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Crystal plasticity and phenomenological approaches for the simulation of deformation behavior in thin copper alloy sheets

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Abstract

In the expanding context of device miniaturization, forming processes of ultra thin sheet metals are gaining importance. Numerical simulation of these processes requires accurate material modeling. In this study, both the phenomenological modeling approach and the crystal plasticity finite element method (CPFEM) are considered. Theoretical definitions of both models, numerical implementation as well as their parameter identification procedures are outlined. Subsequently they are compared on a one to one basis, mainly with regards to their ability to predict mechanical responses for a variety of strain loading paths.

Keywords: A. Sheet metal forming B. Phenomenological modeling B. Crystal plasticity C. Finite elements

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1. Introduction

The current trend for product and device miniaturization has promoted micro-scale manufacturing processes. The demand for microparts has significantly increased in the automotive and electronics industries. Due to their suitability for mass production, sheet metal forming processes are widely used in the manufacturing industry and have been applied to very thin sheets. However, microforming raises a number of challenges related to the size of the parts targeted and the submillimetric thickness of the sheet metals (Geiger et al. (2001)). Finite element based simulations are nowadays common engineering tools. They allow to assess manufacturability of parts and to achieve subsequent time savings at the process design stage. Yet, the quality and predictiveness of these simulations rely on several factors, among which is the material behavior model. Currently, two distinct material modeling approaches can be considered for sheet metal forming. On one hand, the so-called phenomenological models are based on discrete macroscopic experimental observations and the assumption of material homogeneity. The constitutive laws consist of sets of relations with parameters which are adjusted to reproduce the experimental data available. Commercial finite element packages include several phenomenological laws for material modeling. Following the still extensively used von Mises model, quite a number of functions have been proposed to take into account anisotropy in the prediction of plastic yield. Extensive descriptions of most of them can be found in Banabic (2010). Beyond the yield point, the material behavior is described using so-called isotropic and/or kinematic hardening. Power (Swift type) and exponential (Voce type) laws are commonly used isotropic models while kinematic hardening modeling can involve expressions that are linear (Prager type), saturating (Armstrong and Frederick (1966)), a mix of these two types (Chaboche (1991)) or more conceptually complex (Yoshida and Uemori (2003) and Yoshida et al. (2015), Haddadi et al. (2006), Barlat et al. (2011)) depending on the material response during mechanical tests and the required modeling accuracy. On the other hand, the so-called microstructural models are based on the crystal plasticity theory which can be traced back to the works of Taylor and Elam (1923) and Schmid et al. (1934) who related plastic flow in single crystals to crystallographic planes slip. This class of model incorporate material heterogeneity, at microscopic level, in constitutive relations. We will restrict ourselves in this paper to the crystal plasticity finite element method (CPFEM) models which were initiated when Peirce

38 [et al. \(1982\)](#) exploited the versatility of the finite element method to solve
39 boundary value problems with crystal plasticity constitutive relations. Em-
40 bedding the crystal plasticity theory into a finite element formulation enforces
41 the equilibrium of the forces and compatibility of displacements between the
42 grains of a polycrystal in a weak form through the principle of virtual work.
43 The well known limitations (violation of equilibrium, assumptions on grains
44 morphologies and interactions) of mean field models ([Taylor \(1938\)](#), [Kröner
45 \(1961\)](#), [Lebensohn and Tomé \(1993\)](#), [Van Houtte et al. \(2005\)](#)) are overri-
46 den at the expense of a higher numerical cost. The unaltered growth of
47 computing power over the last few decades has triggered an extensive use
48 of CPFEM for various applications, in the research framework, [Erieau and
49 Rey \(2004\)](#), [Kim et al. \(2012\)](#), [Khadyko et al. \(2015\)](#), [Kim and Yoon \(2015\)](#),
50 [Khan et al. \(2015\)](#) as well as for industrial process simulations ([Kalidindi
51 and Anand \(1992\)](#), [Beaudoin et al. \(1994\)](#), [Grujicic and Batchu \(2002\)](#), [Li
52 et al. \(2008\)](#), [Rousselier et al. \(2009\)](#), [Verma et al. \(2013\)](#)). A thorough re-
53 view of CPFEM features, applications and challenges can be found in [Roters
54 et al. \(2010\)](#). Presently, although they provide a physically based simulation
55 framework, CPFEM models are not available in commercial software. The
56 phenomenological approach truly represents the standard constitutive mod-
57 eling choice for industrial process simulations. Indeed, in addition to their
58 computational efficiency, the latter approach has proven reliable and predic-
59 tive enough for most industrial applications.

60 Nevertheless, when it comes to processes involving very thin sheet metals
61 with a few grains in the thickness, usually termed as ultra-thin sheet met-
62 als, phenomenological models often fail to render material behavior ([Engel
63 and Eckstein \(2002\)](#), [Geißdörfer et al. \(2006\)](#), [Peng et al. \(2007\)](#)). As there
64 are few grains, the individual response of each grain which is driven from
65 its orientation, size and shape, strongly influences the behavior of the part
66 being formed. During processing, these sheet metals present a strong hetero-
67 geneity of deformation. In such a context, the phenomenological approach
68 which is based on the assumption of deformation homogeneity becomes ques-
69 tionnable. At the same time, CPFEM based process simulations become
70 computationnally conceivable as the number of grains is lower ([Wang et al.
71 \(2009\)](#)). The present study examines a case where the macroscopical length-
72 scale approaches the microstructural one but where scale separation is still
73 workable. This paper aims at providing a one to one comparison of phe-
74 nomenological and CPFEM based approaches in such a context. The ability
75 of these approaches to predict the response of a copper alloy under differ-

Cu	Be	Co	Ni	Fe
97	1.8 - 2	0.3	0.15	0.15

Table 1: Chemical composition in mass percent of CuBe2.

ent strain paths is investigated. First of all, the mechanical tests performed for the experimental characterisation of the considered material are briefly reported in Section 2. Hill (1948) and Bron and Besson (2004) anisotropic yield functions are considered in the phenomenological model and the developed parameter identification procedure is presented in Section 3. Then the adopted CPFEM framework, its embedment and numerical implementation as well as the parameter identification procedure are presented in Section 4. Comparisons of the two modeling approaches are performed in Section 5 and the main conclusions are presented in Section 6.

2. Experimental characterization

The studied material is the copper alloy CuBe2 whose chemical composition is given in Table 1. The cold rolled sheet is 0.1 mm thick and used in the micro-parts and connectors manufacturing industry. An industrial sheet metal grade is used for the experimental characterization, which is not annealed after cold rolling. The rolling direction of the sheet metal will be indicated RD, the transverse direction TD and the normal direction ND. Quantitative information on the microstructure was obtained from Electron BackScatter Diffraction (E.B.S.D.) scans. The microstructure was measured in the RD-TD and TD-ND sections. The device used in this study is a 7001 Field Electron Gun Scanning Electron Microscope from Jeol equipped with an Oxford EBSD CCD camera. EBSD data were post-treated with the software CHANNEL 5 from Oxford. The scanned areas were $320 \times 240 \mu\text{m}^2$ large and the step size was set to $0.3 \mu\text{m}$. No sensible microstructure gradient was observed and the grains were considered equiaxed. An EBSD map of the microstructure is shown in Fig. 1. The texture is marked and of Goss type. The experimentally measured average grain size was about $4 \mu\text{m}$. Although the maximum grain size was about $50 \mu\text{m}$, little dispersion was observed and the grain size was quite homogeneous leading to an average of 25 grains throughout the sheet thickness. These thin sheet metals provide a convenient framework for the upcoming models comparison as both phenomenological

106 and microstructural based models are still relevant. The influence of the
 107 reduction of the sheet thickness with respect to the grain size (T/D ratio)
 108 on models predictions will be assessed in future work.

109 In order to investigate the sheet behavior under different stress and strain
 110 paths, three types of mechanical tests were performed namely tensile tests,
 111 monotonic and Bauschinger shear tests and balanced biaxial tension. Strains
 112 were measured using Digital Image Correlation (DIC) system Aramis (GOM
 GmbH).

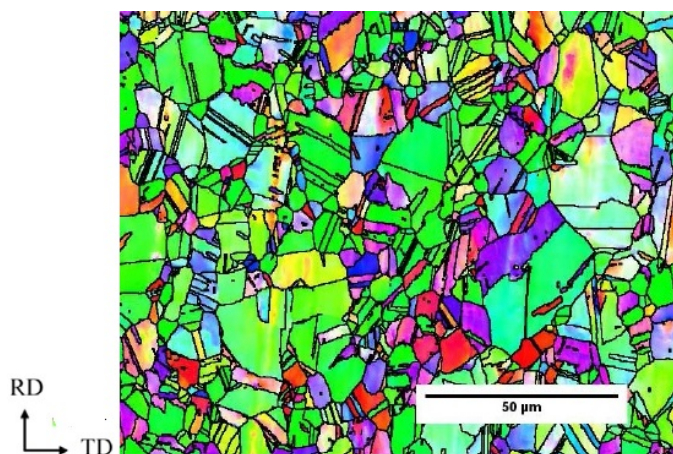


Figure 1: EBSD map of the industrial copper alloy CuBe2.

113

114 2.1. Tensile tests

115 A sample geometry was prepared according to ISO 6892-1 standard, sim-
 116 ilarly to Pham et al. (2015). In order to evaluate material anisotropy, room
 117 temperature monotonous tensile tests were performed at 0°, 45° and 90° from
 118 the RD. The tests were controlled by grip displacement and the strain rate
 119 was around 10^{-4} s^{-1} . Each type of test was performed three times to ensure
 120 reproductibility of the results. Longitudinal ϵ_{xx} and transverse ϵ_{yy} logarith-
 121 mic strains were recorded during the experiments. Plastic anisotropy coeffi-
 122 cients r_α coefficients are used to quantify strain anisotropy. Strain anisotropy
 123 is quite pronounced especially in the sheet plane with a planar anisotropy
 124 coefficient $\Delta r = (r_0 + r_{90} - 2r_{45}) / 2 = 0.28$ and a normal anisotropy coeffi-
 125 cient $\bar{r} = (r_0 + r_{90} + 2r_{45}) / 4 = 0.92$.

126 The influence of viscosity was studied by changing the strain rate during
 127 tensile tests in the RD. The strain rate was increased from 10^{-4} s^{-1} (up to

128 2% of deformation) to 10^{-3} s^{-1} (up to 4% of deformation) and 10^{-2} s^{-1} (up
129 to 8% of deformation). Relaxation times of 60 s were imposed between the
130 first two reloadings. Eventually, a relaxation time of 120 s was imposed and
131 a strain rate of 10^{-4} s^{-1} was applied to the specimen.

132

133 *2.2. Shear tests*

134 Shear tests were performed with the device presented in [Thuillier and](#)
135 [Manach \(2009\)](#). Rectangular samples of dimensions $30 \times 15 \text{ mm}^2$ were used.
136 Monotonic and reversed (Bauschinger) shear tests were performed in the RD
137 at a strain rate of $\dot{\gamma} = 8 \times 10^{-4} \text{ s}^{-1}$. The shear strain γ was obtained from the
138 measured non diagonal component ϵ_{12} of the Green-Lagrange strain tensor
139 as $\gamma = 2\epsilon_{12}$. The shear stress was calculated by $\sigma_{12} = F/S_0$ where F is the
140 load during the test and S_0 the initial gauge section.

141 *2.3. Balanced biaxial tensile test*

142 Material response at large strains and under a balanced biaxial strain
143 state was investigated using a hydraulic bulge test ([Zang et al. \(2011\)](#)). A
144 circular blank of a 60 mm gauge diameter was held on a circular die by
145 a blank-holder fastened with screws. The blank was then deformed up to
146 rupture by water pressure controlled with a sensor. The strain state on the
147 sample surface was measured throughout the test. The curvature radius at
148 the pole R_{pole} was approximated by fitting a sphere over a selected area. A
149 balanced biaxial stress state was assumed and the stress was then calculated
150 using $\sigma = PR_{pole}/2e$ where P is the hydraulic pressure and e the current
151 blank thickness.

152 The experimental stress-strain curves in the tensile test (0° to the RD),
153 balanced biaxial tensile test and simple shear test are shown in [Fig. 2](#). The
154 flow stress and work-hardening rate of the balanced biaxial tensile test and
155 uniaxial tensile test are similar, while much larger strains are reached with
156 the biaxial and shear tests. Detailed experimental results will be further
157 shown in the subsequent sections, in comparison to numerical predictions.

158 **3. Phenomenological modeling and parameter identification**

159 *3.1. Constitutive model*

160 As sheet metal forming involves large strains and material rotation, the
161 constitutive equations need to be written in a finite deformation framework.

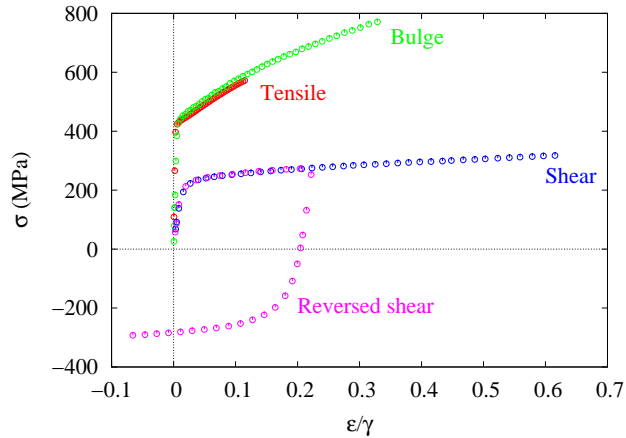


Figure 2: Stress-strain curves of the experimental strain paths investigated.

162 The following relations were implemented in the SiDoLo software ([Cailletaud](#)
 163 [and Pilvin \(1993\)](#)) which enables model development and inverse material pa-
 164 rameter identification. To fulfill the material frame indifference requirement
 165 (principle of objectivity), the orthogonal rotating frame associated to the
 166 objective derivative of Jaumann (the so-called co-rotational frame) is consid-
 167 ered as the main reference ([Sidoroff \(1982\)](#)).

168

169 3.1.1. Elastic-viscoplasticity

170 In its associated rotating frame, an objective derivative corresponds to
 171 a simple time derivative. Thus under the assumption of hypoelasticity, the
 172 constitutive elastic relation is written in an incremental form

$$\dot{\boldsymbol{\sigma}} = \mathbf{C} : \mathbf{D}^e, \quad (1)$$

173 where $\boldsymbol{\sigma}$ is the rotation-compensated Cauchy stress tensor, \mathbf{C} is the isotropic
 174 elastic modulus tensor which is expressed using the Young's modulus E and
 175 Poisson's coefficient ν while \mathbf{D}^e is the elastic part of the strain rate. The
 176 small-strain like splitting of the strain rate is assumed, leading to the follow-
 177 ing rate expression:

$$\mathbf{D}^e = \mathbf{D} - \mathbf{D}^p, \quad (2)$$

178 where \mathbf{D} is the total strain rate and \mathbf{D}^p its viscoplastic part.
 179 The material is assumed to behave elastically in a domain bounded by a yield
 180 surface defined by

$$f(\boldsymbol{\sigma}, \mathbf{X}, R) = \bar{\sigma}(\boldsymbol{\sigma}, \mathbf{X}) - R = 0, \quad (3)$$

181 where $\bar{\sigma}$ is the equivalent stress, \mathbf{X} is the tensorial internal variable pointing
 182 to the centre of the yield surface which is used to model kinematic hardening,
 183 and R is the isotropic hardening.

184 The viscoplastic strain rate tensor \mathbf{D}^p accounting for plastic flow at yield is
 185 derived from a Norton-like viscoplastic potential Ω and the flow rule is then
 186 expressed by

$$\mathbf{D}^p = \frac{\partial \Omega}{\partial \boldsymbol{\sigma}}. \quad (4)$$

187 The stress potential Ω is expressed as

$$\Omega(f) = \frac{K_v}{n_v + 1} \left(\frac{f_+}{K_v} \right)^{n_v + 1} \quad (5)$$

188 where f_+ is the positive part of f , K_v is a weighting coefficient of the viscous
 189 contribution and n_v a strain-rate sensitivity coefficient.

190 Isotropic hardening is chosen as:

$$R = \sigma_{sat} - (\sigma_{sat} - \sigma_0) \exp(-C_R p^{n_r}), \quad (6)$$

191 where n_r is a material parameter, σ_{sat} is the saturation stress, σ_0 the initial
 192 yield stress, C_R the saturation rate and p the cumulated viscoplastic strain
 193 defined by

$$p = \int_0^t \sqrt{\frac{2}{3} \mathbf{D}^p : \mathbf{D}^p} dt. \quad (7)$$

194 The so-called back-stress tensor \mathbf{X} is written as the sum of Armstrong-
 195 Frederick law terms with a Prager type term in order to achieve a good
 196 description both at larger strains and at the onset of plastic flow during
 197 reverse loading. The resulting expression is

$$\mathbf{X} = \frac{2}{3} \left(\sum_{i=1}^N C_{X_i} \boldsymbol{\alpha}_i + H_X \mathbf{d}^p \right); \quad \mathbf{d}^p = \int_0^t \mathbf{D}^p dt, \quad (8)$$

198 with

$$\dot{\boldsymbol{\alpha}}_i = \mathbf{D}^p - B_{X_i} \dot{p} \boldsymbol{\alpha}_i, \quad (9)$$

199 where C_{X_i} is associated to the non-linear Armstrong-Frederick term, B_{X_i} the
 200 parameter related to the recall term (memory effect) and H_X the slope of
 201 the linear Prager term.

202 Material anisotropy is taken into account in the computation of the equivalent
 203 stress by mean of anisotropic yield functions. Hill's 1948 quadratic yield
 204 function and the Bron&Besson yield function are considered in this work.

205 Hill (1948) function is a widely used anisotropic yield function developed for
 206 materials exhibiting orthotropic symmetry just as rolled sheets.

207 The equivalent stress can be expressed as:

$$\bar{\sigma} = \mathbf{T}' : \mathbf{M} : \mathbf{T}', \quad (10)$$

208 where $\mathbf{T} = \boldsymbol{\sigma} - \mathbf{X}$ and $\mathbf{T}' = \text{deviator}(\mathbf{T})$.

209 \mathbf{M} is the fourth order tensor inducing the anisotropy and writes in the ma-
 210 terial orthotropic frame :

$$\mathbf{M} = \frac{1}{3} \begin{bmatrix} 2G + 2H - F & 0 & 0 & 0 & 0 & 0 \\ 0 & 2F + 2H - G & 0 & 0 & 0 & 0 \\ 0 & 0 & 2H + 2G - H & 0 & 0 & 0 \\ 0 & 0 & 0 & N & 0 & 0 \\ 0 & 0 & 0 & 0 & M & 0 \\ 0 & 0 & 0 & 0 & 0 & L \end{bmatrix} \quad (11)$$

211

212 where F, G, H, L, M and N are the anisotropy parameters.

213 Bron and Besson (2004) yield function is an extension of the Karafillis and
 214 Boyce (1993) criterion in which anisotropy is introduced by means of linear
 215 transformations. The equivalent stress is expressed as:

$$\bar{\sigma} = \left(\sum_{k=1}^2 \lambda_k (\psi_k)^{\frac{a}{b_k}} \right)^{\frac{1}{a}} ; \quad \sum \lambda_k = 1. \quad (12)$$

216 The functions ψ_k are first order homogeneous, positive, convex with respect
 217 to their argument and defined by:

$$\psi_1 = \frac{1}{2} \left(|S_2^1 - S_3^1|^{b_1} + |S_3^1 - S_1^1|^{b_1} + |S_1^1 - S_2^1|^{b_1} \right), \quad (13)$$

218

$$\text{and } \psi_2 = \frac{3^{b_2}}{2^{b_2} + 2} \left(|S_1^2|^{b_2} + |S_2^2|^{b_2} + |S_3^2|^{b_2} \right). \quad (14)$$

219 $S_{i=1,3}^k$ are the eigenvalues of the modified deviatoric stress tensor \mathbf{s}'^k . These
 220 tensors are obtained from linear transformations on the tensor \mathbf{T} which are
 221 expressed as:

$$\mathbf{s}'^k = \mathbf{L}^k : \mathbf{T}, \quad (15)$$

222 and where the anisotropy parameters are embedded in the symmetric fourth
 223 order tensors \mathbf{L}^k :

$$\mathbf{L}^k = \frac{1}{3} \begin{bmatrix} c_2^k + c_3^k & -c_3^k & -c_2^k & 0 & 0 & 0 \\ -c_3^k & c_3^k + c_1^k & -c_1^k & 0 & 0 & 0 \\ -c_2^k & -c_1^k & c_1^k + c_2^k & 0 & 0 & 0 \\ 0 & 0 & 0 & 3c_4^k & 0 & 0 \\ 0 & 0 & 0 & 0 & 3c_5^k & 0 \\ 0 & 0 & 0 & 0 & 0 & 3c_6^k \end{bmatrix}. \quad (16)$$

224
 225 Thus through the two linear transformations, twelve parameters $c_{i=1,6}^{k=1,2}$ are
 226 provided to describe anisotropy. The parameters $\lambda = \lambda_1$, a , b_1 and b_2 affect
 227 the yield surface's shape.

228 3.2. Parameter identification

229 A number of material parameters could be determined independently by
 230 specific measurements. The remaining ones were determined by inverse iden-
 231 tification. Young's modulus E and Poisson's ratio ν were determined from
 232 sequential loading-unloadings in the elastic domain. The initial Young mod-
 233 ulus was 120 MPa, then it decreased with plastic strain and rapidly reached
 234 a saturation value of 95 MPa. Consequently, this later value was used for
 235 the simulations throughout the current investigation.

236 The viscous contribution to material behavior is assessed by the increas-
 237 ing strain rate tensile tests and the viscous parameters of Eq (5), K_v and
 238 n_v , were identified based on these experiments. After identification of the
 239 isotropic hardening parameters, consistency of viscosity parameters obtained
 240 beforehand was assessed through the simulation of the increasing strain rate
 241 tensile test. The experimental test and its simulated counterpart are shown
 242 in Fig. 3. The viscosity effect is relatively small, at least under such homo-
 243 geneous loading conditions.

244

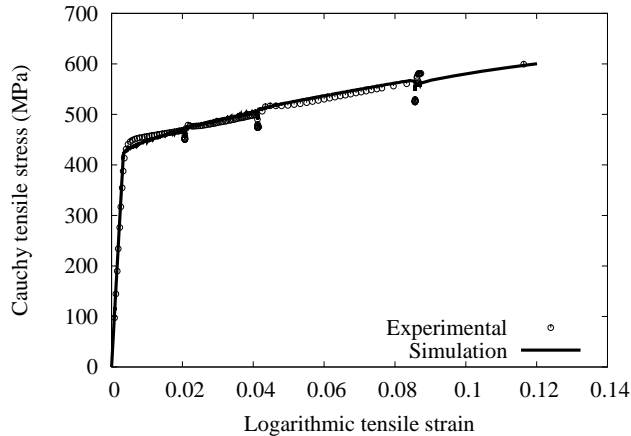


Figure 3: Experimental and predicted increasing strain rate uniaxial tensile test in the RD.

245 *3.2.1. Inverse identification procedure*

246 A set of material parameters was sought to minimize the gap between
 247 the experimental tests and model predictions. Starting from an initial pa-
 248 rameter set, an optimisation process was conducted with a gradient type
 249 algorithm in order to minimize an error function defined in the least square
 250 sense as the weighted gap between experimental and computed data. The
 251 observable variables accounted for in the error function are the stress and
 252 strain components. The weights were chosen according to the uncertainty of
 253 the experimental data and therefore depend on the experimental type of test
 254 (tensile, shear, bulge) as detailed in Zang et al. (2011).

255 Furthermore, anisotropic yield function parameters (L, M) for Hill's 1948
 256 function and $c_{5,6}^k$ for Bron&Besson function are related to shear in the sheet
 257 thickness. Experimental data allowing to identify these parameters was not
 258 available and they were consequently kept constant and equal to isotropic
 259 values, $L=M=3$ and $c_{5,6}^k = 1$. Additionally, since $H+G=2$, only one of these
 260 two parameters needs to be identified.

261 In order to achieve a good description of the overall experimental database
 262 with a single parameter set, the following strategy was used:

- 263 • initial isotropic hardening parameters and a yield stress, σ_{sat} , C_R , n ,
 264 σ_0 , were identified considering the three tensile tests and the monotonic

- 265 shear test,
- 266 • yield surface shape parameters, λ, a, b_1, b_2 of the Bron&Besson yield
 - 267 function were approximated using the balanced biaxial tension test,
 - 268 • as mixed hardening is considered, both kinematic hardening parame-
 - 269 ters, B_X, C_X, H_X and isotropic hardening parameters were identified
 - 270 on cyclic shear tests,
 - 271 • anisotropy parameters, $c_{i=1..4}^{k=1,2}$ for the Bron&Besson yield function and
 - 272 F, G, N for Hill's 1948 yield function were identified considering the
 - 273 whole database.

274 The resulting material parameters for the phenomenological model are listed
 275 in [Table 2](#), [Table 3](#) and [Table 4](#).

E	ν	K_v	n_v	σ_0
95.	0.37	74.2	30.1	290.

Table 2: Material parameters common to both yield functions. E is in GPa, σ_0 in MPa while K_v is in $\text{MPa}\cdot\text{s}^{n-1}$.

276 Up to three Armstrong-Frederick kinematic hardening terms had to be
 277 added to achieve an accurate description as suggested in [Chaboche \(1991\)](#).
 278

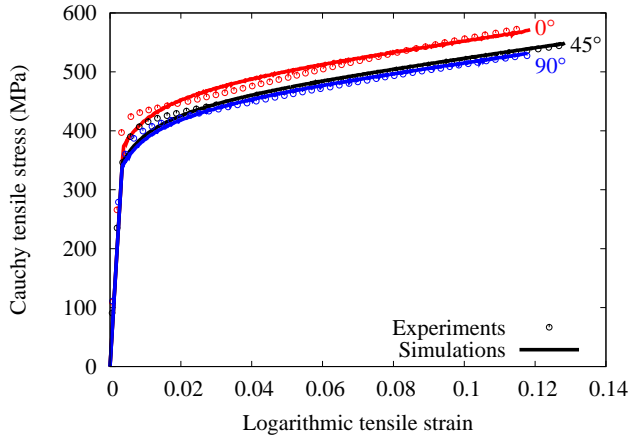
F	G	$H = 2 - G$	N							
1.01	0.83	1.17	3.41							
C_{X_1}	B_{X_1}	C_{X_2}	B_{X_2}	C_{X_3}	B_{X_3}	H_X	σ_{sat}	C_R	n_r	
13400	293	238	0.01	2850	60.4	395	5420	0.16	1.38	

Table 3: Material parameters for Hill's 1948 yield function. C_{X_i}, H_X and σ_{sat} are in MPa.

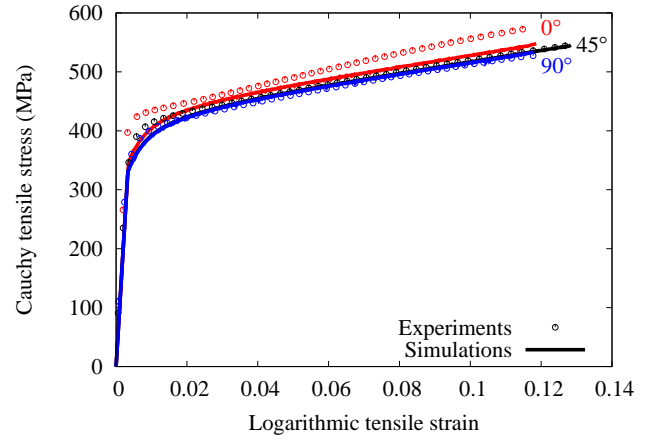
a	b_1	b_2	λ							
49.2	2.1	12.7	0.2							
c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{24}			
1.1	0.8	1.01	1.1	0.5	1.3	0.6	0.9			
C_{X_1}	B_{X_1}	C_{X_2}	B_{X_2}	C_{X_3}	B_{X_3}	H_X	σ_{sat}	C_R	n_r	
5080	243	44.8	0.1	2190	52	336	2820	0.23	1.21	

Table 4: Material parameters for Bron&Besson yield function. C_{X_i} , H_X and σ_{sat} are in MPa.

279 The material presents a noticeable stress anisotropy as tensile strain-stress
280 curves at 45°/RD and 90°/RD are nearly superposed but the stress level at
281 0°/RD is clearly higher. Bron&Besson model captured this feature well while
282 Hill's 1948 model failed to represent the stress anisotropy as shown in Fig. 4.



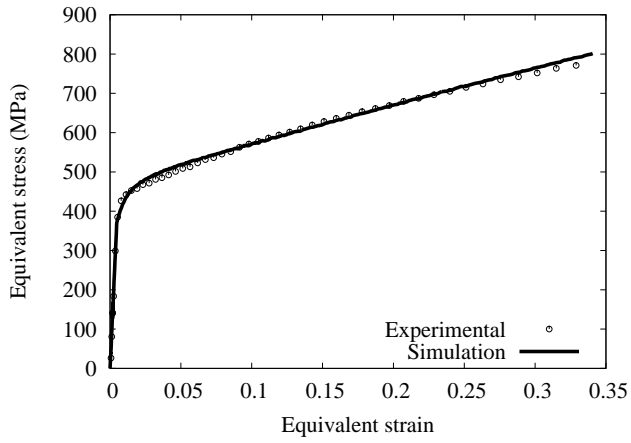
(a) Bron&Besson



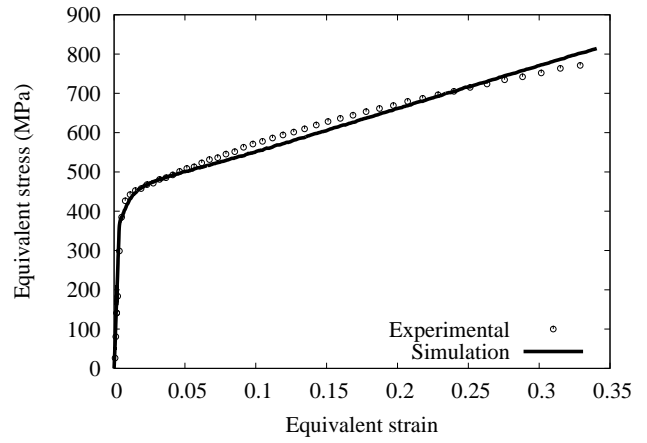
(b) Hill

Figure 4: Prediction of the tensile tests at 0°, 45° and 90° with the phenomenological model using the yield functions of a) Bron&Besson and b) Hill's 1948. Symbols show the experimental values.

283 Both models performed well in balanced biaxial tension and shear test
284 description as illustrated in Fig. 5 and Fig. 6, although the Bron&Besson
285 model predictions were slightly closer to the experiments.

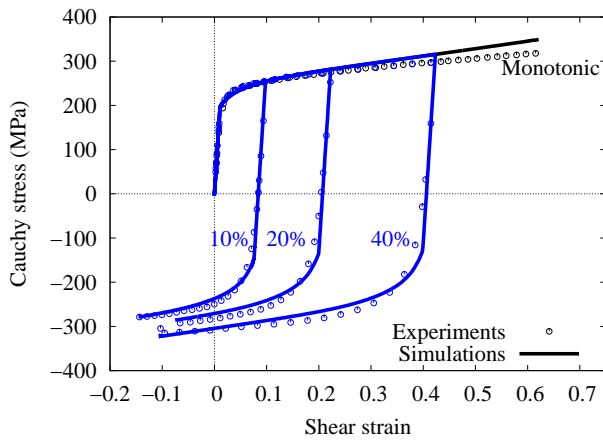


(a) Bron&Besson

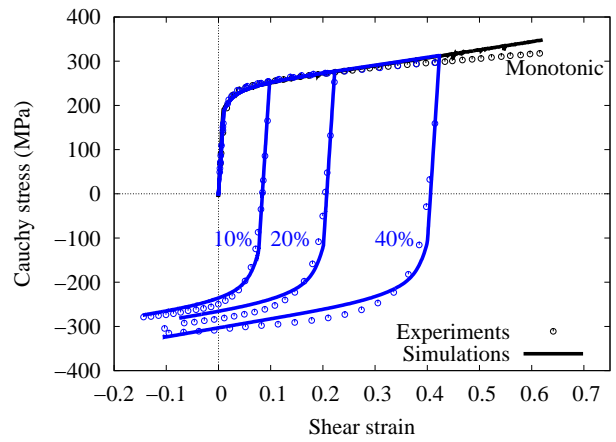


(b) Hill

Figure 5: Prediction of the hydraulic bulge tests with the phenomenological model using the yield functions of a) Bron&Besson and b) Hill's 1948. Symbols show the experimental values.



(a) Bron&Besson



(b) Hill

Figure 6: Prediction of the monotonic and reversed shear tests with the phenomenological model using the yield functions of a) Bron&Besson and b) Hill's 1948. Symbols show the experimental values.

286 Concerning strain anisotropy, Hill’s model did not adequately render the
 287 material feature and only r_{45} was well described. Due to its eight dedicated
 288 anisotropy parameters, Bron&Besson model performed remarkably as well in
 289 the strain anisotropy, see Fig. 7, as in the stress levels description.

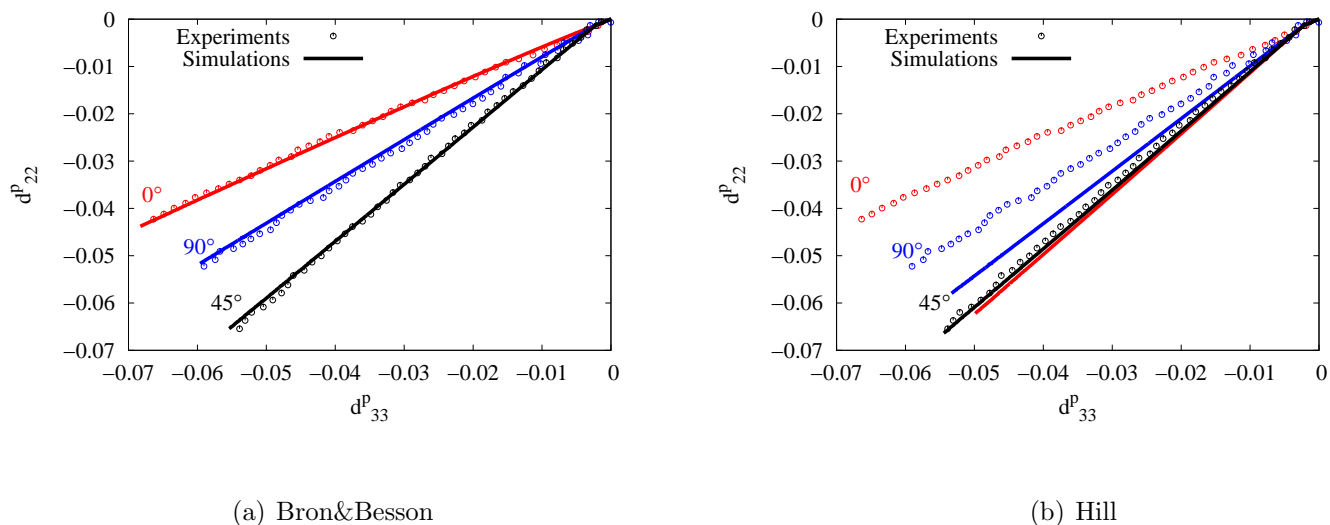


Figure 7: Prediction of the plastic anisotropy ratios with the phenomenological model using the yield functions of a) Bron&Besson and b) Hill’s 1948. Symbols show the experimental values.

290 However in both cases one has to deal with a high number of parameters,
 291 19 for Hill’s 1948 model and 28 for Bron&Besson, throughout a non linear
 292 and relatively complex identification procedure.

293 4. CPFEM model and corresponding parameter identification strate- 294 gies

295 The alternative modeling framework adopted in this work is based on the
 296 rate-dependent theory of crystal plasticity. The rate-dependent or viscoplas-
 297 tic formulation was adopted as the material presents a certain strain rate
 298 sensitivity as shown in Fig. 3 but also because of its numerical efficiency and
 299 robustness when compared to rate-independent formulation.

300 *4.1. Single crystal kinematics and plastic flow*

301 The single crystal plasticity model is based on the works of Rice (1971),
 302 Peirce et al. (1982), Needleman et al. (1985), Raphanel et al. (2004) and
 303 its equations are recalled here for the sake of completeness. The formalism
 304 presented below stands for the general theory of crystal plasticity, regardless
 305 of any aspect of numerical implementation.

306 The classical multiplicative breakdown of the deformation gradient is adopted:

307

$$\mathbf{F} = \mathbf{F}^e \cdot \mathbf{F}^p, \quad (17)$$

308 where \mathbf{F}^p represents plastic deformation of matter through the lattice result-
 309 ing from shearing along activated slip-systems and \mathbf{F}^e corresponds to crystal
 310 lattice rigid rotation and its elastic stretching while all slip movement is as-
 311 summed to be frozen.

312 The velocity gradient $\mathbf{L} = \dot{\mathbf{F}} \cdot \mathbf{F}^{-1}$ can then be expressed as:

$$\mathbf{L} = \mathbf{L}^e + \mathbf{L}^p \quad \text{where} \quad \mathbf{L}^e = \dot{\mathbf{F}}^e \cdot \mathbf{F}^{e-1} \quad \text{and} \quad \mathbf{L}^p = \mathbf{F}^e \cdot \mathbf{L}_i^p \cdot \mathbf{F}^{e-1}. \quad (18)$$

313 The velocity gradient in the intermediate configuration \mathbf{L}_i^p is related to crys-
 314 tallographic slip by:

$$\mathbf{L}_i^p = \dot{\mathbf{F}}^p \cdot \mathbf{F}^{p-1} = \sum_{s=1}^{12} \dot{\gamma}^{(s)} \mathbf{S}_0^{(s)} \quad \text{with} \quad \mathbf{S}_0^{(s)} = \mathbf{m}_0^{(s)} \otimes \mathbf{n}_0^{(s)}, \quad (19)$$

315 where $\dot{\gamma}^{(s)}$ is the shearing slip rate on the slip sytem (s) defined by the slip
 316 direction $\mathbf{m}^{(s)}$ and the slip plane normal $\mathbf{n}^{(s)}$, $\mathbf{S}_0^{(s)}$ being the Schmid tensor
 317 in the reference and intermediate configuration. The subscript '0' stands
 318 for tensorial quantities expressed in the reference configuration. Twelve slip
 319 systems were considered for the studied copper alloy of FCC structure.

320 The rate of deformation \mathbf{D} and spin \mathbf{W} tensors, which are the symmetric
 321 and skew-symmetric part of the velocity gradient \mathbf{L} , can be splitted in elastic
 322 and plastic components as:

$$\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p \quad \text{with} \quad \mathbf{D}^p = \sum_{i=1}^{12} \dot{\gamma}^{(s)} \mathbf{D}^{(s)} \quad (20)$$

323 and

$$\mathbf{W} = \mathbf{W}^e + \mathbf{W}^p \quad \text{with} \quad \mathbf{W}^p = \sum_{i=1}^{12} \dot{\gamma}^{(s)} \mathbf{W}^{(s)}. \quad (21)$$

324 In the above equations, $\mathbf{D}^{(s)} = \text{sym}(\mathbf{S}^{(s)})$ and $\mathbf{W}^{(s)} = \text{skew}(\mathbf{S}^{(s)})$ where $\mathbf{S}^{(s)} =$
 325 $\mathbf{m}^{(s)} \otimes \mathbf{n}^{(s)}$ is the Schmid tensor of slip system (s) in the current configuration,
 326 and 'sym' and 'skew' designate the symmetric and skew-symmetric part of
 327 the given tensor, respectively.

328 The slip direction vector and the slip plane normal in the current configura-
 329 tion are convected from their counterparts in the reference configuration by
 330 means of the elastic deformation gradient:

$$\mathbf{m}^{(s)} = \mathbf{F}^e \cdot \mathbf{m}_0^{(s)} \quad \text{and} \quad \mathbf{n}^{(s)} = \mathbf{n}_0^{(s)} \cdot \mathbf{F}^e. \quad (22)$$

331 Plastic flow occurs on a slip system if the resolved shear stress $\tau^{(s)}$ exerted
 332 on this system is greater than its resistance to slip, the critical resolved
 333 shear stress $\tau_c^{(s)}$. The resolved shear stress is calculated with the generalized
 334 Schmid law:

$$\tau^{(s)} = \mathbf{m}^{(s)} \cdot \boldsymbol{\sigma} \cdot \mathbf{n}^{(s)} = \boldsymbol{\sigma} : \mathbf{D}^{(s)}. \quad (23)$$

335 According to the rate-dependent formulation, the slip rates can be explicitly
 336 computed by the plastic flow rule

$$\dot{\gamma}^{(s)} = \begin{cases} \dot{\gamma}_0^{(s)} \text{sign}(\tau^{(s)}) \left| \frac{\tau^{(s)}}{\tau_c^{(s)}} \right|^n & \text{if } |\tau^{(s)}| \geq \tau_c^{(s)} \\ 0 & \text{if } \tau^{(s)} < \tau_c^{(s)} \end{cases}, \quad (24)$$

337 where $\dot{\gamma}_0^{(s)}$ is the reference shear strain rate and n is the strain rate sensitivity
 338 coefficient.

339 If one assumes an hypoelasticity formalism, the constitutive equation in the
 340 global frame can be written in terms of the rate of deformation tensor and
 341 its work-conjugate stress measure Kirchhoff tensor $\boldsymbol{\tau}$ as:

$$\hat{\boldsymbol{\tau}}^e = \mathbf{C} : \mathbf{D}^e = \mathbf{C} : (\mathbf{D} - \mathbf{D}^p), \quad (25)$$

342 where $\hat{\boldsymbol{\tau}}^e$ is the Jaumann derivative of stress based on crystal lattice spin
 343 tensor \mathbf{W}^e and \mathbf{C} is the elastic modulus tensor.

344 When considering the Jaumann stress rate related to the spin tensor \mathbf{W} , this
 345 equation can be rewritten in terms of Cauchy stress as:

$$\dot{\boldsymbol{\sigma}} = \mathbf{C} : \mathbf{D} + \mathbf{W} \cdot \boldsymbol{\sigma} - \boldsymbol{\sigma} \cdot \mathbf{W} - \boldsymbol{\sigma} \text{tr}(\mathbf{D}) - (\mathbf{C} : \mathbf{D}^p + \mathbf{W}^p \cdot \boldsymbol{\sigma} - \boldsymbol{\sigma} \cdot \mathbf{W}^p) \quad (26)$$

346 where $\dot{\boldsymbol{\sigma}}$ stands for the time derivative of $\boldsymbol{\sigma}$.

347 As straining occurs, material properties evolve. The common generic hard-
 348 ening model on the slip systems is expressed as

$$\dot{\tau}_c^{(s)} = H_{s\beta} \dot{\gamma}^\beta \quad (27)$$

349 where $H_{s\beta}$ are the components of the hardening matrix. In this work, we will
 350 consider the expression proposed by Peirce et al. (1983) and usually termed
 351 as the PAN (for Peirce Asaro Needleman) model. More accurate models are
 352 available in the literature where hardening is explicitly linked to dislocation
 353 density based internal variables (Tabourot (1992)). A comparative study of
 354 such models with PAN-type models can be found in Lee et al. (2010).
 355 The hardening matrix used in this work reads:

$$H_{s\beta} = [q + (1 - q)\delta_{s\beta}] h(\gamma), \text{ with } h(\gamma) = h_0 \operatorname{sech}^2 \left(\frac{h_0 \gamma}{\tau^* - \tau_0} \right), \quad (28)$$

356 where $\delta_{s\beta}$ is the Kronecker delta, q is the latent hardening parameter, $\gamma =$
 357 $\int \sum_s \dot{\gamma}^{(s)} dt$ is the cumulated glide on all slip systems, h_0 is the initial hard-
 358 ening slope, τ^* relates to a saturation shear stress and τ_0 is the initial critical
 359 resolved shear stress.

360 Additionally, straining induces large rotations and subsequent reorientation
 361 of the crystals. Within the formalism used here, the evolution of the crystal
 362 orientation matrix can be expressed in an incremental way as:

$$\dot{\mathbf{Q}} = \mathbf{W}^e \cdot \mathbf{Q} = (\mathbf{W} - \mathbf{W}^p) \cdot \mathbf{Q}. \quad (29)$$

363 4.2. Finite element implementation

364 The elastic-viscoplastic CP model was further implemented in the FE
 365 code ABAQUS/Explicit. In this code, the default coordinate system is the
 366 corotationnal frame (Hibbitt (1992)) associated with the Green-Naghdi ob-
 367 jective derivative. This frame is generated by the rotation tensor \mathbf{R} calculated
 368 from the polar decomposition of the deformation gradient:

$$\mathbf{F} = \mathbf{R} \cdot \mathbf{U}, \quad (30)$$

369 where \mathbf{U} is the right stretch tensor.

370 Thus for crystal plasticity implementation, three coordinate systems should
 371 be distinguished : the global frame, the Crystal lattice Axes Coordinate
 372 System (C.A.C.S) and the additional system defined above termed as the
 373 Material Element Coordinate System (M.E.C.S.), see Fig. 8.

374 Although only the fixed and crystal frames are involved in the theoretical
 375 model, efficiency can be improved by considering an implementation in the
 376 rotating objective space frames, namely the MECS and the CACS (Amirkhizi
 377 and Nemat-Nasser (2007)).

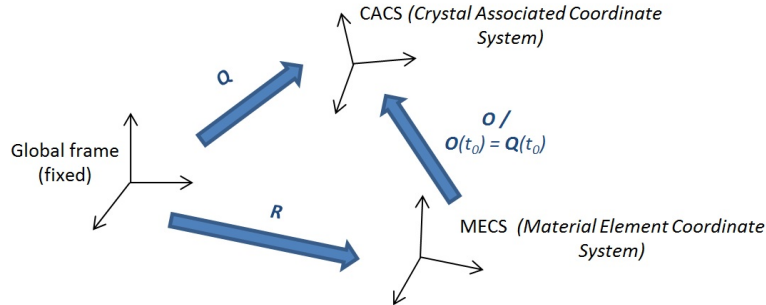


Figure 8: Different frames and associated rotation matrices for crystal plasticity implementation in ABAQUS/Explicit

378 Several other authors Guan et al. (2006), Zamiri et al. (2007), Li et al. (2008),
 379 Rousselier et al. (2009), Segurado et al. (2012), Zhang et al. (2012) have pro-
 380 posed procedures to achieve finite element implementations in objective space
 381 frames. The algorithm adopted here is summarized in Table 5. Except for
 382 orientation update, numerical integration of rate equations was performed
 383 with explicit Runge-Kutta integration schemes of orders 1, 2 and 4. It has
 384 been confirmed, (Haddag et al. (2007), Franz et al. (2009)), that for consti-
 385 tutive models with a large number of internal variables, explicit integration
 386 schemes represent a good compromise between efficiency and accuracy. In
 387 the applications presented in this paper, the first order proved as accurate
 388 and robust as the second and fourth order Runge-Kutta scheme while be-
 389 ing much more efficient. The Euler explicit scheme has also been tested in
 390 CPFEM by Grujicic and Batchu (2002), Dumoulin et al. (2009) and Du-
 391 moulin et al. (2012), Zhang et al. (2014) with very good results. Orientation
 392 update was performed using the method of Raphanel et al. (2004) in order
 393 to ensure that computed rotation matrices remain orthogonal.
 394 In the remaining part of the paper and unless specified otherwise, quantities
 395 expressed in the CACS are indicated with a tilde while those in the MECS
 396 carry no distinctive symbol for the sake of clarity.

397

398 4.3. Parameter identification

399 The parameter identification was conducted while simulating experimen-
 400 tal tests on Representative Volume Elements (RVE). The concept of RVE
 401 (e.g. Kanit et al. (2003), Gitman et al. (2007)) bridges the gap between the
 402 crystal scale and the macroscopical scale. Consistent comparisons can thus

- Initialise rotation matrix from CACS to MECS $\mathbf{O}_{t=0} = \mathbf{Q}_0$

- Input : $\boldsymbol{\sigma}_n, \mathbf{U}_{n+1}, \mathbf{U}_n$
Variables : $\tilde{\boldsymbol{\sigma}}_n, \mathbf{O}_n, \tilde{\mathbf{F}}_n^p, \tilde{\mathbf{F}}_n^e, \tau_{c_n}^{(s=1..12)}, \gamma_n, \mathbf{Q}_n$
- Compute $\mathbf{D}_{n+1} = \text{sym}(\dot{\mathbf{U}}\mathbf{U}_{n+1}^{-1})$
- Rotation to CACS $\tilde{\mathbf{D}}_{n+1} = \mathbf{O}_n^T \mathbf{D}_{n+1} \mathbf{O}_n$; $\tilde{\mathbf{U}}_{n+1} = \mathbf{O}_n^T \mathbf{U}_{n+1} \mathbf{O}_n$
- Update (for s=1..12) $\tilde{\mathbf{S}}_n^s = \tilde{\mathbf{F}}_n^e \tilde{\mathbf{S}}_c^s \tilde{\mathbf{F}}_n^{e-1}$; $\tilde{\mathbf{M}}_n^s = \text{sym}(\tilde{\mathbf{S}}_n^s)$
- Compute (for s=1..12) $\tau_{n+1}^s = \tilde{\boldsymbol{\sigma}}_n : \tilde{\mathbf{M}}_n^s$; $\dot{\gamma}_{n+1}^s$ with Eq(24)
- Update $\tilde{\mathbf{F}}_{n+1}^p = \tilde{\mathbf{F}}_n^p + \Delta t \left(\mathbf{I} + \sum_{s=1}^{12} \dot{\gamma}_{n+1}^s \tilde{\mathbf{S}}_c^s \right)$; $\tilde{\mathbf{F}}_{n+1}^e = \tilde{\mathbf{U}}_{n+1} \tilde{\mathbf{F}}_{n+1}^{p-1}$
- Update (for s=1..12) $\tilde{\mathbf{S}}_{n+1}^s = \tilde{\mathbf{F}}_{n+1}^e \tilde{\mathbf{S}}_c^s \tilde{\mathbf{F}}_{n+1}^{e-1}$;
 $\tilde{\mathbf{M}}_{n+1}^s = \text{sym}(\tilde{\mathbf{S}}_{n+1}^s)$; $\tilde{\mathbf{W}}_{n+1}^s = \text{skew}(\tilde{\mathbf{S}}_{n+1}^s)$
- Compute $\tilde{\mathbf{D}}_{n+1}^p = \sum_{s=1}^{12} \dot{\gamma}_{n+1}^s \tilde{\mathbf{M}}_{n+1}^s$; $\tilde{\mathbf{W}}_{n+1}^p = \sum_{s=1}^{12} \dot{\gamma}_{n+1}^s \tilde{\mathbf{W}}_{n+1}^s$
- Update
 $\tilde{\boldsymbol{\sigma}}_{n+1} = \tilde{\boldsymbol{\sigma}}_n + \Delta t \left[\mathbf{C} : \left(\tilde{\mathbf{D}}_{n+1} - \tilde{\mathbf{D}}_{n+1}^p \right) - \tilde{\boldsymbol{\sigma}}_n \text{tr} \left(\tilde{\mathbf{D}}_{n+1} \right) \right]$
- Update (for s=1..12) $\tau_{c_{n+1}}^s = \tau_{c_n}^s + \Delta t \left(\sum_{p=1}^{12} h^{sp} \dot{\gamma}^p \right)$
- Update $\mathbf{O}_{n+1} = \exp \left[\Delta t \left(\tilde{\mathbf{W}}^p \right) \right] \mathbf{O}_n$ (Euler-Rodrigues formula)
- Rotation to MECS $\boldsymbol{\sigma}_{n+1} = \mathbf{O}_{n+1} \tilde{\boldsymbol{\sigma}}_{n+1} \mathbf{O}_{n+1}^T$

Table 5: Algorithm for the time integration of the crystal plasticity constitutive model.

403 be performed.

404 The numerical microstructure was generated using a Laguerre tessellation
405 (Quey et al. (2011)). Grain diameters were considered as weights and a
406 normal distribution employed with the mean grain size and the standard de-
407 viation as parameters. The microstructure was specified to be periodic. Each
408 grain was meshed with at least 20 solid reduced integration finite elements,
409 in order to guarantee sufficient accuracy for the prediction of the macroscopic
410 response (Diard et al. (2005), Lin et al. (2010), Belkhabbaz et al. (2015)).

411 The orientations attributed to the grains were obtained following a two step
412 procedure. First, the experimental orientation data (4460 triplets of Euler
413 angles) from EBSD were converted in an Orientation Distribution Function
414 (ODF) using the grain diameter as weights (Tarasiuk et al. (2004)). Then the
415 continuous distribution was discretised into the number of desired individual
416 crystallographic orientations. We thereby ensured that our volume elements
417 statistically represented the experimental texture even with few orientations.
418 Experimental pole figures and numerically sampled ones are compared in
419 Fig. 9. The numerical texture replicated the main characteristics of the ex-
420 perimental data.

421 Periodic Boundary Conditions were applied to the virtual volume elements.
422 Tensile tests were simulated on cuboid volume elements of increasing size
423 with 100, 200, 300, 400, 500, 600, 700, 800 grains in order to determine the
424 required size to be representative of the macroscopic behavior. Again, the
425 studied sheet metals were purposely chosen with enough thickness grains to
426 enable analysis with both phenomenological and CPFEM based models.

427 The macroscopic stresses and strains were obtained by averaging the corre-
428 sponding local values over the total volume. The convergence analysis, see
429 Fig. 10, revealed that above 500 grains, the scatter in the volume elements
430 macroscopic responses was negligible and a Representative Volume Element
431 can therefore be set up from a volume element with 500 grains.

432 The parameters of the P.A.N model were identified with respect to the ex-
433 perimental tensile test performed in the rolling direction. Indeed through rep-
434 resentation of material texture, anisotropy features are inherently accounted
435 for. Thus, only a few parameters had to be adjusted and no other exper-
436 imental test was required in the present calibration procedure. Flow rule
437 parameters, $\dot{\gamma}_o$ and n , are usual material values for room temperature Cop-
438 per that can be found in the literature (Tabourot (2001)). Hardening matrix
439 parameters, h_0 , q , τ^* and τ_0 are calibrated with respect to the RD tensile
440 test. As illustrated in Fig. 11, a good agreement was obtained between the

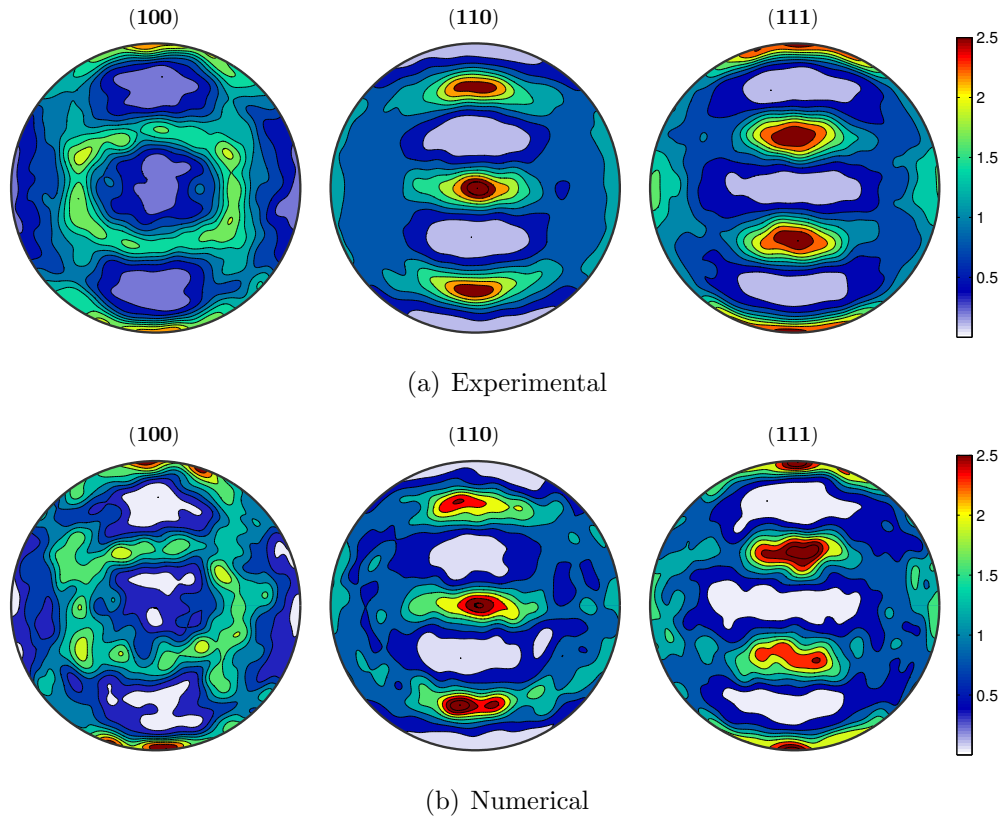


Figure 9: Pole figures obtained from a) EBSD experiments orientation data and b) numerically sampled orientations for the RVE.

441 experiment and the simulated curve although the elastic-plastic transition
 442 could not be perfectly captured. It is worth noting that the phenomenologi-
 443 cal models were also unable to describe this specific detail of the experimen-
 444 tal data set. This feature may be an effect of the cold rolling pre-strain which
 445 is not taken into account in any of the models.

446

447 5. Comparison of both approaches and discussion

448 5.1. Models descriptions for various strain paths

449 Tensile tests at 45° and 90° from the R.D. were simulated with the cali-
 450 brated PAN model. The microstructure was numerically rotated by decreas-
 451 ing the RD corresponding Euler angle of the desired value (45° and 90°) while

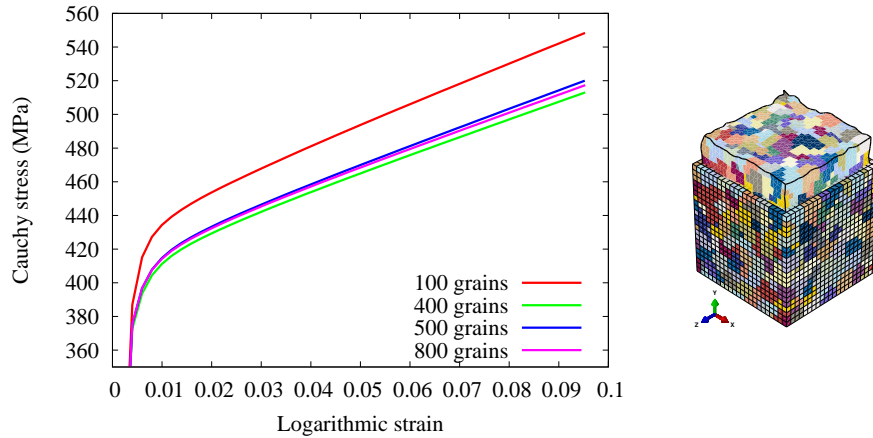


Figure 10: Convergence analysis on Volume Elements for a CPFEM simulated tensile test in the RD. The RVE is shown on the right, prior to straining (with the mesh) and after deformation. The color of the grains is not related to their orientation.

(Model parameters : $h_0=300$ MPa, $\tau^*=750$ MPa, $\tau_0=250$ MPa, $q = 1.4$, $\dot{\gamma}_o=0.001$ s $^{-1}$, $n=50$)

h_0 (MPa)	τ^* (MPa)	τ_0 (MPa)	q	$\dot{\gamma}_o$ (s $^{-1}$)	n
100	700	175	1.4	0.001	50.

Table 6: Material parameters for CPFEM calibrated on a tensile test in the RD.

452 the mesh and the boundary conditions remain unchanged. This methodology
 453 proved relevant for quasi equiaxed grains (Zhang et al. (2015)) as is the case
 454 in the present work.

455

456 The simulated curves match closely with the experimental ones as can be
 457 seen in Fig. 12 and Fig. 13, both in the elastic-plastic transition zone and
 458 at higher strains, even though a little deviation arises at the very end of the
 459 90°/RD curve.

460 Thus, using hardening parameters calibrated on a single tensile test in the
 461 RD, a good description of stress anisotropy was achieved with the CPFEM
 462 approach as the texture was accounted for.

463 The CPFEM model's predictions under balanced biaxial tensile loading are

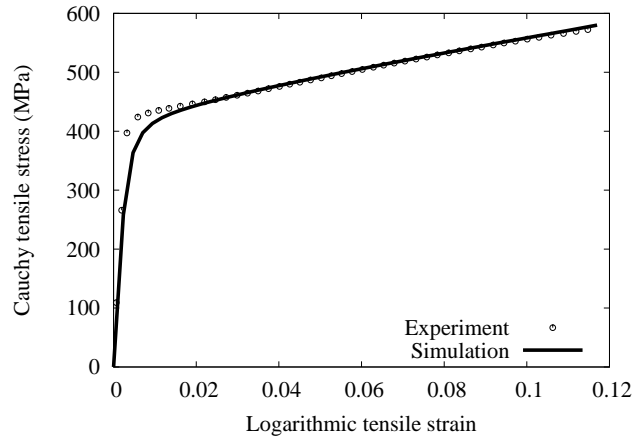


Figure 11: CPFEM model identification on the 0° /RD tensile test.

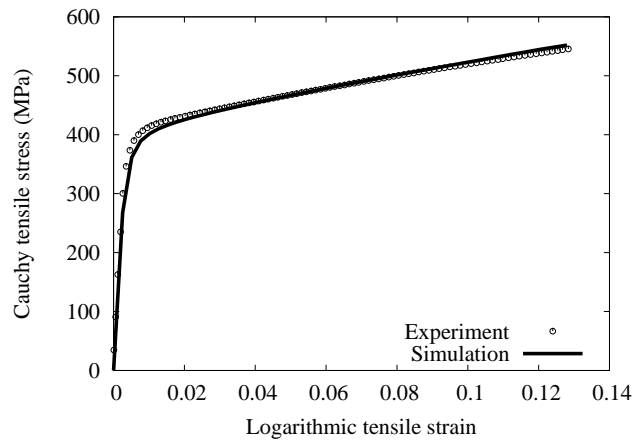


Figure 12: Experimental and CPFEM model simulated tensile curves at 45° /RD.

464 shown in Fig. 14. A good agreement was obtained between experimental
 465 and simulated curves. It is worth noting that the strain level used for this
 466 validation is three times larger than the strain level used for the parameter
 467 identification of the hardening model. Shear test in the RD, as well as reverse
 468 shear tests at different pre-strain ranges were also simulated. The response

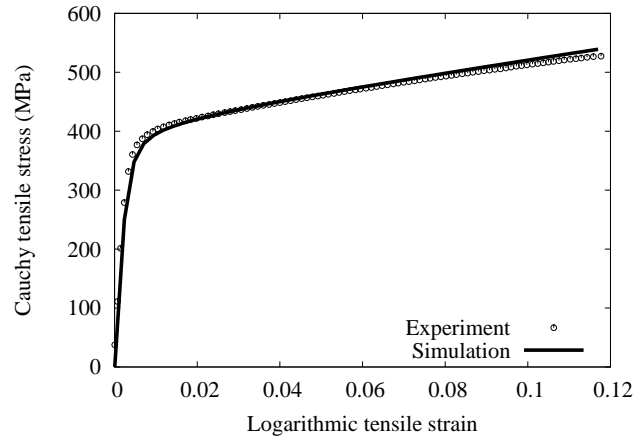


Figure 13: Experimental and CPFEM model simulated tensile curves at 90° /RD.

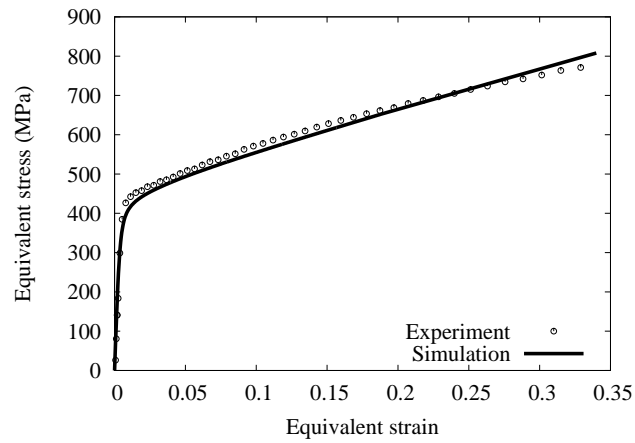


Figure 14: Experimental and CPFEM model simulated responses to equi-biaxial tensile loading.

469 under monotonic shearing was well reproduced, see [Fig. 15](#), nevertheless the
 470 predicted hardening rate being larger than the experimental one. Again,
 471 similar behavior was also predicted with the phenomenological model, see
 472 [Fig. 6](#). The Bauschinger effect and permanent softening induced after strain

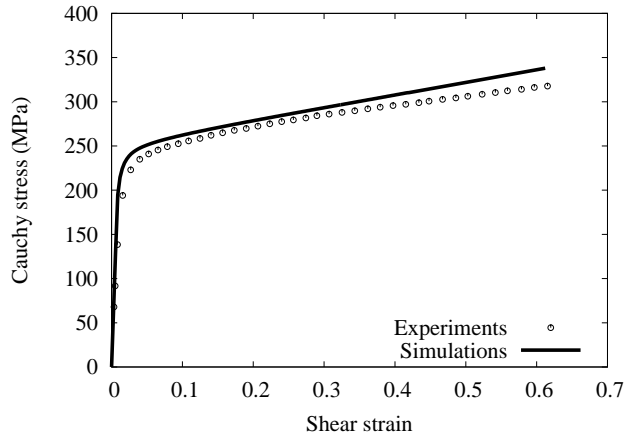


Figure 15: Experimental and CPFEM model simulated responses to monotonic shear loading.

473 path reversal were underestimated quite significantly by the CPFEM model
 474 as shown in Fig. 16.

475 In order to more accurately describe material response during strain reversal,
 476 several authors (Cailletaud (1992), Li et al. (2012)) introduce a kinematic
 477 hardening variable in the flow rule at the slip system level. That variable evo-
 478 lution is governed by an Armstrong-Frederick type law and its material pa-
 479 rameters have to be fitted on cyclic tests. A different approach was proposed
 480 by Balland et al. (2011) which requires relatively less effort for identification.
 481 Different initial critical resolved shear stresses are distributed in the grains
 482 of the aggregate. Thereby stress inhomogeneity, which can be linked to such
 483 material behavior, is introduced in the model. A Rayleigh distribution was
 484 adopted to model an inhomogeneous repartition of the initial critical resolved
 485 shear stress in the aggregate. It is written as:

$$f(\tau_o) = \frac{\tau_o}{(\tau_0^{hom})^2} \exp\left(\frac{-(\tau_o)^2}{2(\tau_0^{hom})^2}\right), \quad (31)$$

486 where the modal of the distribution τ_0^{hom} is chosen equal to the previously
 487 identified initial critical resolved shear stress τ_0 .

488 The latter approach proved successful for pre-strains up to 20 % as can

489 be seen in Fig. 17. However when the pre-strain is larger, the model still
490 underestimates the Bauschinger effect and permanent softening induced.

These results tend to show that intergranular stress inhomogeneity may

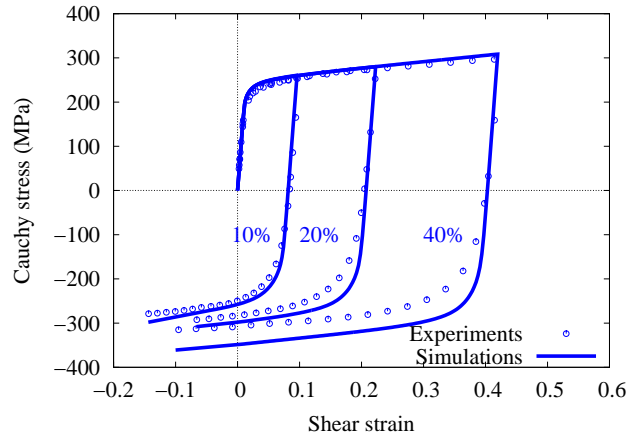


Figure 16: Experimental reversed shear test and CPFEM model simulated curves.

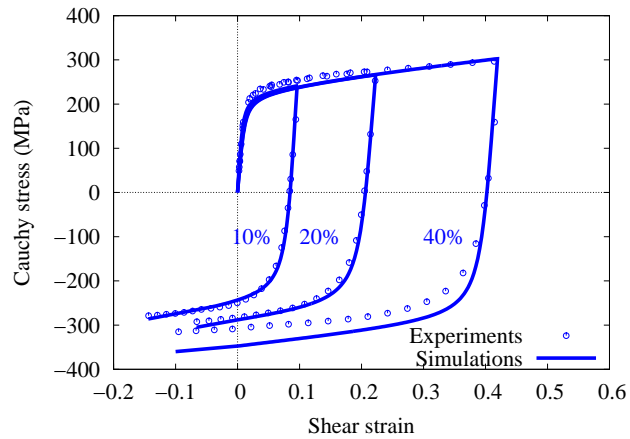


Figure 17: Experimental reversed shear test and CPFEM model simulated curves while accounting for initial stress inhomogeneity.

491 be sufficient to describe strain reversal behavior up to moderate pre-strain
 492 levels. Afterwards, the effect of intragranular backstress cannot be neglected
 493 and the hardening model needs further improvements (Kim et al. (2012), Li
 494 et al. (2014)).
 495

496 *5.2. Phenomenological models calibration with a reduced experimental database*

497 As described in the previous section, the CPFEM model was calibrated
 498 using only a tensile test in the RD as well as the measured texture. In order
 499 to establish a more relevant one-to-one comparison, the phenomenological
 500 models were recalibrated using a reduced experimental database. The latter
 501 consists only of the stress-strain curves for the monotonic tensile tests at 0,
 502 45 and 90° with respect to the rolling direction.

503 The identification procedure was simplified accordingly :

- 504 • isotropic hardening parameters and a reference yield stress were identified considering the three tensile tests
- 505
- 506 • yield function shape parameters of Bron&Besson model retained the reference values ($a=b_1=b_2=2$)
- 507
- 508 • anisotropy parameters were identified on the considered database

509 Calibration of the models was conducted as previously, with the SiDoLo
 510 software, and the obtained parameter sets are given in Table 7 and Table 8.
 511 Then, the entire set of tests was simulated with this new set of parameters.

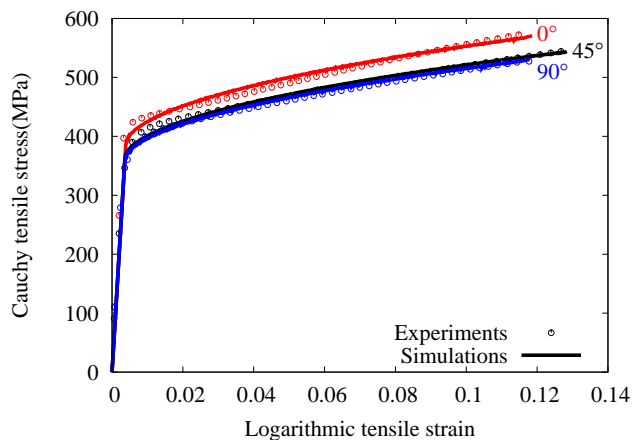
F	G	N	σ_0	σ_{sat}	C_R	n
1.27	1.14	2.94	316	5780	0.11	0.55

Table 7: Material parameters for the phenomenological model and Hill’s 1948 yield function identified on the reduced database. σ_0 and σ_{sat} are in MPa.

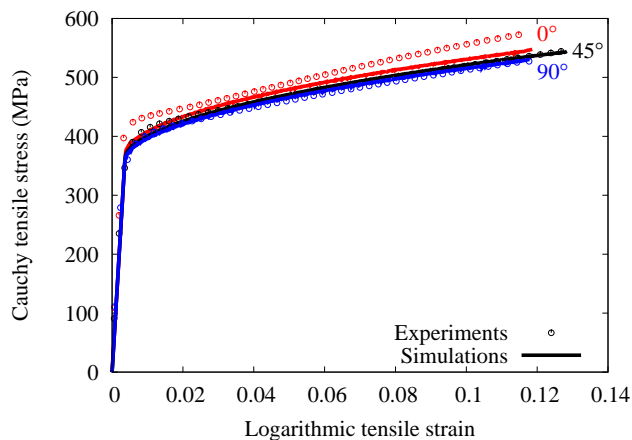
c_{11}	c_{12}	c_{13}	c_{14}	σ_0	σ_{sat}	C_R	n
1.11	0.97	0.95	1.01	316	5780.	0.11	0.55

Table 8: Material parameters for the phenomenological model and Bron&Besson yield function identified on the reduced database. σ_0 and σ_{sat} are in MPa.

512 As expected, this time the predictions of both models were less accurate.
 513 Bron&Besson still performed better than Hill's 1948 model and described
 514 quite well yield stress anisotropy and stress levels, see Fig. 18. The predic-
 515 tions of equi-biaxial tensile loading are shown on Fig. 19. Hill's 1948 model
 516 largely underestimated the stress level while Bron&Besson accurately de-
 517 scribed the hardening behavior only up to 5% of deformation. Both models
 518 provided fairly good predictions of monotonic shear tests. However since
 519 cyclic shear tests were not considered in the database, the two models did
 520 not describe the correct transient behavior after strain reversal, see Fig. 20.
 521 Also, Fig. 21 showed that strain anisotropy could not be captured by the
 522 models.



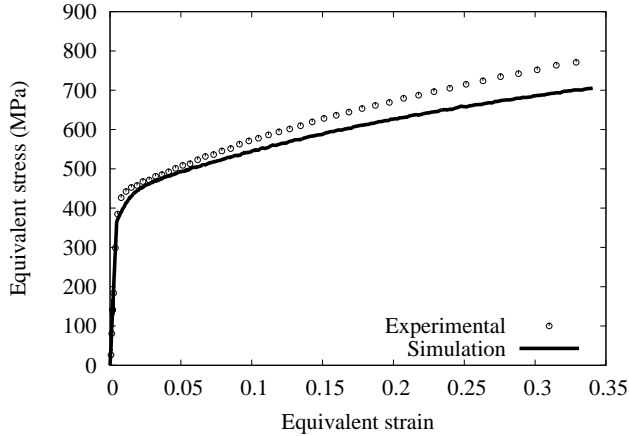
(a) Bron&Besson (reduced database)



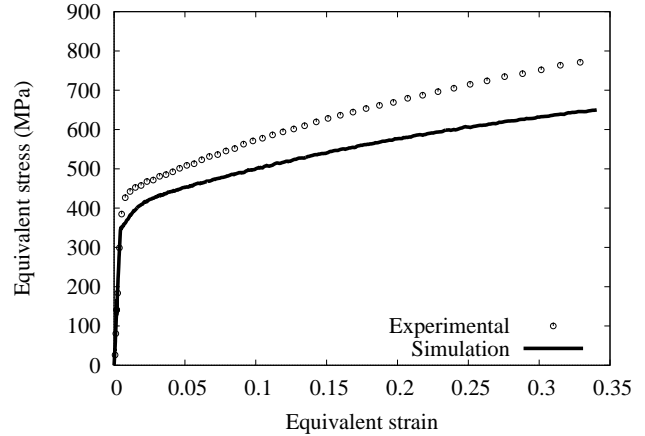
(b) Hill (reduced database)

Figure 18: Prediction of the tensile tests at 0° , 45° and 90° with the phenomenological model based on the reduced database using the yield functions of a) Bron&Besson and b) Hill's 1948. Symbols show the experimental values.

523



(a) Bron&Besson (reduced database)

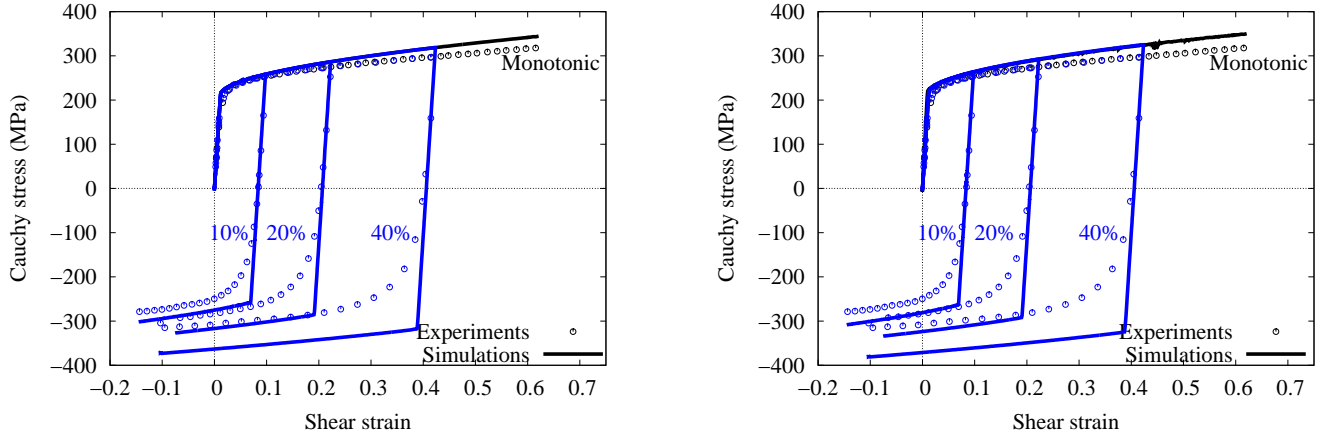


(b) Hill (reduced database)

Figure 19: Prediction of the hydraulic bulge tests with the phenomenological model based on the reduced database using the yield functions of a) Bron&Besson and b) Hill's 1948. Symbols show the experimental values.

524 *5.3. Discussion*

525 When calibrated on the full experimental database, the Bron&Besson
 526 model provided a good overall prediction of all the experiments and strain
 527 paths. Its eight anisotropy parameters enabled an excellent description of
 528 material anisotropy, both in flow stresses and plastic strain ratios. Let us
 529 recall that the parameter identification was performed to optimize simulta-
 530 neously strain and stress anisotropy. Hill's 1948 model could not accurately
 531 capture both features. It couldn't account for the distinctive hardening slope
 532 at 0° from the RD and only the plastic strain ratio at 45° from the RD was
 533 well predicted. Nevertheless, it provided a description of equi-biaxial loading
 534 and shear tests nearly as good as the Bron&Besson model. Stress anisotropy
 535 was still well predicted by Bron&Besson model and failed using Hill's 1948
 536 model. However, this time neither of the two models could predict any of the
 537 plastic strain ratios. Bron&Besson model prediction for equi-biaxial loading
 538 was clearly better than Hill's 1948 model but still it considerably lacked ac-
 539 curacy for large strains. The monotonic shear test description was quite good
 540 for both models while kinematic hardening effects in reversed shear loading



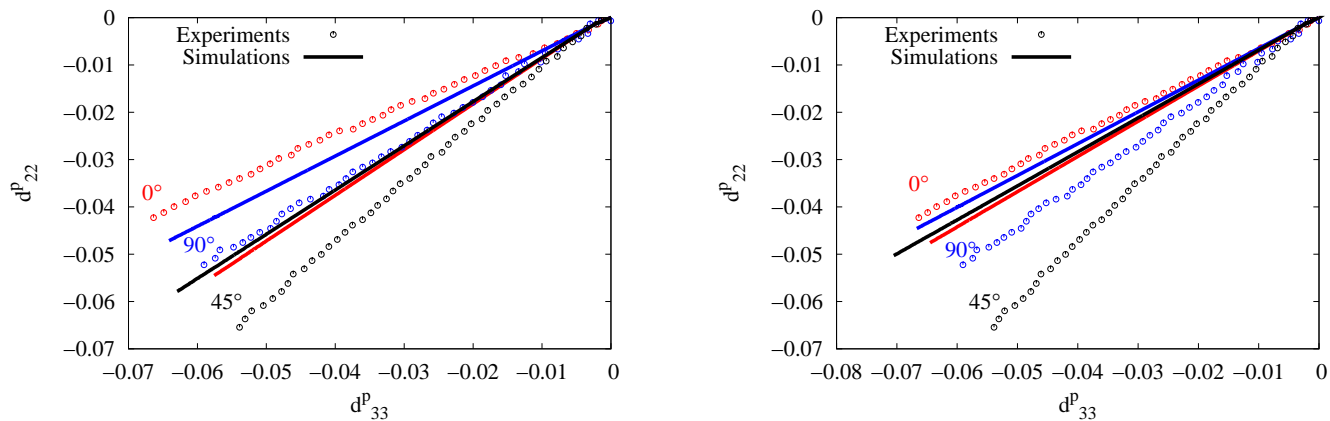
(a) Bron&Besson (reduced database)

(b) Hill (reduced database)

Figure 20: Prediction of the monotonic and reversed shear tests with the phenomenological model based on the reduced database using the yield functions of a) Bron&Besson and b) Hill's 1948. Symbols show the experimental values.

541 were not rendered as expected.

542 The CPFEM model, calibrated on a single tensile test in the RD and ac-
 543 counting for EBSD measured grain orientations, predicted remarkably well
 544 the stress-strain hardening curves. Thus, its predictions on stress anisotropy
 545 compare well with those of Bron&Besson model. Regarding equi-biaxial load-
 546 ing as well as the monotonic and reversed shear tests description, phenom-
 547 ological models calibrated on the reduced database were outperformed by the
 548 CPFEM model. The latter even performed equally well on equi-biaxial load-
 549 ing with the former calibrated on the full experimental database. However,
 550 CPFEM model monotonic and reversed shear tests predictions were not as
 551 accurate as full database calibrated phenomenological models ; especially for
 552 large pre-strained reversed shear loading, Bron&Besson and Hill's 1948 mod-
 553 els provide clearly more accurate predictions. Additionally, Fig. 22 shows
 554 that when it comes to strain anisotropy, the best description was clearly
 555 achieved by Bron&Besson model calibrated on the full database. This time,
 556 simulations were made at every 5° from the RD for the sake of completeness.
 557 The CPFEM model predictions were also in good agreement with experimen-



(a) Bron&Besson (reduced database)

(b) Hill (reduced database)

Figure 21: Prediction of the plastic anisotropy ratios with the phenomenological model based on the reduced database using the yield functions of a) Bron&Besson and b) Hill's 1948. Symbols show the experimental values.

558 tal data and this model gave the best description in the case of the reduced
 559 database.

560 Ultimately, the Bron&Besson model calibrated on the extended database pro-
 561 vided the best description with respect to the existing experiments, demon-
 562 strating the possibilities of advanced phenomenological material models. How-
 563 ever, collecting such an amount of experimental data is time consuming and
 564 not always affordable. Indeed, usual material characterization for industrial
 565 purposes often consist of tensile tests such as the reduced database employed
 566 in this work. If one considers such a database, it is clear that the CPFEM
 567 model stands out as the best modeling choice in terms of accuracy and ro-
 568 bustness. Furthermore, it must be stated that the CPFEM model employed
 569 in the present study was relatively basic and designed to embed the least
 570 possible parameters to fit. This was intended to obtain a model relying fun-
 571 damentally on material microstructure and texture input in the present case.
 572 As a result, only 3 parameters had to be adjusted. Up to 28 and 19 parame-
 573 ters had to be calibrated on the full database respectively for Bron&Besson
 574 model and Hill's 1948 model. The calibration procedure was far more com-

575 plex for phenomenological models and required dedicated procedures and
 576 software. Additionally, it was demonstrated in the reduced database case
 577 that when confronted with strain paths that had not been accounted for in
 578 the calibration step, the phenomenological models could not provide good
 579 results. Consequently, the accuracy of phenomenological models calibrated
 580 on monotonic experiments alone (thus, involving only isotropic hardening)
 581 can be reasonably questioned whenever non-proportional strain paths may
 582 occur. In contrast, the CPFEM model calibrated on a single uniaxial tensile
 583 test proved capable of accurately predicting the mechanical response of a non
 584 annealed copper alloy under various strain paths, although its reverse loading
 585 predictions were not perfect. As it accounts for deformation mechanisms, the
 586 shapes and orientations of the grains, the CPFEM model has the potential
 to predict global material behavior.

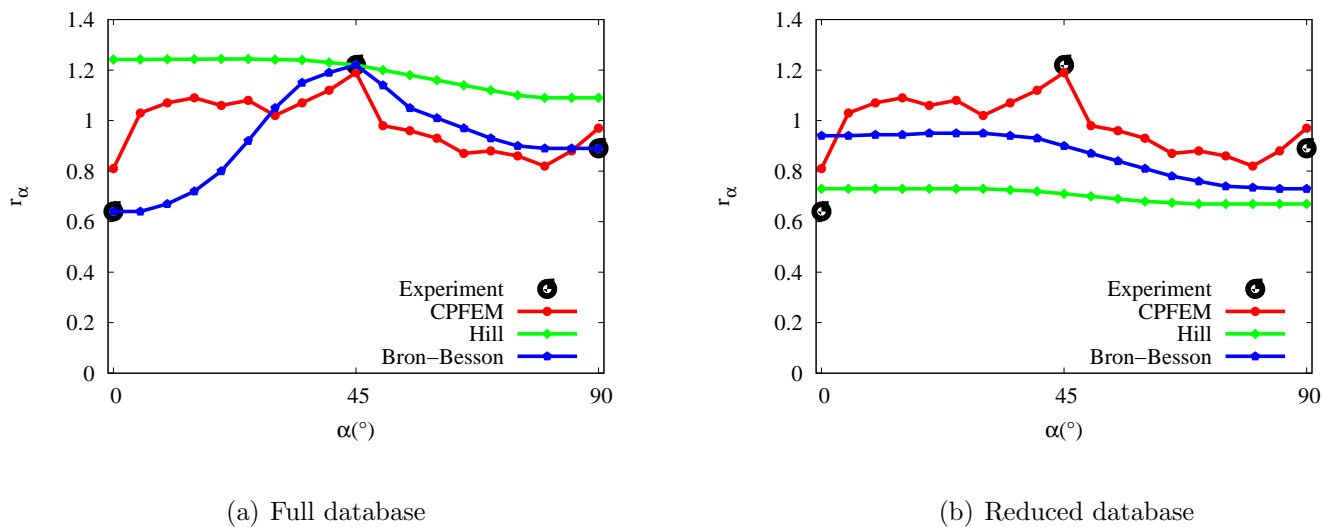


Figure 22: Prediction of the plastic strain ratios at 0° , 45° and 90° with the CPFEM model and the phenomenological models calibrated on the a) full experimental database and b) reduced experimental database.

587

588 6. Conclusion

589 State-of-the-art phenomenological models (Bron&Besson, Hill's 1948) and
 590 a CPFEM classical model (Peirce-Asaro-Needleman type) were compared on

591 their ability to describe the mechanical response of very thin copper alloy
592 sheet metals under several strain paths. Theoretical and numerical imple-
593 mentation aspects as well as material calibration of material parameters of
594 the models were presented. It can be summarized that :

- 595 • The phenomenological model based on the BB yield function provided
596 the best results and accurately predicted all the mechanical tests on
597 which it was calibrated
- 598 • Hill's 1948 could not perform well on the simultaneous description of
599 stress and strain anisotropy
- 600 • both phenomenological models results were very sensitive to the scope
601 of the experimental database considered for calibration ; their predic-
602 tions quality was poor on a reduced database
- 603 • in contrast, a basic CPFEM model calibrated on a single tensile test
604 provided an overall good description of all explored strain paths and
605 strain anisotropy
- 606 • an initial stress inhomogeneity between grains in the CPFEM model im-
607 proved the prediction of the reversed shear test (especially the Bauschinger
608 effect) up to moderate pre-strains

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