

# Cauchy-Lipschitz theory for fractional multi-order dynamics: State-transition matrices, Duhamel formulas and duality theorems

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**Abstract.** The aim of the present paper is to contribute to the development of the study of Cauchy problems involving Riemann-Liouville and Caputo fractional derivatives. First, existence-uniqueness results for solutions of non-linear Cauchy problems with vector fractional multi-order are addressed. A qualitative result about the behavior of local but non-global solutions is also provided. Finally, the major aim of this paper is to introduce notions of fractional state-transition matrices and to derive fractional versions of the classical Duhamel formula. We also prove duality theorems relying left state-transition matrices with right state-transition matrices.

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