Dynamic Control of Soft Robots

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Abstract: Soft robots present several advantages. However, one of the main challenges of this new field of robotics is to control these robots. The methods used to control rigid robots are not directly relevant and new approaches have to be invented or updated to be applied to this kind of robots. This paper introduces control solutions for soft robots studies taking into account dynamics of the system.

Keywords: Soft robotics, Order reduction, Dynamic behaviour, Lyapunov function

1. INTRODUCTION

Soft robots are made of complex deformable structures with designs often inspired by the organic materials we can find in the nature. Soft robotics present several advantages over their rigid counterparts such as being more flexible and compliant to the environment (Trivedi et al. (2008); Kim et al. (2013)). It makes them inherently safer for human-robot interaction. The use of soft robots is a new way to build robotic systems that can deal with uncertainty and dynamic environments, see for instance Majidi (2014).

At the same time, classical control tools used to work with rigid bodies are not suitable anymore. As pointed out by some recent papers, such as Rus and Tolley (2015), new approaches have to be created or adapted to soft robotics. In contrast to rigid ones, soft robots have a theoretical infinite number of degrees of freedom. In practice, the spatial discretization of the structure leads to a huge number of partial ordinary differential equations and state variables. Moreover, soft robots are highly nonlinear systems and the sensitivity of the materials can easily cause a change in the dynamics of the system. That is why the classical tools of control science cannot be used.

Soft robots motion depends on their deformation for which an accurate model can be obtained using numerical methods, such as the Finite Element Method (FEM). Our work is based on the work promoted in Duriez (2013); Largilliere et al. (2015). These papers introduced a feedforward control method that changes the equilibrium point of the studied structure using an inverse model computed by optimization.

These approaches have been extended to real-time closed-loop control in Zhang et al. (2016). However, the control solutions developed were based on a quasi-static model, which limits the control to low velocities trajectories.

In this paper, we focus on the dynamics of the soft robots. We will use the FEM methodology to get a dynamical model of the soft robot. First, we investigate a model order reduction approach. Then, we propose a controller based on the dynamical reduced-order model of the robot. Our method is generic and does not require any geometrical consideration. Finally, we present work tracks to be investigated in the future.

2. PROBLEM STATEMENT

We define $q \in \mathbb{R}^n$ as the position of each nodes of the finite element model. Thus, $v = \dot{q}$ is the velocity vector of the mesh. From the second law of Newton, we get a nonlinear model of the soft robot behaviour :

$$M(q)\dot{v} = P - F_{int}(v, q) + H^T \lambda(t) \tag{1}$$

where $M(q)$ is the mass matrix, P gathers all the known external forces (in practice, we consider only the gravity field), F_{int} are the internal forces and $H^T\lambda$ is the actuator contribution : H^T is the direction and λ is the amplitude of the actuator forces. Through a linearization of the internal forces (see Saupin et al. (2009); Bosman et al. (2015) for details), we obtain the following non-linear approximation of the soft robot behaviour :

$$M(q)\dot{v} + D(q, v)v + K(q, v)(q - q_0) = H^T \lambda(t) \tag{2}$$

where q_0 is one equilibrium point induced by P. The matrices $K(q, v)$ and $D(q, v)$ are respectively the compliance matrix and the Rayleigh damping matrix defined as :

$$D(q, v) = \alpha M(q) + \beta K(q, v) \tag{3}$$

α and β being respectively the mass-proportional and the stiffness-proportional damping coefficients of the material.

We then obtain a large-scale non-linear system described by the following state-space equation :

$$\begin{cases} \dot{x}(t) = A(x)x(t) + B(x)u(t) \\ y(t) = Cx(t) \end{cases} \tag{4}$$

with system matrices defined as :

$$x = \begin{pmatrix} v \\ q - q_0 \end{pmatrix} \; ; \; A = \begin{pmatrix} -M(\cdot)^{-1}D(\cdot) & -M(\cdot)^{-1}K(\cdot) \\ I_n & 0 \end{pmatrix}$$
$$B = \begin{pmatrix} M(\cdot)^{-1}H^T \\ 0 \end{pmatrix} \; ; \; u(t) = \lambda(t)$$

I_n is the identity matrix of the same dimension as the position vector q, the matrix C is a sparse matrix defining the effector coordinates and $(\cdot)$ represent omitted matrix function arguments.

We consider a deformable structure such as the one presented in Fig. 1. This structure does not have a specific shape. It is actuated by three cables pulling on the arms which allows the top of the robot to move on the 2D space. When actuated, each cable creates a deformation on the whole structure so that the three cables are coupled by deformation. The triangular mesh of the robot is made of 157 nodes. Considering both position and velocity in the two dimensions, we will have a system, described by equation (4), of order 628.

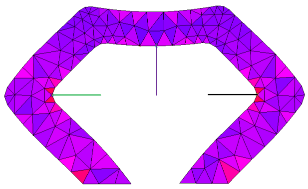

Fig. 1. FEM model of the 2D robot used in simulation.

3. LINEAR REDUCED ORDER CONTROL

3.1 Motivations

Considering the linearization of equation (4) around an equilibrium point, we can assume that the mass, compliance and damping matrices are constant, i.e $M(q) = M$, $K(q, v) = K$ and $D(q, v) = D$ during all the simulation. Thus, the state-space equation (4) becomes a Linear Time Invariant (LTI) model :

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{5}$$

with $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{p \times n}$. In this case, an accurate dynamical control of the soft body could be done with a pole placement feedback. However, the existing algorithms of model reduction in control science are not able to deal with so large scale systems. For instance, the balanced truncation method failed when the order of the system is too high (see Antoulas (2005)).

We could investigate and use different order reduction methods, such as the *Iterative SVD-Krylov based method* (Gugercin (2008)) extended to the multi-input multi-output case with the *Iterative SVD-Tangential Interpolation Method* (see Poussot-Vassal (2011) for details). However, these methods are adapted to the linear case only. Our final objective is to deal with the non-linear representation of the soft-robot, we thus use a method which is more intended for non-linear systems.

3.2 Preliminaries

In this contribution, we will use the snapshot-Proper Orthogonal Decomposition (POD) (see for example Sirovich (1987); Goury (2015)) well known in the computational mechanics community for reducing the computational burden of numerical simulations while keeping an accuracy chosen by the user. The state variables are expressed as a linear combination of modes, which are fixed once and for all. Typically, few modes are required to capture the motion of a mechanical structure, even though this structure is represented by a fine mesh. In consequence, the number of degrees of freedom becomes the magnitude of each mode in the reduced model rather than the displacement of each node of the FEM mesh. The number of degrees of freedom are also greatly reduced. The modes are extracted from a so-called snapshot space using a singular value decomposition. The snapshot space is generated in an offline, expensive stage, where many full order simulations of the structure are performed, under various parametric conditions. These simulations should be as exhaustive as possible, so as to capture any case the structure may encounter in the online stage.

In the context of control of soft robots, this method will provide us with a system of achievable size to apply traditional rigid robotics control techniques. In the case of the robot depicted in Fig. 1, the reduced order is 8.

We can approximate the displacement vector $q - q_0$ by :

$$q - q_0 \approx \sum_{i}^{n} \alpha_i \phi_i = \Phi\alpha \tag{6}$$

where

$$\Phi = (\phi_1 \;\, \phi_2 \;\, \ldots \;\, \phi_n) \;\; and \;\; \alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} \tag{7}$$

Φ being the chosen deformation modes and the corresponding weighting α. The vectors ϕ_i are all orthogonal.

As we are interested in the dynamics of the robot, we store the acceleration of the robot at any time during a first experiment. We also obtain the snapshot matrix S :

$$S = (\dot{v}_{t0} \;\, \dot{v}_{t1} \;\, \dot{v}_{t2} \;\, \ldots \;\, \dot{v}_{tf}) \tag{8}$$

Performing a singular value decomposition of this snapshot matrix, we get Φ, the left-singular matrix. For the reduction model to be as accurate as possible, the singular value decomposition has to be performed on a large number of values, in our case, we store 3360 snapshots of $\dot{v}$.

3.3 Application to soft robots

Starting from the linear state-space representation (5), the following relationship models the transformation between the full state x of dimension n and the reduced state x_r. This new state will be made of the reduced velocity vector and reduced displacement vector, both of dimension $\frac{r}{2}$. The reduced state x_r will consequently be made of r values. This state reduction is made through the reduction matrix T :

$$Tx_r = x \tag{9}$$

where T is a rectangular matrix defined as :

$$T = \begin{pmatrix} \Phi_r & 0 \\ 0 & \Phi_r \end{pmatrix} \tag{10}$$

with $T^T T = I_r$, where I_r is the identity matrix of reduced dimension and Φ_r a matrix made of the $\frac{r}{2}$-first columns of the matrix Φ.

Thus, we have the reduced linear state-space representation :

$$\begin{aligned} &\begin{cases} \dot{x}_r = T^T A T x_r + T^T B u \\ y = C T x_r \end{cases} \\ &\Leftrightarrow \\ &\begin{cases} \dot{x}_r = A_r x_r + B_r u \\ y = C_r x_r \end{cases} \end{aligned} \tag{11}$$

with $A_r \in \mathbb{R}^{r \times r}$, $B_r \in \mathbb{R}^{r \times m}$ and $C_r \in \mathbb{R}^{p \times r}$. From this reduced-order model, we can define a linear feedback $u = -K x_r$. We also obtain the closed-loop state space model :

$$\begin{cases} \dot{x}_r = (A_r - B_r K) x_r \\ y = C_r x_r \end{cases} \tag{12}$$

We use the pole placement method to compute the matrix K. We want to assign the closed-loop poles (denoted by $poles_{CL}$) to desired locations. In order to keep the natural frequencies of the system, we simply delete the imaginary part of the open-loop poles ($poles_{OL}$) :

$$poles_{CL} = Re(poles_{OL}) \tag{13}$$

where $Re(z)$ is the real part of a complex number z.

3.4 Simulation results

The singular values of the model of the 2D robot are shown in Fig. 2. We can easily see that these singular values decrease very fast. For simulation experiments, we keep the first four singular values, which lead to a matrix T of dimension (628×8) and to a reduced-order system of order 8. We get the full state x of the soft robot from a Luenberger observer, which is also based on the reduced linear model. Let $\hat{x}_r$ be the observed state, we thus define a state-feedback control law $u = -K\hat{x}_r$.

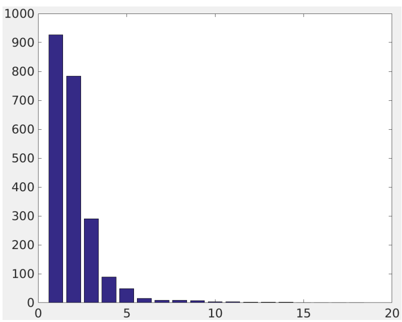

Fig. 2. 20 firsts singular values of the model of the soft robot.

Figures 3 and 4 show the trajectory of the effector of the robot. It is clear that, with actuation, the system converges faster to the desired position. One of the objectives is to design a matrix T that provides controllability and observability properties for the reduced-order system. The challenge is to find the optimal reduced order base. One of the problems is also to find the good excitation signal.

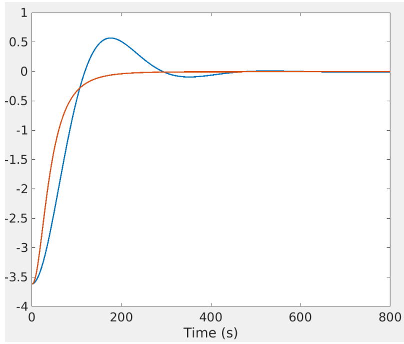

Fig. 3. Abscissa of the effector-node of the mesh in open-loop (blue) and closed-loop (red).

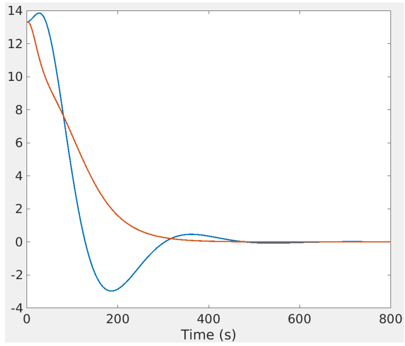

Fig. 4. Ordinate of the effector-node of the mesh in open-loop (blue) and closed-loop (red).

If the matrix Φ is computed with a bad snapshot matrix, the reduced model will not be consistent. The snapshot matrix must be saved while the robot is deformed in the same way as during the control experiment.

For instance, the control law based on the linear model (11), shows limitations in some cases. When the initial geometry of the robot is highly deformed, in a way that has not been taken into account in the snapshot matrix, the control input can destabilize the system. Moreover, as contacts are not handled in this model, when the initial state of the robot presents self-collisions or when the control law leads to self-collision, the robot does not get to the desired position.

4. CONCLUSION AND FUTURE WORK

The control law based on a LTI reduced order model is easy to design. With this method, we can use the classical tools of control theory, such as pole placement, which can be efficiently implemented. One of the challenging parts is to find a suitable reduction matrix to keep interesting properties for the reduced order system. This can be achieved using POD. However, this method only works well while the trajectory of the robot remains in a set around the equilibrium.

The next step of this work is to increase the working area of the robot. To achieve this objective, we first need to bound the uncertainties brought by the order reduction. This work is still under development and several leads are investigated. We will compare the results of the POD to the one brought by the mixed SVD-Krylov approaches, which are implemented in Matlab within the

MORE toolbox (see Poussot-Vassal and Vuillemin (2012) for details).

Once this is achieved, we will design a robust control law (see for instance Zhou and Doyle (1998)), to specify performances while taking into account order reduction uncertainties.

To represent non-linearities more faithfully than with the LTI model, we could use a Linear Parameter-Varying (LPV) model (see Briat (2008)). In the future, we will also compute the linearization of the compliance matrix around different points of interest. We will thus have the possibility to control the robot in a larger working area. To go further, this LPV representation could also help to take into account contacts between the robot and the environment.

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