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Abstract—In this paper, the fundamental limits of simultaneous information and energy transmission in the two-user Gaussian interference channel (G-IC) are fully characterized. More specifically, an achievable and converse region in terms of information and energy transmission rates (in bits per channel use and energy-units per channel use, respectively) are presented. The achievable region is obtained using a combination of rate splitting, power-splitting, common randomness and superposition coding. Finally, the converse region is obtained using some of the existing outer bounds on the information transmission rates, as well as a new outer bound on the energy transmission rate.

I. Introduction

Over the last decade, energy harvesting has been considered as a promising technology with great potential for green technologies, low voltage wearable electronics, and wireless devices. This aligns with the fact that receivers can simultaneously extract information and energy from radio-frequency signals [9], [7]. However, recent research has shown that energy and information transmission are often conflicting tasks. That is, there exists a trade-off between the information transmission rate and the energy transmission rate. This trade-off is observed in point-to-point channels [5] and multi-user channels such as the multiple access channel [1], [4] and the interference channel [8]. However, very little is known about this trade-off in other multi-user channels with energy harvesting. More importantly, very little is known about the fundamental limits of multi-user SIET.

This paper focuses on the Gaussian interference channel with an external energy harvester (EH). This channel models two point-to-point links subject to mutual interference, where both transmitters are engaged with transmitting information to their intended receiver while jointly providing a minimum energy rate at the EH. The fundamental limits of this channel are thoroughly studied. An achievable and converse region in terms of information and energy transmission rates are identified. The achievable region is obtained using a combination of rate splitting, power-splitting, common randomness, and superposition coding. The converse region is obtained using some of the existing outer bounds on the information

transmission rates, as well as a new outer bound on the energy transmission rate.

II. Gaussian Interference Channel with Energy Harvester

Consider the Gaussian interference channel with a non-colocated energy harvester depicted in Fig. 1. Transmitter i, with $i \in \{1, 2\}$, aims to execute two tasks: (a) an information transmission task and (b) an energy transmission task.

A. Information Transmission Task

From the information transmission standpoint, the goal of transmitter i is to convey message index $W_i \in \mathcal{W}_i$ to receiver i using N channel input symbols $X_{i,1}, X_{i,2}, \ldots, X_{i,N}$. The channel coefficient from transmitter k to receiver i, with $k \in \{1, 2\}$, is denoted by $h_{ik} \in \mathbb{R}_+$. For channel use n, input symbol $X_{i,n}$ is observed at receiver i in addition to the interference produced by the symbol $X_{j,n}$ sent by transmitter j, with $j \in \{1, 2\} \setminus \{i\}$, and a real additive Gaussian noise $Z_{i,n}$ with zero mean and variance σ_i^2. Hence, the channel output at receiver i during channel use n, denoted by $Y_{i,n}$, is

$$Y_{i,n} = h_{ii}X_{i,n} + h_{ij}X_{j,n} + Z_{i,n}. \tag{1}$$

At each channel use n, the symbol $X_{i,n}$ sent by transmitter i depends on the message index W_i and a randomly generated index $\Omega \in \mathbb{N}$. The random index Ω is assumed to be independent of both W_1 and W_2 and known by all transmitters and receivers. Let $f_{i,n}^{(N)} : \mathcal{W}_i \times \mathbb{N} \to \mathbb{R}$ be the encoding function at channel use n, such that for all $n \in \{1, 2, \ldots, N\}$:

$$X_{i,n} = f_{i,n}^{(N)}(W_i, \Omega). \tag{2}$$

Channel input symbols $X_{i,1}, X_{i,2}, \ldots, X_{i,N}$ are subject to an average power constraint of the form

$$\frac{1}{N} \sum_{n=1}^{N} \mathrm{E}[X_{i,n}^2] \le P_i, \tag{3}$$

where the expectation is taken with respect to W_i and Ω, which follow uniform probability distributions over their corresponding supports. Receiver i observes the channel outputs $Y_{i,1}, Y_{i,2}, \ldots, Y_{i,N}$ and uses a decoding function

$$\phi_i^{(N)} : \mathbb{N} \times \mathbb{R}^N \to \{1, 2, \ldots, 2^{R_i}\}, \tag{4}$$

to get an estimate $\widehat{W_i}^{(N)} = \phi_i^{(N)}(\Omega, Y_{i,1}, Y_{i,2}, \ldots, Y_{i,N})$ of the transmitted message W_i. The information rate at receiver i is denoted by R_i and it is defined by:
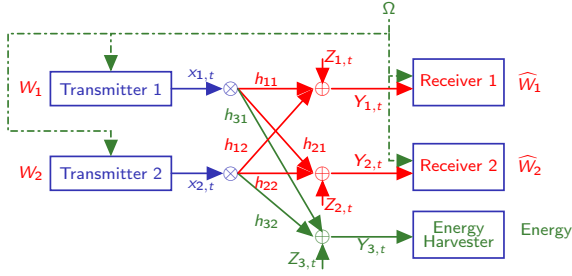
$$R_i = \frac{\log_2 |\mathcal{W}_i|}{N}, \tag{5}$$

Fig. 1: Two-user Gaussian interference channel with a non-colocated energy harvester.

in bits per channel use. The decoding error probability is given by

$$P_{\text{e}}^{(N)}(R_1, R_2)$$
$$= \max\left(\Pr\left(\widehat{W_1}^{(N)} \neq W_1\right), \Pr\left(\widehat{W_2}^{(N)} \neq W_2\right)\right). \quad (6)$$

The signal to noise ratio (SNR) at receiver i is denoted by

$$\text{SNR}_i = \frac{|h_{ii}|^2 P_i}{\sigma_i^2}. \quad (7)$$

Similarly, the interference to noise ratio (INR) at receiver i is denoted by

$$\text{INR}_i = \frac{|h_{ij}|^2 P_j}{\sigma_i^2}, \quad \text{for } j \neq i. \quad (8)$$

B. Energy Transmission Task

Let $h_{3i} \in \mathbb{R}_+$ be the channel coefficient from transmitter i to the energy harvester (EH). The symbols sent by transmitter 1 and 2 to the EH are subject to an additive Gaussian noise $Z_{3,n}$ with zero mean and variance σ_3^2. More specifically, the channel output at the EH, denoted by $Y_{3,n}$, is

$$Y_{3,n} = h_{31}X_{1,n} + h_{32}X_{2,n} + Z_{3,n}. \quad (9)$$

The SNR of transmitter i at the EH is denoted by

$$\text{SNR}_{3i} = \frac{|h_{3i}|^2 P_i}{\sigma_3^2}. \quad (10)$$

Let $b \geqslant 0$ denote the minimum average energy rate that must be guaranteed at the input of the EH. Let $B^{(N)}$ be the average energy transmission rate (in energy-units per channel use) at the end of N channel uses. That is,

$$B^{(N)} \triangleq \frac{1}{N} \sum_{n=1}^{N} Y_{3,n}^2. \quad (11)$$

Note that the maximum average energy rate $B_{\max}$ is

$$B_{\max} = \sigma_3^2 \left(1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}\right). \quad (12)$$

From the energy transmission standpoint, the goal of both transmitters is to jointly guarantee that the average energy rate $B^{(N)}$ is greater than a given operational energy transmission rate B that must satisfy

$$b \leqslant B \leqslant B_{\max}. \quad (13)$$

The probability of energy outage, given an average energy rate B, is defined as follows:

$$P_{\text{outage}}^{(N,\epsilon)}(B) \triangleq \Pr\left[B^{(N)} < B - \epsilon\right], \quad (14)$$

for some $\epsilon > 0$.

C. Simultaneous Information and Energy Transmission (SIET)

Given a minimum energy rate b to be satisfied at the EH, the system is said to be operating at the information-energy rate triplet $(R_1, R_2, B) \in \mathbb{R}_+^3$ when both transmitter-receiver pairs use a transmit-receive configuration such that: (i) reliable communication at information rates R_1 and R_2 is ensured; and (ii) reliable energy transmission at energy rate B is ensured. A formal definition is given below.

Definition 1 (Achievable Rates): The triplet $(R_1, R_2, B) \in \mathbb{R}_+^3$ is achievable if for all $i \in \{1, 2\}$, there exists a sequence of encoding functions $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \ldots, f_{i,N}^{(N)}$, and the decoding functions $\phi_1^{(N)}$ and $\phi_2^{(N)}$, such that both the average error probability $P_{\text{e}}^{(N)}(R_1, R_2)$ and the energy-outage probability $P_{\text{outage}}^{(N,\epsilon)}(B)$ tend to zero as the block-length N tends to infinity. That is,

$$\limsup_{N \to \infty} P_{\text{e}}^{(N)} = 0, \quad \text{and} \quad (15)$$

$$\limsup_{N \to \infty} P_{\text{outage}}^{(N,\epsilon)} = 0. \quad (16)$$

Using Definition 1, the fundamental limits of simultaneous information and energy transmission in the Gaussian interference channel can be described by the information-energy capacity region, defined as follows.

Definition 2 (Information-Energy Capacity Region): The information-energy capacity region given a minimum energy rate b, denoted by $\mathcal{E}_b$, corresponds to the closure of all achievable information-energy rate triplets (R_1, R_2, B).

III. MAIN RESULTS

The main result consists of a description of the information-energy capacity region $\mathcal{E}_b$, for a given $b \geqslant 0$. The following sections show that the information-energy capacity region $\mathcal{E}_b$, with b any positive real number, is approximated by the regions $\underline{\mathcal{E}}_b$ (Theorem 1), which represents an information-energy achievable region, and $\overline{\mathcal{E}}_b$ (Theorem 2), which represents an information-energy converse region.

A. An Achievable Region

The following theorem describes a set of rate-tuples that are achievable (Definition 1).

Theorem 1: Let b be a fixed positive real. Then, the information-energy capacity region $\mathcal{E}_b$ contains all the rate tuples (R_1, R_2, B) that satisfy for all $i \in \{1, 2\}$ and $j \in$

$\{1,2\}\backslash\{i\}$:

$$R_i \le \frac{1}{2} \log \left(1 + \frac{(1-\lambda_{ie})\text{SNR}_i}{1+\lambda_{jp}\text{INR}_i} \right), \tag{17a}$$

$$R_1 + R_2 \le \frac{1}{2} \log \left(\frac{1+(1-\lambda_{ie})\text{SNR}_1+(1-\lambda_{je})\text{INR}_1}{1+\lambda_{jp}\text{INR}_1} \right),$$
$$+ \frac{1}{2}\log\left(1+\frac{\lambda_{jp}\text{SNR}_j}{1+\lambda_{ip}\text{INR}_j}\right), \tag{17b}$$

$$R_1 + R_2 \le \frac{1}{2}\log\left(\frac{1+\lambda_{1p}\text{SNR}_1+(1-\lambda_{2e})\text{INR}_1}{1+\lambda_{2p}\text{INR}_1}\right)$$
$$+\frac{1}{2}\log\left(\frac{1+\lambda_{2p}\text{SNR}_2+(1-\lambda_{1e})\text{INR}_2}{1+\lambda_{1p}\text{INR}_2}\right), \tag{17c}$$

$$2R_i + R_j \le \frac{1}{2}\log\left(\frac{1+(1-\lambda_{ie})\text{SNR}_i+(1-\lambda_{je})\text{INR}_i}{1+\lambda_{jp}\text{INR}_i}\right)$$
$$+\frac{1}{2}\log\left(\frac{1+\lambda_{jp}\text{SNR}_j+(1-\lambda_{ie})\text{INR}_j}{1+\lambda_{ip}\text{INR}_j}\right)$$
$$+\frac{1}{2}\log\left(1+\frac{\lambda_{ip}\text{SNR}_i}{1+\lambda_{jp}\text{INR}_i}\right), \tag{17d}$$

$$b \le B \le \sigma_3^2\Big(1+\text{SNR}_{31}+\text{SNR}_{32}$$
$$+2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}\sqrt{\lambda_{1e}\lambda_{2e}}\Big), \tag{17e}$$

with $(\lambda_{ip}, \lambda_{ie}) \in [0,1]^2$ such that $\lambda_{ip} + \lambda_{ie} \le 1$.

Proof: The sketch of proof of Theorem 1 is presented in the following section. ∎

B. Sketch of Proof of Achievability

The achievability scheme used to obtain Theorem 1 is built upon random coding arguments using rate-splitting [6], superposition coding [2], common randomness and power-splitting [1]. Let $W_i \in \{1,2\ldots,2^{NR_i}\}$ and $\Omega \in \{1,2\ldots,2^{NR_E}\}$ be respectively the message index and the common random index at transmitter i. Following a rate-splitting argument, the index W_i is divided into two sub-indices $W_{i,P} \in \{1,2\ldots,2^{NR_{i,P}}\}$ and $W_{i,C} \in \{1,2\ldots,2^{NR_{i,C}}\}$, where $R_{i,C} + R_{i,P} = R_i$. The message index $W_{i,C}$ must be decoded at both receivers, whereas the index $W_{i,P}$ must be decoded only at the intended receiver. This rate-splitting is reminiscent of the Han-Kobayashi scheme in [6].

Lemma 1: An achievable information rate pair (R_1, R_2) satisfies the following inequalities, for all $i \in \{1,2\}$ and $j \in \{1,2\}\backslash\{i\}$:

$$R_i \le I(X_i; Y_i|U_j, V) \tag{18a}$$
$$R_1 + R_2 \le I(X_i, U_j; Y_i|V) + I(X_j; Y_j|U_i, U_j, V) \tag{18b}$$
$$R_1 + R_2 \le I(X_1, U_2; Y_1|U_1, V) + I(X_2, U_1; Y_2|U_2, V) \tag{18c}$$
$$2R_i + R_j \le I(X_i, U_j; Y_i|V) + I(X_i; Y_i|U_i, U_j, V)$$
$$+ (X_j, U_i; Y_j|U_j, V), \tag{18d}$$

for a given joint distribution $P_{VU_1U_2S_1S_2}(v, u_1, u_2, s_1, s_2)$ that factorizes as $P_V(v)$ $P_{U_1|V}(u_1|v)$ $P_{U_2|V}(u_2|v)$ $P_{S_1|U_1V}(s_1|u_1v)$ $P_{S_2|U_2V}(s_2|u_2v)$ and $X_i = \theta_i(V, U_i, S_i)$, with θ_1 and θ_2 injective functions.

Proof: The proof of Lemma 1 uses standard arguments of weak typicality and is omitted in this paper. ∎

For all $k \in \{1,2\}$ and a fixed triplet $(\lambda_{kc}, \lambda_{kp}, \lambda_{ke}) \in [0,1]^3$ such that $\lambda_{kc} + \lambda_{kp} + \lambda_{ke} = 1$, consider the following random variables: $V \sim \mathcal{N}(0,1)$; $U_k \sim \mathcal{N}(0,\lambda_{kc})$; and $S_k \sim \mathcal{N}(0,\lambda_{kp})$, which are independent of each other. Let the channel input of transmitter k be

$$X_k = \sqrt{P_k}S_k + \sqrt{P_k}U_k + \sqrt{\lambda_{ke}P_k}V. \tag{19}$$

The choice of this input distribution yields

$$I(X_i; Y_i|U_j, V) = \frac{1}{2}\log\left(1+\frac{(1-\lambda_{ie})\text{SNR}_i}{1+\lambda_{jp}\text{INR}_i}\right) \tag{20a}$$

$$I(X_i, U_j; Y_i|V) = \frac{1}{2}\log\left(\frac{1+(1-\lambda_{ie})\text{SNR}_i+(1-\lambda_{je})\text{INR}_i}{1+\lambda_{jp}\text{INR}_1}\right), \tag{20b}$$

$$I(X_j; Y_j|U_i, U_j, V) = \frac{1}{2}\log\left(1+\frac{\lambda_{jp}\text{SNR}_j}{1+\lambda_{ip}\text{INR}_j}\right) \tag{20c}$$

$$I(X_i, U_j; Y_i|U_i, V) = \frac{1}{2}\log\left(\frac{1+\lambda_{ip}\text{SNR}_i+(1-\lambda_{je})\text{INR}_i}{1+\lambda_{jp}\text{INR}_i}\right), \tag{20d}$$

$$I(X_i, U_j; Y_i|U_i, V) = \frac{1}{2}\log\left(\frac{1+\lambda_{ip}\text{SNR}_i+(1-\lambda_{je})\text{INR}_i}{1+\lambda_{jp}\text{INR}_i}\right), \tag{20e}$$

$$I(X_j, U_i; Y_j|U_j, V) = \frac{1}{2}\log\left(\frac{1+\lambda_{jp}\text{SNR}_j+(1-\lambda_{ie})\text{INR}_j}{1+\lambda_{ip}\text{INR}_2}\right). \tag{20f}$$

Finally, plugging (20) into (18) completes the proof of (17a) - (17d).

The average received energy rate $\bar{B}$ achieved by using the input signals in (19) is given by

$$\bar{B} = E[Y_{3,n}^2]$$
$$= h_{31}^2 E[X_{1,n}^2] + h_{32}^2 E[X_{2,n}^2] + 2h_{31}h_{32}E[X_{1,n}X_{2,n}] + \sigma_3^2$$
$$\le h_{31}^2 P_1 + h_{32}^2 P_2 + 2|h_{31}||h_{32}|\sqrt{\lambda_{1e}P_1\lambda_{2e}P_2} + \sigma_3^2$$
$$= \sigma_3^2\left(1+\text{SNR}_{31}+\text{SNR}_{32}+2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}\sqrt{\lambda_{1e}\lambda_{2e}}\right).$$

From the weak law of large numbers, it holds that $\forall \epsilon > 0$,

$$\lim_{N\to\infty} \Pr\left(B^{(N)} < \bar{B} - \epsilon\right) = 0. \tag{21}$$

From (21), it holds that for any energy rate B that satisfies $0 < B \le \bar{B}$, it holds that

$$\lim_{N\to\infty} \Pr\left(B^{(N)} < B - \epsilon\right) = 0, \tag{22}$$

which proves (17e) and completes the sketch of proof.

C. Converse

The following theorem describes a converse region denoted by $\overline{\mathcal{E}}_b$.

Theorem 2: Let b be a fixed positive real. Then, the information-energy capacity region $\mathcal{E}_b$ is contained into the set

of all the rate tuples (R_1, R_2, B) that satisfy for all $i \in \{1,2\}$ and $j \in \{1,2\}\backslash\{i\}$:

$$R_i \leq \frac{1}{2}\log(1 + \beta_i \mathrm{SNR}_i), \tag{23a}$$

$$R_1 + R_2 \leq \frac{1}{2}\log(1 + \beta_i \mathrm{SNR}_i + \beta_j \mathrm{INR}_i)$$
$$+ \frac{1}{2}\log(1 + \frac{\beta_j \mathrm{SNR}_j}{1 + \beta_j \mathrm{INR}_i}), \tag{23b}$$

$$R_1 + R_2 \leq \frac{1}{2}\log\left(1 + \frac{\beta_1 \mathrm{SNR}_1 + \beta_2 \mathrm{INR}_1 + \beta_1 \beta_2 \mathrm{INR}_1 \mathrm{INR}_2}{1 + \beta_1 \mathrm{INR}_2}\right)$$
$$+ \frac{1}{2}\log\left(1 + \frac{\beta_2 \mathrm{SNR}_2 + \beta_1 \mathrm{INR}_2 + \beta_1 \beta_2 \mathrm{INR}_1 \mathrm{INR}_2}{1 + \beta_2 \mathrm{INR}_1}\right), \tag{23c}$$

$$2R_i + R_j \leq \frac{1}{2}\log(1 + \frac{\beta_i \mathrm{SNR}_i}{1 + \beta_i \mathrm{INR}_j})$$
$$+ \frac{1}{2}\log(1 + \beta_i \mathrm{SNR}_i + \beta_j \mathrm{INR}_i)$$
$$+ \frac{1}{2}\log\left(1 + \frac{\beta_j \mathrm{SNR}_j + \beta_i \mathrm{INR}_j + \beta_i \beta_j \mathrm{INR}_i \mathrm{INR}_j}{1 + \beta_j \mathrm{INR}_i}\right), \tag{23d}$$

$$b \leq B \leq \sigma_3^2\Big(1 + \mathrm{SNR}_{31} + \mathrm{SNR}_{32}$$
$$+ 2\sqrt{(1-\beta_1)\mathrm{SNR}_{31}(1-\beta_2)\mathrm{SNR}_{32}}\Big), \tag{23e}$$

with $(\beta_1, \beta_2) \in [0,1]^2$.

Proof: The sketch of the proof of Theorem 2 is presented in the following section. ∎

D. Sketch of Proof of the Converse

Fix an information-energy rate triplet (R_1, R_2, B) achievable with a given coding scheme (Definition 1). Denote by $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ the channel inputs resulting from transmitting the independent message W_1 and W_2 using such coding scheme. Denote by $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$ the corresponding channel outputs. Define the following random variables:

$$U_1 = h_{21}X_1 + Z_2', \text{ and}$$
$$U_2 = h_{12}X_2 + Z_1',$$

where, Z_1' and Z_2' are real Gaussian random variables independent of each other with zero means and variances σ_1^2 and σ_2^2, respectively. Using assumption (15) and Fano's inequality, it follows that the information rates R_1 and R_2 must satisfy the following inequalities:

$$NR_i \leq \sum_{n=1}^{N} h(Y_{i,n}|X_{j,n}) - Nh(Z_i) + \mathrm{o}(N) \tag{24a}$$

$$N(R_1 + R_2) \leq \sum_{n=1}^{N} h(Y_{i,n}) + \sum_{n=1}^{N} h(Y_{j,n}|U_{j,n}, X_{i,n})$$
$$- Nh(Z_j) - Nh(Z_i') + \mathrm{o}(N) \tag{24b}$$

$$N(R_1 + R_2) \leq \sum_{n=1}^{N} h(Y_{1,n}|U_{1,n}) + \sum_{n=1}^{N} h(Y_{2,n}|U_{2,n})$$
$$- Nh(Z_1') - Nh(Z_2') + \mathrm{o}(N) \tag{24c}$$

$$N(2R_i + R_j) \leq \sum_{n=1}^{N} h(Y_{i,n}) + \sum_{n=1}^{N} h(Y_{i,n}|U_{i,n}, X_{j,n})$$
$$+ \sum_{n=1}^{N} h(Y_{j,n}|U_{j,n}) - N(h(Z_1) + h(Z_2))$$
$$+ h(Z_1') + h(Z_2')) + \mathrm{o}(N), \tag{24d}$$

where $\frac{\mathrm{o}(N)}{N}$ tends to zero as N tends to infinity. Using assumption (16), for a given $\epsilon_N > 0$ and an $\eta > 0$, there exist $N_0(\eta)$ such that for any $N \geq N_0(\eta)$ it holds that

$$\Pr\left(B^{(N)} < B - \epsilon_N\right) < \eta. \tag{25}$$

Equivalently,

$$\Pr\left(B^{(N)} \geq B - \epsilon_N\right) \geq 1 - \eta. \tag{26}$$

From Markov's inequality, the following holds:

$$(B - \epsilon_N)\Pr\left(B^{(N)} \geq B - \epsilon_N\right) \leq E[B^{(N)}]. \tag{27}$$

Combining (26) and (27) yields

$$(B - \epsilon_N)(1 - \eta) \leq E[B^{(N)}], \tag{28}$$

which can be written as

$$(B - \delta_N) \leq E[B^{(N)}], \tag{29}$$

for some $\delta_N > \epsilon_N$ and a sufficiently large N. In the following, for all $n \in \mathbb{N}$, the bounds in (24) and (29) are evaluated assuming that the channel inputs $X_{1,n}$ and $X_{2,n}$ are independent random variables with mean and variance:

$$\mu_{i,n} \triangleq \mathrm{E}[X_{i,n}], \tag{30}$$
$$\gamma_{i,n}^2 \triangleq \mathrm{Var}[X_{i,n}]. \tag{31}$$

The input sequences must satisfy the input power constraint (3) which can be written for $i \in \{1,2\}$, as

$$\frac{1}{N}\sum_{n=1}^{N} \mathrm{E}[X_{i,n}^2] = \left(\frac{1}{N}\sum_{n=1}^{N} \gamma_{i,n}^2\right) + \left(\frac{1}{N}\sum_{n=1}^{N} \mu_{i,n}^2\right) \leq P_i. \tag{32}$$

Using these elements, the terms in the right-hand side of (24) can be upper-bounded as follows:

$$h(Y_{i,n}|X_{j,n}) \leq \frac{1}{2}\log\left(2\pi e(\sigma_i^2 + h_{ii}^2\gamma_{i,n}^2)\right), \tag{33a}$$

$$h(Y_{i,n}) \leq \frac{1}{2}\log\left(2\pi e(\sigma_i^2 + h_{ii}^2\gamma_{i,n}^2 + h_{ij}^2\gamma_{j,n}^2)\right), \tag{33b}$$

$$h(Y_{i,n}|U_{i,n}, X_{j,n}) \leq \frac{1}{2}\log\left(1 + \frac{\frac{h_{ii}^2\gamma_{i,n}^2}{\sigma_i^2}}{1 + \frac{h_{ji}^2\gamma_{i,n}^2}{\sigma_j^2}}\right)$$
$$+ \frac{1}{2}\log(2\pi e\sigma_i^2\sigma_j^2), \text{ and} \tag{33c}$$

$$h(Y_{i,n}|U_{i,n}) \leq \frac{1}{2}\log(2\pi e\sigma_i^2\sigma_j^2)$$
$$+ \frac{1}{2}\log\left(1 + \frac{\frac{h_{ii}^2\gamma_{i,n}^2}{\sigma_i^2} + \frac{h_{ij}^2\gamma_{j,n}^2}{\sigma_i^2} + \frac{\gamma_{i,n}^2\gamma_{j,n}^2 h_{ij}^2 h_{ji}^2}{\sigma_i^2\sigma_j^2}}{1 + \frac{\gamma_{i,n}^2 h_{ji}^2}{\sigma_j^2}}\right). \tag{33d}$$

The expectation of the average received energy rate is given by

$$\mathrm{E}\left[B^{(N)}\right] = \mathrm{E}\left[\frac{1}{N}\sum_{n=1}^{N}Y_{3,n}^{2}\right] =$$

$$h_{31}^{2}\left(\frac{1}{N}\sum_{n=1}^{N}(\gamma_{1,n}^{2}+\mu_{1,n}^{2})\right) + h_{32}^{2}\left(\frac{1}{N}\sum_{n=1}^{N}(\gamma_{2,n}^{2}+\mu_{2,n}^{2})\right)$$

$$+2h_{31}h_{32}\frac{1}{N}\sum_{n=1}^{N}\mu_{1,n}\mu_{2,n}+\sigma_{3}^{2}. \tag{34}$$

Using Cauchy-Schwarz inequality, combining (29) and (34) yields the following upper-bound on the energy rate B:

$$B \leqslant \sigma_{3}^{2} + \frac{h_{31}^{2}}{N}\sum_{n=1}^{N}(\gamma_{1,n}^{2}+\mu_{1,n}^{2}) + \frac{h_{32}^{2}}{N}\sum_{n=1}^{N}(\gamma_{2,n}^{2}+\mu_{2,n}^{2})$$

$$+2h_{31}h_{32}\left(\frac{1}{N}\sum_{n=1}^{N}\mu_{1,n}^{2}\right)^{1/2}\left(\frac{1}{N}\sum_{n=1}^{N}\mu_{2,n}^{2}\right)^{1/2}+\delta_{N}. \tag{35}$$

Consider the following definitions, for all $i \in \{1,2\}$:

$$\mu_{i}^{2} \triangleq \frac{1}{N}\sum_{n=1}^{N}\mu_{i,t}^{2}, \tag{36a}$$

$$\gamma_{i}^{2} \triangleq \frac{1}{N}\sum_{n=1}^{N}\gamma_{i,n}^{2}, \text{ and} \tag{36b}$$

$$\beta_{i} \triangleq \frac{\gamma_{i}^{2}}{P_{i}}. \tag{36c}$$

Plugging (33) in (24) and after some manipulations using the definitions in (36) and using Jensen's inequality complete the sketch of the proof.

E. Approximation of the Information-Energy Capacity Region

Using the inner region $\underline{\mathcal{E}}_{b}$ and the outer region $\overline{\mathcal{E}}_{b}$, described respectively by Theorem 1 and Theorem 2, the information-energy capacity region $\mathcal{E}_{b}$ can be approximated according to the following theorem.

Theorem 3 (Approximation of $\mathcal{E}_{b}$): *Let $\underline{\mathcal{E}}_{b} \subset \mathbb{R}_{+}^{3}$ and $\overline{\mathcal{E}}_{b} \subset \mathbb{R}_{+}^{3}$ be the sets of tuples (R_{1}, R_{2}, B) described by Theorem 1 and Theorem 2, respectively. Then,*

$$\underline{\mathcal{E}}_{b} \subset \mathcal{E}_{b} \subset \overline{\mathcal{E}}_{b}, \tag{37}$$

and for all $(R_{1}, R_{2}, B) \in \overline{\mathcal{E}}_{b}$ it follows that $((R_{1} - 1/2)^{+}, (R_{2} - 1/2)^{+}, (B - \frac{B_{\max}}{2})^{+}) \in \underline{\mathcal{E}}_{b}$.

Proof: Following similar steps as in [3], it can be shown that for all $(R_{1}, R_{2}, 0) \in \overline{\mathcal{E}}_{b}$ it follows that $((R_{1} - 1/2)^{+}, (R_{2} - 1/2)^{+}, 0) \in \underline{\mathcal{E}}_{b}$. Note also that for all $(R_{1}, R_{2}, B) \in \overline{\mathcal{E}}_{b}$ and for all $(R_{1}, R_{2}, B') \in \underline{\mathcal{E}}_{b}$, there always exists a tuple $(\beta_{1}, \beta_{2}, \lambda_{1e}, \lambda_{2e})$ such that: $\frac{B - B'}{B_{\max}}$

$$= \frac{2|h_{31}||h_{32}|\sqrt{P_{1}P_{2}}\left(\sqrt{(1-\beta_{1})(1-\beta_{2})}-\sqrt{\lambda_{1e}\lambda_{2e}}\right)}{\sigma_{3}^{2}+h_{31}^{2}P_{1}+h_{32}^{2}P_{2}+2|h_{31}||h_{32}|\sqrt{P_{1}P_{2}}}$$

$$\leq \frac{2\sqrt{\mathrm{SNR}_{31}\mathrm{SNR}_{32}}}{1+4\sqrt{\mathrm{SNR}_{31}\mathrm{SNR}_{32}}}$$

$$\leq \frac{1}{2},$$

which completes the proof. ∎

IV. EXAMPLE

(a) 3-D representation of $\underline{\mathcal{E}}_{b}$.   (b) 3-D representation of $\overline{\mathcal{E}}_{b}$.
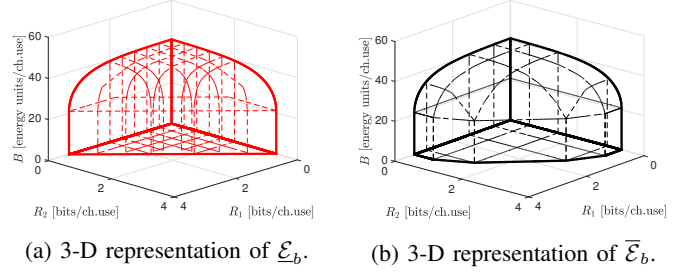
Fig. 2: 3-D representation of $\underline{\mathcal{E}}_{b}$ and $\overline{\mathcal{E}}_{b}$.

Consider a Gaussian interference channel with an external EH with parameters $\mathrm{SNR}_{1} = \mathrm{SNR}_{2} = 20$ dB, $\mathrm{INR}_{1} = \mathrm{INR}_{2} = \mathrm{SNR}_{31} = \mathrm{SNR}_{32} = 10$ dB and $\sigma_{3}^{2} = 1$.

Figure 2a and Figure 2b show $\underline{\mathcal{E}}_{b}$ and $\overline{\mathcal{E}}_{b}$, respectively, with $b = 0$. Note that for all $B \in [0, 1 + \mathrm{SNR}_{31} + \mathrm{SNR}_{32}]$, transmitting information with independent codewords is enough to satisfy the energy rate constraints. This implies that $\beta_{1} = \beta_{2} = 1$ is optimal in this regime. Alternatively, for all $B \in [1 + \mathrm{SNR}_{31} + \mathrm{SNR}_{32}, B_{\max}]$, transmitters deal with trade-off between the information and energy rate. Increasing B reduces the information region and makes the information-energy capacity region shrink.

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