Beyond realistic considerations: modeling conceptions and controls in task examples with simple word problems

Irit Peled, Nicolas Balacheff

307

Beyond realistic considerations: Modeling conceptions and controls in task examples with simple word problems

Irit Peled*[1] and Nicolas Balacheff**

* *University of Haifa, Israel*

Phone 972-4-8288352

Fax 972-4-8240911

ipeled@construct.haifa.ac.il

** *CNRS, laboratoire d'informatique de Grenoble, France*

Nicolas.Balacheff@imag.fr

Abstract. Using simple word problems, we analyze possible teacher conceptions on the process of problem solving, its goals and the choices that a problem solver can make in problem mathematization. We identify several possible teacher conceptions that would be responsible for the different didactical contracts that teachers create in the mathematics class. Using especially chosen and designed task examples we demonstrate the diagnosis of teacher own controls in solving problems and in evaluating problem solutions. We also discuss characteristics of task examples that might promote a shift from a problem solving perspective to a modeling perspective that goes beyond merely accepting alternative solutions due to realistic considerations. This shift in perspective would be exhibited through a new understanding of the process of fitting mathematical models in problem situations.

Keywords*: Modeling, problem solving, teacher conceptions, examples, didactical contract, control*

1 Introduction

The purpose of this paper is to understand the didactical complexity of using simple word problems and to design task examples for prospective and in-service teachers. The goal of these tasks is to elicit, identify, and possibly make a change in teacher conceptions about the processes involved in solving these problems and especially about the role of mathematics.

[1] This study was supported by a grant from the Israeli Science Foundation (59/06)

The motivation for embarking on this venture emerged from a discussion between a mathematics education researcher (the first author) and her doctoral student on some solutions to a problem suggested by the graduate student's sixth grade son. There are actually four participants in this episode, two who are present in the discussion and two who are not present: The researcher, the graduate student, her son (Ron), and his teacher.

Ron had solved the following problem:

Andrea and Bill bought a $5 lottery ticket together. Andrea paid $3 and Bill paid $2. They won $40. How will they split it?

Ron suggested three solutions:

1. Split the winnings evenly: Andrea gets $20 and Bill gets $20.
2. Split it so that the difference is close to the difference between the "investments": Andrea gets $21 and Bill gets $19.
3. Split it proportionally: Use the investment ratio 3:2. Andrea gets $24 and Bill gets $16.

Finally, he commented: I think the first solution is the "most fair", but the third solution is the "most right" because it uses ratio.

What Ron might be saying is that in real life, if he were in a situation like that, he would have used an even split. Since he knows that the teacher expects him to solve this problem using proportion, this is what he has to accept as what is considered a good mathematics class solution.

308

The doctoral student brought this story about her son with the intention to demonstrate interesting alternative solutions that children bring when they use realistic considerations in solving a problem.

Her own attitude towards his solution was combined, on the one hand, of pride in her son's creativity and, on the other hand, of identification with what the teacher would have thought. That is, being a teacher educator herself, she thought that each of the first two solutions: *might be ok in life, and can be considered an alternative solution. However, when I give it in class the solution that uses proportion is the correct mathematical solution.*

At this point the story could serve as a basis for discussing value and moral issues, viewing the three different answers as representing different values, and arguing

about the consequences of suppressing certain alternatives.[2] However, as further discussed, we take a different direction.

The doctoral student supervisor (the mathematics education researcher) disagreed with her student. In her opinion, all three solutions should have the same status as a mathematical solution (whatever "a mathematical solution" means, for that matter), and there was no reason to "mathematically accept" the solution that uses proportion over any of the others.

In her opinion, this was so not because of realistic or moral considerations. That is, not on the grounds that this is how one might behave in life, but on much deeper grounds that involve the information given in the problem and what could be deduced from it (in terms of mathematical structures). As the problem was phrased, she saw nothing that could lead to the conclusion that one should use proportion in solving this problem.

The graduate student listened to the claim made by her research supervisor. She was not convinced. It was at this point that the researcher felt she had to get to the bottom of the gap between them and construct some argumentation to support her view.

2 Theoretical Background

The introductory episode demonstrates common beliefs about problem solving in the mathematics class, and about the role of mathematics. The two discussants have an entirely different view on both. In fact, as we will further discuss, they represent different beliefs, a problem solving view and a modeling view.

More than two decades ago Nesher (1980) and Schoenfeld (1987) increased awareness of the stereotyped nature of word problems and the effect of this nature on student beliefs. As it seemed, students, developing a "sense for teachers" and for the hidden (or explicit) curriculum, had captured the sense of problem solving activity as an exercise in using the just-taught mathematical structure without any actual attention to the context.

---

[2] To use one of our reviewer's terms, the three answers could represent a "socialist", "almost socialist", and "capitalist" approaches.

While mathematics educators might have felt uneasy about these findings, students did not have a reason to develop any such feelings, because their problem solutions to classroom tasks were correct (especially in the case of simple elementary school problems discussed by Nesher (1980)).

This phenomenon of ignoring the realistic aspect was identified in the early 80's and named the "Captain age" effect (IREM de Grenoble, 1980). Later, in a collection of connected studies by Verschaffel and his colleagues (Verschaffel and De Corte, 1997; Verschaffel et al., 1997; Reusser and Stebler, 1997; Yoshida et al., 1997; Greer, 1997; to mention just a few) simple word problems especially designed by these researchers resulted in what they considered problematic solutions. The problems (termed Problematic or, in short, P-problems) were characterized by having the structure of a stereotyped word problem, albeit presenting a situation in which the use of the associated conventional solution led to a realistically problematic solution that was also considered an incorrect solution. For example, in a problem involving cutting a few (n) wooden planks (of length L) into smaller pieces (of length l), students and teachers, for the same matter, multiplied n by L and divided by l even though L/l did not yield a whole number of pieces.

The reason these P-problems failed problem solvers was that they were "disguised" as traditional problems in which students identify a certain structure with hardly any situation analysis. In order to get an acceptable solution, these problems require a relatively simple analysis of the situation and the use of realistic considerations.

Similar automatic behavior was found by Peled and Hershkovitz (2004) in a word problem involving crowdedness. Although teacher and student solutions were correct in applying proportional reasoning (as with the earlier elementary school problems), they did it without giving themselves any account on the meaning and reason for using proportion. This fact became evident when the researchers asked the teachers to argue with an alternative (incorrect) solution. The need to convince others caused a special effort to analyze the situation resulting with better understanding of their own original (automatic) solution.

Many of the efforts to improve and increase the use of realistic considerations failed because they were often conducted within traditional instruction, i.e. within the traditional didactical contract, as termed by Brousseau (translated 1997). More

successful were studies that changed the style of the given word problems, giving them a more authentic formulation or involving an activity (making a phone call to order buses) that can facilitate the appropriation of the real context (DeFranco and Curcio, 1997).

There was also some feeling that children should be told the rules of the game. In a reflection by Verschaffel et al (2002) on their own work, the researchers realize that the didactical contract should be clear, and children should be able to tell whether they are expected to solve a given problem with or without realistic considerations. This suggests that there was no fundamental change in problem solving goals. The "default option" was that children *were* expected to apply a structure without much analysis of the reality involved. The "activation" of a situation analysis was expected in a variety of special cases such as exemplified by the P-problems, when there is some hint from the teacher or the curriculum to do so. In contrast, the earlier mentioned bus-crowdedness example (Peled and Hershkovitz, 2004) tried to demonstrate the benefits of activating a situation analysis as a habit.

While Verschaffel et al. (ibid) and Peled and Hershkovitz (ibid) used simple traditional-style problems, and while the former focused on realistic considerations with some extension of the existing didactical contract, other researchers promoted a more dramatic change both in goals and in problem style. Earlier goals of word problem solving involved the construction of mathematical concepts with some experiences in applying them for the purpose of developing richer concepts. As described by Blum and Niss (1991), the new goals viewed problem solving not as a mean but as a goal in its own right and acknowledged the importance of developing modeling skills. Proponents of this perspective suggest changing problem nature towards more complex problems that are real (in the sense of asking a question that has a reason to be asked) and have several other special characteristics as specified by Lesh et al. (2000) and discussed by Lesh and Doerr (2003).

Problems that are designed according to these specifications are called "model eliciting problems" (MEA) and involve "a modeling process" requiring an analysis and organization of problem situation and then a mathematization process consisting of choosing, integrating and fitting mathematical concepts and representations. Their name also indicates that through their work on these

problems problem solvers (children as well as teachers) exhibit their mathematical and their problem solving conceptions and beliefs. Thus, although the shift towards modeling is not easily accepted by teachers (Kaiser, 2006; Verschaffel et al., 2010), and implemented differently in different countries (Ikeda and Kaiser, 2005), some teachers eventually realize its benefits and learn from their observations of children's modeling processes (Doer and English, 2006).
While the abovementioned studies focus on the development of modeling skills and on teacher difficulties with implementing specially designed modeling problems, this article focuses on task examples for the development of the meaning of modeling and problem solving, using examples that consist of simple traditional problems.

3 Conceptions in and about problem solving

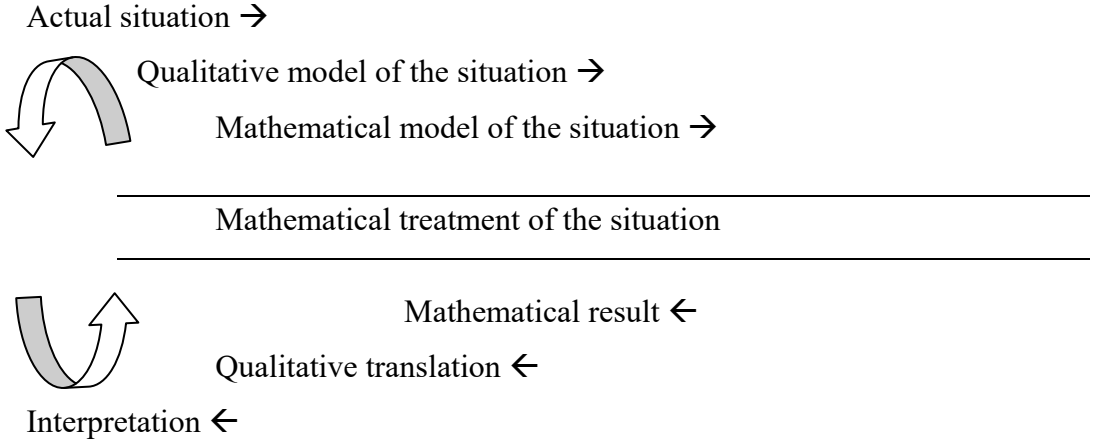
Going back to the argument between the doctoral student and her supervisor, presented in the introduction, the differences of opinion with regard to the correctness and legitimacy of problem solutions are surprising. Is there not a clear and determined answer to a mathematical word problem, especially a relatively simple problem?
In the following sections we analyze the process of solving a problem with a focus on the mathematization process. Although problem solving tasks and modeling tasks have very different characteristics (as mention earlier, cf. Lesh et al. (2000)), both involve an act of fitting a mathematical model to a given situation. We use the term modeling in a relaxed way for describing this mapping, even in cases where there is very little situation organization. We believe that understanding the nature of this mapping in relatively simple cases can promote better teacher understanding of modeling and facilitate the use of modeling tasks.
In the first section we present the process in general, while highlighting the *problématique* of solving problems "automatically", as exhibited in the research described earlier (Peled and Hershkovitz, 2004). This automatic act of fitting a mathematical model is represented through the collapse of steps in the process. Then in the consecutive section we look at teacher solution to the lottery problem, observing their preference for the automatic fit of proportion, and their reactions towards alternative solutions. In light of these observations, the next chapter will

describe our efforts to design task examples that might facilitate change in teacher conceptions and attitudes.

3.1 The problem solving process, controls and didactical contracts

Solving a problem can be seen as constructing a bridge between a situation evoked or lived, and mathematics. This bridge consists of a representation the semantic of which is provided by the situation and which is accessible to a mathematical treatment which result can be interpreted back in the situation.
A classical and relatively simple example is the Buses and Soldiers problem in which one has to decide how many buses are needed to transport soldiers, given the total number of soldiers and the number of people that each bus can carry. In this case the mathematical operation that is used is division, but when the result in the mathematical world is transformed and interpreted back into the real world, many children fail to adjust the numerical solution that might have a remainder into its realistic meaning which requires that in reality another "whole" bus will be needed for the "remainder".
Although they look like modeling situations, these problems are often used missing the core of the scientific approach. In a scientific context the first step consists of analyzing the situation to identify the key entities (variables, parameters, objects) and their relations (properties). This is made possible by an integrated and coherent theoretical framework in which objects and relations are specified following a specific *problématique* (a way of looking at the situation). In the case of everyday life, the process is less rigorous or systematic, the model is often constructed on the spot, without looking at it as being such; it seems to be induced naturally by experience and/or a cultural reference.
In a classroom, mathematical word problems are often proposed to students by means of a text which tells a story. This means that linguistic skills and an initial knowledge are needed to make sense of the story and of the question posed. The initial treatment, the results of which are the coding of objects, variables, parameters and their relations is not a mathematical activity in the first place; it relies on the solver prior knowledge in the field of reference and practice at stake. It is an activity recognized from within the specific sphere of practice to which the situation refers. This step produces a first level of modeling, a first model which could be called a *qualitative model of the situation*. It is on this model that the

construction of the *mathematical model of the situation* is based. Then comes the mathematical treatment the outcome of which is an object (a number, a statement, a diagram) which the solver considers as the solution of the problem. This object must be localized back in the qualitative model, being translated as an object relevant to the situation, expressing its relations with the other objects, and then be interpreted within the context of the actual situation. The following schema sketches this process:



This schema does not systematically correspond to steps one can observe, some of these steps can be left implicit. For example, the congruence between the text describing the actual situation and the expression of the corresponding mathematical model could be so close that the qualitative modeling is not perceived as such.

This is the case in simple word problems such as "Joshua had five marbles. He won two more marbles. How many marbles does he have now?" On the other hand, it can be the origin of difficulties when there is no direct linguistic congruence. For example, in a problem that includes an expression such as: "Joshua lost two more marbles" (for more about semantic and congruence see Duval 1988).

The process is not always linear. Loops can be created in the course of the construction of one of the models or translation/interpretation of the results. Modeling is a dialectical activity which implies constant confrontation of the situation of reference and the mathematics.

Thus, although the solving of a problem should have the general structure we have described, the actual process does not necessarily include these steps explicitly.

Moreover, the problem solver brings into the situation his controls and representations that are related to his conception of the current didactical contract. These inputs can lead to very different processes and thus also to different results. This can happen not only in the solution of a word problem, where the situation has to be analyzed, but also in solving mathematical problems. Observing two students solving a geometry problem Balacheff (2010) points out that although they are supposedly solving the same problem, they have different conceptions leading to different controls. While one of them is using algebraic equations leading him easily to a correct solution, the other is using a symbolic-arithmetical approach that leads him to a conflict with the visual evidence.

A similar observation can be made about the doctoral student and her supervisor: Although they are supposedly solving the same problem, their conceptions are so different that one might say they are, practically, solving different problems. In the following section we investigate what conceptions teachers develop on the solution of a simple word problem.

3.2 The Lottery Problem and teacher conceptions

Two research directions emerged following the encounter with the doctoral student: A theoretical investigation of the meaning of mathematizing, and an investigation of teacher conceptions.

A part of the theoretical investigation is described by Peled (2010) in an article that raises the issue of authority in the sense of trying to identify sources for determining the choice of a mathematical model. In a classical example of what is considered a proportion problem about fish food (with given information on fish length and fish feeding) Peled (ibid) claims that there is not enough information for deducing a proportion model,

Using this analysis and looking for the source of information for determining the mathematical model in the lottery problem, one realizes that here, too, there is no basis for thinking that proportion is the model that should be used. Still, as observed in Ron's story, Ron was convinced that his teacher would only accept the solution that used a proportion model as the right solution. Do other teachers have similar conceptions and beliefs?

In our work with prospective teachers and in-service teachers we have used several problems including the lottery problem to identify teacher conceptions.

The lottery problem was often accompanied with a request to evaluate alternative solutions similar to the additional solutions suggested by Ron.

In one of our studies, part of which is described by Peled and Bassan-Cincinatus (2005), one of the groups we worked with consisted of 43 elementary school teachers who were asked to solve the problem and react to three answers: 1. Equal sharing (20 each). 2. Same difference as between the "investments" (20½, 19½). 3. Proportional sharing (24, 16). While all 43 teachers thought that the proportional split is a correct solution, they differed with regard to their opinions on the two other solutions. For example, with regard to the equal share solution, only 7 teachers were ready to accept it as a correct solution, and 23 thought it was incorrect. A third group of 13 teachers thought it was correct and incorrect at the same time, meaning that "it is ok in real life, but not as a mathematical solution" and that the two partners have the right "to share the money anyway they choose, but in principle they should share their winnings using the 3:2 ratio". It should be noted that the second alternative solution was much less popular with only 1 teacher viewing it as a correct solution and 6 as acceptable but not as a mathematical solution.

As it seems, these reactions and similar experience with prospective and in-service teachers suggest that most of the teachers have a traditional conception towards the solution of the problem and at best are ready to accept alternative solutions as "A good real-life solution but not a good mathematical solution". This latter view is similar to the view expressed in the introduction by the doctoral student. Very few teachers have expressed a view that exhibits the viewpoint, presented in the introduction by the advisor, that the problem does not impose a specific mathematical structure. This viewpoint was expressed by a teacher following some reflection: "On a second thought, nowhere in the problem does it say that they will receive according to their investment ratio".

The different viewpoints can be summarized as the following conceptions:

1. A traditional view of problem solving
2. A traditional view of problem solving with openness towards realistic considerations.
3. A modeling view of problem solving

Table 1. Different conceptions exhibited in the solution of the Lottery Problem.

	1. A traditional PS conception	2. A traditional PS conception + realistic considerations	3. A modeling conception
The solution	proportion	proportion	No preference
Attitude towards alternative solutions	Does not accept as legitimate problem solutions	Accepts as proper solutions because "that's how it is in life", but doesn't view them as "mathematical solutions"	Attributes the same "mathematical status" to all well explained solutions

Table 1 details the essence of these three conceptions including the opinion whether there is a solution that can be considered *the* correct solution, and the attitude towards any other alternative solutions. The first two conceptions seem to be relatively close in their viewpoints, while the third looks more fundamentally different than the others.

4 Designing task examples to diagnose and change conceptions

The findings on teacher conceptions strengthen our conviction that allowing for realistic considerations is not the complete story on what modeling is about, and that we should design task examples that give teachers the opportunity to move beyond realistic considerations.

As it turns out, the traditional didactical contract has not only resulted in making problem solvers avoid making a serious analysis of the given problem situation, but has also caused them to develop a wrong conception on the meaning of fitting a mathematical model in a given situation, i.e. on the role of mathematics.

312

As mentioned earlier, the main problem according to Peled (2010) is in attributing too much authority to certain mathematical models in the process of choosing and fitting a mathematical structure. In the Lottery Problem solvers accept a specific model as a must, without realizing that the participants in this "game" can make a

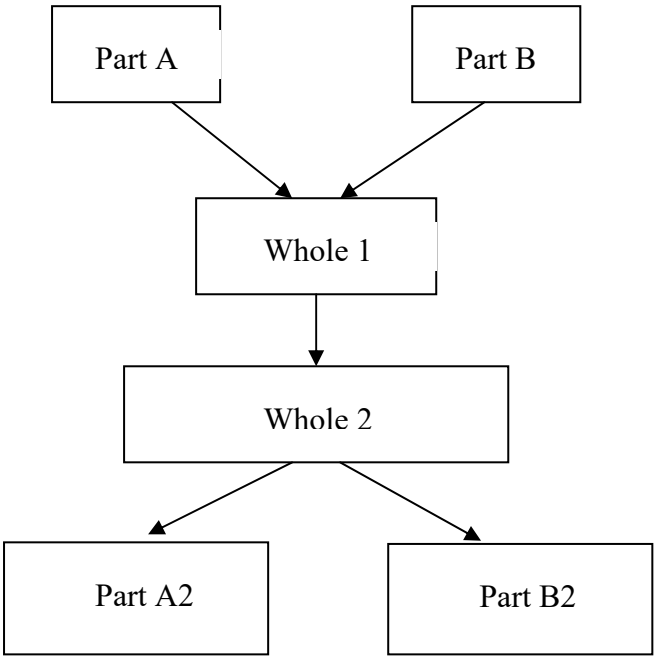
variety of sharing decisions. They do not bother contemplating about the source of this decision and never raise any doubt, or pause and ask: why proportion?
In the following sections we present tasks designed to lead to a new didactical contract. This contract is supposed not only to encourage situation analysis and the use of realistic considerations, but also to facilitate constructing the habit to search for the relevant sources of knowledge for fitting mathematical models in different and specific contexts.

4.1 Starting to ask questions: Creating a cognitive conflict

Obeying the traditional didactical contract teachers identify the Lottery Problem immediately as belonging to the collection of ratio and proportion problems. They have most likely, encountered similar problems in the mathematics textbook and have used it in class.
As detailed earlier, even teachers who accept alternative solutions as legitimate answers do not view them as having "a mathematical status". Therefore, in the design of this task we searched for a problem they would tend to solve not using proportion, and would still consider it "a mathematical solution". Another design requirement was that the problem will have a structure that would be analogical (or rather pseudo-analogical) to the lottery problem's structure.
The goal was to create a cognitive conflict as a result of teacher realization that they do not use proportion in a problem that is similar to a proportion problem, and that they regard this other solution as a mathematical solution and not as [a mathematically inferior] real life solution.
The following Lemonade Stand Problem was composed as an example of a problem that obeys these specifications:

> *During the Country Fair Patricia and Max put up a lemonade stand. Max bought disposable cups for $5 and Patricia bought some concentrated juice for $10. These were all their expenses. They sold lemonade for a total of $300. How should they split the money?*

As can be seen, this is not a common textbook problem, but it has a structure that is similar to the lottery problem:



In both problems two partners are paying ("investing") different amounts of money, depicted by Part A and Part B, adding up to the needed price, Whole 1. Then there is some total revenue, Whole 2, which has to be split up between the two partners.

It is expected that at least part of the teachers will solve the Lemonade Stand problem by using a "reimburse and split" model. That is, reimburse each partner for the expenses and split the rest evenly. Thus, Max would get the amount of $5 + $ ((300-15)/2) and Patricia would get the rest, $10 + ((300-15)/2).

It is also expected that they will regard the solution as a sound mathematical solution.

Class discussion can then ask for comparison with the lottery problem and raise different issues. The structure similarity can be brought up followed by possible discomfort with the earlier attitude towards one of the alternative solutions (the "keep the same difference" solution is, in fact, equivalent to the "reimburse and split" solution) as non mathematical.

It should be noted that the order of task implementation might influence teacher solution for the Lemonade Stand Problem. If the Lemonade problem follows the Lottery Problem too closely, more teachers would perceive it as similar to the

Lottery Problem, impose a similar mathematical structure on it, and solve it using proportion.

Obviously, the use of proportion spoils the whole point and stands in the way of creating a cognitive conflict. However, if there is a diversity of solutions, class discussion can still evoke interesting argumentations and raise enough doubt to start creating a change. This was evident in our work with a group of elementary school teachers, who came up with three different solutions to the Lemonade problem:

a. split the income evenly.

b. reimburse expenses and split the rest evenly.

313 c. divide the income proportionally (using the expenses ratio).

The teacher educator asked whether they feel that one of these three solutions is *the* correct solution. The following excerpt details the reaction of three of the teachers:

Anna: *The answer that uses proportion is the correct answer. The other two answers can be correct only in a social-studies class. In a mathematics class I expect a mathematical solution.*

The teacher educator asked her to try and convince the group that they should use proportion. Turning to the whole class she asked them to think about the question: *Who determines the mathematical model we should use, and how do we know that the proportion is the right model here?*

Leora: *I think that in the two solutions, half-half and reimburse and split, we made some assumptions on the basis of which the solution was given.*

TE: *and what about the proportion solution – have we had no assumption there?*

Leora: *No. There was no assumption there. This is given in the problem: This is the investment and this is the income.*

Molly (turns to Leora abruptly): *Who said so?! It isn't written here (in the problem) 'please split the income using the investment ratio'*

4.2 Knowledge about modeling and the role of mathematics

The comparison of the Lemonade Stand problem with the Lottery Problem raises questions on whether proportion is the only solution of the Lottery Problem, and

what should be the status of alternative solutions. Additional tasks will help in resolving some of the conflicts that are expected to be created.

In an effort to understand the different roles of mathematics in a variety of contexts, Peled and Bassan-Cincenatos (2005) designed a task that helps highlight these differences by comparing between problems. The comparison was done between three analogical problems especially constructed for this purpose:

The Lottery Problem

The Paint-mixing Problem

The Car-assembly Problem

All the problems appear in a shorter version in Table 2.

This same task can be given to teachers with the goal of facilitating the re-invention of the analysis presented in the original research. Following problem comparison, the article differentiates between three types of problem contexts: Social-moral problems, scientific problems and computational-technical problems.

These problem categories differ in the freedom the problem solver has in choosing mathematical models to fit given situations. While the scientific context involves a scientific phenomenon the behavior of which dictates the choice of the mathematical model, in the social-moral situation the problem solver has a higher degree of freedom.

As discussed by Peled (2010), these categories involve different types of authorities that serve as sources for determining the mathematical model. The Lottery Problem falls in the social-moral category, where the solver can take responsibility for the model's choice, being free to choose a way of sharing which is not necessarily proportion.

Obviously the freedom to choose a mathematical model is also determined by the wording of the problem. When the problem composer explicitly states assumptions or details that constraint and direct towards a specific choice, no decisions might be left. In the problems that we use here, such as the lottery problem, no explicit assumptions have been made, and yet there is some consensus on the way they should be solved. This consensus or automatic map is what we are trying to break by our analysis while at the same time offering problem categorization as a mental artifact, a tool to think with, to use Norman's (1993) terms.

The design of the following task is based on this categorization.

4.3 Diagnosing conceptions

Following the identification of two main teacher conceptions, a traditional problem solving conception and a modeling conception, and with the categorization of problems into three types of contexts, a diagnostic task was designed.

The design idea was to compose problems that enable two categorizations corresponding to each of the teacher conceptions. The six problems depicted in Table 2 are expected to be sorted by a teacher holding a traditional problem solving conception into two categories: problems that are solved using ratio and proportion, and problems that are solved using an average. A teacher who has a modeling perspective and has acquired some understanding about model fitting and the role of mathematics is expected to categorize the same problems into three categories: social-moral, scientific, and computational-technical.

Table 2: A problem sorting task.

Sort the following problems into at least two disjoint categories.
1. **Paint**: Tom is painting his garden fence. He mixed 3 cans of yellow 2 cans of blue creating a green shade. He needs 30 more cans. How many of them Yellow and how many blue to get the same shade of green? 2**. Packages**: Volunteers are collecting give away food packages. They have 30 packages of 10 Kg each and 20 packages of 18 Kg. each. They have to move some products to get equal weight. What will be the weight of each package? 3. **Lottery**: Andrea and Bill are office mates. They bought a lottery ticket together. The ticket's price was $8 Andrea paid $5 Bill paid $3. They won $40. How much money would each get? 4. **Grades**: The chemistry teacher is preparing the students' grades. Oren's test grades during the course: 4 tests his grade was 6, one test grade: 8, and in 2 tests grade 10. Will Oren's grade be below or above 8? 5. **Car Assembly**: A car is assembled. Each car requires 2 parts called Om and 5 parts called Al. In a shipment the parts for each car are put in a plastic bag. In the last shipment the plastic bags got torn. There is a total of 112 parts. How many Om parts and Al parts should there be in the shipment? 6. **Temperature**: The food engineers are testing different ways of producing some fruit juice. They were going to mix 4 cups of juice the temperature of which was 20°c with 4 cups of juice in 80°c. What should they expect the mixture temperature to be?

Table 3 depicts the two different categorizations at the same time. The lines differentiate between the two categories expected to be created by a teacher holding a traditional conception, while the three columns indicate the three context types within the modeling conception. As can be seen, each problem was composed with specific features so that all the six table cells would have one problem assigned to them.

The task can be used in a process of working with teachers on developing understanding of modeling to evaluate the effect of the process by comparing the way teachers sort in the "pre-test" to the way they sort in the "post-test". A desired outcome would involve a shift from sorting the problems as in Table 3 lines into sorting the problems as depicted in Table 3 columns.

Table 3: The two problem categorizations.

	Social-moral	Scientific	Computational-Technical
Ratio and proportion	Lottery	Paint	Car-Assembly
average	Grades	Temperature	Packages

5 Concluding remarks

With the changing of goals of problem solving from focusing on the construction of mathematical tools to a modeling perspective that includes the construction of modeling skills, teacher educators need new tools. These tools would include tasks that diagnose teacher conceptions and teacher knowledge about modeling. They might also include tasks that help the teacher educator facilitate change from a traditional problem solving conception to a modeling conception.

Within the modeling approach and the new desired mathematical contract, we have chosen to focus on understanding the meaning of modeling, i.e. the meaning of fitting a mathematical model in a given situation, rather than on the development of modeling skills. Thus, although proponents of the modeling approach tend to use complex tasks, we have chosen to use more simple and traditional problems believing that, for our purposes, these "quiet problems" would "run deeply".

On the one hand this choice of simple problems might be difficult because of the strong power of the traditional didactical contract. On the other hand, a successful change in conception with traditional problems might make the shift more complete avoiding the impression that modeling and situation analysis should be used only with especially designed complex modeling problems.

References

Balacheff, N. (2010). Bridging knowing and proving in mathematics: A didactical perspective. In G. Hanna, H. N. Jahnke, H. Pulte (Eds.) *Explanation and Proof in Mathematics: Philosophical and Educational Perspectives* (pp. 115-136). New York: Springer.

Blum, V., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects – state, trends and issues in mathematics instruction. *Educational Studies in Mathematics*, *22*, 37-68.

Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics*. Dordrecht: Kluwer Academic Publishers.

DeFranco, T. C., & Curcio, F. R. (1997). A division problem with a remainder embedded across two contexts: Children's solutions in restrictive versus real-word settings. *Focus on Learning Problems in Mathematics, 19*(2), 58-72.

315

Doerr, H. M., & English, L. D. (2006). Middle grade teachers' learning through students' engagement with modeling tasks. *Journal of Mathematics Teacher Education*, *9*, 5-32.

Duval R. (1988) Ecarts sémantiques et coherence mathématiques : introduction aux problèmes de congruences. *Annales de didactique et de sciencse cognitives*. Vol.1, IREM de Strasbourg, pp.7-23.

Ikeda, T., & Kaiser, G. (2005). The role and the relevance of applications and modelling in Japan and Germany – a comparative study. *Proceedings of the Third International ICMI East Asia Regional Conference on Mathematics Education*, Shanghai, August 7-12.

IREM de Grenoble (1980) Quel est l'âge du capitaine. Bulletin de l'APMEP, 59, 323, 235-243.

Kaiser, G. (2006). The mathematical beliefs of teachers about applications and modelling – results of an empirical study. In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (eds), *Proceedings of the 30th International Conference for the Psychology of Mathematics Education, 3*, 393-400.

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modelling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: A model and modelling perspective on teaching, learning, and problem solving in mathematics education* (pp. 3-33). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought-revealing activities for students and teachers. In A. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 591-645). Mahwah, NJ: Lawrence Erlbaum.

Nesher, P. (1980). The stereotyped nature of school word problems. *For the Learning of Mathematics*, *1*(1), 41-48.

Norman, D. A. (1993). *Things that make us smart*. Addison-Wesley Publishing Company: New York.

Peled, I. (2010). (Fish) food for thought: Authority shifts in the interaction between mathematics and reality. *Mathematics Education Research Journal. 22* (2), 108-120.

Peled, I., & Bassan-Cincinatus, R. (2005). Degrees of freedom in modeling: Taking certainty out of proportion. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th International Conference for the Psychology of Mathematics Education, 4,* 57-64.

Peled, I., & Hershkovitz, S. (2004). Evolving research of mathematics teacher educators: The case of non-standard issues in solving standard problems. *Journal of Mathematics Teacher Education*. *7*(4), 299-327.

Reusser, K., & Stebler, R. (1997). Every word problem has a solution – The social rationality of mathematical modeling in schools. *Learning and Instruction, 7*(4), 309-327.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In Alan H. Schoenfeld (Ed.), *Cognitive science and mathematics education*. Hillsdale, NJ: Erlbaum.

Verschaffel, L., & De Corte, E. (1997). Teaching realistic mathematical modeling and problem solving in the elementary school: A teaching experiment with fifth graders. *Journal for Research in Mathematics Education*, 28, 577-601.

Verschaffel, L., De Corte, E., & Borghart, L. (1997). Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematical modeling of school word problems. *Learning and Instruction*, *7*(4), 339-359.

Verschaffel, L., Greer, B., & De Corte, E. (2002). Everyday knowledge and mathematical modelling of school word problems. In K. Gravemeijer, R. Lehrer, B. van Oers & L. Verschaffel (Eds.), *Symbolizing, modelling and tool use in mathematics education* (pp. 257-276). The Netherlands: Kluwer Academic Publishers.

Verschaffel, L., Van Dooren, W., Greer, B., Mukhopadhyay, S. (2010). Reconceptualising word problems as exercises in mathematical modeling. *Journal fur Mathematik Didaktik (ZDM), 31*, 9-29.