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Deterioration estimation for remaining useful lifetime prognosis in a friction drive system

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Abstract: This paper presents a method for on-line estimation of contact surfaces deterioration in a friction drive system. It is based on a recent developed linear parameter-varying model which includes both the mechanical device and the actuator deterioration dynamics in the same framework. In this work an Extended Kalman Filter is explored to estimate the current state of deterioration assuming the knowledge of the operating conditions, input signals, and sensor information. A simulated example illustrates the potential integration of the deterioration estimation into the prognostics of Remaining Useful Lifetime.

Keywords: Remaining lifetime prediction, nonlinear observers and filter design, fault detection and diagnosis, parameter-varying systems, mechatronic systems, modeling

1. INTRODUCTION

Nowadays, manufacturers or end users are becoming increasingly motivated to manage the complete life-cycle of an asset using proactive maintenance strategies. In this framework, Reliability Adaptive Systems (RAS), are a kind of approaches which can autonomously manage their state of health according to their current condition and by considering the influence of the system input over them, see for instance Meyer and Sextro (2014) and Rakowsky (2006). Management of the health state of a component needs an acceptable and efficient diagnosis of the current state of deterioration, which is often difficult due to the stochastic nature of deterioration phenomena. Existent solutions demand high computational costs which increase the difficulty to implement on-line condition monitoring with high accuracy.

Deterioration estimation on electromechanical devices represents a key issue for condition-based and predictive maintenance. Estimation of any state of a dynamical system often requires the availability of a mathematical model and enough accurate measurements to perform a reliable estimate. In several motion applications based on friction, the measurements providing angular positions, speeds and/or accelerations are often available. In addition, pure mechanical models can be considered as well-known models and their parameters are relatively simple to obtain. However, dynamical models which can characterize the deterioration phenomena (useful on state estimation) are rarely presented in the literature.

A recent model of deterioration, on friction drives systems, has been proposed in Rodriguez Obando et al. (2016a). Its potential use for fault-detection and for Remaining Useful Lifetime (RUL) prediction has been presented in Rodriguez Obando et al. (2016b). That model can be used

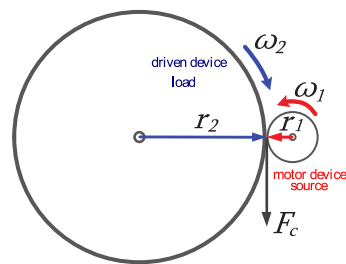


Fig. 1. Roller-on-tire system.

to estimate the state of the deterioration of a friction drive system. In particular, the quality of the contact surfaces (which allows the transmission of mechanical power) decreases as the deterioration increases. At the same time, the deterioration model describes the rate of change of the quality of the contact surfaces. This rate of change depends on the current state of health, the current operational conditions and the characteristics of the material. Since the model uses a few number of unknown parameters describing the deterioration phenomena, the state estimation process can be performed with low computational costs.

In this paper, it is proposed to use an augmented non-linear model for simultaneously estimate the current state of deterioration and the mechanical system states of a friction drive system, here a roller-and-tire actuator. The augmented model includes *i*) a deterministic model (mechanical motion equations), *ii*) a dynamical model of deterioration and *iii*) an unknown input model (modeling the rate of change of the material properties).

The remainder of this paper is organized as follows. Section 2 presents the description of the roller-and-tire actuator model. Section 3 presents the dynamical model of the actuator deterioration and Section 4 explains the un-

known input model and the design process of an Extended Kalman Filter using the augmented non-linear system model, for a suitable estimation of the deterioration of the friction drive. In Section 5, the performance of the proposed observer is evaluated. Lastly, in Section 6 a simulated example illustrates the potential integration of the deterioration estimation into the prognostics of the Remaining Useful Lifetime. Conclusions and future work are given in Section 7.

2. DESCRIPTION OF ROLLER-ON-TIRE SYSTEM

The considered system is called roller-on-tire actuator. It is shown in Fig. 1 and its nomenclature in Table 1. This is a friction drive system composed by a driver device (dc motor) and a driven device (wheel). The actuator is modeled as an Uncertain Linear System in a previous work, see Rodriguez Obando et al. (2016b). As depicted in Fig. 1, both devices are affected by the contact force F_c . It is produced by the motor and causes a torque which drives the wheel and depends on the tangential speeds produced for both motor and wheel, denoted as v_1 and v_2 respectively. Therefore, the main assumption in the model is that $F_c(t)$ is proportional to the relative tangential speed at the contact level, denoted Δ_v . That is, $F_c(t) = \alpha \Delta_v = \alpha(r_1\omega_1 - r_2\omega_2)$, where $\Delta_v = v_1(t) - v_2(t)$ and $\alpha \geq 0$ is an uncertain parameter, called here as the *contact quality coefficient*.

Using Newton's laws of motion, the roller-on-tire dynamics can be written in the state space representation:

$$\dot{x} = A(\alpha)x + Bu \quad (1)$$

$$y = Cx \quad (2)$$

where $x := [\omega_1(t) \ \omega_2(t)]^T$ is the system state, $u = I(t)$ is the control input (the electrical motor current) and α stands for the uncertain parameter (or the scheduling parameter in the case of a linear parameter varying model interpretation), with matrices:

$$A(\alpha) = \begin{bmatrix} (-\alpha r_1^2 - B_1)/J_1 & \alpha r_1 r_2/J_1 \\ \alpha r_2 r_1/J_2 & (-\alpha r_2^2 - B_2)/J_2 \end{bmatrix}, \quad (3)$$

$$B = \begin{bmatrix} K_m/J_1 \\ 0 \end{bmatrix} \quad (4)$$

and C an identity matrix that means that both: angular speed of the motor and angular speed of the driven device are measured, i.e. $y = [\omega_1(t) \ \omega_2(t)]^T$.

Table 1. Nomenclature

Symb.	Value	Units	Physical meaning
v_1		[m/s]	Tangential speed of the motor
v_2		[m/s]	Tangential speed of the driven device
ω_1		[rad/s]	Angular speed of the motor
ω_2		[rad/s]	Angular speed of the driven device
$\dot{\omega}_1$		[rad/s ²]	Angular acceleration of the motor
$\dot{\omega}_2$		[rad/s ²]	Angular acceleration of the driven device
I		[A]	Electrical motor current
r_1	0.0315	[m]	External radius of the motor
r_2	0.35	[m]	External radius of the driven device
B_1	6.36x10 ⁻³	[Kgm ² /s]	Viscous damping coefficient of the motor
B_2	1.76x10 ⁻³	[Kgm ² /s]	Viscous damping coefficient of the driven device
J_1	3.47x10 ⁻⁴	[Kgm ²]	Moment of inertia of the motor
J_2	0.2	[Kgm ²]	Moment of inertia of the driven device
K_m	0.0477465	[V s/rad]	Motor back-electromotive force constant

3. DYNAMICAL MODEL OF DETERIORATION

3.1 Definition of deterioration for the roller-on-tire system

The deterioration is defined here as a measure of the loss in the actuator ability to transfer power to the load device. The power performed by the motor is transformed into mechanical power on the load side by means of the contact force F_c .

In this paper, the parameter α characterizes the quality of the contact (e.g. the inter-surface adhesion and the surface roughness) between both rotational devices. In addition, we consider that this parameter will monotonically decrease in time for modeling the deterioration of the roller-on-tire actuator.

3.2 Dissipation-energy based model of deterioration

The dissipated power at the contact level can be computed as $P_c(t) = \alpha(r_1\omega_1 - r_2\omega_2)^2 = \alpha \Delta_v^2$. The dissipated energy could be considered as an image of the heat and the material worn at the contact level during traction. This assumption is very similar to the Archard's equation that is more commonly used in railway industry for wear prediction (see Bevan et al. (2013) and Cremona et al. (2016)). Thus, an index of the deterioration is obtained:

$$D(t) := \int_0^t P_c(t)dt = \int_0^t \alpha(r_1\omega_1 - r_2\omega_2)^2 dt \quad (5)$$

In addition, by assumption, the contact quality coefficient $\alpha(t)$ decreases as $D(t)$ increases. Thus, a first order linear variation of α with respect to D , with initial value $\alpha(0) > 0$, is defined as:

$$\alpha(t) = -mD(t) + \alpha(0) \quad (6)$$

where m and $\alpha(0) \in \mathbb{R}^+$, and are considered as *unknown* parameters, but belonging to a given known interval.

Therefore, using (5) and (6) we can compute the dynamics of the parameter $\alpha(t)$, as follows:

$$\dot{\alpha}(t) = -mp(x) \alpha(t) \quad (7)$$

where $p(x) \geq 0$, called here the sliding factor, is given by $p(x) := (r_1\omega_1 - r_2\omega_2)^2 = \Delta_v^2$. The contact quality deterioration-rate (7), depends on the relative tangential speed, which could be controlled by the input $u = I(t)$ if the uncertain system (1)-(2) is *controllable*.

From (6) we obtain the *normalized deterioration*, defined as $\bar{D}(t) := (m/\alpha(0))D(t)$, where $0 \leq \bar{D}(t) \leq 1$. Thus, for a given initial condition $\alpha(0)$, $\bar{D}(t)$ can be computed at every time-instant using $\alpha(t)$:

$$\bar{D}(t) = 1 - \frac{\alpha(t)}{\alpha(0)} \quad (8)$$

The deterioration $\bar{D}(t)$ tends to 1 as the quality coefficient $\alpha(t)$ tends to 0. This normalized deterioration has the advantage to depend only on $\alpha(t)$ and $\alpha(0)$. Estimation of those variables is unavoidable for estimation of the current condition of the friction drive deterioration, but also for the prediction of its RUL. The latter point requires the knowledge of the possible evolution of the contact quality coefficient $\alpha(t)$. This can be possible by using the dynamics (7) but it requires an estimation of the current and possible futures values of the parameter m .

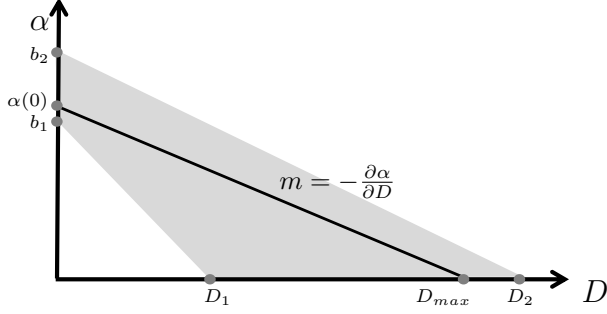


Fig. 2. α as a function of D .

The shaded area in Fig. 2 depicts the domain of possible trajectories of the parameter $\alpha(t)$ for different possible values of the parameter m , with an initial condition $\alpha(0) \in (b_1, b_2)$, $\alpha(0) > 0$. The bold line corresponds to a case where m is constant and the reached maximum level of deterioration $D = D_{max}$. Remark that this value can be a priori computed as $D_{max} = \alpha(0)/m$.

In this paper, the main problem is to estimate these two parameters for condition monitoring and RUL prediction. A suitable non-linear state-observer can be designed for this goal. This is addressed in Section 4.

4. DETERIORATION ESTIMATION

Using (1)-(2) and (7), consider the augmented system:

$$\dot{x} = A(\alpha) x + B u \quad (9)$$

$$\dot{\alpha} = -m p(x) \alpha \quad (10)$$

$$\dot{m} = 0 \quad (11)$$

and the system output $y = x$. Suppose this nonlinear system is observable, then it is possible to design an *Extended Kalman Filter* to estimate the states x , the contact quality coefficient α and the parameter m , by considering the knowledge of the input $u = I(t)$ and the available signals $\omega_1(t)$ and $\omega_2(t)$.

4.1 Observability properties of the system

Considering the fact that parameter α affects the matrix $A(\alpha)$ in an affine way in (3), and the availability of measurements $y = x$, the estimation of the state α can be possible. The estimation of α surely requires enough degree of variation concerning y and u . The latter follows the notion of Persistence of Excitation, see for instance Besançon (2007). Thus, the electrical current $u = I(t)$ has to be different from zero and suitably varying in time to increase the observability of the state α in the nonlinear system (9)-(11).

In terms of the observability of the parameter m , remark that it also appears into the dynamical equation characterizing the evolution of α in (10). There, the variation of the parameter α depends on the parameter m in an affine way. Thus, this parameter can be also estimated using previous estimations of α and its time-derivative. As a consequence, the estimation of m requires persistence of the excitation on α . In other words, since m will be used for predicting the RUL, it is necessary to deteriorate the system to better estimate its future behavior.

4.2 Synthesis of an Extended Kalman Filter

Defining the vector state of the augmented system as $\mathbf{x} := [\omega_1(t) \ \omega_2(t) \ \alpha(t) \ m]^\top$, the control input $u = I(t)$, and assuming that at every time instant $\omega_1(t)$ and $\omega_2(t)$ are available from the sensors, the state transition and the system output in continuous time are respectively:

$$\dot{\mathbf{x}} = f(\mathbf{x}) + B u + \mathbf{w} \quad (12)$$

$$\mathbf{y} = \mathbf{C} \mathbf{x} + \mathbf{v} \quad (13)$$

with

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (14)$$

and where \mathbf{w} and \mathbf{v} are the process and measurement noises which are both assumed to be Gaussian noises with zero mean and covariance Q and R respectively.

In order to synthesize an Extended Kalman filter, the following covariance matrices are selected:

$$Q = \text{diag}([0 \ 0 \ 0 \ \sigma_m^2]); \quad R = \text{diag}([\sigma_1^2 \ \sigma_2^2]) \quad (15)$$

where σ_m^2 stands for the disturbance variance affecting the behavior of the state m . The symbols σ_1^2 and σ_2^2 represent the sensor noise variances in speed sensors measuring ω_1 and ω_2 , respectively.

The chosen matrix Q takes into account the fact that in the model (9)-(11) the state m (a parameter that models the speed of the deterioration) can be affected by neglected and/or unmodelled dynamics. In other words, we accept that the model is far from the real process but this model error is only associated to the misknowledge on the behavior of the variable m . On the other hand, the matrix R considers that both sensors are affected by the same level of measurement noise, and this level noises are relatively smaller than possible state disturbances and/or model errors. The estimation process is performed as follows: assuming the availability of discrete-time measurements at every time-instant, with a sample time t_s , the a priori prediction of the state estimate can be calculated using the continuous-time state transition model:

$$\hat{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1|k-1}) + B u_{k-1} \quad (16)$$

and the estimated output: $\hat{\mathbf{y}}_{k|k-1} = \mathbf{C} \hat{\mathbf{x}}_{k|k-1}$

The prediction of the a priori covariance estimate matrix \mathbf{P} is calculated at every time instant as:

$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k-1} \mathbf{P}_{k-1|k-1} \mathbf{F}_{k-1}^\top + Q \quad (17)$$

where \mathbf{F}_{k-1} is the Jacobian of the function $f(\mathbf{x})$ in discrete time. That is, $\mathbf{F}_{k-1} = \exp(F t_s)$ with

$$F = \left. \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_{k|k-1}} \quad (18)$$

the Jacobian of the function $f(\mathbf{x})$ in continuous time, calculated as:

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & 0 \\ F_{21} & F_{22} & F_{23} & 0 \\ F_{31} & F_{32} & F_{33} & F_{34} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (19)$$

where $F_{11} = -(\alpha r_1^2 + B_1)/J_1$, $F_{12} = (\alpha r_1 r_2)/J_1$, $F_{13} = (r_1 r_2 w_2 - r_1^2 w_1)/J_1$, $F_{21} = (\alpha r_1 r_2)/J_2$, $F_{22} = -(\alpha r_2^2 +$

$B_2)/J_2$, $F_{23} = (r_1 r_2 w_1 - r_2^2 w_2)/J_2$, $F_{31} = -2\alpha m r_1 (r_1 w_1 - r_2 w_2)$, $F_{32} = 2\alpha m r_2 (r_1 w_1 - r_2 w_2)$, $F_{33} = -m(r_1 w_1 - r_2 w_2)^2$, and $F_{34} = -\alpha(r_1 w_1 - r_2 w_2)^2$.

The innovation covariance, denoted \mathbf{S}_k , will be:

$$\mathbf{S}_k = \mathbf{C}\mathbf{P}_{k|k-1}\mathbf{C}^\top + R \quad (20)$$

and the Kalman Gain:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{C}^\top\mathbf{S}_k^{-1} \quad (21)$$

Considering the prediction error: $\tilde{\mathbf{e}}_k = \mathbf{y}_k - \mathbf{C}\hat{\mathbf{x}}_{k|k-1}$ (the innovation), the updating of the state estimate is calculated as $\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\tilde{\mathbf{e}}_k$.

Finally, the a posteriori covariance matrix can be updated with $\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{C})\mathbf{P}_{k|k-1}$.

Then, the estimation process re-starts again, by considering all the updated and estimated state vectors and covariance matrices. The estimation process requires the initialization of the estimated state at instant $k = 0$, and an initial a priori covariance matrix $\mathbf{P}_{0|0}$.

4.3 Stochastic bounds for the state estimation

Define the estimation error as $\tilde{\mathbf{x}}_{k|k} := \mathbf{x}_k - \hat{\mathbf{x}}_{k|k}$. Considering that the expected value of $\tilde{\mathbf{x}}_{k|k} \in \mathbb{R}^n$ is equal to zero, its covariance equal to $\mathbf{P}_{k|k}$ and $\mathbf{c} > 0$ any real number, we can use the multidimensional *Chebyshev's inequality*:

$$\Pr\left(\tilde{\mathbf{x}}_{k|k}^T \mathbf{P}_{k|k}^{-1} \tilde{\mathbf{x}}_{k|k} > \mathbf{c}^2\right) \leq \frac{n}{\mathbf{c}^2} \quad (22)$$

for computing a stochastic ellipsoidal set and then compute bounds of the state estimation error. Inequality (22) can be used when there is not knowledge of the probability distribution of the estimation error $\tilde{\mathbf{x}}_{k|k}$. Otherwise, it is possible to use a more accurate description, for instance in the case where the estimation error presents a normal distribution (that corresponds to the case studied in this paper), it is possible to bound the estimation error (with a given probability), as follows:

$$\Pr\left(\tilde{\mathbf{x}}_{k|k}^T \mathbf{P}_{k|k}^{-1} \tilde{\mathbf{x}}_{k|k} \leq \mathbf{c}^2\right) = \text{erf}\left(\frac{\mathbf{c}}{\sqrt{2}}\right) \quad (23)$$

where $\text{erf}(\cdot)$ corresponds to the Gauss error function.

Even if there is a probability that some trajectories of the estimation error $\tilde{\mathbf{x}}_{k|k}$ go out this set, we can use this set to establish an interval of possible values of the state \mathbf{x}_k with a given probability.

Using geometrical properties of the ellipsoids, bounds on the estimation error $\tilde{\mathbf{x}}_{k|k}$, denoted $\bar{\mathbf{x}}_k$, can be obtained as follows:

$$\bar{\mathbf{x}}_k = \text{diag}\left(\mathbf{P}_{k|k}^{1/2}\right)\mathbf{c} \quad (24)$$

These bounds together with the estimated value of the system state $\hat{\mathbf{x}}_{k|k}$ will be used as initial conditions for predicting the RUL. In particular, for the element corresponding to the estimation of the parameter α we have:

$$\hat{\mathbf{x}}_{k|k}(3) - \bar{\mathbf{x}}_k(3) \leq \alpha_k \leq \hat{\mathbf{x}}_{k|k}(3) + \bar{\mathbf{x}}_k(3) \quad (25)$$

with a probability greater than $1 - (n/\mathbf{c}^2)$, for an unknown probability distribution, or equal to $\text{erf}(\mathbf{c}/\sqrt{2})$ for normal probability distributions. Here $n = 4$ because $\mathbf{x} \in \mathbb{R}^4$. That means that with $\mathbf{c} = 3$ we can expect that the real value is within the interval given by the estimates with a probability higher than 55.5% (for unknown distribution) or 99.7% (for normal distribution).

4.4 Checking consistence of the innovations

Since in practice we can not measure the performance of the observer with respect to the state error measures (since we do not know the true state values), we can check if the observer is performing correctly in terms of the innovation.

It is known that if the observer is working correctly then is zero mean and white $\tilde{\mathbf{e}}_k$ with a covariance \mathbf{S}_k . Thus, we can verify that the observer is consistent by applying the following two procedures: i) check that the innovations are consistent with their covariance and ii) check that the innovations are unbiased and white noise. The first test can be performed by using the following bounds on the innovation signal:

$$\bar{\mathbf{e}}_k = \text{diag}\left(\mathbf{S}_{k|k}^{1/2}\right)\mathbf{c} \quad (26)$$

where $\mathbf{c} > 0$ can be chosen to guarantee that the innovations will be bounded by the above values with a given probability.

If those tests are not verified, it is possible that there exist an under-estimate or an over-estimate of the chosen variances of the disturbances. Thus, the chosen matrices \mathbf{Q} and \mathbf{R} have to be reformulated or adapted.

5. OBSERVER PERFORMANCE EVALUATION

Some scenarios are built to validate in simulation the obtained estimations of the parameters α and m . Table 1 summarizes the used system parameters.

5.1 Predefined operating conditions.

In this case the observer is tested in scenarios with known and predefined operating conditions. The purpose of these scenarios is to evaluate the quality of the estimates $\hat{\alpha}$ and \hat{m} with known variations of the input. Here the chosen input signal $I(t)$ is a square wave with a predefined amplitude (20A) and predefined values of duty cycle (period: 2s with the 50% in this case) and the parameters of the model (9)-(11) are considered as constant parameters. For these scenarios $\alpha(0) = 10$ and nominal $m = 0.01$ were chosen.

5.2 Tuning the matrices \mathbf{Q} and \mathbf{R} .

The observer is designed in the framework of the Sections 4.3 and 4.4, in order to build a consistent design. Assuming a known variance of the measurement noises \mathbf{v} , matrix \mathbf{R} is selected as in (15) with $\sigma_1^2 = 1.0063 \times 10^{-8}$ and $\sigma_2^2 = 1.0083 \times 10^{-10}$.

Concerning the matrix \mathbf{Q} in (15), the chosen value for σ_m^2 is obtained by assuming possible abrupt variations on values of m . This variations can be modeled as impulse disturbances (a discrete-time Dirac delta), affecting the dynamics of the state m and taking values in the interval $(a, b) = (0.00, 0.02)$. If we assume, for instance, that these disturbances are random variables with an uniform probability distribution, for all $k > 0$, their variance can be calculated as:

$$\sigma_m^2 = \text{var}(\delta(k)) = \frac{1}{12}(a - b)^2 \quad (27)$$

which provides $\sigma_m^2 = 3.3333 \times 10^{-5}$. This value is also used to initialize the covariance matrix $\mathbf{P}_{0|0}$. It is chosen as a

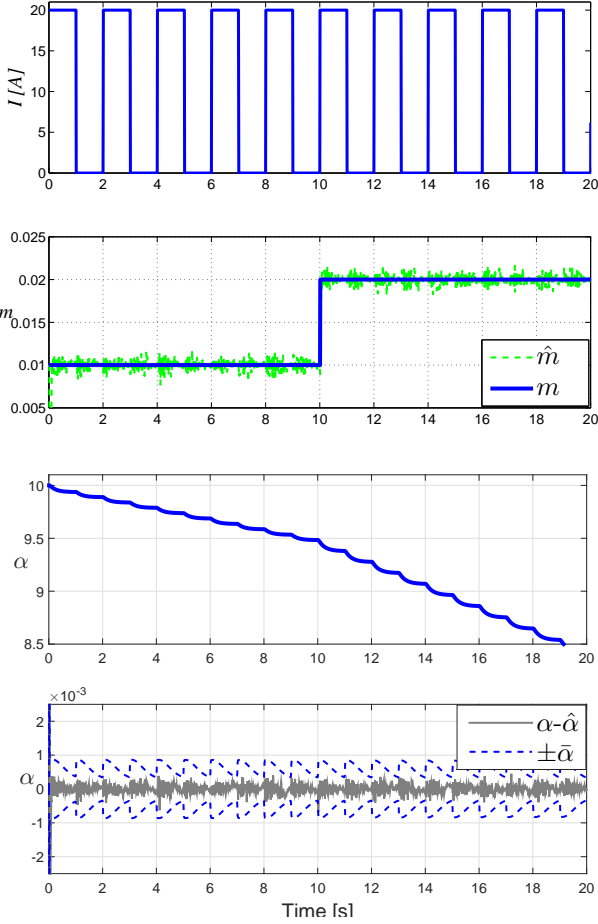


Fig. 3. Input sequence and estimation of the current state of \hat{m} and $\hat{\alpha}$ with an abrupt variation of m at $t = 10s$.

diagonal matrix containing in its diagonal constant values equal to 3.3333×10^{-5} .

Additionally, in this numerical example, the Jacobian (19) has been modified by including $F_{44} = -100$. This allows us to consider a band-pass filter model for m which reduces the effect of noise during the estimation of this variable. That is, the Extended Kalman filter considers the following dynamics $\dot{m} = -100m$ instead of $\dot{m} = 0$ in (11).

5.3 Analysis of the uncertainties in the model

The variations of the parameter m are assumed to be equal to 0 in the augmented system (9)-(11). Nevertheless, the purpose of this scenario is to assess the proposed observer for possible variations on m in real applications. The variations of the parameter m represents changes in the time-derivative of the quality of the contact α . These changes could depend on the material properties and are not produced by operational conditions modeled by the function $p(x)$. In (28) three different assumptions on the dynamics of m are presented: (i) the parameter m is always constant, (ii) the parameter m is piece-wise constant, and an abrupt change in the value of m can appears at the instant $k = t^*$ (a Dirac delta function models this aspect), and (iii) the parameter m can suffer a progressive change with a rate of change equal to ε (a possible random but *a priori* bounded input).

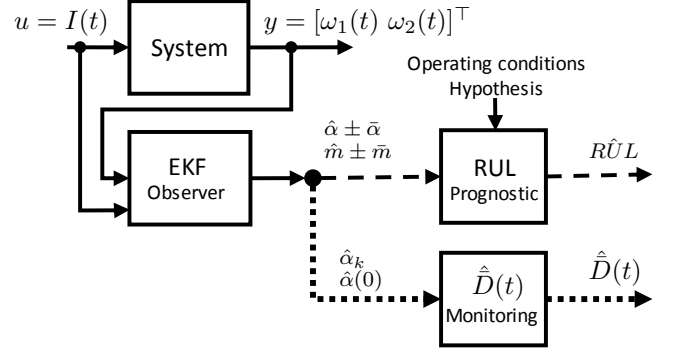


Fig. 4. Condition monitoring and RUL prognosis.

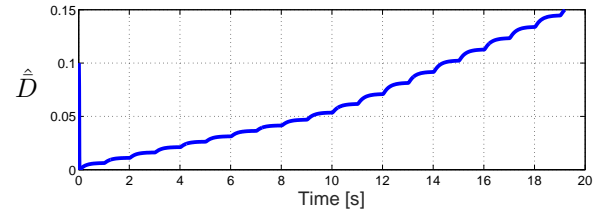


Fig. 5. Condition monitoring of deterioration. Here \hat{D} is calculated using (8) for a progressive variation of m .

$$\text{Assumption (i)} : \dot{m} = 0 \quad (28)$$

$$\text{Assumption (ii)} : \dot{m} = \delta(t^*)$$

$$\text{Assumption (iii)} : \dot{m} = \varepsilon$$

Fig. 3 shows the performance of the observer in a scenario which considers assumption (i) before $t = 10s$. The variance of the obtained estimations on m are consistent with the variances chosen for tuning the Extended Kalman Filter. Additionally, Fig. 3 shows the scenario type (ii), i.e. with an abrupt variation of the nominal value m at $t = 10s$. Notice the change in the slope at this time, in the curve that depicts the behavior of $\hat{\alpha}$. Thus, this observer is able to estimate the possible variations on m even if it was not designed for this purpose. This type of change in m can be understood as a fault or a material change at the contact level. As consequence, the observer can be useful for fault detection applications. These results also confirm that the estimation of α is accurate, as it can be evaluated using the bounds calculated using (24) and (25).

6. INTEGRATION OF THE OBSERVER INTO THE PROGNOSTIC OF REMAINING USEFUL LIFETIME

The proposed observer in Section 4 provides an estimate of the current state of α and m . Given those estimates, we can address two tasks: (i) monitoring the current condition of the deterioration and (ii) prognosticate the RUL of the actuator. Fig. 4 depicts the condition monitoring and the RUL prognosis architecture proposed in this paper.

Condition monitoring of deterioration. The normalized deterioration \hat{D} is calculated at every time-instant using the estimated value of α and (8). This value has to be obtained after assuring that the estimations errors have converged into the confidence intervals computed according to Section 4.4. Fig. 5 sketches the normalized estimated deterioration \hat{D} for the scenario of Fig. 3.

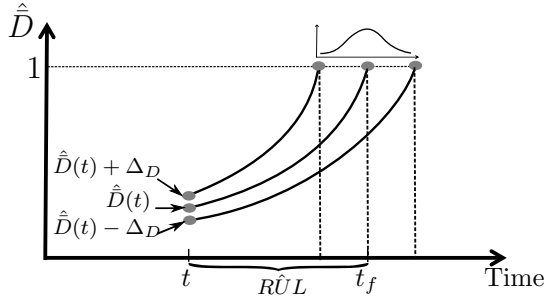


Fig. 6. Prognostic of the RUL at time t , by considering the current estimation uncertainties.

Prognostics of the RUL. To perform a prognostic of the RUL, we can use the estimated values $\hat{\alpha}$ and \hat{m} and their available confidence intervals. The prognostic has additionally to include the assumptions about the future operating conditions, for instance the possible behavior of the electrical current $I(t)$. In this work, the prognosis is performed by considering a constant electrical current $I(t) = 20A$ and constant parameter m .

Fig. 6 shows the schema of the RUL prognostic. At a given time t , an estimation of the normalized deterioration, denoted $\hat{D}(t)$, can be performed by using the dynamical model (9)-(10) and the output equation (8). The model (9)-(10) is initialized with the available estimations $\hat{\mathbf{x}}_{k|k}$ and their confidence intervals, at time t . The resultant uncertainty in the estimation of $\hat{D}(t)$ is denoted as Δ_D . The prognostic is stopped once the normalized deterioration reaches the maximum value, i.e. when $\hat{D}(t)=1$, for a given time t_f . The estimated RUL is calculated as $\hat{RUL} = t_f - t$.

The uncertainties in the pair $(\hat{\alpha}, \hat{m})$ at time t , produce a dispersion on the estimation of the RUL . In this way, it is possible to obtain a central value (the mean) and two extreme values (a constant number of standard deviations).

Fig. 7 shows the trajectory of the estimated deterioration (the bold continuous line). The dashed lines correspond to the trajectory of deterioration for the optimistic initial conditions, i.e. $D_{max}^{opt} = (\hat{\alpha} + \bar{\alpha})/(\hat{m} - \bar{m})$, and for the pessimistic initial conditions, i.e. $D_{max}^{pes} = (\hat{\alpha} - \bar{\alpha})/(\hat{m} + \bar{m})$. Here we use $c = 3$ in (24) which implies the following bounds: $\bar{\alpha} = 3 \cdot \sigma_{\alpha}$ and $\bar{m} = 3 \cdot \sigma_m$, where σ_{α}^2 and σ_m^2 correspond to the estimated variances of α and m obtained from the Extended Kalman Filter. In addition, Fig. 7 shows 100 trajectories of \hat{D} (the gray lines), by considering initial conditions $\hat{\mathbf{x}}(t)$, $\hat{\alpha}(t)$ and $\hat{m}(t)$ with estimation errors belonging to the following normal distribution:

$$\tilde{\mathbf{x}}_{k|k} \sim \mathcal{N}(0, \mathbf{P}_{k|k}) \quad (29)$$

where $\mathbf{P}_{k|k}$ stands for the available estimated covariance matrix at time t .

Fig. 8 shows that the obtained data of the estimated RUL fit a normal distribution. It was found a mean value of $\hat{RUL} = 106.09h$ and a standard deviation of $\sigma_{\hat{RUL}} = 3.36h$.

7. CONCLUSIONS AND FUTURE WORK

In this paper a non-linear state-observer is presented for estimation of the state of deterioration in a friction drive system. The estimator provides the current state of deterioration of the contact surfaces with high precision.

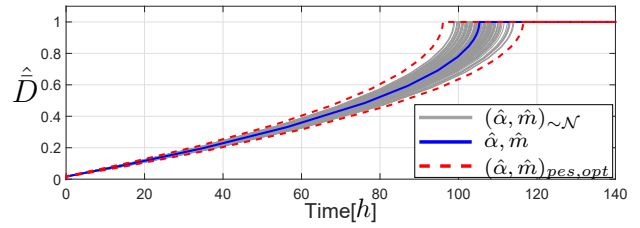


Fig. 7. Deterioration \hat{D} for prognostic of RUL. The figure shows 100 possible trajectories of \hat{D} within the intervals given by the observer in the time t .

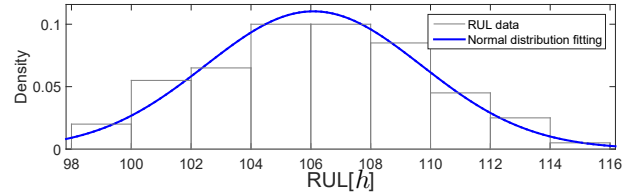


Fig. 8. Probability density function for a normal distribution fitting. $\hat{RUL} = 106.09h$ and a standard deviation of $\sigma_{\hat{RUL}} = 3.36h$.

The estimator was evaluated in simulation by taking into account known operating conditions. The estimations about the state of health of the contact surfaces allow us to make a prognostic of the RUL with a confidence level linked to the quality of the estimations.

Future work concerns the use of the proposed condition monitoring and the RUL estimation for designing a Reliability Adaptive System.

REFERENCES

- Besançon, G. (2007). *Nonlinear observers and applications*, volume 363. Springer.
- Bevan, A., Molyneux-Berry, P., Eickhoff, B., and Burstow, M. (2013). Development and validation of a wheel wear and rolling contact fatigue damage model. *Wear*, 307, 100–111.
- Cremona, M.A., Liu, B., Hu, Y., Bruni, S., and Lewis, R. (2016). Predicting railway wheel wear under uncertainty of wear coefficient, using universal kriging. *Reliability Engineering & System Safety*, 154, 49–59.
- Meyer, T. and Sextro, W. (2014). Closed-loop control system for the reliability of intelligent mechatronic systems. In *European Conference of the Prognostics and Health Management Society - 2014 - Paderborn, Germany*, 53–57.
- Rakowsky, U.K. (2006). Modelling reliability-adaptive multi-system operation. *International Journal of Automation and Computing*, 3(2), 192–198.
- Rodriguez Obando, D.J., Martinez Molina, J.J., and Berenguer, C. (2016a). Deterioration modelling of contact surfaces for a friction drive system. In *Proc. of the 26th European Safety and Reliability Conference-ESREL 2016 - 26-29 Sep. 2016 - Glasgow, Scotland*, 1961–1968.
- Rodriguez Obando, D.J., Martinez Molina, J.J., and Berenguer, C. (2016b). Set-invariance analysis for deterioration prediction on a roller-on-tire actuator. In *Proc. 3rd Conference on Control and Fault-Tolerant Systems - SysTol2016 - 7-9 Sep. 2016 - Barcelona, Spain*, 87–92.