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# On the Tradeoff between Resolution and Ambiguities for Non-Uniform Linear Arrays

François Vincent, Olivier Besson, Souleymane Abakar-Issakha, Laurent Ferro-Famil and Frantz Bodereau

**Abstract**—The performance of most direction of arrival (DoA) estimation algorithms is driven by two different kinds of errors: small, local errors in the main lobe and possibly large errors due to sidelobe peaks ambiguities. Reducing these two kinds of errors simultaneously is not possible and therefore a compromise has to be made. Motivated by automotive applications with small arrays, we consider solving this tradeoff by modifying the positions of the sensors of the array. In this letter, two new criteria are proposed to solve the above mentioned tradeoff. Optimal solutions are derived and illustrations are provided with a 3-element non-uniform linear array.

## I. INTRODUCTION

DoA estimation is a crucial issue for many radar applications. This is especially the case for automotive systems designed to early detect and avoid pedestrians, cyclists or other cars. In the most advanced automotive applications, arrays with a small number of elements are used, typically 3 or 4. Moreover, these systems usually use a large bandwidth, resulting in a very high range resolution and hence the number of targets belonging to the same range-Doppler bin is typically one. Therefore, the main issue here is to have a good DoA estimation accuracy in order to precisely estimate the targets trajectory and decide whether there is a collision risk or not. DoA estimation errors can be classified in two types: small errors in the main lobe of the array and possibly large errors due to sidelobe peaks or ambiguities. Both of them are governed by the array geometry.

Most often, uniform linear arrays (ULA) are employed, which consists of equidistant sensors, usually spaced a half-wavelength apart in order to avoid wavefront under sampling. In order to reduce the front-end system cost and hardware complexity for a given resolution, non-uniform linear arrays (NULA) have been investigated in many domains, see [1] and references therein. Most of the time, the so-called minimum redundancy linear array (MRLA) geometry is considered when dealing with NULA because this kind of array allows to measure all correlation lags as the same size ULA, yet with a minimum of sensors. However, the MRLA geometry does not guarantee any optimality with respect to resolution or sidelobe level.

In [2] Chambers *et al.* showed that the Cramér-Rao Bound (CRB) is inversely proportional to the spatial variance of the sensors. This dependence is also confirmed in [3] where the positions of the sensors are optimized with respect to

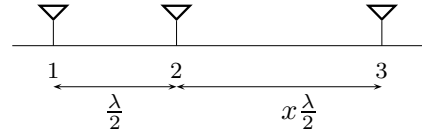


Fig. 1. Three-sensors non uniform linear array

array resolution using genetic algorithms. Briefly speaking, the larger the spatial variance of the sensors of the linear array, the better the resolution. In this letter, we focus on the simplest case of a three-element NULA where the two first elements are placed a half-wavelength apart to prevent DoA aliasing, as illustrated on figure 1. Spatial resolution is improved when the distance  $x\frac{\lambda}{2}$  between the last two sensors increases, i.e. the spatial variance maximized, while keeping the array unambiguous. Obviously as  $x$  increases, resolution improves but the increase of sidelobes level could result in large DoA estimation errors. In this paper, we try to define the best antenna size  $((x+1)\frac{\lambda}{2})$  that satisfies the compromise between high resolution and low sidelobes level. Towards this end, two different criteria are proposed and optimized with respect to  $x$ . Observe that such a compromise depends on signal to noise ratio (SNR). Indeed, at high SNR, it is unlikely that a sidelobe peak be mistaken for a true DoA and therefore focus will be on optimizing the main lobe width. In contrast, with low SNR and adaptive beamforming or DoA estimation techniques using a finite number of snapshots, high sidelobe levels might be expected, and optimization should be carried out with this potential problem in mind. In practice though, it is impossible to adapt in real time the antenna geometry to the source SNR. Moreover, SNR is a-priori unknown in many array antenna applications. Accordingly, we propose to find the value of  $x$  that maximizes the directivity of the array or, equivalently, that minimizes the integral of the beam pattern over all DoA. Indeed, this integral is proportional to the main lobe width and inversely proportional to the sidelobes levels. Additionally, we also consider a variance-type criterion of the array beampattern and compare it to the first criterion.

This paper is organized as follows. We first introduce the framework at hand. Then, we present the criterion to be minimized in order to satisfy the resolution/sidelobe levels compromise and solve it with respect to  $x$ . We also introduce another possible optimization criterion which leads to a size close to the standard ULA. To assess the validity of the analysis, we compare the RMSE for different antenna sizes and for a classical DoA estimation algorithm, namely MUSIC.

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## II. DATA MODEL AND ANTENNA SIZE OPTIMIZATION

As stated in the introduction, we consider the three-sensors array represented in figure 1. Considering spatial frequencies, the steering vector model is as follows :

$$\mathbf{a}(f) = [1 \quad e^{i\pi f} \quad e^{i\pi f(1+x)}]^T \quad (1)$$

where  $\cdot^T$  stands for the vector transpose operator. The beam pattern for a given spatial frequency  $f_0$  is defined as  $g_{f_0}(f) = |\mathbf{a}(f_0)^H \mathbf{a}(f)|^2$ , where  $\cdot^H$  stands for transpose and conjugate.

We do not consider here any specific DoA estimation algorithm and our analysis is based on the beam pattern characteristics. In fact, evaluating the maximum of the latter corresponds to maximum likelihood (ML) estimation under the single source hypothesis and, as stated in the introduction, this will be the case in the majority of the situations encounter with an automotive radar. Furthermore,  $g_{f_0}(f)$  corresponds to the square modulus of the cross-correlation between any steering vector and the steering vector of interest. Hence, its shape will impact the performance of any DoA estimation method: ambiguities in case of high sidelobes and a good precision in case of sharp main lobe.

As displayed in figure 2, it is well known that increasing  $x$  results in a main lobe width reduction but a sidelobes level increase, two effects that have opposite consequences on the final DoA estimation performance.

The first criterion that comes in mind to manage the compromise is to maximize the directivity. Indeed this criterion aims to minimize the main lobe width while maintaining low sidelobes. Notice that this technique has been widely used to derive weights that minimize the sidelobe levels in case of NULA, see [4], [5], [6], [7] for example. The directivity is defined as [1]

$$D = \left( \frac{1}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} g_{f_0}(f) df \right)^{-1}. \quad (2)$$

Hence maximizing the directivity consists in minimizing the area under the beam pattern diagram. If we consider  $f_0 = 0$  as the source position, we are simply looking for  $x_D$  that minimizes

$$C_1(x) = \int_{-\frac{1}{2}}^{\frac{1}{2}} |\mathbf{a}(0)^H \mathbf{a}(f)|^2 df = \mathbf{a}(0)^H \mathbf{\Gamma}_1 \mathbf{a}(0) \quad (3)$$

with

$$\mathbf{\Gamma}_1 = \begin{bmatrix} 1 & \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-i\pi f} df & \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-i\pi f(1+x)} df \\ \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{i\pi f} df & 1 & \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-i\pi f x} df \\ \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{i\pi f(1+x)} df & \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{i\pi f x} df & 1 \end{bmatrix} \\ = \begin{bmatrix} 1 & 0 & \frac{\sin(\pi(1+x))}{\pi(1+x)} \\ 0 & 1 & \frac{\sin(\pi x)}{\pi x} \\ \frac{\sin(\pi(1+x))}{\pi(1+x)} & \frac{\sin(\pi x)}{\pi x} & 1 \end{bmatrix} \quad (4)$$

so that

$$x_D = \arg \min_x \left[ \frac{\sin(\pi(1+x))}{\pi(1+x)} + \frac{\sin(\pi x)}{\pi x} \right]. \quad (5)$$

This function is plotted on figure 3 together with the directiv-

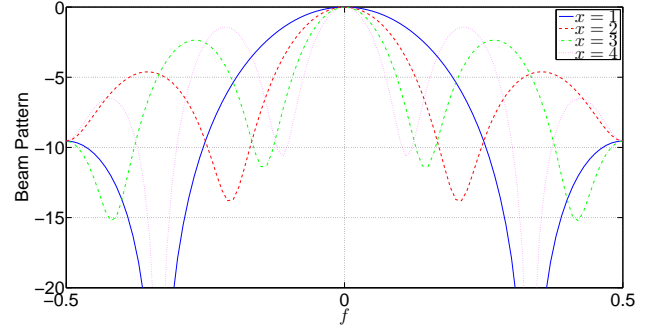


Fig. 2. Examples of beam pattern for  $x = 1, x = 2, x = 3, x = 4$

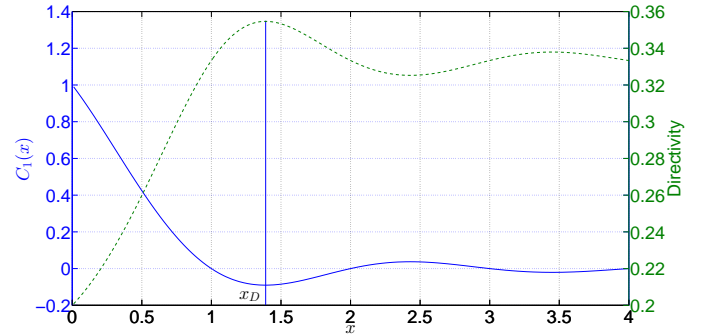


Fig. 3.  $C_1(x)$  and the corresponding directivity vs.  $x$

ity. The global minimum is obtained for  $x_D \simeq 1.39$  so that the global NULA antenna size is  $1.195\lambda$  (to be compared with the  $\lambda$  width of the three-sensors ULA equivalent array).

The second approach consists in minimizing the following variance-like criterion:

$$C_2(x) = \int_{-\frac{1}{2}}^{\frac{1}{2}} |\mathbf{a}(0)^H \mathbf{a}(f)|^2 f^2 df = \mathbf{a}(0)^H \mathbf{\Gamma}_2 \mathbf{a}(0) \quad (6)$$

Actually, the distance between a high sidelobe and the main lobe has a direct influence on the final RMSE. Indeed, the farther the sidelobe, the larger the spatial frequency error. The above criterion can be viewed as the variance of the error  $(f - f_0)$  considering a distribution for  $f$  proportional to the beampattern. In this case, we have

$$\mathbf{\Gamma}_2 = \begin{bmatrix} \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 df & \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 e^{-i\pi f} df & h(1+x) \\ \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 e^{i\pi f} df & \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 df & h(x) \\ h(1+x)^* & h(x)^* & \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 df \end{bmatrix} \quad (7)$$

and

$$h(x) = \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 e^{-i\pi f x} df \\ = \frac{1}{4} \frac{\sin(\pi x)}{\pi x} + \frac{1}{2\pi^2 x^2} \left( \cos(\pi x) - \frac{\sin(\pi x)}{\pi x} \right)$$

so that minimizing (6) is equivalent to

$$\min_x [h(x) + h(1+x)] \quad (8)$$

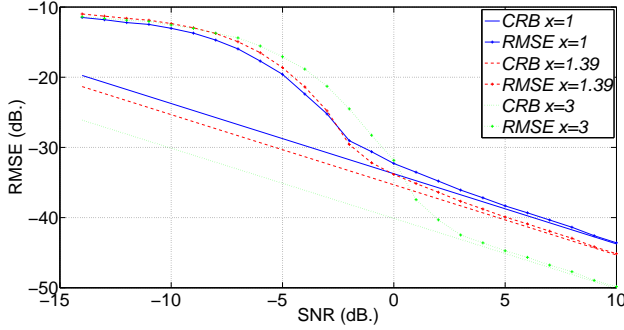


Fig. 4. Music and Capon performance analysis for 4 antenna geometries

whose global minimum is obtained for  $x_D \simeq 0.96$ , a value very close to the standard ULA size composed of sensors placed a half-wavelength apart. The standard ULA configuration is close to optimal when considering this variance criterion, and hence, in the sequel, we compare the ULA configuration with the one obtained by maximizing directivity.

### III. NUMERICAL ILLUSTRATIONS

In this section, we compare the performance of a very popular DoA estimation algorithm, namely spectral MUSIC, for 4 different antenna geometries. We consider the case of a single source impinging on the array with spatial frequency  $f_0 = 0$ , so that the snapshot received at time  $t = 0, \dots, (N-1)$  can be written as

$$\mathbf{x}_t = \mathbf{a}(f_0)s_t + \mathbf{n}_t \quad (9)$$

where  $\mathbf{n}_t$  is assumed to be a white Gaussian noise. The sample covariance matrix (SCM) ( $\hat{\mathbf{R}} = \frac{1}{N} \sum_{t=0}^{N-1} \mathbf{x}_t \mathbf{x}_t^H$ ) is calculated from  $N = 15$  snapshots. Its eigenvalue decomposition is as follows:

$$\hat{\mathbf{R}} = \lambda_1 \mathbf{u}_s \mathbf{u}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^H. \quad (10)$$

MUSIC estimates the DoA as

$$\hat{f}_{MUSIC} = \arg \max_f \left[ \frac{1}{\mathbf{a}(f)^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{a}(f)} \right]. \quad (11)$$

We only consider here MUSIC algorithm as the majority of the other procedures result approximately in the same performance for this single source case. Figure 4 represents the corresponding RMSE as well as the Cramér-Rao Bound (CRB) for each array configuration. As expected, the larger  $x$ , i.e., the larger the array aperture, the smaller the CRB. Also, as it is well known, MUSIC is efficient at high SNR where its RMSE is equal to the CRB. At very high SNR, it is thus preferable to increase the antenna size in order to favor the main lobe width reduction against the sidelobes level. However, the SNR required for MUSIC to achieve the CRB is also increased. In fact, in most applications where a moderate SNR is encountered, the main objective is to decrease the SNR threshold at which most methods depart from the CRB. With this respect, the solution which optimizes the antenna directivity ( $x_D = 1.39$ ) offers a very good compromise as it allows a 2dB gain in the asymptotic zone compared with the non-ambiguous ULA and departs from the CRB approximately

at the same SNR. Hence, the RMSE will always be better with this antenna size excepted in the no-information zone, where unfortunately none of the solutions is valid.

### IV. CONCLUSIONS

In this letter, we proposed an analysis of the geometry of a 3-sensors NULA, the two first sensors position being fixed so as to fulfill the non-aliasing constraint. This kind of antenna is extremely interesting for automotive collision avoidance radars or for any low-cost applications. We propose to maximize the directivity of such an antenna to achieve the compromise between precision (thin mainlobe) and ambiguities (low side-lobes). This size optimization led to a distance between the last 2 sensors of  $1.39 \frac{\lambda}{2}$ . Numerical simulations show that this configuration could be a good choice compared to the standard ULA.

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