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# Topological superconductivity in the one-dimensional interacting Creutz model

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**Abstract.** We consider one-dimensional topological insulators characterized by zero energy end states. In presence of proximity induced pairing, those end states can become Majorana states. We study here the fate of those various end states when Hubbard electron-electron repulsive interactions are added, using a combination of mean-field theory and density matrix renormalization group techniques.

## 1. Introduction

In the context of quantum field theory, Jackiw and Rebbi [1] introduced a general mechanism to generate zero modes with fractional charges. In condensed matter, this mechanism manifests itself by the apparition of topologically protected states localized at the end of some atomic chain, like polyacetylene [2, 3]. More recently, another type of zero-energy excitation, the Majorana fermion, has attracted a lot of attention from the condensed matter community [4, 5]. A Majorana fermion is its own antiparticle [6], and can appear at defects of topological superconductors, such as  $p$ -wave superconductors [7] or superconducting hybrid systems mimicking them, like nanowires with strong spin-orbit deposited on a singlet  $s$ -wave superconductor [8, 9, 10, 11].

This paper aims at understanding the physics of systems that host fractionally charged solitons in their normal state and Majorana modes in some of their superconducting phases. In the whole



paper, superconductivity refers to pairing correlations transferred from a standard s-wave singlet superconductor towards the 1D fermionic chain. We focus on the Creutz model [12, 13, 14, 15], and address two kinds of issues: 1) the evolution from zero-energy solitonic modes towards Majorana states under the addition of proximity-induced superconductivity, 2) the robustness of these zero energy states with respect to strong Hubbard repulsive interactions. The effect of a repulsive Hubbard interaction is studied using a combination of mean-field theory and density matrix renormalization group (DMRG). Globally, the parameter range, where topological superconductivity is obtained, grows upon increasing the Hubbard interaction, as recently obtained in nanowire models [16]. Nevertheless, there is a region in parameter space, where interactions prove detrimental to the Majorana states (cf. Fig. 1).

The paper is organized as follows: Section 2 reviews the Creutz model and its topological properties. Section 3 investigates the transition from chiral bound states to Majorana bound states upon increasing the pairing. Section 4 takes into account the effects of repulsive interactions in the Creutz-Majorana-Hubbard model (CMH).

## 2. Creutz model

Here, we present the noninteracting part of the lattice Creutz model, first in absence of superconductivity and then in presence of proximity induced pairing.

### 2.1. Normal state

In the absence of superconductivity and interactions, our starting point is the lattice Hamiltonian

$$H_C = \frac{1}{2} \sum_j [w c_j^\dagger \sigma_1 c_j + c_j^\dagger (it\sigma_3 - g\sigma_1) c_{j-1}] + \text{H.c.}, \quad (1)$$

where the sum runs over all sites indexed by  $j$ . The electronic spin is represented by the standard Pauli matrices  $\sigma_i$  ( $i = 1, 2, 3$ ). The spin indices for the electron annihilation operators  $c_j = (c_{j\uparrow}, c_{j\downarrow})$  and spin matrices are implicit. The electrons can jump from one site to the nearest-neighboring site while conserving their spin: this process has a complex amplitude  $\pm it$ , i.e., the electrons gain or lose a phase  $\pi/2$  when hopping between the same spin states. The electrons can also hop between sites with amplitude  $g$  while flipping their spin, which mimics a spin-orbit coupling. Finally, there is an onsite mass term  $w$  which favors the polarization of the electronic spin along the  $x$  direction. The Hamiltonian is diagonalized by Fourier transforming it into momentum space  $H_C = \sum_k c_k^\dagger \mathcal{H}_C(k) c_k$ :

$$\mathcal{H}_C(k) = t \sin k \sigma_3 + (w - g \cos k) \sigma_1. \quad (2)$$

Consequently, there are two bands with the energy dispersion

$$E_\pm = \pm \sqrt{(t \sin k)^2 + (w - g \cos k)^2}. \quad (3)$$

From Eq. (3) it follows that the two energy bands can touch either at  $k = 0$ , or at  $k = \pi$ . At these momenta, the energy dispersion is linear and there is a Dirac cone band touching. For  $g = w$ , the band touching takes place at  $k = 0$ , and for  $g = -w$ , at  $k = \pi$ . When both  $g$  and  $w$  vanish, the system exhibits two Dirac cones.

When  $g = w$  (resp.  $g = -w$ ), the system is gapless with a Dirac cone at momentum  $k = 0$  (resp.  $k = \pi$ ). A particular point is the case of vanishing  $g$  and  $w$ , when the system recovers the time-reversal symmetry, and exhibits two Dirac cones at  $k = 0$  and  $\pi$ . For all other values of the parameters ( $g, w$ ), the chain is a fully gapped insulator, whose topological properties are encoded in its associated winding number  $\mathcal{W}$ ,

$$\mathcal{W} = \frac{1}{2} [\text{sgn}(w + g) - \text{sgn}(w - g)]. \quad (4)$$

When  $|g/w| < 1$ , the system is a topologically trivial insulator ( $\mathcal{W} = 0$ ) and when  $|g/w| > 1$ , it is a topologically nontrivial insulator characterized by  $\mathcal{W} = \text{sgn}(g)$  [15].

In the topologically nontrivial state there are chiral zero-energy bound states (CBS) at the edges of an open system. These localized states are protected by the chiral symmetry  $\sigma_2$  and the bulk gap. For a finite chain, the two edge CBS overlap through the insulating bulk. This overlap is exponentially small for chains longer than the spatial extension of the CBS wave function [13].

## 2.2. Proximity induced superconductivity

Let us modify the Creutz Hamiltonian (1) by adding an  $s$ -wave singlet superconducting pairing with amplitude  $\Delta$ . This leads to the Creutz-Majorana model described by the Hamiltonian

$$H_{\text{CM}} = H_{\text{C}} + H_{\Delta}, \quad H_{\Delta} = \sum_j \Delta c_{j\uparrow}^\dagger c_{j\downarrow}^\dagger + \text{H.c.}, \quad (5)$$

where the sum is taken over all the lattice sites. The order parameter  $\Delta$  can be considered real without any loss of generality. The momentum-space Hamiltonian is quadratic in the standard BdG form. Let us choose the basis  $C_k^\dagger = (c_{k\uparrow}^\dagger, c_{k\downarrow}^\dagger, c_{-k\uparrow}, c_{-k\downarrow})$ . In this basis, the Hamiltonian is written as  $H_{\text{CM}} = \frac{1}{2} \sum_k C_k^\dagger \mathcal{H}_{\text{CM}}(k) C_k$  with

$$\mathcal{H}_{\text{CM}}(k) = t \sin k \sigma_3 \tau_0 + (w - g \cos k) \sigma_1 \tau_3 - \Delta \sigma_2 \tau_2. \quad (6)$$

The  $\sigma$  are the spin Pauli matrices and  $\tau$  are Pauli matrices in particle-hole space. The products of two Pauli matrices from different spaces is understood as a tensor product.

By diagonalizing the BdG Hamiltonian  $H_{\text{CM}}(k)$ , it follows that there are four energy bands  $\pm E_{\pm}$  satisfying

$$E_{\pm} = \sqrt{(t \sin k)^2 + (w - g \cos k \pm \Delta)^2}. \quad (7)$$

There are four possible gap closings in the system at  $k = 0$ , for  $\Delta = \pm(w - g)$ , and at  $k = \pi$ , for  $\Delta = \pm(w + g)$ . As it will be shown, all these lines mark topological transitions between different gapped topological phases.

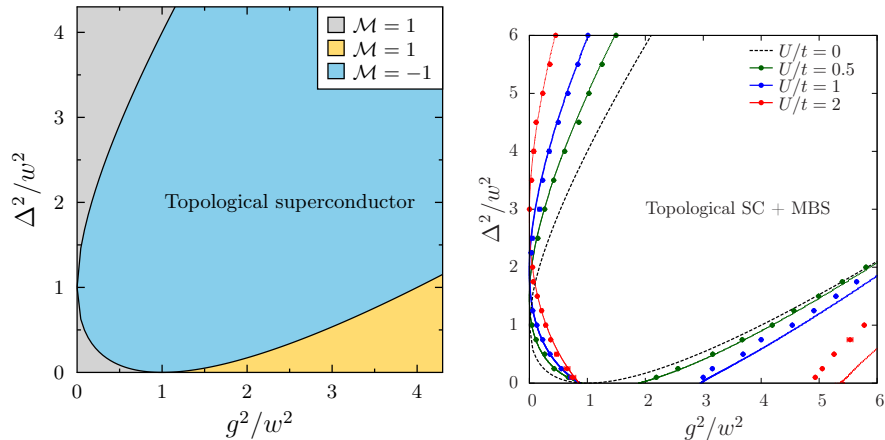
Following Ref. [17], the Majorana number is defined at the time-reversal-invariant momenta,  $k = 0$  and  $\pi$

$$\mathcal{M} = \text{sgn}\{\text{Pf}[\tau_1 \mathcal{H}_{\text{CM}}(0)] \text{Pf}[\tau_1 \mathcal{H}_{\text{CM}}(\pi)]\} = \text{sgn}\left[\left(1 + \frac{g^2}{w^2} - \frac{\Delta^2}{w^2}\right)^2 - 4 \frac{g^2}{w^2}\right], \quad (8)$$

where Pf denotes the Pfaffian of a matrix. The ensuing phase diagram is illustrated in Fig. 1. Both the phase without in-gap states (gray) and the CBS hosting phase (yellow) are trivial, with respect to the Majorana number ( $\mathcal{M} = 1$ ). In fact, these two phases are actually distinct with regards to a different topological invariant. When  $|g/w| = 1$ , an infinitesimal  $\Delta$  is sufficient to open a superconducting gap at the Dirac cone, and produces a topologically nontrivial superconductor. However, when the Creutz model is deep in a nontrivial insulating state with solitons at its end, the addition of superconducting pairing does not destroy these modes. Since the CBS are protected by the bulk gap and a chiral symmetry, they survive the induced superconducting pairing.

## 3. From chiral bound states to Majorana fermions

Here, we further analyze the transition between the trivial superconductor with CBS and the topological superconductor with MBS (respectively the yellow and blue regions in Fig. 1). To this aim, we consider the Creutz model on an infinite line, and an additional spatial twist is introduced in the mass of the Dirac fermion  $v(x) = (g - w) \text{sgn}(x) = v \text{sgn}(x)$ , where  $v$  is a



**Figure 1.** (Color online) Left panel: Phase diagram of the non interacting Creutz-Majorana model. The blue region represents the topological superconductor with zero-energy Majorana bound states ( $\mathcal{M} = -1$ ). The gray and yellow regions represent phases with  $\mathcal{M} = 1$ . The yellow region hosts chiral bound states at zero energy, even if it is trivial with respect to Majorana number  $\mathcal{M}$ . Right panel: Topological phase diagram of the Creutz-Majorana-Hubbard model at different interaction strengths. The dotted line represents transitions between topological phases obtained at the mean-field level, while the dots represent critical points from the DMRG calculations.

positive number, and  $x$  is the spatial coordinate. According to Eq. (4), the trivial phase is realized to the left ( $x < 0$ ), and the nontrivial phase, to the right ( $x > 0$ ). At the interface there will be a single solitonic mode (CBS), eigenstate of the chiral symmetry operator  $\sigma_2$ .

Now, let us investigate the more interesting case of the Creutz-Majorana model, when a superconducting pairing  $\Delta$  is added to the nontrivial insulating phase in order to study the effect of the superconducting pairing on the bound state from the topological insulator. The continuum Hamiltonian which models the interface between the trivial Creutz insulator and the CM model is readily obtained by linearizing the lattice Hamiltonian near  $k = 0$  and neglecting second-order contributions in momentum

$$\mathcal{H} = v_F \sigma_3 \tau_0 p - v(x) \sigma_1 \tau_3 - \theta(x) \Delta \sigma_2 \tau_2, \quad (9)$$

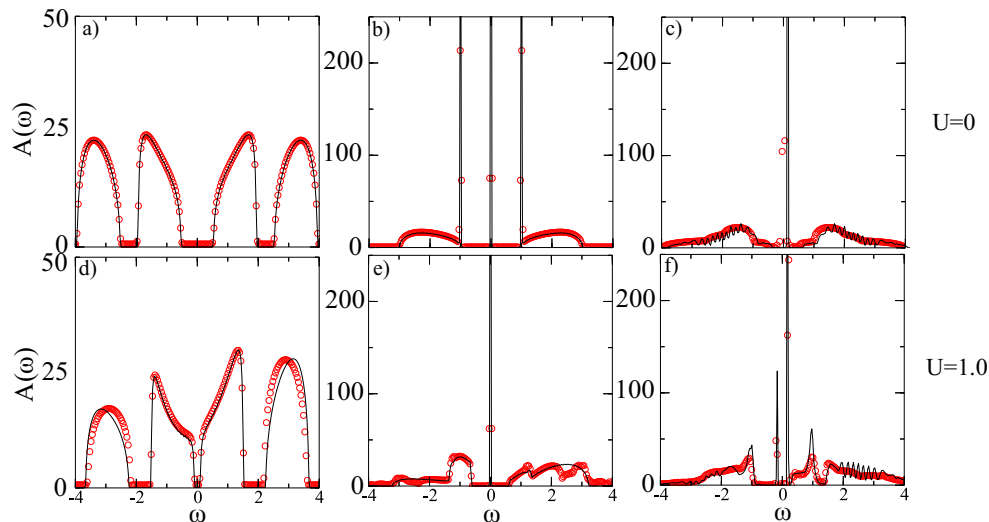
with  $v(x) = v \operatorname{sgn}(x)$  and  $v = g - w > 0$ . The function  $\theta(x)$  is the Heaviside step function. The Fermi velocity  $v_F$  is determined from the lattice model  $v_F = ta/\hbar$ , with  $a$  the lattice constant.

It is possible to solve analytically this problem and obtain in particular the wave function at zero-energy. Details are provided in Ref. [18]. In conclusion, we have described the transition between the trivial superconducting phase with doubly-degenerate zero-energy states protected by the chiral symmetry to a phase with Majorana fermions.

#### 4. Interaction effects

The Creutz-Majorana model on a finite-size chain hosts either zero-energy solitonic (CBS) states, or Majorana modes, depending on the strength of the superconducting pairing  $\Delta$ . The aim of this section is to investigate the effects of repulsive onsite interactions on these edge modes and to obtain the topological phase diagram of the model. To that end, we consider the following Hamiltonian of the Creutz-Majorana-Hubbard (CMH) model:

$$H = H_{\text{CM}} + U \sum_j n_{j\uparrow} n_{j\downarrow}. \quad (10)$$



**Figure 2.** (Color online) Local spectral function  $A_1(\omega)$  at the edge of an open wire for  $U = 0$  (left column) and  $U = 1$  (right column). (a), (d) The trivial phase at  $g^2 = 0.5$  and  $\Delta^2 = 5$  has no quasiparticle peak at zero energy. (b), (e) A Majorana edge fermion in the topological phase at  $g^2 = \Delta^2 = 1$  is seen as a sharp peak at zero energy, which survives finite interactions. (c), (f) The superconducting phase at  $g^2 = 5$  and  $\Delta^2 = 0.5$ . (e) The presence of zero-energy states is reflected in the spectral function peak at zero energy. (f) The CBS are not robust to interactions  $U$ , which removes them from zero energy. Each subfigure combines TEBD (red open points) with mean-field results (lines), scaled by an overall factor.

The first term,  $H_{\text{CM}}$ , represents the Hamiltonian of the Creutz-Majorana model from Eq. (5), and the second term contains the Hubbard interaction between onsite electronic densities of opposite spin, with  $U$  being the interaction strength.

This CMH model has been studied using a combination of self-consistent Hartree-Fock theory and extensive DMRG simulations. Here we present the results while technical details (mean field procedure and DMRG) are provided in Ref. [18].

#### 4.1. Phase diagram

Figure 1 shows the topological phase diagram for different values of interaction  $U$ , combining results from the mean-field analysis and DMRG simulations. The agreement between both approaches is overall very good. The main standout feature of the phase diagram is that the topological phase and its associated Majorana bound states at zero energy are robust to interactions. In fact, the parameter range for which a topological Majorana phase is stabilized globally expands, upon increasing the Hubbard coupling. However, there is also a region in the phase diagram ( $|\Delta/w|, |g/w| < 1$ ), where the Hubbard interaction is detrimental to the Majoranas. The mean-field phase diagram captures the topological transitions of the model rather accurately for small and moderate interactions. At strong interactions, the mean field keeps a very good estimate of the topological transition at small  $g/w$ , while at large  $g/w$  it tends to overestimate the extension of the Majorana phase. The “spin-orbit” coupling  $g$  tends to delocalize the electrons and leads to quantum fluctuations in the particle number. This explains, at a qualitative level, the divergence of the mean field results from the “exact” DMRG results at large “spin-orbit” coupling. *A priori*, this overall very good agreement may seem surprising in a quantum 1D system, where fluctuations are expected to be pronounced. However, let us recall that  $U(1)$  charge and the  $SU(2)$  spin-rotation symmetries are broken in this model, and hence

the system acquires a certain stiffness to fluctuations, as observed in nanowire models [16] which also break these symmetries.

#### 4.2. Spectral properties

We now turn our attention to the edge physics, and evaluate the *local* spectral function  $A_1(\omega) = A(i=1, \omega)$  for open chains of size  $L = 140a$  (Fig. 2) using both mean-field results and DMRG approaches. In the trivial phase, the interactions simply renormalize the energy gap [cf. Figs. 2(a) and 2(b)]. In the topological phase, a clear signature of the Majorana zero-energy mode is shown for both  $U = 0$  and 1 in Figs. 2(c) and 2(d). Without interactions, this mode is well separated by a symmetric gap of width  $\omega = 1$  from the remainder of the excitations. The overall structure survives the addition of interactions, reflecting once again the robustness of the Majorana states. The edge CBS at large  $g$  are also clearly evidenced by the presence of a zero-energy peak in the noninteracting regime [Fig. 2(e)]. In order to perform this simulation, the ground-state MPS is biased to one of the four degenerate states by applying a small pinning potential to an edge of the chain. Turning interactions to  $U = 1$  splits this peak into two peaks at nonzero energies, showing that the CBS have moved away from zero energy, as described in the preceding sections.

## 5. Conclusions

Using a specific model, we have shown that Majorana states are robust to Hubbard interactions. Moreover the Hubbard interaction can extend the parameter range where topological superconductivity and Majorana modes are stabilized.

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