



Benchmark on the 3D Numerical Modeling of a Superconducting Bulk

Kévin Berger¹, Guillaume Escamez², Loïc Quéval³,
Abelin Kameni³, Lotfi Alloui^{3,4}, Brahim Ramdane²

¹GREEN, Université de Lorraine, 54506 Vandœuvre-lès-Nancy, France

²Univ. Grenoble Alpes, CNRS, Grenoble INP, G2Elab, F-38000 Grenoble, France

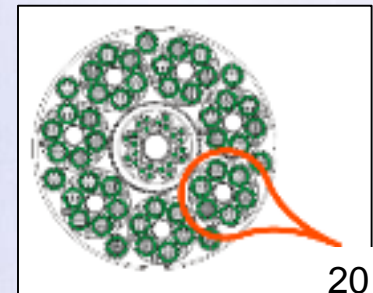
³Group of Electrical Engineering - Paris (GeePs), CNRS UMR 8507,
CentraleSupélec, UPSud, UPMC, France

⁴Laboratoire de Modélisation des Systèmes Energétiques, LMSE, Université de
Biskra, Algeria



Context : superconducting applications

- High Temperature Superconducting (HTS) applications are growing rapidly.
- Superconducting materials are a viable option for high power applications (the carrying current densities are **10 times higher than copper**).
- Devices are getting powerful and efficient



Source: American Superconductor Corp

20 kA

Discovery
of HTS

First HTS cable
30 kV 2.4 kA

LIPA cable
138 kV/2.4 kA

MgB₂ cable

1986

1990

2008

2014

To design and optimize
the superconducting
systems

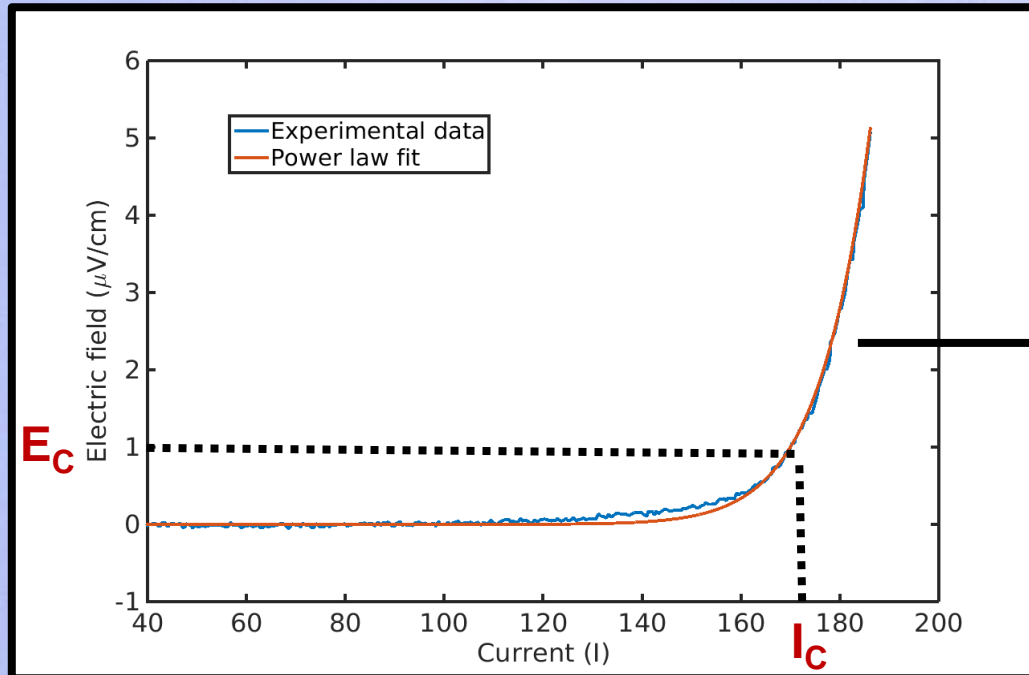
Need to develop reliable
models: AC losses
computation,...

Main challenge in modeling superconducting devices

Nonlinear Constitutive law

$$\vec{E} = \rho(\vec{J}, \vec{B}, T) \vec{J}$$

- highly nonlinear
- Dependence on J , T and B



Power-law model

$$E = E_c \left(\frac{J}{J_c} \right)^n$$

J_c : Critical current density
 $n \sim$ high values

Study and development of well-suited methods and formulations

Main challenge in modeling superconducting devices

Nonlinear Constitutive law

$$\vec{E} = \rho(\vec{J}, \vec{B}, T) \vec{J}$$

→ highly nonlinear

→ Dependence on J , T and B

Goal of this work

- Introduction of a benchmark on the 3D numerical modeling of a superconducting bulk
- Make a brief inventory of the main methods and formulations used by the laboratories participating in this benchmark
- Comparison of the performance of some software based on:
 - The formulation
 - The numerical method
 - Mesh sensitivity
 - The Transient nonlinear solver

Outline

- **Context: superconducting applications and main challenges in modeling superconductors**
- **3D Numerical Modeling of AC losses in superconducting cables: State of the art**
- **Benchmark on the 3D Numerical Modeling of a Superconducting Bulk**
 - **Benchmark presentation**
 - **Participants in the benchmark**
- **Results**
- **Conclusion and future works**

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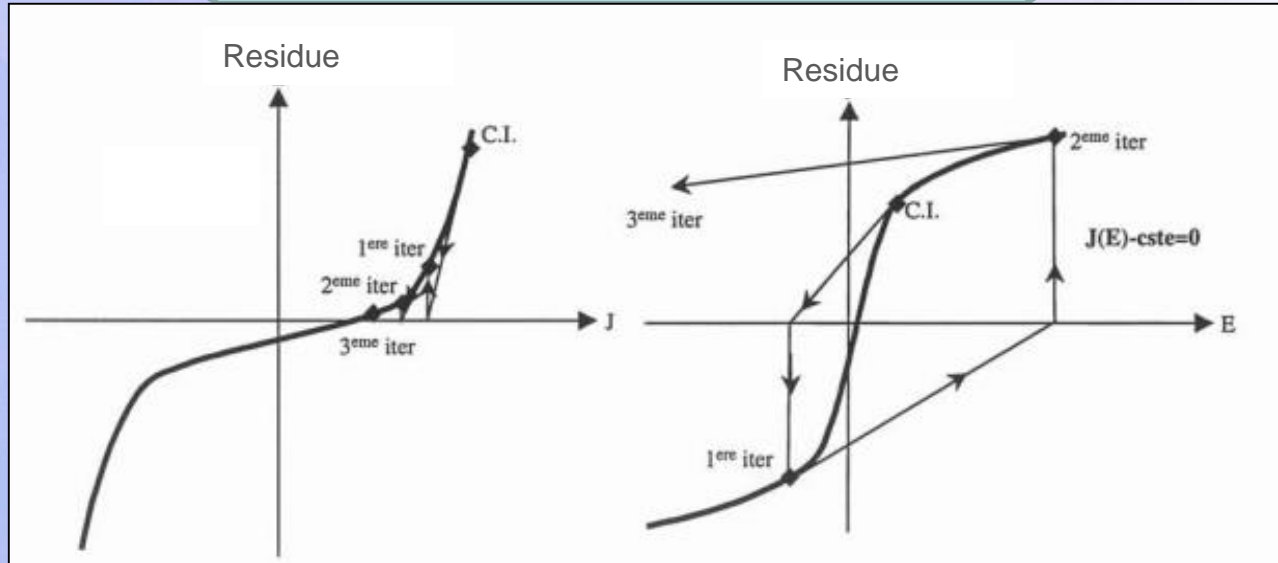
3D Numerical Modeling of AC losses in superconducting cables: formulations

Superconducting constitutive law

$E = \rho(J) \cdot J$
Magnetic Formulations
 $H, H - \Omega - \Psi, T - \Omega$

$J = \sigma(E) \cdot E$
Electric Formulations
 $E, A - V$

Influence on the convergence problem



3D Numerical Modeling of AC losses in superconducting cables: Magnetic formulation

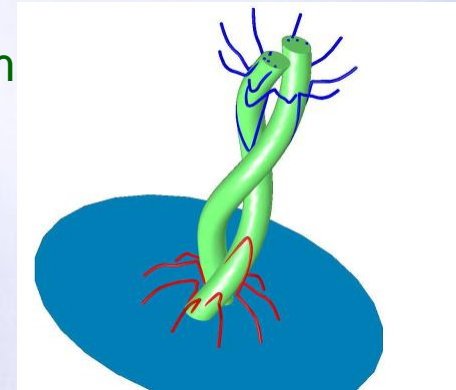
$$\text{With } \rho(\mathbf{J}) = \left(\frac{E_c}{J_c} \cdot \left(\frac{\|\mathbf{J}\|}{J_c} \right)^{(n-1)} + \rho_0 \right)$$

H formulation $\nabla \times \rho(\mathbf{J}) \nabla \times \mathbf{H} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} = \mu_0 \frac{\partial \mathbf{H}_0}{\partial t}$

- » The most frequently used by the superconducting comm
- » Simple to implement
- » Uses the E(J) behavior law

But

- » H -formulation uses finite air resistivity leading to:
- » **Unphysical current leaks in the air regions**
- » **Ill-conditioning of the matrix system**



A. Stenvall, V. Lahtinen, and M. Lyly, Superconductor Science & Technology, vol. 27, no. 10, p. 104004, 2014.

3D Numerical Modeling of AC losses in superconducting cables: Magnetic formulation

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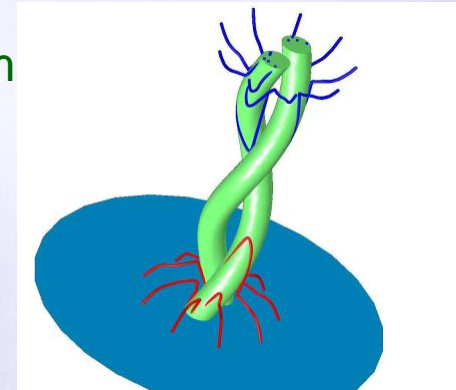
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T-Ω formulation

$$\begin{cases} \nabla \times \rho(\mathbf{J}) \nabla \times \mathbf{T} + \mu_0 \left(\frac{\partial \mathbf{T}}{\partial t} + \nabla \Phi \right) = - \frac{\partial}{\partial t} \mu_0 \mathbf{T}_0 \\ \nabla \cdot \mu_0 (\mathbf{T} + \mathbf{T}_0 - \nabla \Phi) = 0 \end{cases}$$

- » Don't need an artificial high resistivity in air
- » Uses the E(J) behavior law

But

- » Need to treat multiply connected conductors

3D Numerical Modeling of AC losses in superconducting cables: formulations and methods

$$\text{With } \sigma(\mathbf{E}) = \left(\frac{E_c}{J_c} \cdot \left(\frac{\|\mathbf{E}\|}{E_c} \right)^{\frac{(n-1)}{n}} + \rho_0 \right)^{-1}$$

Electric formulation A-V formulation

$$\begin{cases} \nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} + \sigma(\mathbf{E}) \cdot \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V \right) = \mathbf{J}_s \\ \nabla \cdot \left(\sigma(\mathbf{E}) \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V \right) \right) = 0 \end{cases}$$

- » Relatively simple to implement
- » Don't need an artificial high resistivity in air
- » Don't need to treat multiply connected conductors

But

- » Uses the J(E) behavior law (convergence problem)

Numerical methods:

- Finite element method
- Finite volume method
- Integral methods

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Benchmark presentation

Geometry

The superconductor domain is a cube with a side length of 10 mm

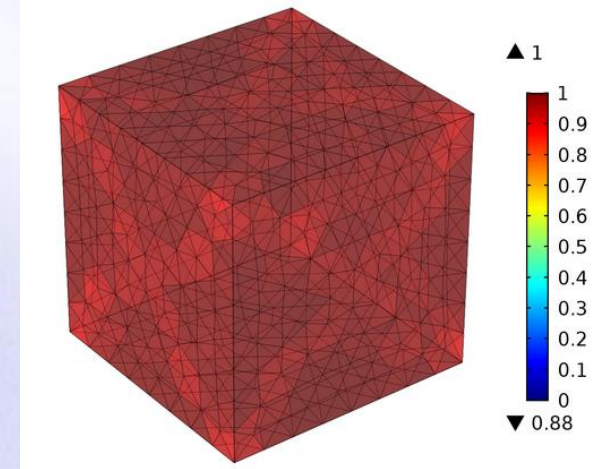
Mesh

Several meshes have been performed in order to study the mesh sensitivity.

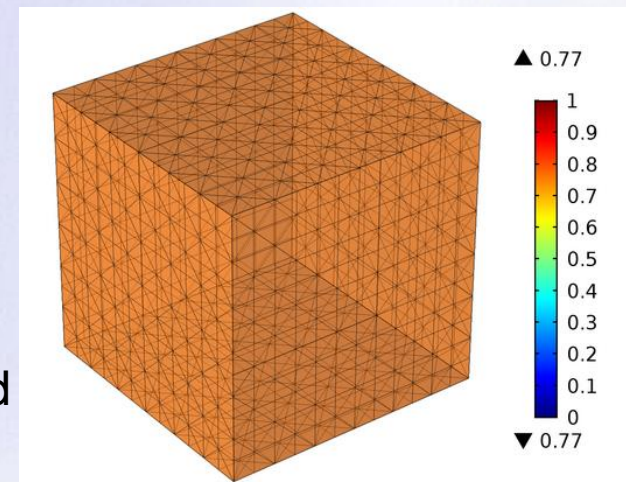
| | Number of elements in the bulk | |
|--------|--------------------------------|----------------|
| | Regular mesh | Irregular mesh |
| Cube6 | 1080 | 1076 |
| Cube12 | 8640 | 8677 |
| Cube24 | 69120 | 69227 |
| Cube36 | 233280 | 233277 |

Physical properties

- For the superconductor, the power law is used ($E_c=1.0E^{-4}V.m^{-1}$, $J_c=2.5E^6 A.m^{-2}$ and $n=25$).
- A sinusoidal external magnetic field is imposed (5mT, 10mT, 15mT and 20 mT).



Irregular mesh



Regular mesh

Participants in the benchmark

- **Goal :**

- Comparison of the performance of some software based on different formulations

- **Software**

FEM, T- Ω formulation: FLUX, commercial code (ALTAIR-G2ELab)



FEM, H formulation:

- **COMSOL Multiphysics®**, commercial code
- **Daryl-Maxwell**: homemade software (Polytechnique Montréal, Canada)
- **GetDP** (Liège University), free software



FVM, A-V formulation: homemade software (Université de Biskra, Algérie)

Some numerical considerations

- FEM methods are based on the first edge elements.
- All methods use an implicit time discretization scheme.

| Label | FEM, H | | | FEM, T- Ω | FVM, A-V |
|---------------------------|-------------------------------------|------------|----------------|--------------------|-------------------------------|
| | Comsol | GetDP | Daryl | FLUX3D | Home code |
| Nonlinear solver | Newton-Raphson | Point fixe | Newton-Raphson | Newton-Raphson | |
| Relaxation | No relaxation | | | Optimal relaxation | No relaxation |
| Transient solver | implicit time discretization scheme | | | | |
| Adaptive step size | Yes | No | No | No | No |
| Linear solver | Mumps | Mumps | Pardiso | Mumps | Gauss-Seidel it rative method |

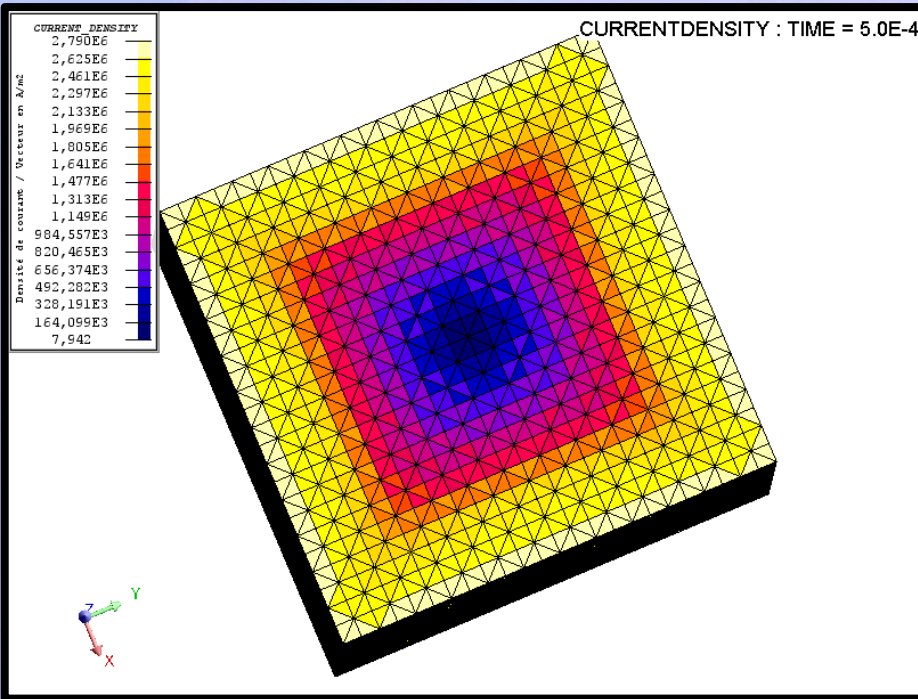
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Results

AC losses induced by an External Magnetic Field

Good concordance between the results is observed



Current density cartography

| Formulation (software) | AC losses (mW) | |
|---------------------------|----------------|-------|
| | 5 mT | 20 mT |
| T – Ω (Flux) | 0,8354 | 14 |
| H (Comsol) | 0,7933 | 13,01 |
| H (GetDP) | 0,7933 | 13,90 |
| H (Daryl Maxwell) | 0,7969 | 14,34 |
| A-V (home code) | 0,8396 | 13,84 |

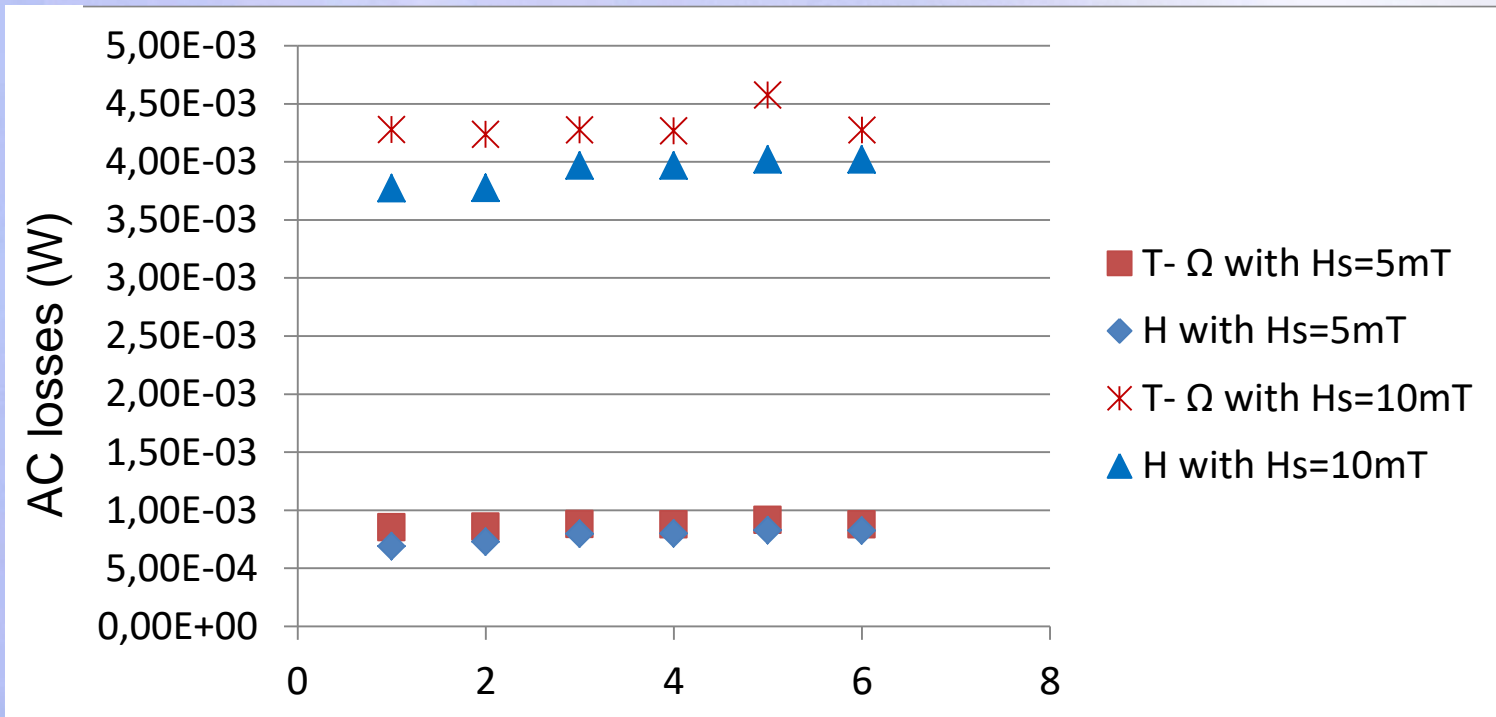
The average AC losses with the regular mesh of Cube6

Results

AC losses induced by an External Magnetic Field

Comparison between two formulations:

- AC losses are slightly higher in the case of $\mathbf{T} - \Omega$ formulation especially when the values of the source field increase.



The average AC losses in the case of $\mathbf{T} - \Omega$ and \mathbf{H} formulation for different meshes at 5 mT and 10 mT

Results

- A. First investigations on the mesh sensitivity
- B. First investigations on the nonlinear solver
- C. Investigation on the performance of the transient solver

Results

A. First investigations on the mesh sensitivity

B. First investigations on the nonlinear solver

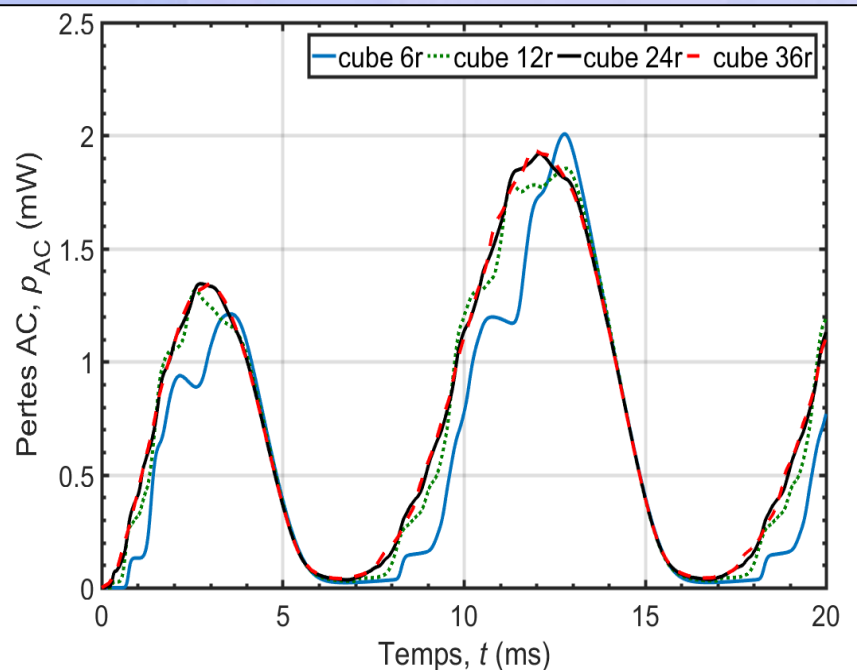
C. Investigation on the performance of the transient solver

The results show:

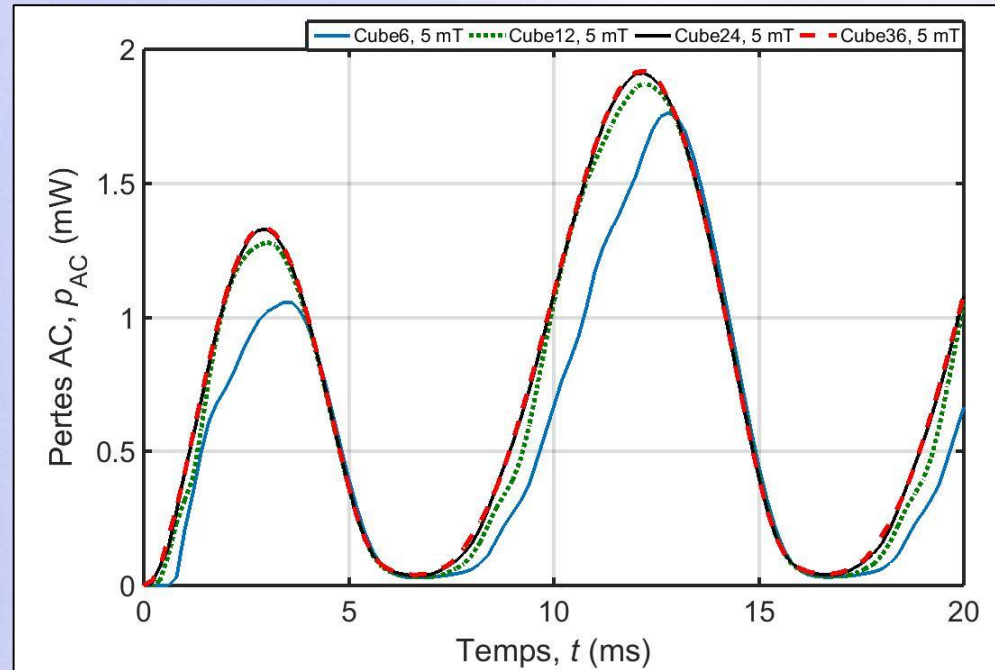
- The average AC losses are close for regular and irregular meshes.

But

- The instantaneous AC losses behavior is impacted by the mesh quality
- These disturbances are reduced with the refinement of the mesh



(a) Regular meshes



(a) Irregular meshes

Results

A. First investigations on the mesh sensitivity

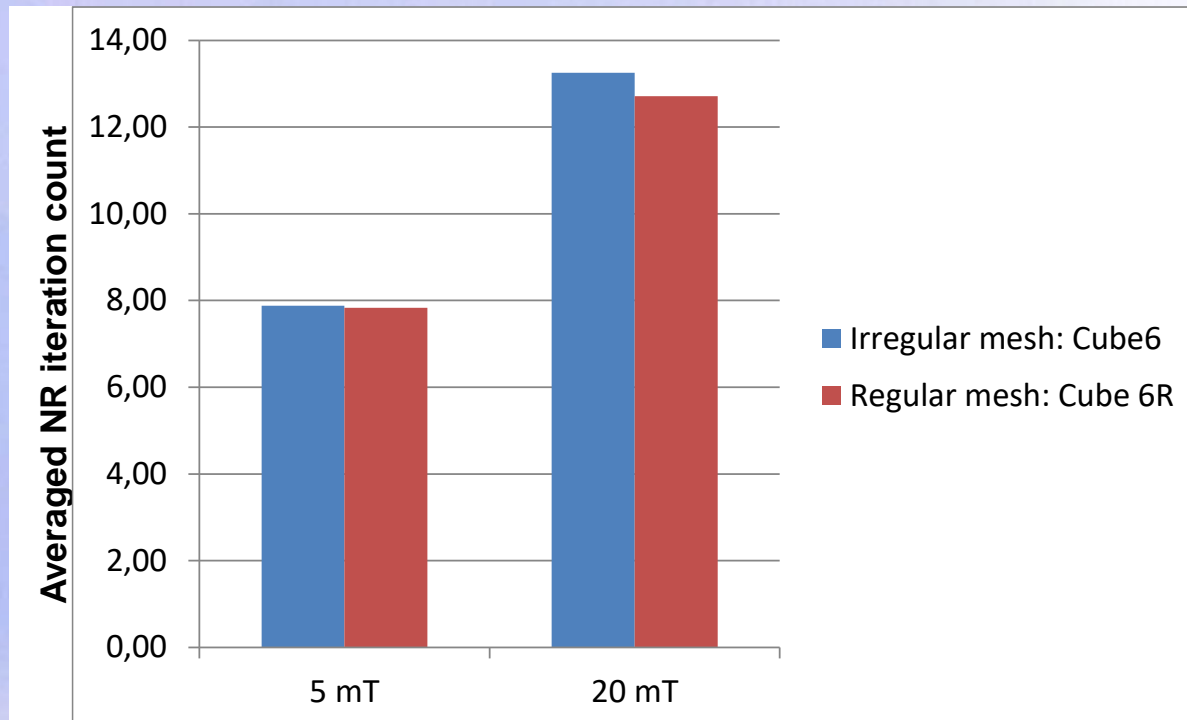
B. First investigations on the nonlinear solver

C. Investigation on the performance of the transient solver

2 values of an external magnetic field compared to the depth field B_p .

The results show:

- Without relaxation, 400 transient iterations are needed for the convergence.
- With optimal relaxation, the problem converge even with 40 transient iterations.
- The quality of the mesh doesn't influence significantly the convergence of nonlinear problem.



Results

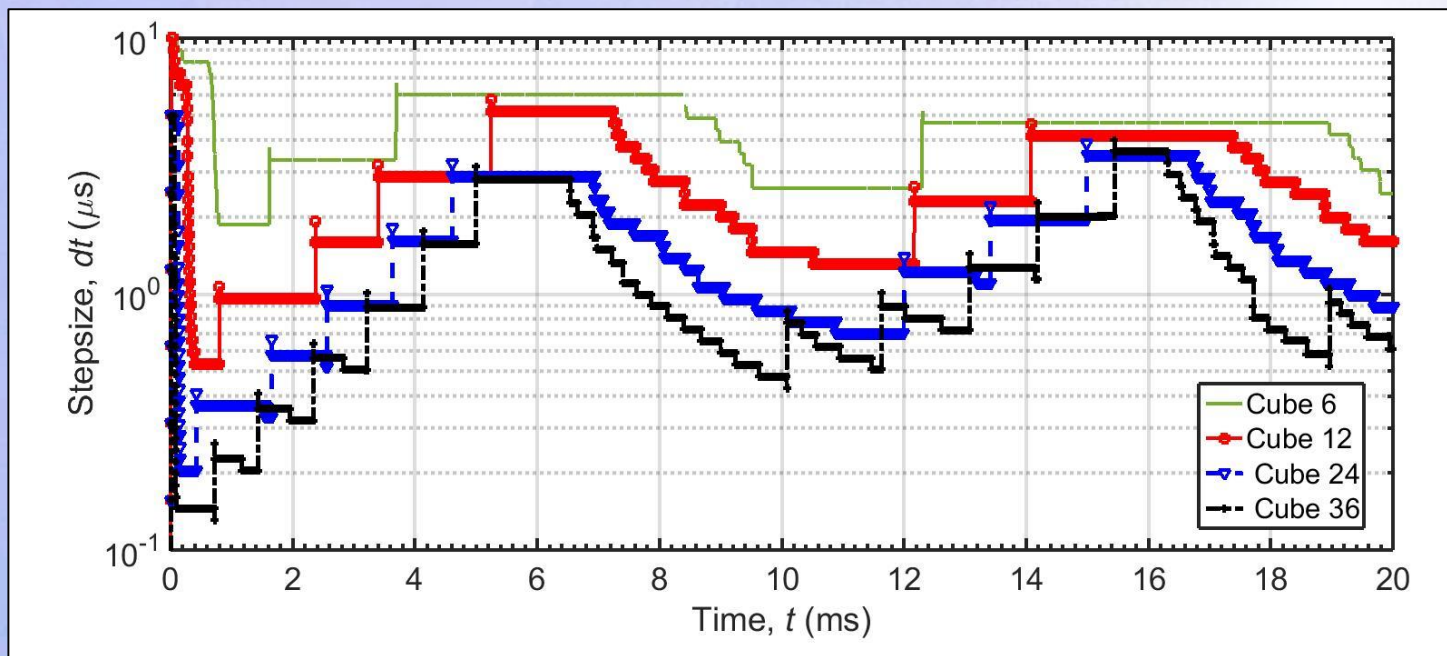
A. First investigations on the mesh sensitivity

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Adaptive time step size

- The time step size decreases when the mesh is finer.
- The finer mesh generates more time step size variations chosen by the solver.



Time step size vs time with $H_s = 5$ mT

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Conclusion

- A benchmark on the 3D numerical modeling of a superconducting bulk is introduced.
- The results show a good agreement between the models when considering the AC losses.
- First investigations have been made on the mesh sensitivity and on the solver strategies.
- A research group have been created including the participants in the benchmark.

Future works

- Deepen comparisons by including the results of different models (time computation,...).
- Add to the benchmark the case of the transport current.
- Integration of other methods in the benchmark: integral methods
 - B. Ramdane, G. Meunier, B. Rozier and A. Badel, "3D Volume Integral Formulation Based on Facet Elements for the Computation of AC Losses in Superconductors" ISEM2017.
- More accurate model of the behavior law should be included (dependence on B and T).



Paris

15-17 Novembre, 2017

Thank you !!
Questions ??



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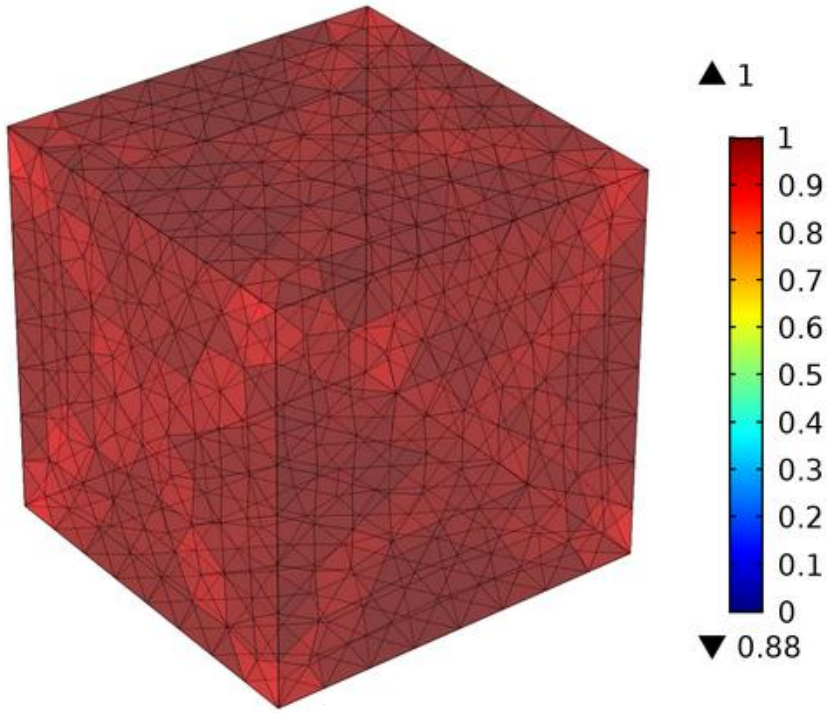
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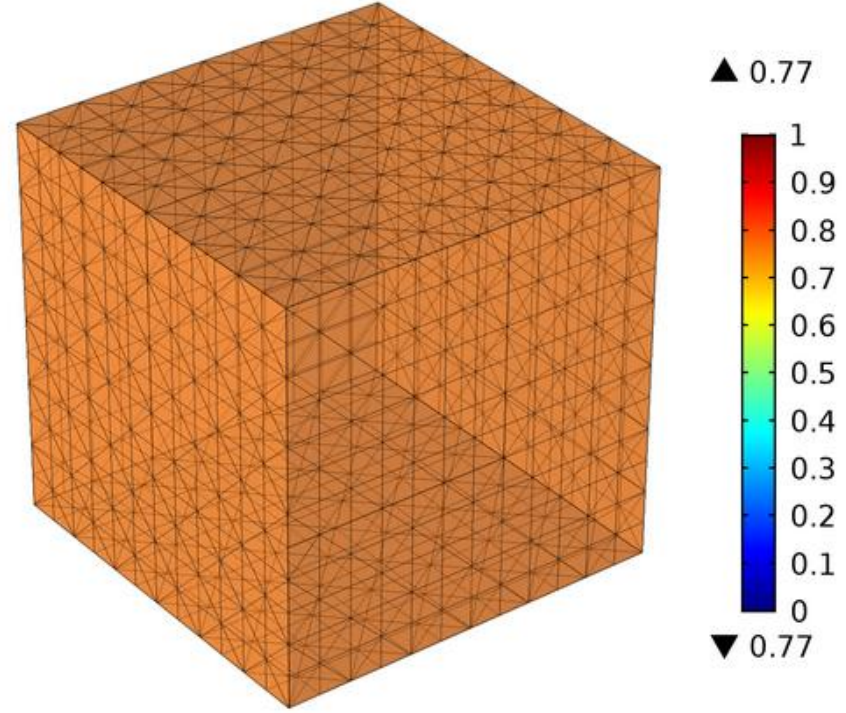
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Irregular mesh



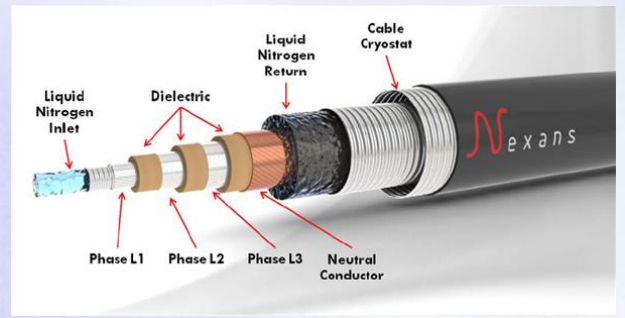
Regular mesh

Main challenge in modeling superconducting devices

- Multi-scale problem

The characteristic quantities over several orders of magnitude

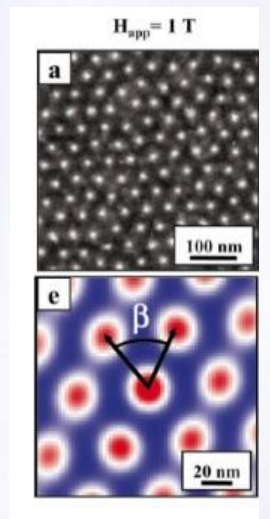
Development of homogenization techniques



- Nonlinear Constitutive law

$$\vec{E} = \rho(\vec{J}, \vec{B}, T) \vec{J} \quad \begin{array}{l} \rightarrow \text{highly nonlinear} \\ \rightarrow \text{Dependence on } J, T \text{ and } B \end{array}$$

Study and development of well-suited methods and formulations



Results

First investigations on the transient solver

Adaptative step size

