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Conical maximal regularity for elliptic operators via Hardy spaces

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We give a technically simple approach to the maximal regularity problem in parabolic tent spaces for second order, divergence form, complex valued elliptic operators. By using the associated Hardy space theory combined with certain $L^2 - L^2$ off-diagonal estimates, we reduce the tent space boundedness in the upper half-space to the reverse Riesz inequalities in the boundary space. This way, we also improve recent results obtained by P. Auscher et al.

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1. Introduction

Let \mathbb{R}_+^{1+n} be the upper half-space $\mathbb{R}_+ \times \mathbb{R}^n$ with $\mathbb{R}_+ = (0, \infty)$ and $n \in \mathbb{N}_+ = \{1, 2, \dots\}$. Define the tent space T_{par}^p , $\frac{n}{n+1} < p < \infty$, as the space of all

locally square integrable functions on \mathbb{R}_+^{1+n} such that

$$(1) \quad \|F\|_{T_{\text{par}}^p} := \left(\int_{\mathbb{R}^n} \left(\iint_{\mathbb{R}_+^{1+n}} \frac{\mathbf{1}_{B(x, t^{\frac{1}{2}})}(y)}{t^{\frac{n}{2}}} |F(t, y)|^2 dt dy \right)^{\frac{p}{2}} dx \right)^{\frac{1}{p}} < \infty.$$

The scale T_{par}^p , $\frac{n}{n+1} < p < \infty$, is a parabolic analogue of the tent spaces introduced by R. R. Coifman, Y. Meyer and E. M. Stein in [CMS85].

Let $A = A(x)$ be an $n \times n$ matrix of complex L^∞ coefficients, defined on \mathbb{R}^n , and satisfying the ellipticity (or “accretivity”) condition

$$(2) \quad \lambda |\xi|^2 \leq \operatorname{Re} A \xi \cdot \bar{\xi} \quad \text{and} \quad |A \xi \cdot \bar{\zeta}| \leq \Lambda |\xi| |\zeta|$$

for $\xi, \zeta \in \mathbb{C}^n$ and for some λ, Λ such that $0 < \lambda \leq \Lambda < \infty$. Let

$$L := -\operatorname{div} A \nabla$$

(its precise definition will be recalled in next section). Consider the associated forward maximal regularity operator \mathbf{M}_L^+ given by

$$(3) \quad \mathbf{M}_L^+(F)_t := \int_0^t L e^{-(t-s)L} F_s ds,$$

originally defined on $F \in L^2(\mathbb{R}_+; \mathbf{D}(L))$. Here $\mathbf{D}(L)$ is the domain of L in $L^2(\mathbb{R}^n)$ and $F_s = F(s, \cdot)$. By a classical result of L. de Simon [dS64], \mathbf{M}_L^+ extends to a bounded operator on $L^2(\mathbb{R}_+; L^2(\mathbb{R}^n))$. By Fubini’s theorem

$$(4) \quad T_{\text{par}}^2(\mathbb{R}_+^{1+n}) \simeq L^2(\mathbb{R}^n; L^2(\mathbb{R}_+)).$$

For p different from 2, the analogous equivalence of (4) between $T_{\text{par}}^p(\mathbb{R}_+^{1+n})$ and $L^p(\mathbb{R}^n; L^2(\mathbb{R}_+))$ breaks down. We shall refer to the maximal regularity (namely, the boundedness of \mathbf{M}_L^+) in T_{par}^p as *conical* maximal regularity for the reason that (parabolic) *cones* are involved in defining tent spaces in (1).

The maximal regularity operator \mathbf{M}_L^+ is a typical example of singular integral operators with operator valued kernels. Let $1 \leq p \leq 2$. Let

$$\operatorname{dist}(E, E') := \inf \{|x - y| : x \in E, y \in E'\}.$$

We shall say that a class of uniformly $L^2 = L^2(\mathbb{R}^n)$ bounded kernels $\{T(t)\}_{t>0}$ satisfies the $L^p - L^2$ off-diagonal decay with some order $M \in \mathbb{N}_+$, if we have

$$(5) \quad \|\mathbf{1}_{E'} T(t) \mathbf{1}_E f\|_{L^2} \lesssim t^{-\frac{n}{2}(\frac{1}{p}-\frac{1}{2})} \left(1 + \frac{\text{dist}(E, E')^2}{t}\right)^{-M} \|\mathbf{1}_E f\|_{L^p}$$

for all Borel sets $E, E' \subset \mathbb{R}^n$, all $t > 0$ and all $f \in L^p \cap L^2$. We shall say $\{T(t)\}_{t>0}$ satisfies the $L^p - L^2$ off-diagonal decay if it satisfies the $L^p - L^2$ off-diagonal decay with any order $M \in \mathbb{N}_+$. Denote by $p_- = p_-(L)$ the infimum of p for which the heat semigroup $\{e^{-tL}\}_{t>0}$ satisfies the $L^p - L^2$ off-diagonal decay. Define the index

$$(6) \quad (p_-)_* := \frac{np_-}{n + p_-}.$$

For $L = -\Delta = -\text{div} \nabla$, one has $p_- = 1$ and $1_* = \frac{n}{n+1}$.

Our main result in this letter reads as follow.

Theorem 1.1. *Let $L = -\text{div} A \nabla$ with A satisfying (2) and p_- defined as in (6). Then, for $p \in ((p_-)_*, 2]$, the maximal regularity operator \mathbf{M}_L^+ defined as in (3) extends to a bounded operator on T_{par}^p .*

We end the introduction with several remarks.

Remark 1.2. Under the assumption $(p_-)_* < 1$ Theorem 1.1 is first proved in [AKMP12, Theorem 3.1] (with $m = 2$, $\beta = 0$ and q close to p_- in their statement). Indeed, we note that $(p_-)_* < 1$ is equivalent to $(p_-)' > n$, where $(p_-)'$ is the dual exponent of p_- . A threshold condition essentially the same as $(p_-)' > n$ is used in [AKMP12].

A general framework of singular integral operators on tent spaces is also presented in [AKMP12]. Their method is heavily based on the $L^p - L^2$ off-diagonal decay of the family $\{tLe^{-tL}\}_{t>0}$ for $p \in (p_-, 2)$. Note that they already improved the previous result in [AMP12], the T_{par}^p -boundedness of \mathbf{M}_L^+ for $p \in (2_*, 2]$, which assumes $L^2 - L^2$ off-diagonal decay only.

Here we shall give a technically simple approach to Theorem 1.1 by using the well-established L -associated Hardy space theory combined (mainly) with $L^2 - L^2$ off-diagonal decay of $\{tLe^{-tL}\}_{t>0}$.

Remark 1.3. The motivation of the following reduction scheme

$$(\text{Operator theory on tent spaces}) \longrightarrow (\text{Hardy space theory}),$$

which is involved in our proof of Theorem 1.1, comes from the study of conical maximal regularity (in elliptic tent spaces) for first order perturbed Dirac operators. See [Hua15, Chap. 5]. Furthermore, the motivation of considering such conical (elliptic) maximal regularity estimates is suggested by their applications to boundary value elliptic problems (see [AA11] for example). In parabolic case, the conical maximal regularity results have already proven to be useful in various settings (see for example [AvNP14, AF15]).

Remark 1.4. Though the singularity of the integral operator \mathbf{M}_L^+ is at $s = t$, the most involved part turns out to be the estimation of tent space norms when $s \rightarrow 0$. For more explanations concerning the “singularity” pertaining to singular integral operators and maximal regularity operators on tent spaces, see [AKMP12, Remark 3.6] and [AF15, Remark 5.23].

Remark 1.5. Theorem 1.1 also extends to higher order elliptic operators. Then one changes correspondingly the homogeneity of tent spaces and off-diagonal decay in (5). We leave this issue to the interested reader.

2. Elliptic operators and Hardy spaces

We give some preliminary materials needed in the proof of Theorem 1.1.

Let A satisfy (2). We define the divergence form elliptic operator

$$Lf := -\operatorname{div}(A\nabla f)$$

which we interpret in the sense of maximal-accretive operators via a sesquilinear form. That is, $\mathbf{D}(L)$ is the largest subspace contained in $W^{1,2}$ for which

$$\left| \int_{\mathbb{R}^n} A\nabla f \cdot \nabla g \right| \leq C\|g\|_2$$

for all $g \in W^{1,2}$ and we set Lf by

$$\langle Lf, g \rangle = \int_{\mathbb{R}^n} A\nabla f \cdot \overline{\nabla g}$$

for $f \in \mathbf{D}(L)$ and $g \in W^{1,2}$. Thus defined, L is a maximal-accretive operator on L^2 and $\mathbf{D}(L)$ is dense in $W^{1,2}$. Furthermore, L has a square root, denoted by $L^{1/2}$ and defined as the unique maximal-accretive operator such that

$$(7) \quad L^{1/2} L^{1/2} = L$$

as unbounded operators (see [Kat76, p. 281]).

For L as formulated above, the development of an L -associated Hardy space theory, was taken in [HM09] (and independently in [AMR08] in a different geometric setting), in which the authors considered the model case $H_L^1(\mathbb{R}^n)$. In presence of pointwise heat kernel bounds, see [DY05]. The definition of H_L^1 given in [HM09, AMR08] can be extended immediately to $\frac{n}{n+1} < p \leq 2$ (see [HMM11]). To this end, consider the (conical) square function associated with the heat semigroup generated by L

$$\mathbf{S}_L(f)(x) := \left(\iint_{\Gamma(x)} \left| t^2 L e^{-t^2 L} f(y) \right|^2 \frac{dt dy}{t^{1+n}} \right)^{\frac{1}{2}}, \quad x \in \mathbb{R}^n,$$

where, as usual,

$$\Gamma(x) = \{(t, y) \in \mathbb{R}_+^{1+n} : |x - y| < t\}$$

is a non-tangential cone with vertex at $x \in \mathbb{R}^n$. As in [HM09, HMM11], we define $H_L^p(\mathbb{R}^n)$ for $\frac{n}{n+1} < p \leq 2$ as the completion of

$$\{f \in L^2(\mathbb{R}^n) : \mathbf{S}_L(f) \in L^p(\mathbb{R}^n)\}$$

in the quasi-norm

$$\|f\|_{H_L^p(\mathbb{R}^n)} := \|\mathbf{S}_L(f)\|_{L^p(\mathbb{R}^n)}.$$

We will not get into the dual side ($p > 2$) of the Hardy space theory.

For $L^2 - L^2$ off-diagonal decay related to $\{e^{-sL}, sL e^{-sL}, \sqrt{s} \nabla e^{-sL}\}_{s>0}$, and other holomorphic functions of L (for example $(I - e^{-sL})^\sigma$ with $\sigma > 0$), we refer to Chapter 2 of the memoir [Aus07].

3. Proof of Theorem 1.1

Note that the extension of \mathbf{M}_L^+ will be divided into two steps: first from $F \in L^2(\mathbb{R}_+; \mathbf{D}(L))$ to T_{par}^2 , and then for $\frac{n}{n+1} < p < 2$, from $T_{\text{par}}^2 \cap T_{\text{par}}^p$ to T_{par}^p .

First we split the operator \mathbf{M}_L^+ : for $\ell \in \mathbb{N}_+$ large, set

$$(8) \quad \mathbf{R}_L^\ell := \mathbf{M}_L^+ - \mathbf{V}_L^\ell,$$

where for $F \in L^2(\mathbb{R}_+; \mathbf{D}(L))$, the singular part \mathbf{R}_L^ℓ is given formally by

$$(9) \quad \mathbf{R}_L^\ell(F)_t = \int_0^t L e^{-(t-s)L} (I - e^{-2sL})^\ell F_s ds,$$

and the regular part is defined by

$$\mathbf{V}_L^\ell = \sum_{k=1}^{\ell} \binom{\ell}{k} \mathbf{V}_{L,k}$$

with

$$\mathbf{V}_{L,k}(F)_t := \int_0^t L e^{-(t+(2k-1)s)L} F_s ds, \quad t \in \mathbb{R}_+.$$

For the above binomial sum \mathbf{V}_L^ℓ , it suffices to consider $\mathbf{V}_L := \mathbf{V}_{L,1}$.

Let $2\mathbb{N}_+ = \{2, 4, \dots\}$. We have the following observation.

Lemma 3.1. *For $\ell \in 2\mathbb{N}_+$ and $\frac{\ell}{2} > \frac{1}{2} + \frac{n}{4}$, the operator \mathbf{R}_L^ℓ , as given in (9) through (8), extends to a bounded operator on T_{par}^p for any $\frac{n}{n+1} < p \leq 2$.*

Proof. The T_{par}^2 -boundedness is de Simon’s theorem plus the uniform L^2 -boundedness of $\left\{ (I - e^{-2sL})^\ell \right\}_{s>0}$. By interpolation it suffices to consider $\frac{n}{n+1} < p \leq 1$, and this follows from Lemma 3.4 and Lemma 3.5 of [AKMP12] in the particular case $m = 2$, $\beta = 0$ and $q = 2$.¹ Indeed, first we can decompose the operator \mathbf{R}_L^ℓ as in [AKMP12] in the way

$$\begin{aligned} \mathbf{R}_L^\ell(F)_t &= \int_{t/2}^t L e^{-(t-s)L} (I - e^{-2sL})^\ell F_s ds \\ &\quad + \int_0^{t/2} L e^{-(t-s)L} (I - e^{-2sL})^\ell F_s ds =: I + II. \end{aligned}$$

¹We point out that one can also prove this lemma by adapting directly the arguments for Lemma 3.4 of [AKMP12] (see [Hua15] for details).

Here we view $\mathcal{T}_1 = \left\{ (I - e^{-2sL})^\ell \right\}_{s>0}$ as an operator on T_{par}^p given by

$$\mathcal{T}_1 : F \mapsto \mathcal{T}_1(F)_s := (I - e^{-2sL})^\ell F_s,$$

with the similar interpretation for $\mathcal{T}_2 = \left\{ (I - e^{-2sL})^\ell / (sL)^{\frac{\ell}{2}} \right\}_{s>0}$ in

$$\begin{aligned} & L e^{-(t-s)L} (I - e^{-2sL})^\ell \\ &= \left(\frac{s}{t-s} \right)^{\frac{\ell}{2}} L ((t-s)L)^{\frac{\ell}{2}} e^{-(t-s)L} \frac{(I - e^{-2sL})^\ell}{(sL)^{\frac{\ell}{2}}}. \end{aligned}$$

Note that $t-s \sim t$ when $s < t/2$. Therefore, to obtain the T_{par}^p -boundedness of \mathbf{R}_L^ℓ for $\frac{n}{n+1} < p \leq 1$, we can use Lemma 3.4 of [AKMP12] together with the T_{par}^p -boundedness of \mathcal{T}_1 to estimate I , and use Lemma 3.5 of [AKMP12] together with the T_{par}^p -boundedness of \mathcal{T}_2 to estimate II . The latter tent space boundedness results on \mathcal{T}_i , $i = 1, 2$, are implied by their $L^2 - L^2$ off-diagonal decay with order at least $\frac{\ell}{2}$, which satisfies the condition

$$\frac{\ell}{2} > \frac{1}{2} + \frac{n}{4} = \frac{n}{2} \left(\frac{1}{\frac{n}{n+1}} - \frac{1}{2} \right).$$

This implication can be easily verified via the extrapolation method on tent spaces through atomic decompositions. Note that we also need the condition $\frac{\ell}{2} > \frac{1}{2} + \frac{n}{4}$ in $(\frac{s}{t-s})^{\frac{\ell}{2}} \sim (\frac{s}{t})^{\frac{\ell}{2}}$ when applying Lemma 3.5 of [AKMP12]. \square

Next we rewrite the operator \mathbf{V}_L in the following way:

$$(10) \quad \mathbf{V}_L(F)_t = -\tilde{\mathbf{V}}_L(F)_t + \mathbf{I}_L(F)_t, \quad t \in \mathbb{R}_+,$$

where for $F \in L^2(\mathbb{R}_+; \mathbf{D}(L))$, the backward part $\tilde{\mathbf{V}}_L$ is defined by

$$(11) \quad \tilde{\mathbf{V}}_L(F)_t := \int_t^\infty L e^{-(t+s)L} F_s ds, \quad t \in \mathbb{R}_+$$

and the trace part \mathbf{I}_L is defined by

$$\mathbf{I}_L(F)_t := \int_0^\infty L e^{-(t+s)L} F_s ds = \sqrt{L} e^{-tL} \int_0^\infty \sqrt{L} e^{-sL} F_s ds.$$

We used the square root property $\sqrt{L}\sqrt{L} = L$ recalled in (7).

Lemma 3.2. *The integral operator $\tilde{\mathbf{V}}_L$ as given in (11) extends to a bounded operator on T_{par}^p for any $\frac{n}{n+1} < p \leq 2$.²*

Proof. This is a consequence of a more general claim found in [AKMP12, Proposition 3.7], again corresponding to the case $m = 2$, $\beta = 0$ and $q = 2$. Indeed, [AKMP12, Proposition 3.7] deals with a counterpart to \mathbf{M}_L^+ , namely, the backward maximal regularity operator

$$\mathbf{M}_L^-(F)_t := \int_t^\infty L e^{-(s-t)L} F_s ds,$$

where $F \in L^2(\mathbb{R}_+; \mathbf{D}(L))$ and they use the splitting

$$\begin{aligned} \mathbf{M}_L^-(F)_t &= \int_t^{2t} L e^{-(s-t)L} F_s ds \\ &\quad + \int_{2t}^\infty L e^{-(s-t)L} F_s ds =: III + IV. \end{aligned}$$

We only need to use those arguments in proving [AKMP12, Proposition 3.7] with IV involved, since $s - t \sim s$ when $s > 2t$, which is equivalent to that $s + t \sim s$ when $s > t$ in our setting. We omit the details. \square

Now we use the L -associated Hardy spaces, which we recalled in Section 2, to treat the trace part \mathbf{I}_L . First, from the conical square function estimates (see [HMM11, Proposition 4.9]), one has, for $\frac{n}{n+1} < p \leq 2$

$$\left\| \sqrt{L} e^{-tL} \int_0^\infty \sqrt{L} e^{-sL} F_s ds \right\|_{T_{\text{par}}^p} \lesssim \left\| \int_0^\infty \sqrt{L} e^{-sL} F_s ds \right\|_{H_L^p}$$

for $F \in L^2(\mathbb{R}_+; \mathbf{D}(L))$. Next, from the reverse Riesz inequalities (see [HMM11, Proposition 5.17]), one has, for $p \in ((p_-)_*, 2]$

$$\|\sqrt{L}f\|_{H_L^p} \lesssim \|\nabla f\|_{H^p}$$

²As we will see in the proof, the lemma also holds for any $0 < p \leq 2$. But that does not help in proving Theorem 1.1.

for $f \in L^2$, hence one further has, for $p \in ((p_-)_*, 2]$

$$\left\| \int_0^\infty \sqrt{L} e^{-sL} F_s ds \right\|_{H_L^p} \lesssim \left\| \int_0^\infty \nabla e^{-sL} F_s ds \right\|_{H^p}.$$

Here, as usual, we make the convention $H^p = L^p$ for $p > 1$.³

For $F \in T_{\text{par}}^2$ consider the sweeping operator

$$\pi_L(F) := \int_0^\infty \nabla e^{-sL} F_s ds.$$

An equivalent formulation of the Kato square root estimate for L^* (see [AHL+02]) is the following square function estimate

$$\iint_{\mathbb{R}_+^{1+n}} \left| e^{-tL^*} \operatorname{div} \vec{F}(y) \right|^2 dt dy \lesssim \|\vec{F}\|_2^2$$

for all $\vec{F} \in L^2(\mathbb{R}^n; \mathbb{C}^n)$, hence the mapping given by

$$\mathbb{Q}_{L^*} : \vec{F} \mapsto \mathbb{Q}_{L^*}(\vec{F})(t, y) := \left(e^{-tL^*} \operatorname{div} \vec{F} \right)(y)$$

is bounded from $L^2(\mathbb{R}^n; \mathbb{C}^n)$ to T_{par}^2 . Thereby, we see that $\pi_L : T_{\text{par}}^2 \rightarrow L^2$ is a bounded operator by duality with \mathbb{Q}_{L^*} .

Recall that, a T_{par}^p -atom A supported in the parabolic Carleson cylinder

$$\text{Cyl}(B) := (0, r_B^2) \times B$$

for some ball $B \subset \mathbb{R}^n$ (with radius r_B), satisfies the size estimate

$$(12) \quad \|A\|_{T_{\text{par}}^2} \leq |B|^{-(\frac{1}{p} - \frac{1}{2})}.$$

We have the following result on π_L .

³We remark that in [AF15, Lemma 5.21] a variant of \mathbf{I}_L is treated in a similar way, with informative connections to the Hardy space theory associated with the first order perturbed Dirac operators as alluded in Remark 1.3.

Lemma 3.3. *For any $\frac{n}{n+1} < p \leq 1$ and any T_{par}^p -atom A with $\text{supp } A \subset \text{Cyl}(B)$ for some ball $B \subset \mathbb{R}^n$ (with radius r_B), then we have that*

$$m := \pi_L(A) = \int_0^{r_B^2} \nabla e^{-sL} A_s ds$$

satisfies the uniform estimate

$$(13) \quad \|m\|_{H^p} \lesssim 1.$$

Hence π_L extends to a bounded operator from T_{par}^p to H^p for $\frac{n}{n+1} < p \leq 2$.

Proof. For $m = \pi_L(A)$ with A being T_{par}^p -atoms, $\frac{n}{n+1} < p \leq 1$, and by adapting [CMS83, Théorème 3] and [CMS85, Theorem 6], (13) follows from the $L^2 - L^2$ off-diagonal decay for the heat semigroup $\{e^{-sL}\}_{s>0}$ and the gradient family $\{\sqrt{s}\nabla e^{-sL}\}_{s>0}$, the size estimate (12) and the Coifman-Weiss molecular theory for H^p . Then for $\frac{n}{n+1} < p \leq 1$, π_L extends to a bounded operator from T_{par}^p to H^p , and by interpolation, π_L extends to a bounded operator from T_{par}^p to H^p for $\frac{n}{n+1} < p \leq 2$. \square

With the splittings (8) and (10), together with the conditions $\ell \in 2\mathbb{N}_+$ and $\frac{\ell}{2} > \frac{1}{2} + \frac{n}{4}$, and using Lemmata 3.1, 3.2 and 3.3 in order, the proof of Theorem 1.1 (with $p \in ((p_-)_*, 2]$) is then concluded.

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