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Non-parametric Bayesian estimation for intensity function in Hawkes processes



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Introduction

Hawkes process is the most basic self-exciting counting process. It has been widely used to modelize earthquakes [1-3], financial or economical data but also in neurobiology. The Bayesian estimation of the conditional intensity function has been tackled by [4] in a parametric context. In this work, we propose to consider a non-parametric Bayesian estimation

Hawkes processes

- The Hawkes process $(N_t)_{t \in \mathbb{R}}$ is defined by its conditional intensity

$$\lambda(t) = \nu + \int_{-\infty}^t h(t-u) dN_u = \nu + \sum_{T \in \mathcal{N}, T < t} h(t-T) \quad (1)$$

where

- ν is a positive parameter,
- h a nonnegative function with support on \mathbb{R}^+ and $\int h < 1$
- dN_u is the point measure associated to the process.
- Interpretation** : $\lambda(t)dt = \mathbb{P}(\text{point} \in [t; t+dt] \text{ conditionally to the past before } t)$.
So from (1) : constant rate ν and also *all the previous occurrences condition the apparition of an occurrence at t* .
- Stationarity** :
 - We assume that h has a bounded support $[0, s_{max}]$ such that $\int_0^{s_{max}} h(u)du < 1$
 - This condition guarantees the existence of a stationary version of the process
 - And the number of events on $[T^*, T^* + \Delta]$:

$$E[N^*] = E[N_{[T^*, T^* + \Delta]}] = \Delta^* \frac{\nu}{1 - \int_0^{s_{max}} h(u)du}$$

Likelihood and prior distributions

- Parameters of interest : h and ν
- Observations and likelihood : computation of the likelihood on $[T^* - s_{max}, T^* + \Delta]$

$$\log \ell(N; h, \nu) = \sum_{T_i \in [T^*, T^* + \Delta]} \log \lambda(T_i) - \int_{T^*}^{T^* + \Delta} \lambda(t) dt$$

- Prior distributions

- h step function. $h(t) = \sum_{k=0}^{K-1} \alpha_k \mathbb{1}_{[s_k, s_{k+1})}(t) = l_h \sum_{k=0}^{K-1} \bar{\alpha}_k \mathbb{1}_{[s_k, s_{k+1})}(t)$ with
 - $s_K = s_{max}$ known and $s_0 = 0$
 - $l_h = \int_0^{s_{max}} h(t) dt < 1$
 - $\sum_{k=0}^{K-1} \bar{\alpha}_k (s_{k+1} - s_k) = 1$
- $l_h \sim \mathcal{B}(a_h, b_h)$
- Prior on s :

$$\left(\frac{s_0}{s_{max}}, \dots, \frac{s_K}{s_{max}} \right) \sim \text{Dir}(a_1, \dots, a_K) \quad (2)$$
- Prior on $\bar{\alpha}_1, \dots, \bar{\alpha}_K$: $\bar{\alpha}_0(s_1 - s_0), \dots, \bar{\alpha}_K(s_K - s_{K-1}) \sim \text{Dir}(b_1, \dots, b_K)$
- $\nu \sim \Gamma(a_\nu, b_\nu)$

Sampling of the posterior distribution : MCMC

At iteration (l) ,

- Sampling of ν with a classical random walk on $\log \nu$ to guaranty the positivity
- Sampling of l_h with a random walk on $[0, 1]$

$$l_h^c = \Phi^{-1}(\Phi(l_h)^{l-1} + \varepsilon), \quad \varepsilon \sim 0.5\mathcal{N}(0, \rho_1) + 0.5\mathcal{N}(0, \rho_2), \quad \rho_1 < \rho_2$$

- Sampling of $\bar{\alpha}_k$

- $k \sim \mathcal{U}_{\{0, \dots, K-1\}}$ and $\bar{\alpha}_k^c = \bar{\alpha}_k^{l-1} + \xi$, with $\xi \sim \sum_{r=1}^4 0.25\mathcal{N}(0, \rho_r)$
- Correction of the others $\bar{\alpha}$ to keep $\sum_{k=0}^K \bar{\alpha}_k (s_{k+1} - s_k) = 1$:

$$\Delta_k = \bar{\alpha}_k^c (s_{k+1} - s_k) - \bar{\alpha}_k^{l-1} (s_{k+1} - s_k)$$

$$\bar{\alpha}_j^c (s_{j+1} - s_j) = \bar{\alpha}_j^{l-1} (s_{j+1} - s_j) - \frac{\Delta_k}{K-1} \quad \forall j \neq k.$$

- Sampling of s

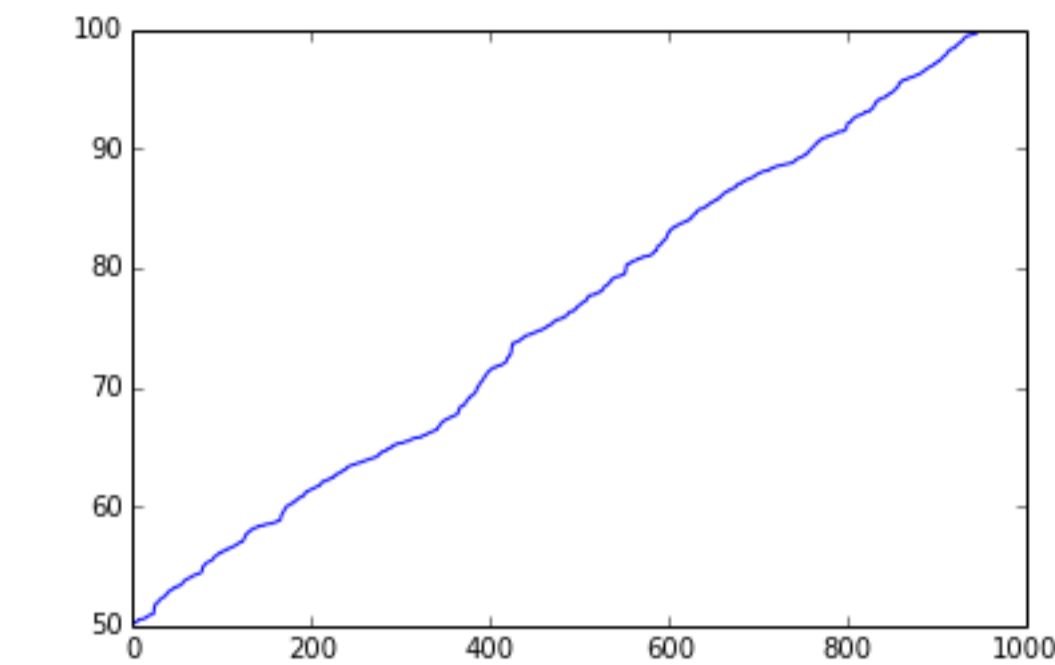
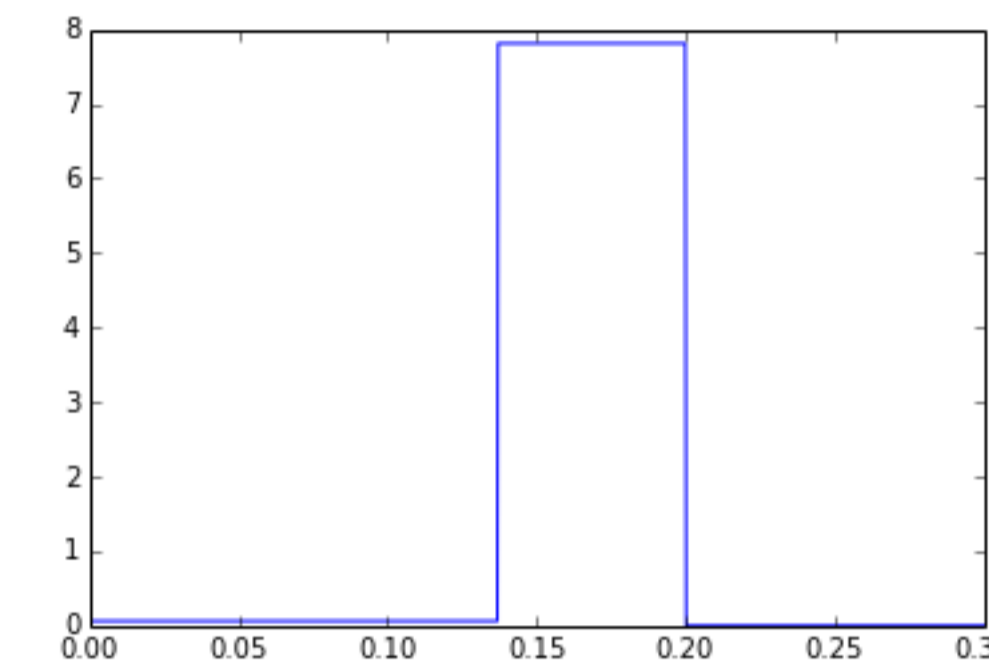
- $k \sim \mathcal{U}_{\{1, \dots, K-1\}}$ and $s_k^c \sim \text{Triang}(s_{k-1}^{l-1}, s_l - 1, s_{k+1}^{l-1})$
- Correction of the $\bar{\alpha}_{k-1}^c$ and $\bar{\alpha}_k^c$ to keep the integral equal to 1:

$$\bar{\alpha}_{k-1}^c = \bar{\alpha}_{k-1}^{l-1} \frac{s_k^{l-1} - s_{k-1}^{l-1}}{s_k^c - s_{k-1}^{l-1}} \quad \text{and} \quad \bar{\alpha}_k^c = \bar{\alpha}_k^{l-1} * \frac{s_{k+1}^{l-1} - s_k^{l-1}}{s_{k+1}^c - s_k^c}$$

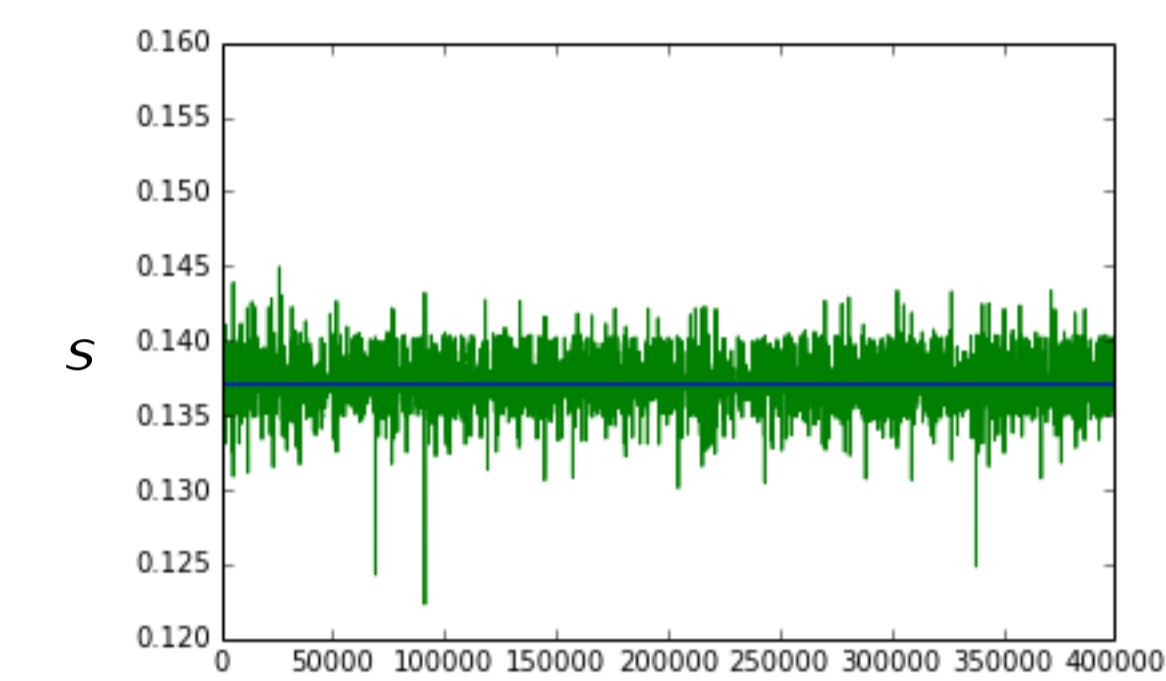
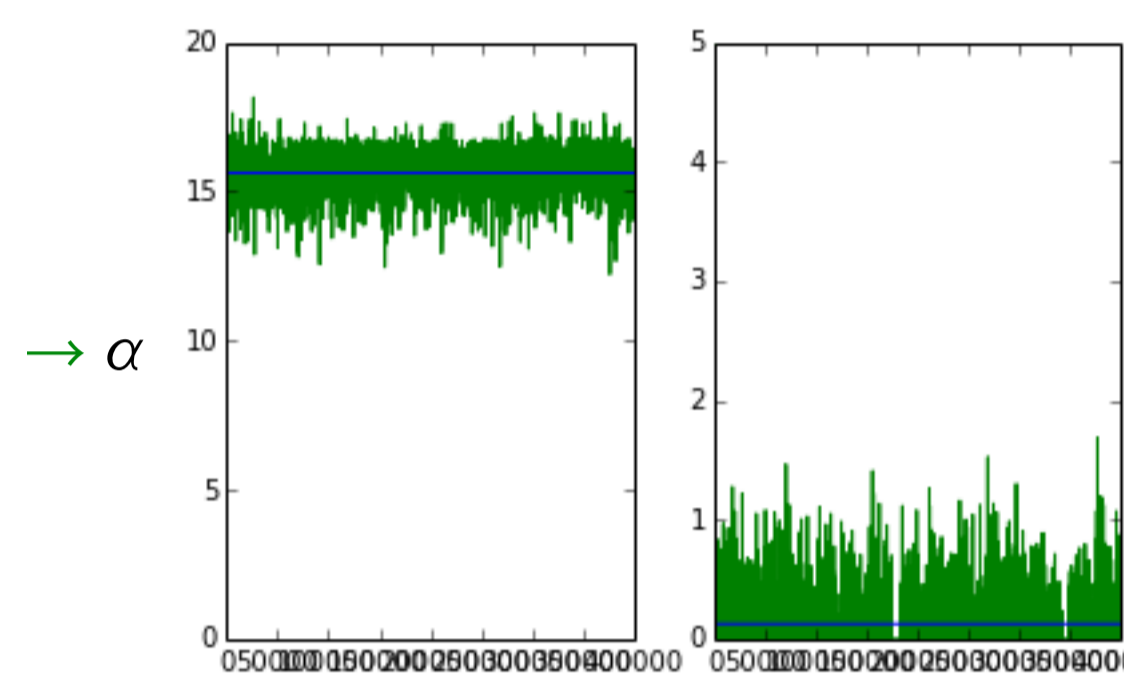
First numerical results

$T^* = 50 \quad \Delta = 50, \quad l_h = 0.5, \quad \nu = 10$

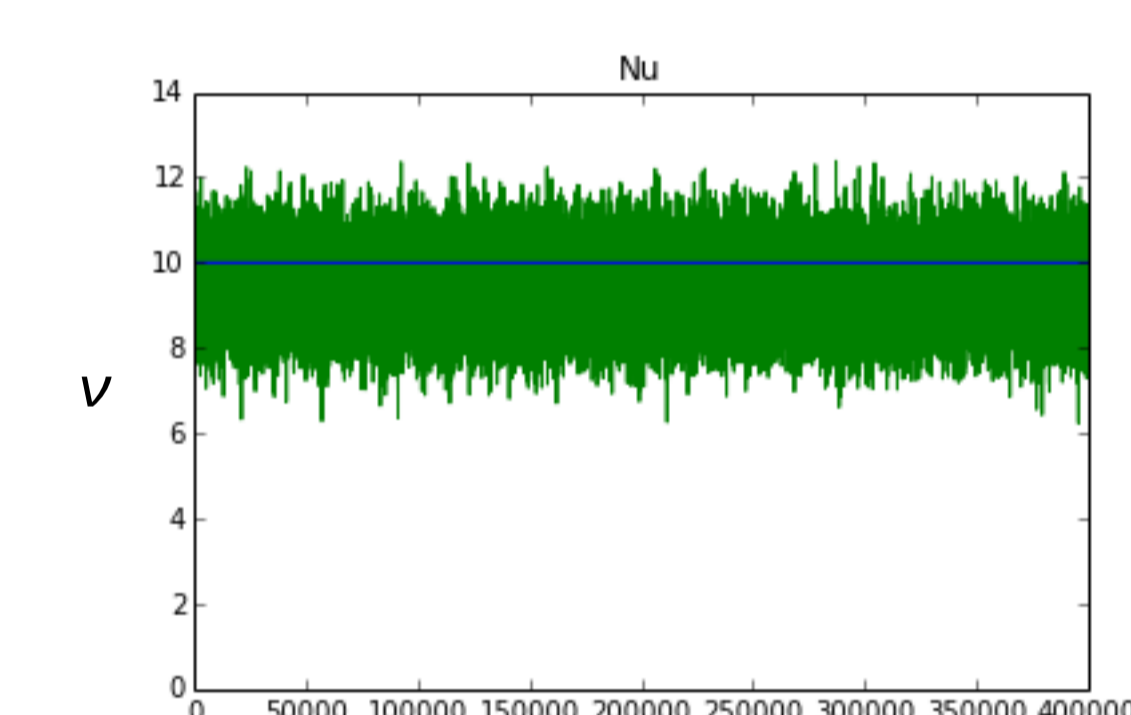
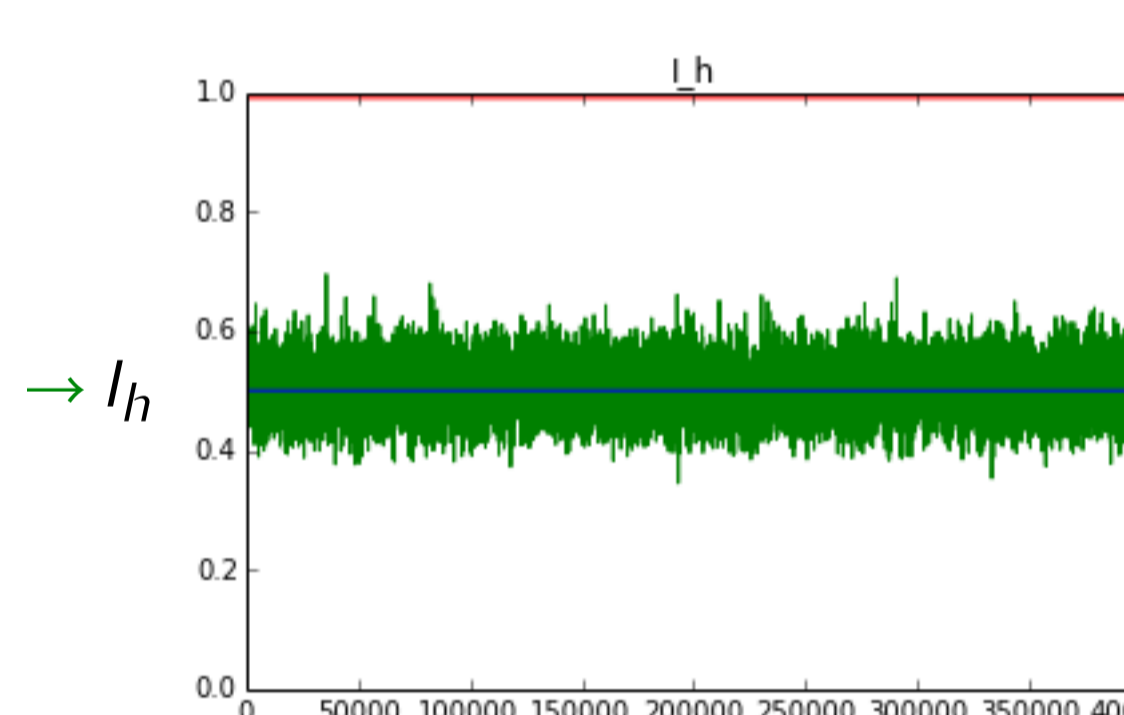
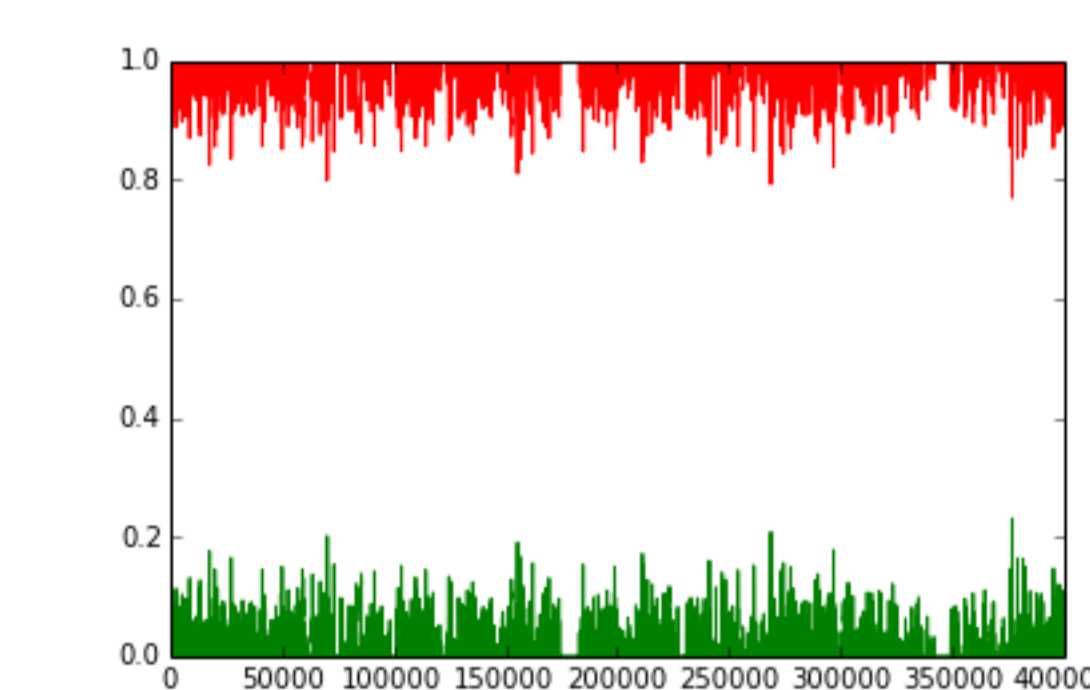
- Data



- MCMC - 200 000 iterations



$\rightarrow \bar{\alpha}_k^{(l)}(s_{k+1}^{(l)} - s_k^{(l)})$



Perspectives of work

- Unknown K : reversible jump
- $\pi(\bar{\alpha}_k = 0) > 0$
- $\pi(\bar{\alpha}_k < 0) > 0$ to modelize inhibition
- Multi-dimensional Hawkes processes to modelize interactions

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