



HAL
open science

Squaring the Circle

Philippe Woloszyn

► **To cite this version:**

Philippe Woloszyn. Squaring the Circle: Diffusion Volume and Acoustic Behaviour of a Fractal Structure. Miroslav M. Novak. Paradigms of Complexity. Fractals and Structures in the Sciences, pp.299, 2000, 981-02-4292-1. hal-01552932

HAL Id: hal-01552932

<https://hal.science/hal-01552932>

Submitted on 3 Jul 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/235178384>

Squaring the Circle: Diffusion Volume and Acoustic Behaviour of a Fractal Structure

Article · November 2012

CITATIONS

2

READS

21

1 author:



[Philippe Woloszyn](#)

French National Centre for Scientific Research

99 PUBLICATIONS 209 CITATIONS

SEE PROFILE

Some of the authors of this publication are also working on these related projects:



HAUP HyperAmbiotopes Urbains Participatifs [View project](#)

All content following this page was uploaded by [Philippe Woloszyn](#) on 03 July 2017.

The user has requested enhancement of the downloaded file. All in-text references [underlined in blue](#) are added to the original document and are linked to publications on ResearchGate, letting you access and read them immediately.

UNCLASSIFIED

Defense Technical Information Center
Compilation Part Notice

ADP010924

TITLE: Squaring the Circle: Diffuison Volume and
Acoustic Behaviour of a Fractal Structure

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: Paradigms of Complexity. Fractals and
Structures in the Sciences

To order the complete compilation report, use: ADA392358

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, ect. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:

ADP010895 thru ADP010929

UNCLASSIFIED

SQUARING THE CIRCLE : DIFFUSION VOLUME AND ACOUSTIC BEHAVIOUR OF A FRACTAL STRUCTURE

PHILIPPE WOLOSZYN

*CERMA Laboratory, UMR CNRS 1563, Ecole d'Architecture de Nantes, rue Massenet,
F-44300 Nantes, France.*

E-mail : woloszyn@cerma.archi.fr

The main topic presented in this paper concerns the reduction of the diffusion volume to convex isotropic Euclidean structures as polygons and convex polyhedrons, in order to estimate the diffusive acoustic behavior of a fractal structure.

The volume V and the surface S of an Euclidean structure are defined with both quadratic and cubic relationships to the perimeter P with $S = F_q P^2$, and $V = F_c P^3$. Their geometrical nature depends on the values of the *prefactors* in those relationships, the *shape factors* F_q and F_c [1]. Extension of this relationship to the structure's generalized volume V_x [2] gives us the equation :

$$V_x = F_x P_x^D. \quad (1)$$

Calculation of the corresponding shape factors respectively gives $1/4\pi$ for a circular and $3/32\pi$ for a spherical structure. In order to generalize this expression to the whole platonician 2-D family, we operate a polygonal circle quadrature with the convexes from square to n-sided polygons, $n \rightarrow \infty$.

The quadrature factor Q represents the ratio between the circumscribed and inscribed circles of the structural polygons of the analyzed objects. They are varying from $Q = 2$ for the square (minimal quadrature) to $Q = 1$ for the circle (total quadrature). The general expression is given by :

$$Q = \frac{S_{cir}}{S_{ins}} = \frac{\frac{R_{cir}^2 \sin \frac{2\pi}{n}}{2}}{r_{ins}^2 \tan \frac{\pi}{n}} \quad (2)$$

Taking into account D , fractal dimension and d , Euclidean dimension of the structure, the F_x factor expression becomes :

$$F_x = \frac{d}{2^{dQ} (d-1) 2\pi^{|2-Q|} \frac{d^{Q-1}}{Q^{D-1}}} \quad (3)$$

Eq. (3) gives the F_x value of the square ($F_x = 1/16$), the cube ($F_x = 1/64$), the circle ($F_x = 4\pi$) and the sphere ($F_x = 3/32\pi$). It gives also the intermediate values of the regular polygonals which square the circle, noticing that for dodecagon, the prefactor F_x reaches the circle constant value $F_x = 1/4\pi$. This confirms the quadrature convergence.

By replacing F_x with its expression in equation (3), one can write the perimeter-surface relationship (P/S_x) for irregular 2-D shapes and to closed 2-D fractals, with its quadrature factor Q and its Euclidean dimension d as follows :

$$S_x = \frac{d}{2^{dQ} (d-1) 2\pi^{12-Q} \frac{d^{Q-1}}{Q^{D-1}}} P^D \quad (4)$$

We are using the Minkowski analysis method to scrutinize the structure with perimeter P and real diffusion section S_x . The structuring element of radius Λ we use, gives the fractal dimension of the diffusion volume depending on perimeter P , true diffusion section S_x and the structure's fractal dimension D as follows [3] :

$$D = 1 - \frac{S_x}{P\Lambda} = 1 - \frac{\nu S_x}{P} \quad (5)$$

This leads to the definition domain of the *diffusion coefficient* δ which evolves [4]:

$$\delta = \frac{1}{2\nu\tau} = \frac{S_x(1-D)}{2P\Delta t} \quad (6)$$

where Δt represents the diffusion time-share in the structure. This leads to the diffusion time expression, depending on the fundamental length of acoustic diffusion process λ_{\max} , and the roughness parameter n (number of indentations per length unit) expressed as :

$$\tau = \lambda_{\max}^{-D} f^{d-1} = \left(F_x(1-D) \frac{P^{d(1+D-d)-1}}{n\Lambda^2} \right)^{-D} f^{d-1} \quad (7)$$

by replacing diffusion time τ with its expression in (7), the diffusion coefficient becomes :

$$\delta = \frac{1}{2\nu} \left(F_x(1-D) \frac{P^{d(1+D-d)-1}}{n\Lambda^2} \right)^D \frac{1}{f^{d-1}} \quad (8)$$

As we can see equation (8), this coefficient is defined with only the structural morphologic diffusion parameters and the sound frequency f .

That result leads us to confirm that the diffusion coefficient is a geometrical-dependant parameter, confirming the rule of the structure complexity on acoustics.

Thus, after a multiscale renormalisation, the diffusion coefficient has to be spectrally defined by adjusting a simulation procedure to experimental results, in order to implement this diffusive acoustic model for any complex configuration : those perspectives constitutes the next step of our research works.

References

1. Mandelbrot B. : " Les objets fractals ", Paris, Flammarion, 1992
2. Pfeiffer P., Obert M. & Cole M.W. : " Fractal BET and FHH theories of adsorption: a comparative study ", Proc. R. Soc. Lond. A423, pp. 169-188, 1989
3. Woloszyn P. : " Mesures multiéchelles du tissu urbain et paramétrage d'un modèle de diffusion acoustique en milieu construit ", *Symposium Saint-Venant : " Analyse multiéchelle et systèmes physiques couplés "*, Marne-la-Vallée, août 1997
4. Chandrasekhar : " Stochastic Problems in Physics and Astronomy ", *Review of Modern Physics*, 15 (1) : 1-89, 1943