velocity field and p the pressure, \hat{u} and \hat{p} their coarse-grained
135 component at scale ℓ , $E^\ell = \frac{\hat{u}_i \hat{u}_i}{2}$ is the kinetic energy per unit mass at scale ℓ (such that $\lim_{\ell \rightarrow 0} E^\ell =$

136 $u^2/2)$, \mathcal{D}_ℓ is expressed in terms of velocity increments $\delta\vec{u}(\vec{r}, \vec{x}) \stackrel{def}{=} \vec{u}(\vec{x} + \vec{r}) - \vec{u}(\vec{x}) \equiv \delta\vec{u}(\vec{r})$ (the
 137 dependence on ℓ and \vec{x} is kept implicit) as:

$$\mathcal{D}_\ell(\vec{u}) = \frac{1}{4\ell} \int_{\mathcal{V}} d\vec{r} (\vec{\nabla} G_\ell)(\vec{r}) \cdot \delta\vec{u}(\vec{r}) |\delta\vec{u}(\vec{r})|^2, \quad (6)$$

138 where G is a smooth filtering function, non-negative, spatially localized and such that
 139 $\int d\vec{r} G(\vec{r}) = 1$. The function G_ℓ is rescaled with ℓ as $G_\ell(\vec{r}) = \ell^{-3} G(\vec{r}/\ell)$. As shown in Duchon
 140 and Robert (2000), the choice of G has no impact on the value of \mathcal{D}_ℓ , in the limit $\ell \rightarrow 0$, as long
 141 as it satisfies the properties specified previously. In the sequel, we therefore choose a spherically
 142 symmetric function of x given by:

$$G_\ell(r) = \frac{1}{N} \exp(-1/(1 - (r/(2\ell))^2)), \quad (7)$$

143 where N is a normalization constant such that $\int d^3r G_\ell(r) = 1$.

144 As noticed by Duchon and Robert, the average of $\mathcal{D}_\ell(\vec{u})$ can be viewed as a weak form of the
 145 transfer term $-\frac{1}{4} \vec{\nabla}_\ell \cdot \langle \delta\vec{u}(\delta u)^2 \rangle$ in the anisotropic version of the KHM equation Eq. (1), the
 146 divergence being taken not on the term itself, but instead on the test function G_ℓ . Without taking
 147 the average, we see then that $\mathcal{D}_\ell(\vec{u})$ is a local energy transfer, the sign of which provides the
 148 direction of the fluxes in the scale space: a positive sign implies transfer towards the scales smaller
 149 than ℓ .

150 By construction, the intrinsic weak formulation of $\mathcal{D}_\ell(\vec{u})$ makes it less sensitive to noise than
 151 classical gradients, or even than the usual KHM relation: indeed, the derivative in scale is not
 152 applied directly to the velocity increments, but rather on the smoothing function, followed by a
 153 local angle averaging. This guarantees that no additional noise is introduced by the procedure.
 154 Even more, the noise coming from the estimate of the velocity is naturally averaged out by the
 155 angle smoothing as shown in Kuzzay et al. (2015). In the same study, the authors argued that the

156 Duchon and Robert approach was a better alternative to the widespread large eddies simulation
 157 based method for the computation of energy fluxes, since it relies on very few arbitrary hypotheses.
 158 Experimentally, in the von Karman set-up, the DR formula provided a better estimate of the energy
 159 dissipation than a LES method: in particular, estimates of the injected and dissipated powers were
 160 within 20% of the measured value using the LES-PIV method, whereas reached 98% of the actual
 161 dissipation rate of energy with the DR formula Kuzzay et al. (2015).

162 In order to use this approach for atmospheric dynamics requires taking into account density
 163 stratification, and considering Boussinesq equations instead of Navier-Stokes equations. We have
 164 adapted the Duchon-Robert formalism to the Boussinesq equations. The equation for the kinetic
 165 energy is simply restated as

$$\partial_t E^\ell + \vec{\nabla} \cdot \vec{J}_K^\ell = -\nu \partial_j u_i \partial_j \hat{u}_i - \mathcal{D}_\ell + \frac{1}{2} (b \hat{u}_z + \hat{b} u_z). \quad (8)$$

166 Using the point-split buoyancy perturbation as fundamental variable, we can then obtain an equa-
 167 tion related to the local variance of the buoyancy perturbation (details are given in the appendix)

$$\partial_t E_T^\ell + \vec{\nabla} \cdot \vec{J}_T^\ell = -\mathcal{D}_\ell^T - \frac{1}{2} (b \hat{u}_z + \hat{b} u_z) - \kappa \partial_j \hat{b} \partial_j b / N^2, \quad (9)$$

168 where $E_T^\ell = \frac{\hat{b} b}{2N^2}$ is the available potential energy at scale ℓ , \mathcal{D}_ℓ^T is expressed in terms of the
 169 increments $\delta b(\vec{r}, \vec{x}) \stackrel{def}{=} b(\vec{x} + \vec{r}) - b(\vec{x}) \equiv \delta b(\vec{r})$ (the dependence on ℓ and \vec{x} is kept implicit) as

$$\mathcal{D}_\ell^T = \frac{1}{4\ell} \int_{\mathcal{V}} d\vec{r} (\vec{\nabla} G_\ell)(\vec{r}) \cdot \delta \vec{u}(\vec{r}) |(\delta b)|^2 / N^2. \quad (10)$$

170 Considering now that the energy for stratified flows is given by expression (4), we can sum equa-
 171 tion (8) and (9), to get the total local energy balance

$$\partial_t E^\ell + \vec{\nabla} \cdot \vec{J}^\ell = -\mathcal{D}_\ell(\vec{u}, b) - \nu \partial_j \hat{u}_i \partial_j u_i - \kappa \partial_j \hat{b} \partial_j b / N^2, \quad (11)$$

172 where

$$\vec{J}^\ell = \vec{J}_K^\ell + \vec{J}_T^\ell, \quad (12)$$

173 is the spatial energy flux, and

$$\mathcal{D}_\ell(\vec{u}, b) = \frac{1}{4\ell} \int_{\mathcal{V}} d\vec{r} (\vec{\nabla} G_\ell)(\vec{r}) \cdot \delta\vec{u} \left[(\delta u)^2 + \frac{(\delta b)^2}{N^2} \right], \quad (13)$$

174 is the total local scale to scale energy flux. It is easy to see that the average of $\mathcal{D}_\ell(\vec{u}, b)$ is a
 175 weak formulation of the energy transfer terms of the generalized KHM equation of Augier et al
 176 Eq. (3). The DR indicator $\mathcal{D}_\ell(\vec{u}, b)$ is thus a local energy transfer term, that can be split into a
 177 kinetic (dynamical) part $\mathcal{D}_\ell(\vec{u})$ (the original DR indicator) and a potential (thermodynamic) part
 178 (the remaining part, implying the field b). In order to easily implement the expression of $\mathcal{D}_\ell(\vec{u}, b)$
 179 in climate models, the buoyancy parameter has been rewritten as a function of temperature T using
 180 the equation of state for dry air: $\delta b = -\delta p / \rho_0 R \cdot 1 / \delta T$, where ρ_0 is a reference density at surface
 181 pressure and δp is a pressure horizontal perturbation, which is set to be about 10 hPa each 100
 182 km. Furthermore, in Eq. 13, we set a constant Brunt-Väisälä frequency, amounting to 1.2×10^{-1}
 183 s^{-1} Holton and Hakim (2012), is chosen. In this way, the computation of $\mathcal{D}_\ell(\vec{u}, b)$ only requires
 184 the numerical 3D velocity u and T fields.

185 The sign and geometry of the zones associated with high and low values of $\mathcal{D}_\ell(\vec{u}, b)$ will then
 186 provide interesting information about the dynamics of the energy exchange in the atmosphere. For
 187 example, a study of the occurrence of high and low values of $\mathcal{D}_\ell(\vec{u})$ in the von Kármán swirling
 188 flow has revealed that such events are associated with well defined, characteristic geometry of the
 189 velocity field Saw et al. (2016). For the kinetic (dynamical) part, positive values of $\mathcal{D}_\ell(\vec{u})$ are
 190 measured whenever there is a strong convergence of the flow. Divergent flows are instead associ-
 191 ated to negative values of $\mathcal{D}_\ell(\vec{u})$, and they point to injection of energy from the sub-filter scales.
 192 This simple description is not valid anymore when we also consider the potential (thermodynamic)
 193 component. For all these reasons, we cannot reduce the computation of $\mathcal{D}_\ell(\vec{u})$ to only that of the

194 divergence/vorticity. As we will see in Section 3.1 for real atmospheric flows, dipolar $\mathcal{D}_\ell(\vec{u}, 0)$
195 structures may appear with 3D structures as those observed during hurricanes.

196 **3. Analysis**

197 For this study, outputs of the ERA-Interim and NCEP-NCAR Reanalysis 1 have been
198 used. ERA-Interim is the currently operational Reanalysis product at the European Center for
199 Medium-Range Weather Forecasting (ECMWF) (Dee et al. 2011). Released in 2007, it provides
200 reanalyzed data from 1979 to nowadays, stored at an original T255 spectral resolution (about
201 80 km horizontal resolution), with 60 vertical hybrid model levels. A 12h four-dimensional
202 variational data assimilation (4D-Var) is adopted. As a forecast model, the Integrated Forecast
203 Model (IFS), Cy31r2 release, is used, fully coupling modules for the atmosphere, ocean waves
204 and land surface. Sea-surface temperatures (SST) and sea-ice concentration (SIC) are ingested
205 as boundary conditions and interpolated on a reduced-Gaussian grid as needed. In our case
206 zonal, meridional and vertical wind components are considered at a $0.75^\circ \times 0.75^\circ$ horizontal
207 resolution over 12 pressure levels between 1000 and 100 hPa. A 12h time-step is considered.
208 Known problems concerning these datasets are the lack of dry mass conservation (Berrisford et al.
209 2011) and the slight asymmetry between evaporation and precipitation (Dee et al. 2011). The
210 turbulent fluxes are based on the tiled ECMWF scheme for surface exchanges over land (Viterbo
211 and Beljaars 1995; Viterbo and Betts 1999). Each gridbox is divided into up to six fractions
212 (over land) depending on the type of surface, having different transfer coefficients based on a
213 Monin-Obukhov formulation. Similarly, over oceans, two different coefficients are used for stable
214 and unstable conditions (Beljaars 1995).

215

216 NCEP-NCAR Reanalysis 1 has been developed in a joint effort by the National Center for Envi-
217 ronmental Prediction (NCEP) and the National Center for Atmospheric Research (NCAR) (Kalnay
218 et al. 1996). The simulation is operational since January 1995, covering a period from 1948 to
219 nowadays. Data assimilation is performed via a 3D variational scheme (Parrish and Derber 1992).
220 The NCEP model system, operational in 1994, has been used for forecasting. It features a T62
221 spectral resolution, corresponding to a $2.5^\circ \times 2.5^\circ$ horizontal grid (about 200 km horizontal reso-
222 lution), with 28 sigma levels. Most of the major physical processes involving the climate system
223 are parametrized. SST, SIC, snow cover, albedo, soil wetness and roughness length are ingested as
224 boundary conditions. Data are archived at an original 6h time-step, and such a temporal resolution
225 is retained for our analysis. The atmospheric model which provides the NCEP/NCAR reanalysis
226 data, uses bulk aerodynamic formulas to estimate the turbulent fluxes, with exchange coefficients
227 depending on empirical profiles extending the Monin-Obukhov similarity relationship (Miyakoda
228 and Sirutis 1986). For more details on the comparison between different subgrid parametrization
229 of surface fluxes, one might refer to Brunke et al. (2011).

230 *a. Analysis of local energy transfers*

231 1) YEARLY AND SEASONAL AVERAGE LOCAL ENERGY TRANSFERS

232 We begin the analysis by studying the latitudinal averages and the spatial features of the DR
233 indicator for both the ERA-Interim reanalysis and the coarser NCEP-NCAR reanalysis. To enable
234 comparison between the two datasets, one has to choose the analysis length larger than the reso-
235 lution scale of NCEP-NCAR (200km) since going below the resolution size introduces spurious
236 effects dependent on the filter design. On the other hand, since we want to have as much details as
237 possible, we have to choose the smallest scale consistent with those requirements. Here, we thus

238 adopt a scale of $\ell = 220$ km, this scale being the smallest that provide reliable estimates of DR
239 indicator. A further discussion of the dependence of the results with scale is done in section 3.a.3.

240 Results obtained for both reanalysis are consistent with each other, as can be checked from
241 Figure 1 (ERA) and Fig. 2 (NCEP), and do not depend on whether one undertakes a yearly
242 average (a,d), or seasonal (b,c,e,f): in the panels (a,b,c), which show height dependence of the
243 longitudinally averaged $\langle \mathcal{D}_\ell(\vec{u}, b) \rangle_{long}$, one observes the the total local energy transfers $\mathcal{D}_\ell(\vec{u}, b)$
244 are mostly positive in the troposphere, about zero at the tropopause and negative in the lower
245 stratosphere. By looking at cuts at different pressure levels, one can look more precisely about
246 the spatial distribution of the yearly and seasonal averages of $\langle \mathcal{D}_\ell(\vec{u}, b) \rangle_{time}$. Close to the ground
247 ($P = 1000$ hPa), the DR indicator is approximately zero except in proximity of sharp elevation
248 gradients (Antarctica costs, Himalaya, Greenland and Andes mountain ranges). By splitting the
249 local energy transfers is their kinetic $\langle \mathcal{D}_\ell(\vec{u}) \rangle_{time}$ and thermodynamic part $\langle \mathcal{D}_\ell^T \rangle_{time}$ (Fig. S1-S4),
250 one sees that this effect is mostly due to the the density fluctuations (i.e. the thermodynamic
251 component of the DR indicator) that produce these negative fluxes.

252 In the middle troposphere ($P = 500$ hPa), the behavior of $\mathcal{D}_\ell(\vec{u}, b)$ is associated to that of the jet
253 stream, since the most intense positive patterns are observed in winter for the northern hemisphere
254 and summer for the southern hemisphere. In the lower stratosphere ($P = 100$ hPa), $\mathcal{D}_\ell(\vec{u}, b)$ is
255 negative at the middle latitudes, and become slightly positive in polar regions and in the intertrop-
256 ical convergence zone. Overall, the splitting between the kinetic and thermodynamic component
257 detailed in the Supplementary material suggests that the dynamical component dominates with
258 respect to the thermodynamic one, although the DR thermodynamic contributions are significant
259 especially in the proximity of the ground.

260 2) CORRELATION WITH ENERGY SPECTRUM

261 The above result shows that the kinetic energy flux are globally positive in the troposphere, indi-
262 cating a *direct* kinetic energy cascade, while they are negative in the lower stratosphere indicating
263 a *inverse* kinetic energy cascade. Our results are therefore consistent with those found by Peng
264 et al. (2015) who also found upscale transfer in the lower stratosphere at outer mesoscale length
265 scales and downscale transfers at scales smaller than 360 km (KE) or 200 km (APE).

266 To get some insight on these cascades, we have further computed the kinetic horizontal energy
267 spectra where k is the inverse of the wavelength from the horizontal velocity fields at different
268 pressure levels in the two reanalysis. They are reported in Figure 5. One sees that for $P \leq 500$
269 hPa (corresponding to the stratosphere), the energy spectrum is mostly scaling like k^{-3} , while for
270 $P \geq 500$ hPa (middle troposphere), the energy spectrum scales like $k^{-5/3}$, at least for scales larger
271 than $\ell = 220$ km-in agreement with the Nastrom-Gage spectrum in the lower stratosphere at scales
272 between 10^3 and 10^2 km. In the ERA Interim data, the spectrum steepens below this scale and is
273 closer to k^{-2} . These values are however to be taken with caution, since our resolution does not
274 able one to distinguish clearly between a slope of $-5/3$ and $-7/5$ or $-11/5$ and -3 , which are
275 classical spectral slope that appear in rotating stratified or quasi 2D turbulence. The difference in
276 spectra between the troposphere and stratosphere can be explained by different values of ratio of
277 f the rotation frequency, to N the Brunt-Vaissala frequency, that was shown to influence strongly
278 the spectral slope and the magnitude or sign of the energy transfer (Pouquet et al. 2017). By
279 changing the value of f/N , one may also change the value of the crossover between the large
280 scales involving an inverse energy cascade, and the small scale, involving a direct energy cascade.

281 3) PROBABILITY DISTRIBUTION FUNCTIONS OF INSTANTANEOUS LOCAL ENERGY TRANS-
282 FERS

283 In addition to time average, it is also interesting to study the probability distribution function of
284 instantaneous local energy transfers, $\mathcal{D}_\ell(\vec{u}, b)$ at a different height (pressure level), and see how
285 it varies with scale and height. This is provided in Figure 3 for ERA-Interim and Figure 4 for
286 NCAR reanalyses. Panels (b,d,f) show the distributions at each level for $\ell = 220$ km. Panels (a,b)
287 show the kinetic component $\mathcal{D}_\ell(\vec{u}, 0)$, panels (c,d) the thermodynamic component \mathcal{D}_ℓ^T and (e,f) the
288 total $\mathcal{D}_\ell(\vec{u}, b)$. Tables S1 and S2 of the supplemental material report the values of mean, standard
289 deviation, skewness and kurtosis as a function of the height for the total DR indicator.

290 Overall, all distributions are skewed, and exhibit fat tails. The sign of the skewness depends
291 on the height: for both the total and kinetic component, it is positive in the lower troposphere,
292 and negative for $P < 500$ hPa, in agreement with the time averages. For the thermodynamic part,
293 the behaviour is opposite, with a negative skewness at low altitude ($P > 700$ hPa) and positive
294 skewness at large altitude. In such case, the distribution is totally asymmetric, and includes only
295 positive transfer, indicating that in the high part of the atmosphere, the density fluctuations only
296 contribute to a downscale energy transfer. Although there is agreement between the ERA interim
297 and the NCEP-NCAR data, the latter shows fatter tails. This might be due either to the different
298 resolution of the datasets and/or on the different physical parametrizations.

299 Looking now at the dependence with scale at fixed height, we see that both the kinetic and total
300 local energy transfer display similar behaviour, with a tendency to have fatter tails with decreasing
301 scales. This means that the energy imbalance of the reanalysis is reduced when we look at motions
302 whose characteristic scales are larger. This type of behaviour was also observed in local energy
303 transfers measured in a laboratory turbulent von Karman flow (Saw et al. 2016), and therefore

304 appear generic of highly turbulent flows. Regarding the thermodynamical part of the transfer, the
305 scale dependence is much more mild on the positive side of the distribution, and even absent in
306 the negative part of the distribution.

307 4) POSSIBLE INTERPRETATION

308 A possible way to explain the sign of the DR indicator is to invoke the relation between baroclinic
309 and barotropic flows and direct and inverse cascades. In Tung and Orlando (2003), it is argued
310 that the baroclinic motions responsible for the genesis and decay of extratropical cyclones are
311 mostly associated to direct cascades (corresponding to positive $\mathcal{D}_\ell(\vec{u}, b)$), while the essentially
312 barotropic motions governing the lower stratosphere atmospheric dynamics, are associated to an
313 inverse energy cascade (that would correspond to negative values of $\mathcal{D}_\ell(\vec{u}, b)$). To check such
314 interpretation, we have analyzed the maps of $\mathcal{D}_\ell(\vec{u})$ collected each 6 or 12h depending on the
315 datasets. They are collected for NCEP-NCAR in the supplementary video. Large positive and
316 negative values of the DR indicator are found as dipoles in baroclinic eddies. When increasing
317 the scale ℓ of the analysis, the tails become lighter as the local positive and negative contributions
318 get averaged out. To investigate better the role of baroclinic eddies in the development of large,
319 positive, DR values, we analyze two tropical cyclones (Katrina and Jolina) whose life-cycle ended
320 up as extratropical storms.

321 *b. Local energy transfers during two case-studies hurricanes*

322 Some aspects of DR indicator evolution in Katrina and Jolina (Nabi) tropical cyclones case
323 studies are shown in Figures 6,7 and 8. More time snapshots of 3D DR indicator are reported in
324 Figure S5 and S6 of the supplemental material. These extreme events have been well documented
325 in the literature (Wang and Oey 2008; Harr et al. 2008; Hsiao et al. 2009). Here we observe

326 that hurricanes correspond to large values of the DR indicator and are extremely localized in
327 space and time. To have a finer understanding of their dynamics, we have computed $\mathcal{D}_\ell(\vec{u}, b)$ in
328 proximity of the cyclonic structures and we use the dataset at higher resolution (ERA-Interim).
329 Figures 6,7 show a single snapshot where the dynamical $\mathcal{D}_\ell(\vec{u})$ (a), \mathcal{D}_ℓ^T thermodynamic (b) and
330 total $\mathcal{D}_\ell(\vec{u}, b)$ component (c). At every time step, we have isolated the cyclone as the region
331 bordered by sea-level pressure (SLP) values lower than 1005 hPa (in the case of Katrina),
332 lower than 1000hPa (in the case of Jolina). The threshold difference is motivated by the
333 presence of other low pressure structures in the region corresponding to Jolina's trajectory.
334 As the cyclones follow their path, a transition occurs from a condition characterized by the
335 alternation of negative and positive DR values, to one characterized by dominant positive values.
336 This is particularly evident in the middle levels of the troposphere (between 850 hPa and 250 hPa).

337

338 Some relevant differences are found, comparing the two cyclones. On one hand, $\mathcal{D}_\ell(\vec{u}, b)$ de-
339 creases as Katrina moves northward, and the patterns of DR at higher levels apparently lose their
340 co-variability with the surface structure of the cyclone. On the other hand, the DR indicator re-
341 mains large as Jolina moves northward, preserving its vertical coherence. Furthermore, the land-
342 sea contrasts seems to have a role in shaping the DR patterns at lower levels, with Jolina possibly
343 deriving its strength from its permanence over a sea surface, even at mid-latitudes. While Katrina
344 loses its energy source as cyclonic system as soon as it move northward in the continental areas,
345 Jolina develops into a powerful extra-tropical storm, as also noticeable looking at the SLP values
346 at the center of the system (Harr et al. 2008).

347 The DR evolution during the lifetime of the two systems suggests the different natures of energy
348 transfers in a tropical cyclone: Dominant positive $\mathcal{D}_\ell(\vec{u}, b)$ values reflect an unresolved energy
349 transfer towards the smaller scales, that is coherent with the downscale enstrophy transfer by

350 means of eddy-eddy interaction in baroclinic eddies (Burgess et al. 2013). The jet stream, which
351 is sustained by the eddy-mean flow interactions over mid-latitudes, is indeed denoted by large
352 positive DR values (cfr. Figure 1), enforcing the hypothesis that the downscale enstrophy transfer
353 is associated with an upscale kinetic energy transfer (Held and Hoskins 1985; Straus and Ditlevsen
354 1999). The thermodynamic structure of DR indicator (Panels b in Figure6-7) is interesting because
355 the positive energy transfers in the middle troposphere can be associated to the presence of the
356 warm core of the tropical cyclones, transferring energy to small scale proceses. A dipole vertical
357 structure emerges, as shown in Figure 1, with opposite sign DR values in coincidence of the
358 jet stream. This is consistent with previous findings from general circulation models (Koshyk
359 and Hamilton 2001) and higher resolution reanalysis (Burgess et al. 2013), showing a decay of
360 synoptic scale rotational kinetic energy and a downscale transfer of divergent kinetic energy above
361 the jet stream and in correspondence to the tropopause. As a caveat, hurricanes are associated with
362 strong horizontal divergence and convergence (and generation of kinetic energy) which can affect
363 the interpretation of $\mathcal{D}_\ell(\vec{u}, b)$ for such low resolution reanalysis datasets. It will definitely be worth
364 to compare these computations with those obtained at higher spatial and temporal resolutions.

365 4. Discussion

366 Weather and climate models do not resolve the viscous scales, which for the atmospheric mo-
367 tions are order of 0.1 mm (Priestley 1959). Up to date, their resolution ranges from $\simeq 2$ km of
368 regional weather models to $\simeq 100$ km of global climate models. To correctly represent dissipation
369 effects at a scale ℓ , the turbulent cascade needs to be parametrized at each grid point depending on
370 the type of motion and the geographical constraints. Despite the importance of such energy trans-
371 fers, their distribution and their time and spatial behavior is known only partially through field
372 campaigns (Lübken 1997) or by global averages (Sellers 1969; Seinfeld and Pandis 2016). This

373 does not ensure a global coverage and does not tell the direction of the energy transfers in the free
374 troposphere. In this paper, we have used Duchon and Robert (2000) to compute and characterize
375 the distribution of instantaneous and local sub-filter energy transfers in the atmosphere using 3D
376 velocity fields obtained in NCEP-NCAR and ERA-Interim reanalysis. Those energy transfers are
377 highly correlated with the baroclinic eddies occurring at mid-latitudes and with severe tropical
378 cyclones. Our computation of local energy transfer provides the direction of the local energy cas-
379 cade at a certain scale ℓ in physical space. At the grid resolution l , the value of $\mathcal{D}_\ell(\vec{u}, b)$ is an exact
380 measure of the amount of energy *that must be transferred to subgrid scales* (positive DR contribu-
381 tions) or *that must be injected from the subgrid scales* (negative contributions) to the scale ℓ of the
382 analysis. This information could be used to interactively adjust the energy fluxes to account for
383 the energy conservation laws in the atmosphere (Lucarini and Ragone 2011). Furthermore, the ex-
384 pression of $\mathcal{D}_\ell(\vec{u})$ is separable in a dynamical and a thermodynamic contributions. Although most
385 of the total $\mathcal{D}_\ell(\vec{u}, b)$ contribution is due to the dynamical component, negative fluxes are found
386 at the ground in presence of mountain ranges and sharp temperature/pressure gradients, positive
387 fluxes in the middle troposphere reinforce the dynamic contributions. We have also observed that
388 extreme events as tropical and extratropical storms are associated with large values of $\mathcal{D}_\ell(\vec{u}, b)$,
389 even at the ground.

390 The quantity $\mathcal{D}_\ell(\vec{u}, b)$ could also be a proxy of the flux of energy that can be exploited in wind
391 turbines (Miller et al. 2011, 2015). Although our analysis is performed for large scale general
392 circulation models, the Duchon and Robert (2000) formula can be applied to regional climate and
393 weather prediction models. At smaller scales, it will be extremely interesting to analyze the rela-
394 tion between $\mathcal{D}_\ell(\vec{u}, b)$ and the genesis of extreme wind gusts or even tornadoes. At such scales, one
395 could investigate the distributions of $\mathcal{D}_\ell(\vec{u}, b)$ to the instantaneous subgrid scales dissipation ob-
396 tained by field measurements (Higgins et al. 2003). It will also be worth to investigate whether

397 adaptive asymptotic methods, as those proposed by Klein et al. (2001) or the Lagrangian scale-
 398 dependant models for the subgrid scales in Large Eddy Simulations (Bou-Zeid et al. 2004), afford
 399 better energy balances, i.e. the spatial and temporal average of $\mathcal{D}_\ell(\vec{u}, b)$ is closer to zero.

400 It is evident that the resolution plays an important role in determining spurious energy fluxes by
 401 looking at the difference in the $\mathcal{D}_\ell(\vec{u}, b)$ indicator near the ground (NCAR vs ERA-Interim re-
 402 analysis). However, it is positively surprising that the average spatial and vertical structure of the
 403 indicators is very similar in both reanalysis.

404 Several waves phenomena in the atmospheric - gravity and Rossby waves - are tied to these hor-
 405 izontal density variations, and are associated with energy conversion between available potential
 406 and kinetic energy. One key question is whether this diagnostic may incorrectly assess such en-
 407 ergy conversion as an energy transfer across scale. For future research directions, it might be worth
 408 applying the diagnostic to a simple gravity or Rossby wave model.

409 APPENDIX

410 **A1. Derivation of the local Duchon-Robert equation for Boussinesq equations**

411 We start from the Boussinesq equation Eqs. (2) for the buoyancy perturbation and write it at two
 412 different position, \vec{x} and \vec{x}' for $b(\vec{x})$ and $b' = b(\vec{x}')$ and $u(\vec{x})$ and $u' = u(\vec{x}')$:

$$413 \quad \partial_t b + \partial_j (u_j b) = -N^2 u_z + \kappa \partial_j^2 b, \quad (\text{A1})$$

$$\partial_t b' + \partial_j (u'_j b') = -N^2 u'_z + \kappa \partial_j^2 b', \quad (\text{A2})$$

414 Multiplying the equation (A1) by b' and equation (A2) by b and adding the results we obtain

$$\partial_t (bb') + b \partial_j (b' u'_j) + b' \partial_j (b u_j) = -N^2 (b' u_z + b u'_z) + \kappa (b' \partial_j^2 b + b \partial_j^2 b'). \quad (\text{A3})$$

415 To simplify the equation, we can write the diffusive term as

$$b' \partial_j^2 b + b \partial_j^2 b' = \partial_j^2 b b' - 2 \partial_j b \partial_j b', \quad (\text{A4})$$

416 while the nonlinear can be written as

$$b \partial_j b' u'_j + b' \partial_j b u_j = b \delta u_j \partial_j b' + \partial_j b u_j b', \quad (\text{A5})$$

417 where $\delta u_j = u'_j - u_j$ as before. Considering the term $(u'_j - u_j)(b' - b)^2 = \delta u_j (\delta b)^2$, it reads as

$$\delta u_j (\delta b)^2 = b'^2 (\delta u_j) + b^2 (\delta u_j) - 2b'(u'_j - u_j)b \quad (\text{A6})$$

418 Using now the identities $\nabla_r \cdot (\delta \mathbf{u}) = \nabla_r \cdot \mathbf{u}' = 0$, and after some manipulations we have:

$$b \delta u_j \partial_j b' = \frac{1}{2} \left[\partial_j (b'^2 \delta u_j) - \partial_j (\delta u_j (\delta b)^2) \right] + \partial_j b u_j b' \quad (\text{A7})$$

419 Substituting the results from the equations (A4) and (A7) and multiplying both the sides by 1/2
420 and simplifying gives:

$$\begin{aligned} \partial_t \left(\frac{1}{2} b b' \right) + \frac{1}{2} \partial_j \left((u_j b') b + \frac{1}{2} b'^2 \delta u_j - \kappa \partial_j (b b') \right) &= \frac{1}{4} \nabla_r \cdot \delta \vec{u} (\delta b)^2 - \kappa \partial_j b \partial_j b' \\ &\quad - \frac{N^2}{2} (b u'_z + b' u_z); \end{aligned} \quad (\text{A8})$$

421 Applying the filter operator G_ℓ , and noting $\hat{f} = f * G_\ell$ (* being the convolution), we get:

$$\begin{aligned} \partial_t \left(\frac{1}{2} \hat{b} \hat{b} \right) + \vec{\nabla} \cdot \left(\frac{1}{2} (\vec{u} \hat{b}) b + \frac{1}{4} \widehat{(b^2 \vec{u})} - \frac{1}{4} \widehat{(b^2)} \vec{u} - \kappa \vec{\nabla} \left(\frac{1}{2} \hat{b} \hat{b} \right) \right) \\ = - \frac{1}{4\ell} \int d\vec{r} (\vec{\nabla} G)_l \cdot \delta \vec{u}(r) (\delta b)^2 - \kappa \vec{\nabla} b \cdot \vec{\nabla} \hat{b} - \frac{N^2}{2} (b \hat{u}_z + \hat{b} u_z). \end{aligned} \quad (\text{A9})$$

422 Introducing $E_T^\ell = \hat{b} \hat{b} / 2N^2$, the available potential energy at scale ℓ , and the terms

$$\vec{\mathcal{J}}_T^\ell = \left(\frac{1}{2} (\vec{u} \hat{b}) b + \frac{1}{4} \widehat{(b^2 \vec{u})} - \frac{1}{4} \widehat{(b^2)} \vec{u} - \kappa \vec{\nabla} \left(\frac{1}{2} \hat{b} \hat{b} \right) \right) / N^2, \quad (\text{A10})$$

$$\mathcal{D}_\ell^b = \frac{1}{4\ell} \int d^d r (\vec{\nabla} G)_l \cdot \delta \vec{u}(r) (\delta b)^2 / N^2, \quad (\text{A11})$$

423 we get the equation , Eq (9) of section 2.

424 Now, taking the limit $\ell \rightarrow 0$ and introducing the available potential energy $E_T = b^2/2N^2$, the
 425 equation finally simplifies to:

$$\partial_t E_T + \vec{\nabla} \cdot \left(\frac{1}{2} \vec{u} E_T \right) - \kappa \nabla^2 E_T = -b u_z - \mathcal{D}^b - \kappa (\nabla b)^2 / N^2, \quad (\text{A12})$$

426 with

$$\mathcal{D}^b = \lim_{l \rightarrow 0} \mathcal{D}_\ell^b. \quad (\text{A13})$$

427 The equation for the kinetic energy has been derived in Duchon and Robert (2000), without the
 428 term due to buoyancy which can be simply included, and writes as:

$$\begin{aligned} \partial_t E^\ell + \partial_j \left(\hat{u}_j E^\ell + \frac{1}{2} (u_j \hat{p} + \hat{u}_j p) + \frac{1}{4} \left(\widehat{u^2 u_j} - \frac{1}{4} \widehat{u^2} u_j \right) - v \partial_j E^\ell \right) = \\ - v \partial_j u_i \partial_j \hat{u}_i - \mathcal{D}_\ell + \frac{1}{2} (b \hat{u}_z + \hat{b} u_z), \end{aligned} \quad (\text{A14})$$

429 with \mathcal{D}_ℓ being given by Eq. 6. Introducing the KE spatial flux:

$$\vec{J}_K^\ell = \hat{u} E^\ell + \frac{1}{2} (\vec{u} \hat{p} + \hat{u} p) + \frac{1}{4} \left(\widehat{(u^2 \vec{u})} - \frac{1}{4} \widehat{(u^2)} \vec{u} \right) - v \vec{\nabla} E^\ell, \quad (\text{A15})$$

430 we get Eq. (8) of section 2.

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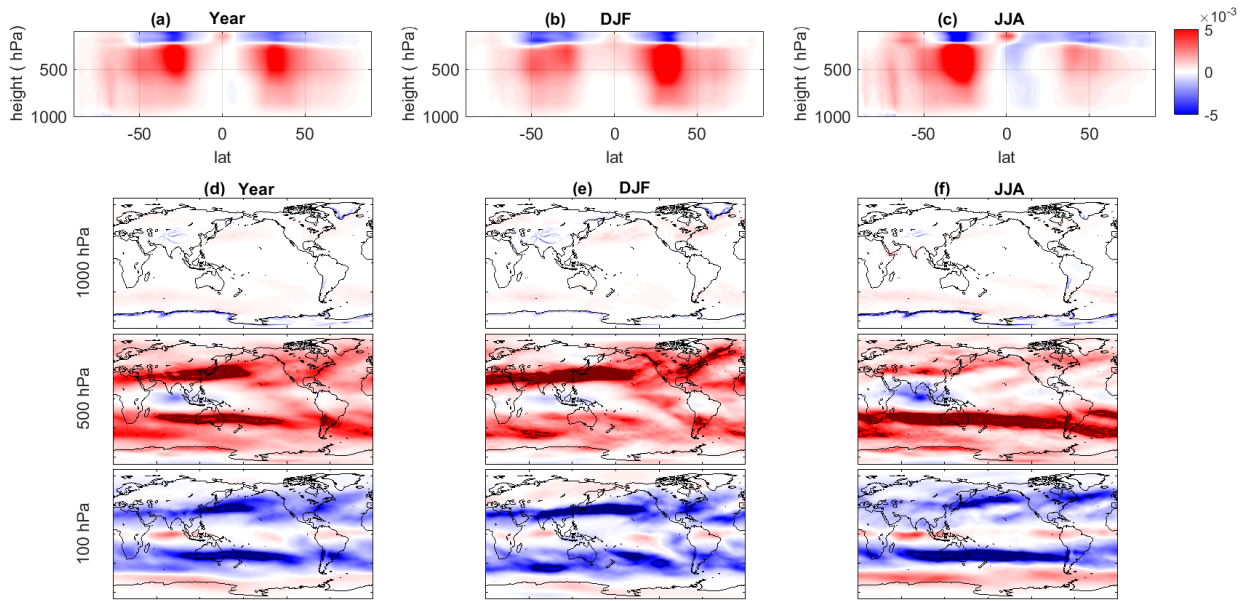
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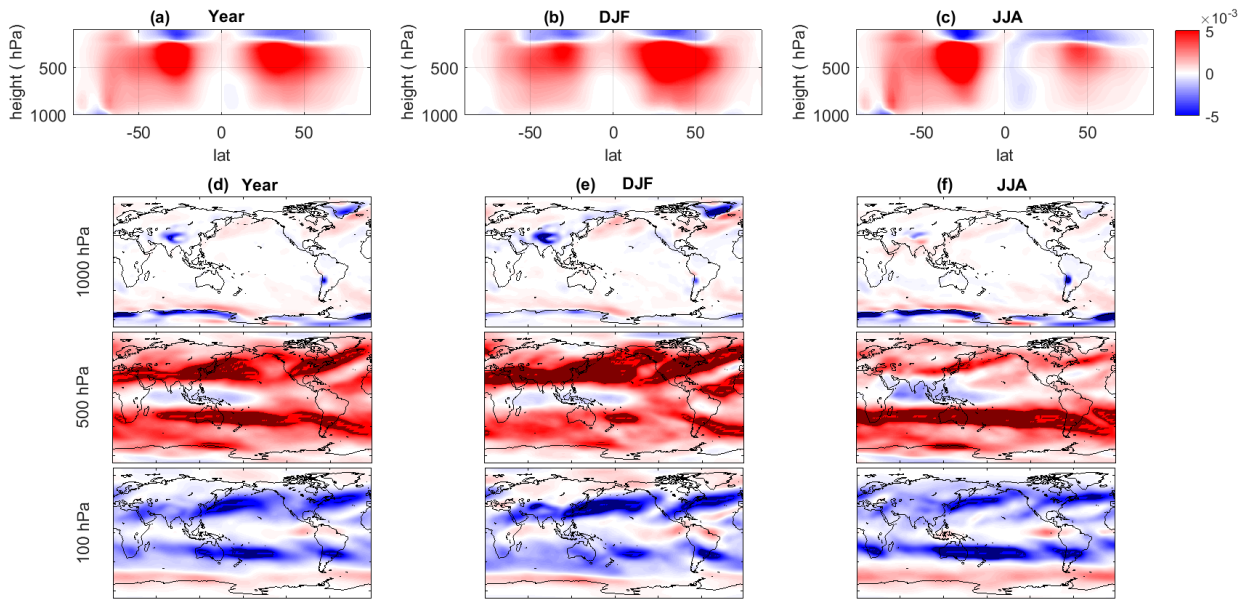
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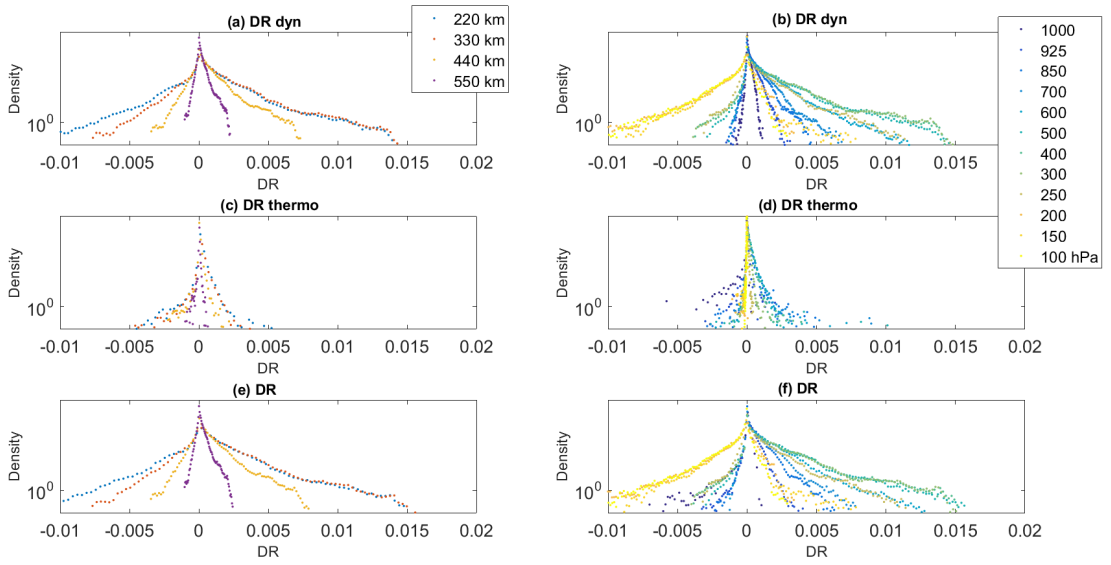
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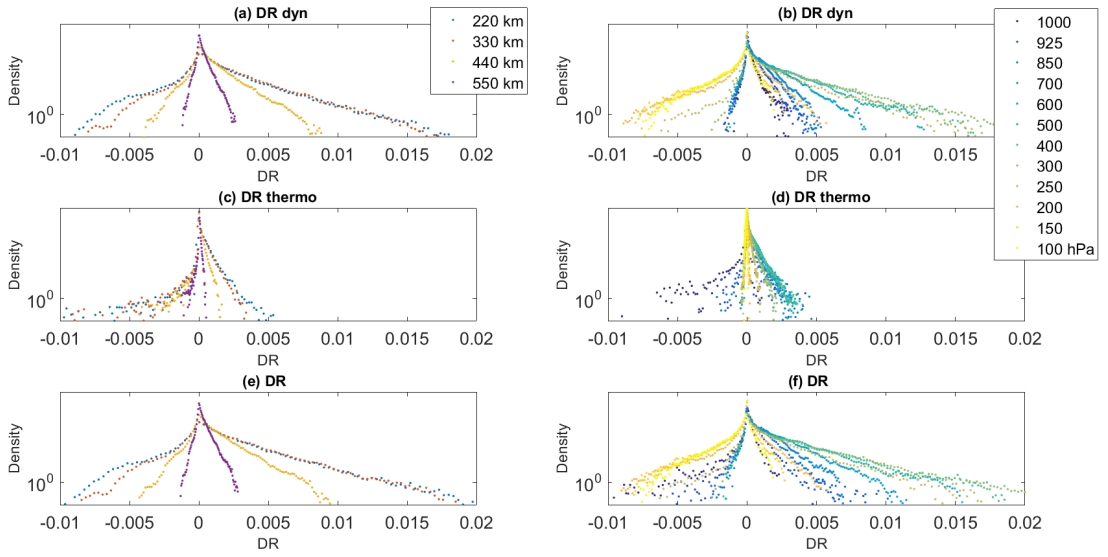
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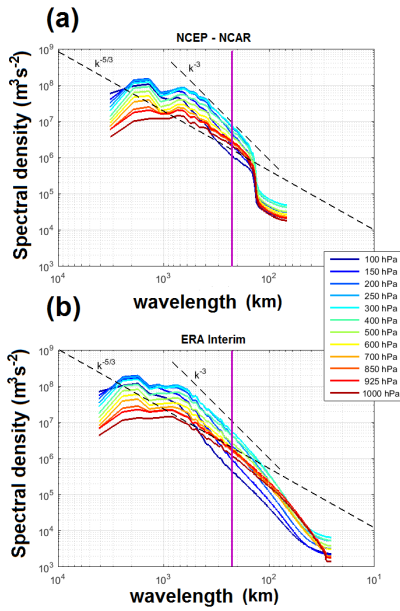
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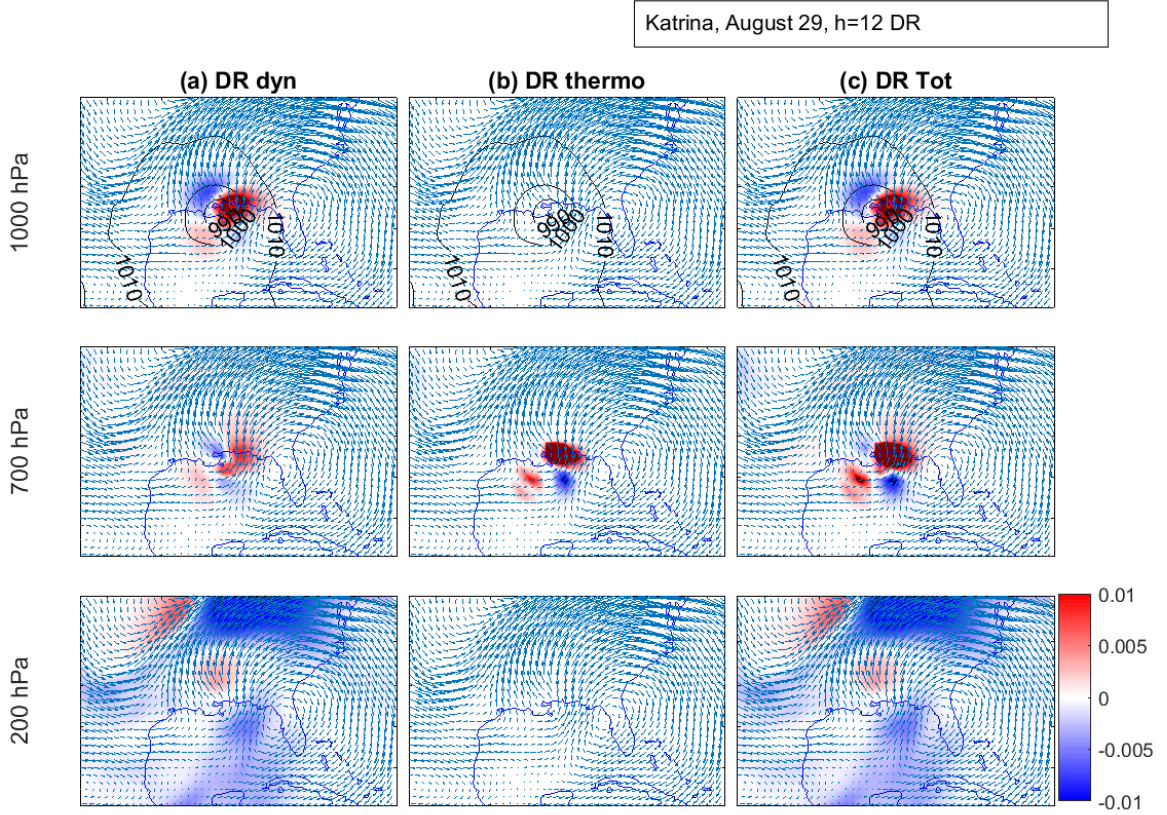
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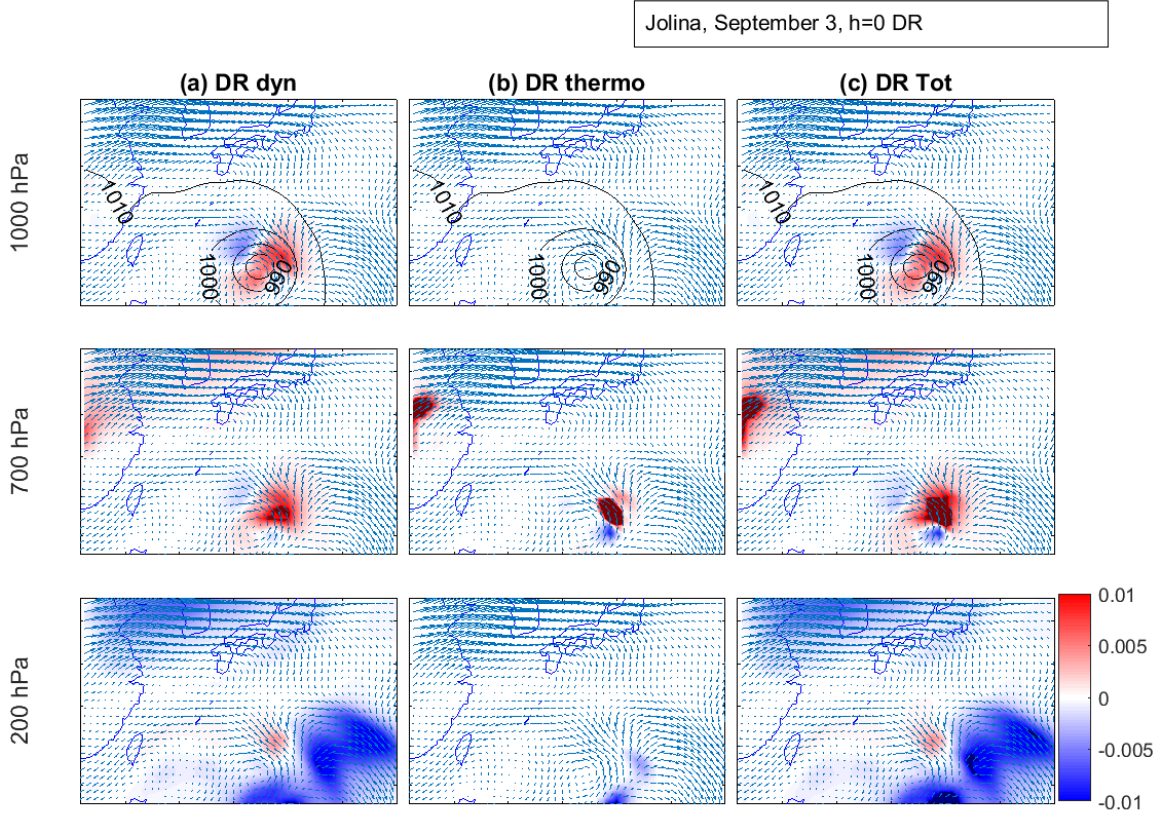
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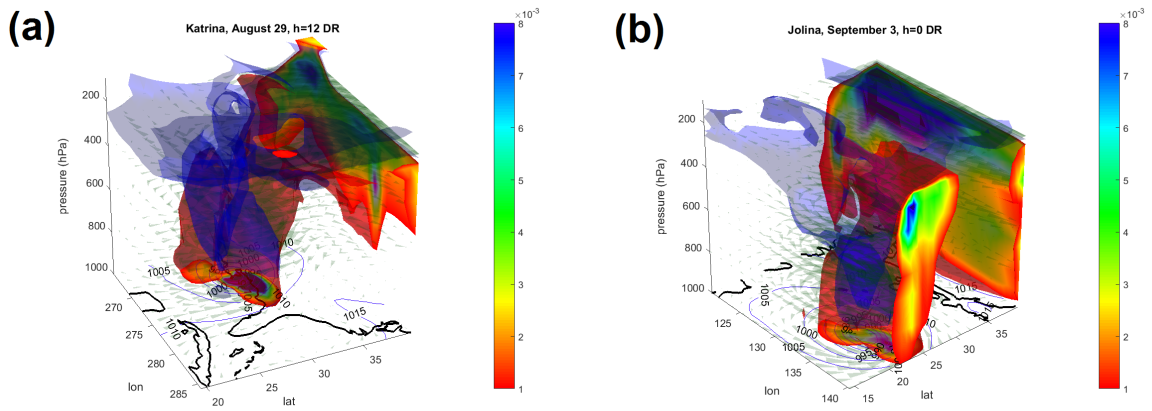
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