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► **To cite this version:**

Luc Trouche. New technological environments: new constraints, new opportunities for the teacher. International Journal of Computer Algebra in Mathematics Education, 2000, 7 (3), pp.167-180. hal-01552317

HAL Id: hal-01552317

<https://hal.science/hal-01552317>

Submitted on 2 Jul 2017

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New technological environments:

new constraints, new opportunities for the teacher

by Luc Trouche

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French experiments carried out over the last three years on mathematics teaching in complex calculator environments -- in particular with TI-92 -- allow to draw some lessons:

- the introduction of this kind of calculators does not simplify the teacher's work, nor the pupils' ;*
- it requires a new teaching organization of learning space and a new management style of time.*

We shall argue that, under these conditions, work in complex calculator environments can lead to a transformation of students' relationships to mathematics (giving priority to the creative side of work). Moreover, one can observe a transformation of students' collective relationships to knowledge (giving greater importance to social aspects).

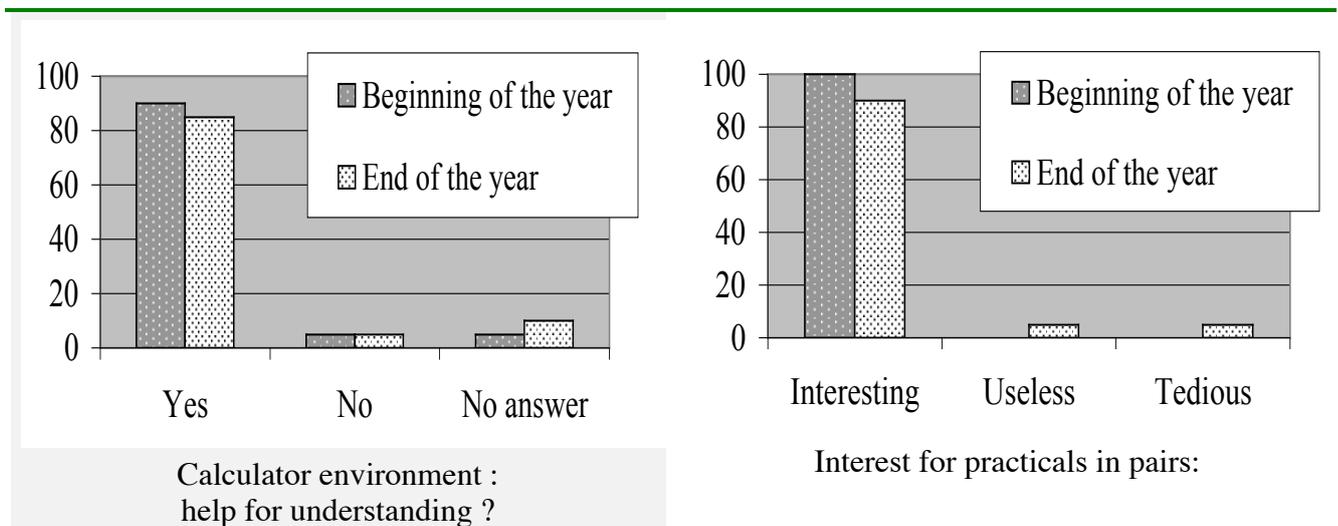
1. About a common idea

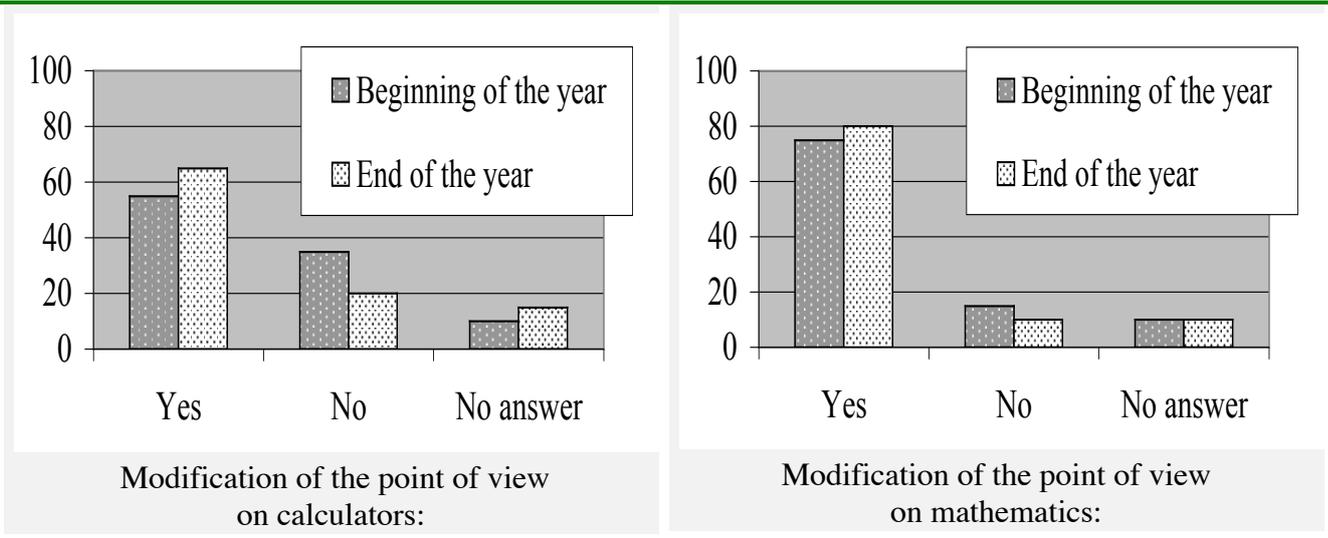
New technological environments make teaching and learning easier or more interesting : it's a belief appearing in some papers about experiments with computers or calculators. Shoaf (1997) describes students's behaviour in such environments as « a process in which the student is silently conversing with himself through the calculator, asking questions of himself as he manipulates the concrete screen image. This leads him

to have the knowledge to conjecture not only what is actually occurring with the image, but what it is happening. By using the graphic calculator students are more likely to construct their own mathematical understanding through conscious reflection ».

Such point of view can also appear from inquiries among students. I teach mathematics for five years in the last level of a french highschool (students 18 years old) in an experimental environment : each student has a TI-92 which is given by the school (from september to june). He can use it both at school and at home, in normal courses as in practicals (Trouche, 1997). We can see below these students' answers at four questions :

- does this calculator environment help you for understanding mathematics ?
- what do you think about doing practicals in pairs ?
- does this kind of environment modify your point of view on calculators ?
- does this kind of environment modify your point of view on mathematics ?



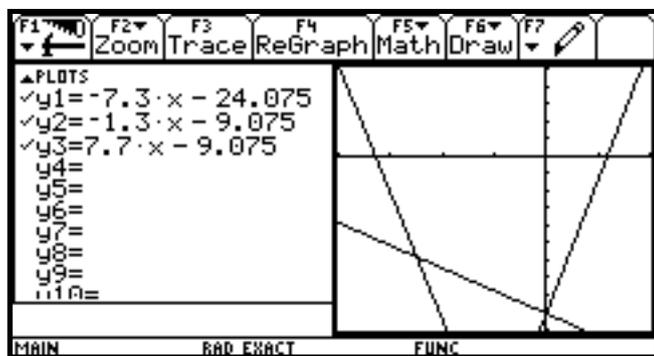


The students' answers, very positives, do'n't allow to know what they have actually learnt in these environment. Knowing it needs to study what they precisely **do** with a calculator and what they precisely take off their own activity.

2. Seeing is reality (Trouche & Guin, 1996)

The first lesson of our experiment is about he perception of mathematical objects throughout calculators. As Noss and Hoyles (1996) wrote, « these tools wrap up some of the mathematical ontology of the environment and form part of the web of ideas and actions embedded in it ».

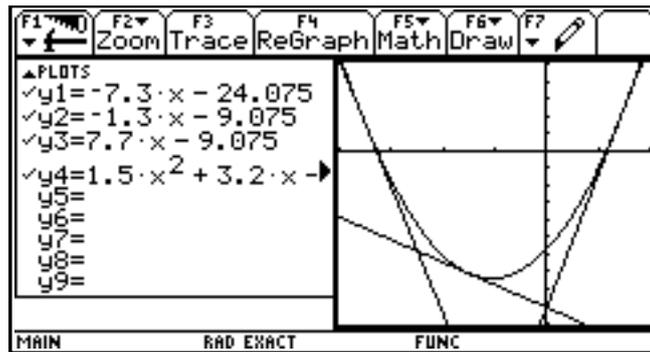
An example : the students had to find the equation of a quadratic curve tangential to three given lines.



They try various coefficients and

find an approximate solution. To check this solution, they made successive zooms.

The teacher asked : « What is your definition of a line tangential ? ».



One student answered : « A line is all the more tangential to a curve that it shares the higher number of points with it ». This isn't a definition possible in a paper and pencil environment... But in a calculator environment, it's a definition created by the zooming action.

Frequent students' use of calculators bears on the conceptualization process :it can lead to a confusion between mathematical objects and their representations by the tool.

3. Particular environment, particular action.

The second lesson of our experiment is the influence of the environment on the students' mathematical actions. This kind of influence exists also for the teacher, or the mathematician himself...

3. a. Auto-observation.

The following question comes from a personal problem, arrived during the ICTM4 (Plymouth, August 1999). My bag was lost by the airline company and arrives at Plymouth four days after me (quite at the end of the conference). I imagined that an airline company had a lot of ways to bring a bag from Montpellier to Plymouth :

- a direct way (that means carrying my bag in the same airplane than me, from Montpellier to Plymouth) ;
- an indirect way (that was the way choosed by the airline compagny : Montpellier-Paris-London-Philadelphia USA-London-Plymouth...) ;
- a random way. That is the way I would like to study in an TI-92 environment, and in the same time, I would like to look at me solving this kind of problem, in order to analyse the influence of this kind of environment.

What is for me a random way ? Look at the figure 1 below.

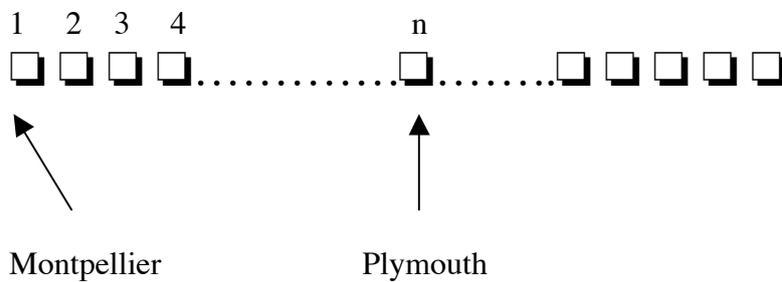


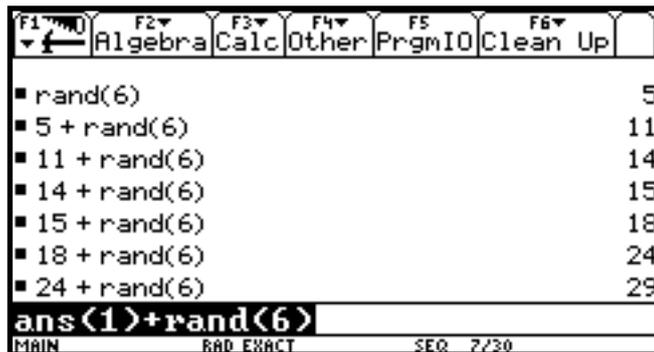
Figure 1

Montpellier is at the square 0. Plymouth at the square n. Each time I throw a dice. my bag can progress (between one and six squares obviously). I ask (to me) three questions :

- what is, in such conditions, the probability to reach Plymouth ?
- if I call $P(n)$ the probability to reach the square n, what is the limit of $P(n)$ when n tends to infinity ?
- has this problem something to do with technology (why and how) ?

Let's try to answer to the first question.

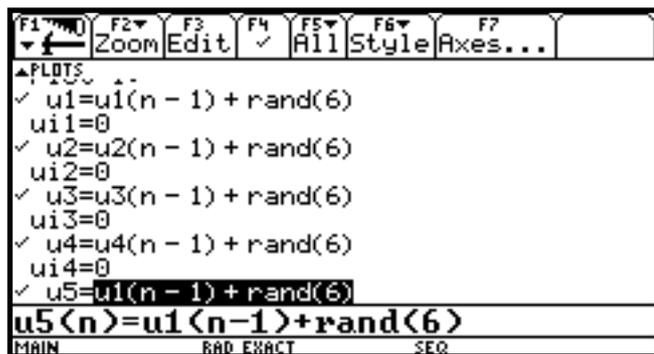
We could modelize this walk with the command « random » of a TI-92 (see first screen below); we could so realize a simulation of a lot of walks and have a statistic



answer for each value of n. It's a « tentation » in this kind of environment to use immediatly the powerfull of the calculator and see what happens. I call this first movement a **movement to calculator concretisation** : to try, to calculate something is easy with a calculator, so we do it, even if it is not the most usefull...

It is also possible to use sequences syntax : we can have quicker some « random walks » (see screens below).

Little problem : although these sequences are (from their definition) growing, the table of values shows some irregular phenomenas... It happens often in a calculator environment : an amazing event arises, and attracts our attention. It is perhaps an interesting thing, but it is far from our problem...



n	u1	u2	u3	u4	u5
0.	0.	0.	0.	0.	0.
1.	3.	4.	1.	2.	5.
2.	4.	10.	7.	4.	8.
3.	7.	11.	11.	11.	9.
4.	14.	13.	16.	20.	13.
5.	21.	20.	14.	22.	19.
6.	19.	14.	15.	17.	18.
7.	29.	23.	28.	24.	31.

n=0.

I call this a **movement of distraction**.

I suppose that I don't follow this movement. So I leave my calculator and I calculate the first values of P(n). It is quite obvious than $P(0) = 1$ (my bag start surely from Montpellier...). The only possibility to reach the square 1 is obtaining 1 when throwing

the dice : $P_1 = \frac{1}{6}$. To reach the square 2, two possibilities : obtaining 2 with a dice or obtaining twice 1 (when throwing two dices). Then $P_2 = \frac{1}{6} + \frac{1}{36}$. To reach the square 3, one can obtain 3 (with only one dice), or 1-2, or 2-1 (with two dices), or 1-1-1 (with three dices) ; then $P_3 = \frac{1}{6} + \frac{2}{6^2} + \frac{1}{6^3}$;

It becomes more and more complicated. Here takes place, in a calculator environment, a second movement, **a movement of abstraction** : it is necessary to give a mathematical form to the problem if one wants to be helped by the calculator. So I'll try to detect a regularity in the first results, to find a general formula for $P(n)$.

It's easy to observe at the denominators the successive powers of 6 (because one throws one dice, then two dices, etc...) ; it's necessary to think a little more to remarque at the numerators the « Pascal triangle » : 1, puis 1-1, puis 1-2-1...

Then I have two possibilities :

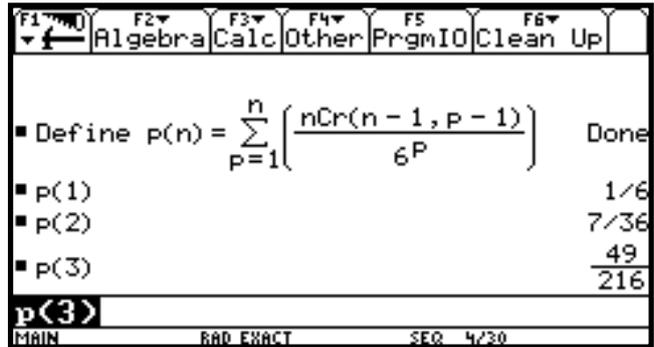
- if I follow the movement of calculator concretisation, I'll try immediatly to enter the new formula in my calculator « to see what happens » ;
- if I follow the movement of abstraction, I'll try to understand the reasons of such a formula. It needs to think again : to reach the square n with p throwing of a dice, I need to share n in p non empty pieces. It means I have to choose (p-1) « partitions » among the (n-1) spaces separating the n squares. In fact, I do obtain C_{n-1}^{p-1} possibilities.

So my temporary result is :for $n > 0$: $P_n = \sum_{p=1}^n \frac{C_{n-1}^{p-1}}{6^p}$.

Now I'll see what happens on my calculator :

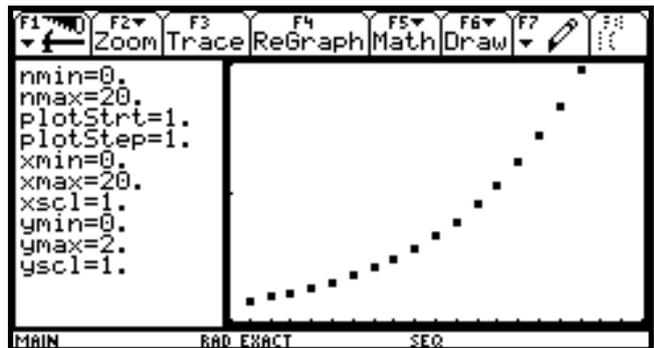
The first results are well convenient : we obtain the results already known.

Let's see further : the sequence graph (see below) is a little surprising. The probabilities overtake 1...



So our formula is perhaps true for the first values of n, then it's false. Why ?

For a numerical reason first. We can transform our result by using the Newton binomial formula :



$$P_n = \sum_{p=1}^n \frac{C_{n-1}^{p-1}}{6^p} = \frac{1}{6} \sum_{p=1}^n C_{n-1}^{p-1} \frac{1}{6^{p-1}} = \frac{1}{6} \sum_{k=0}^{n-1} C_{n-1}^k \frac{1}{6^k} = \frac{1}{6} \left(1 + \frac{1}{6}\right)^{n-1}$$

of our result is now obvious...

For a theoretical reason also ! It's always useful to think about the validity domain of a result. Our formula for P_n suppose that we can reach the square n with 1 dice, or 2 dice... or n dice. From the square 7, our formula is false, because 7 cannot be obtained

with a dice alone ! Then $P_7 = \sum_{p=2}^7 \frac{C_6^{p-1}}{6^p}$. For the same reason, the square 13 can be

reached with at less three dices. One can see here how the two movements of abstraction and concretisation are linked, and how it's difficult to come out the calculator and engage a theoretical reflexion.

I have now two possibilities :

- looking for another strategy ;
- trying to modify my first formula.

For some « economical thinking reasons », I'll choose the second possibility. I have a good formula for the first values, I spent a lot of time to write this formula for my

calculator, I have understood the origine of my mistake, so I can hope doing a good « reparation » of it.

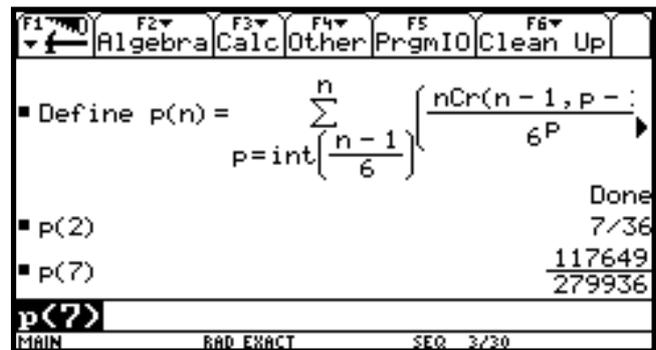
My sum has to be made from the integer part of $(n-1)/6$. I obtain a new formula :

$$P_n = \sum_{p=\text{E}\left(\frac{n-1}{6}\right)}^n \frac{C_{n-1}^{p-1}}{6^p}$$

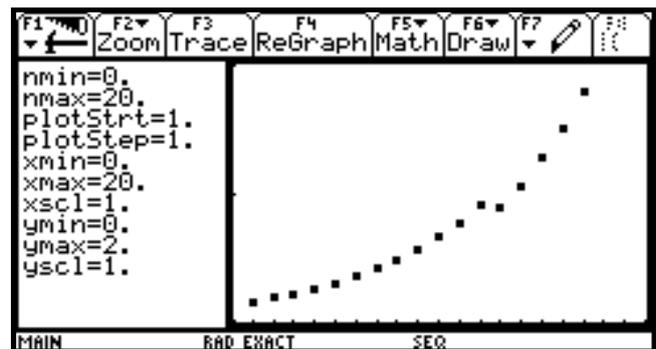
I try straight away this new formula.

It's OK for $n=2$, $n=7$.

Let's try further. Alas (see below) ! The probabilities become also bigger than 1...



Thinking again : for $n=9$ (for example), if one shares the walk in two parts, one can obtain a part of 1 (possible with a dice) and a part of 8 (impossible with a dice !).

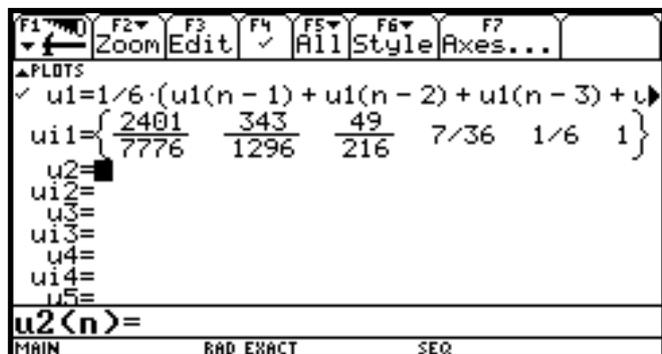


Our new formula is true till 7, false further. Again two possibilities, looking for a new strategy or trying to treat our poor formula...

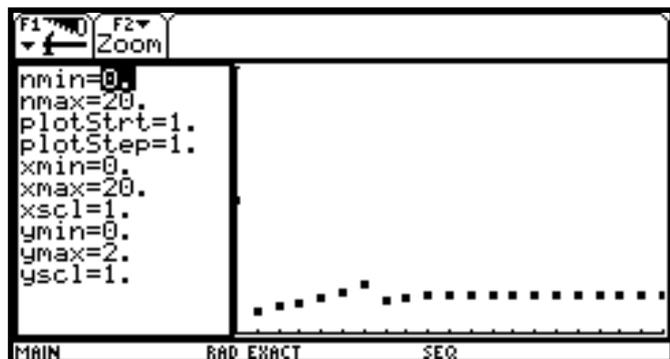
Our choice : changing our point of view. It's very important for a scientific thinking (cf. Robert et Tenaud 1989). Many authors have showed how difficult it was to change a representation register, even to change, inside a same representation register, a problem form (cf. Duval, 1995). This **movement of diversification** is not natural in any mathematical environment. It's even more difficult in a calculator environment : one can often observe in such environment a **movement of fixation** on a same application (graphical for example) and then on the same form of a given problem.

So, about our problem. Let us try to come back on our walk : because we know the first results, could we find a « recurrence » (I need help from reviewers, I don't know the english word !) formula ? We suppose known the results till square (n-1), let us calculate P_n . If we are on square n, where were we just before ? Of course, on square n-1, or on square n-2, or on square n-3, or on square n-4, or on square n-5, or at last on square n-6. Each of these possibilities has the same probability (if our dice is not fixed...). Then $P_n = \frac{1}{6}(P_{n-1} + P_{n-2} + P_{n-3} + P_{n-4} + P_{n-5} + P_{n-6})$.

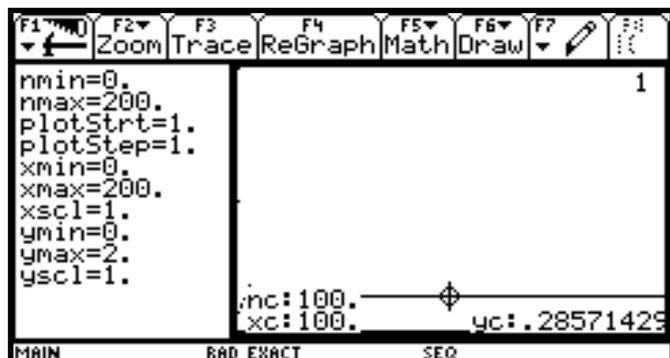
Let's have a glance on such a defined sequence (I use the TI-92 sequences editor, which provides approximate values). The graphic representation, with always the same window, seems to show a convergent phenomena. It seems quite « normal », because of the formula barycentric form.



The opposite screen shows a « stabilisation » of the sequence at about 0,28571429. We are at the end



of the movement to calculator concretisation. We can see the sequence, know the differentes values of $P(n)$, have a quite precise idea of the sequence limit.



To go further, it's necessary to follow both a movement of abstraction and diversification :

- our sequence is a classical linear sequence, with classical ways of studies (study of the characteristic polynomial, research of its roots, etc...). It would be too long to do this here, but it would need both theoretical calculation and calculator use ;
- the apparent sequence limit, about 0,28571429, can also arouse some thinking. If it was a rational number, it could be $\frac{2}{7}$. Why such a limit ? It's quite easy to understand (after asking to himself the question !). With a dice, we can obtain 1, 2, 3, 4, 5 or 6, with an equal probability. So the arithmetical mean of a step is $\frac{1+2+3+4+5+6}{6} = \frac{7}{2}$. It means that, at the limit, we can reach two squares among seven squares. This is our limit for P(n) !

End of my bag's random walk and of a mathematician thinking about it...

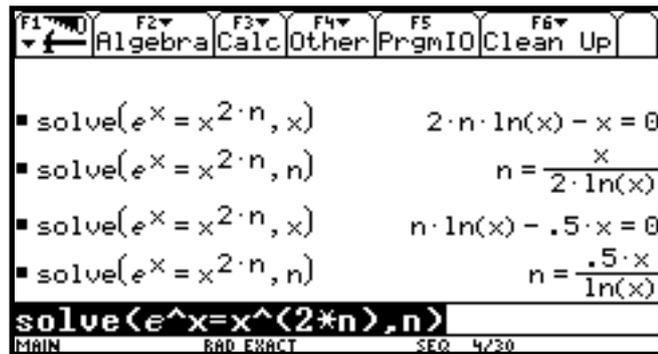
One can see, through this example, both the virtualities of a symbolic calculator and the difficulties to stand back the first results of the screen. The movements of distraction (to follow each new calculator suggestion), of concretisation (to see at once something) and of fixation on a single application (by trying and trying again, by zooming and zooming again) are very strong in a calculator environment. Following a movements of abstraction or diversification requires a conscious effort of a mathematician. Seeing and thinking is absolutely not the same thing. And what is true for a teacher is all the more true for a learner !

3.b. A second example, about students' behaviour

Let's look at various students work about a particular problem solving. The students have to find, according to the different values of n, the number of solutions of the equation : $e^x = x^{2n}$.

One can see a « weak » student's realization :

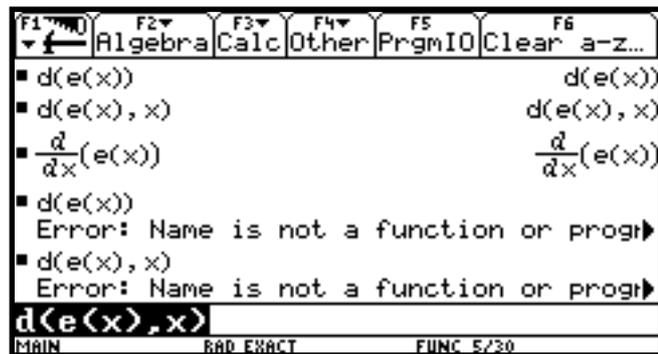
- he tries to give the problem to the calculator, before thinking about what is the unknown ?



He tries x, then n, without any success. At last, he repeats the same question, in the approximate mode. No more success... We recognize here a movement of **calculator concretisation** (to see what happens with the calculator) and of **fixation** in the same application, but in a quasi **random way** (it's quite similar as throwing a dice...): trying to translate the question for the calculator on each possible (or imaginative) form. Artigue (1995) calls this kind of behaviour a « fishing behaviour ».

After this first attempt, a second one, after changing the problem form.

This student remember it's possible to solve a problem by studying some functions variations. He tries to derivate the exponential function with the calculator.



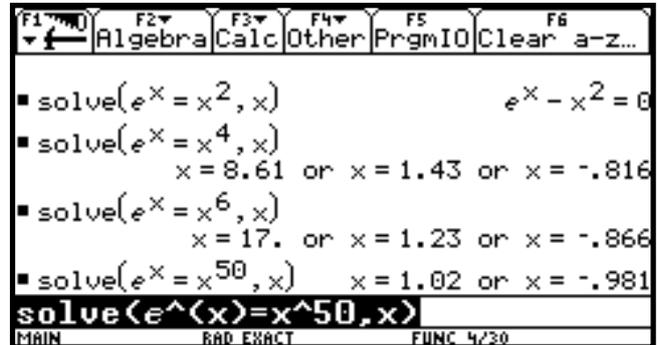
(I ask to him : « you don't know this derivative ? », he answers : « yes, but I prefer to check it ! »). Alas, he uses a false syntax, and obtains some answers he can't understand (false syntax for the derivative, false syntax for the exponential function- he uses the letter e). Again the same movements of **calculator concretisation** and of **fixation** one a same application, but in an **approximative form**.

On the contrary, a « good » student of the same class follows first a **movement of abstraction**. With a small drawing (cf. figure 2 below), he sets the differents elements of the problem. He reminds the theoritical result : the exponential function increases

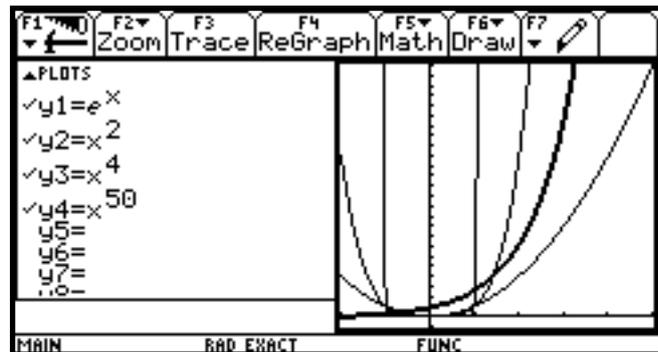
faster than each power function. Then he supposes the existence of three solutions (we can observe the deformation of the curve in order to obtain the third solution).

Figure 2

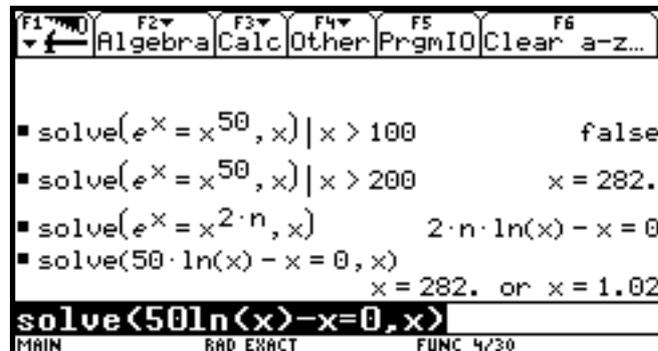
After (and only after this theoretical thought, he tries some particular resolutions in the TI-92 numerical application and in the graphic one.



He knows there is a third solution. Then (see the third screen right) he precises $x > 100$: no solution. He looks for the lost solution further : $x > 200$: he finds this solution : **thus he doesn't know because he sees, but he sees because he knows.**



He says : «I cannot seek each third solution for each value of n at random, I have to find an easier way...». A possible explanation of the TI-92 difficulties is the magnitude of numbers like e^x . So he changes the equation form by using logarithms.



We recognize a **movement of diversification**, under a **theoretical control**.

Next stage of his work, to win some time, he chooses the sequences editor, writes three sequences according to the three previous solutions (one can notice a **great syntaxe precision**).

```

F1  F2  F3  F4  F5  F6  F7
Zoom Edit All Style Axes...
▲PLOTS
✓ u1=nSolve(e^x = x^2·n, x) | x < 0
  u1=
✓ u2=nSolve(e^x = x^2·n, x) | x > 0 and x < 2
  u2=
✓ u3=nSolve(x = 2·n·ln(x), x) | x > 2
  u3=
  u4=
  u4=
  u3=
MAIN          RAD EXACT          SEQ
  
```

It's a manifestation of the **articulation** of the movement of abstraction and the movement of calculator concretisation.

n	u1	u2	u3
0.	-1.E-38	1.E-38	undef
1.	-.703467	undef	undef
2.	-.815553	1.42961	8.61317
3.	-.86565	1.22689	16.9989
4.	-.894241	1.15537	26.0935
5.	-.912765	1.11833	35.7715
6.	-.925754	1.0956	45.9238
7.	-.935371	1.08021	56.4726
n=0.			

MAIN RAD EXACT SEQ

He ends this exercise by interpretation of the first results (for n=0 and n=1).

The differences between these two students' kind of work are very significant. It shows a real danger : a complex calculator environment may increase disparities among pupils' behaviour, favouring the better students, hampering the weaker ones.

4. Some theoretical elements

In order to understand what kind of knowledge can be learnt in these computer environment, it is usefull to give due consideration to cognitive ergonomy. As langage and thought are related, Vygotsky (1962) points out the fundamental relationship between gestures and thought. Verillon and Rabardel's (1995) studies focusing on learning processes in the area of cognitive ergonomy are based on this idea. If cognition evolves through interaction with the environment, accomodating to artefacts may have an effect on cognitive development, knowledge construction and processing, and the

nature itself of the knowledge generated. They suggest models to analyse the instrumented activity of students confronted with tasks involving artefacts :

- an instrument does not exist in itself, an artefact (like a calculator) becomes an instrument when the subject has been able to appropriate it for himself and has integrated it with his activity ;
- this appropriation is not only an individual process ; as in each human activity, it's a social process, in wich both the teacher 's responsibility and the peers'one are engaged.

This model gives us some guiding ideas to control the calculator appropriation process :

- the teacher is the designer of activities where mathematics is the central focus. Dreyfus (1993) stresses the importance of the choice and the way activities are promoted by the teacher for making an effective learning tool. Situations have to be carefully designed in order to take advantage of the constraints and discrepancies caused by calculators, which may be considered as new learning potential ;
- the choice of the applications involved in the activity and their articulation aims to improve investigation by enhancing varying points of view (to favour **the movement of diversification**) ;
- one must develop situations aiming to foster experimental work with interaction between graphic observations and theoritical calculation, in order to tackle the distorsion between the paper and machine environment (to favour the **articulation** of the calculator concretisation movement and of the abstraction one) ;
- it is important to buid classroom devices to allow the control by the teacher of the pupils'instrumented activity (in order to limit the movement of distraction, the random research way, to correct the approximate syntax) ;

- it's at last important to build classroom collectives work devices, to favour social learning process : students' various behaviours can compensate each to each (for example it can favour a diversification of the action, a combination of calculator concretisation and abstraction).

So new environments require a new teaching organization of learning space and a new management style of time.

5. Some ideas from our experimental work with a TI-92 highschool class.

We shall present here only two elements of this new organization (Guin & Trouche, 1999).

1. A new organization of the ordinary learning space

As the figure 3 shows it, the teacher, throughout his courses, combines both blackboard and screen to enhance interactions between paper/pencil and screen/keyboard classwork ; the data from one student's calculator are overhead projected on the class screen : it gives a particular go-between role to him (and it allows the control by the teacher and by the other students of this student's instrumented activity).

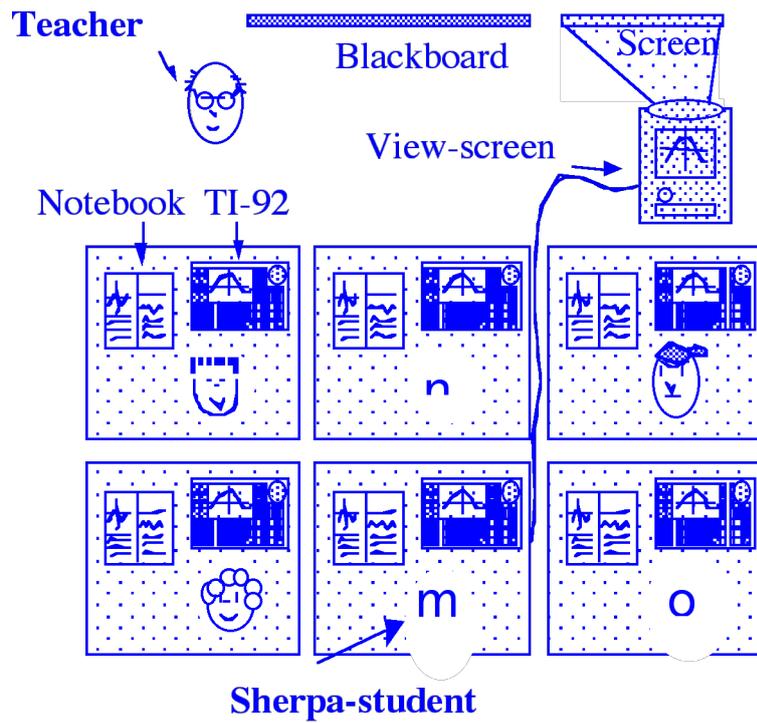


Figure 3

2. A new management style of time

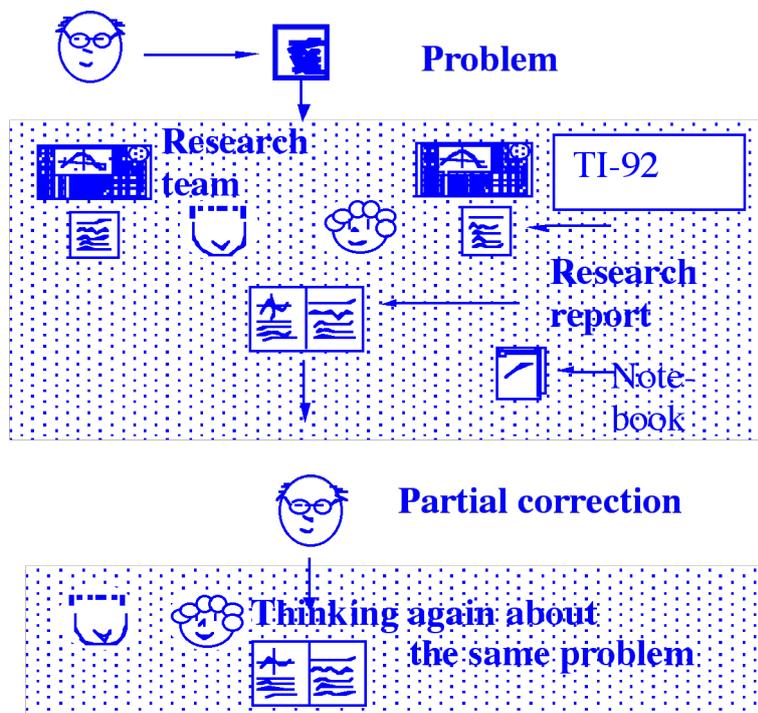


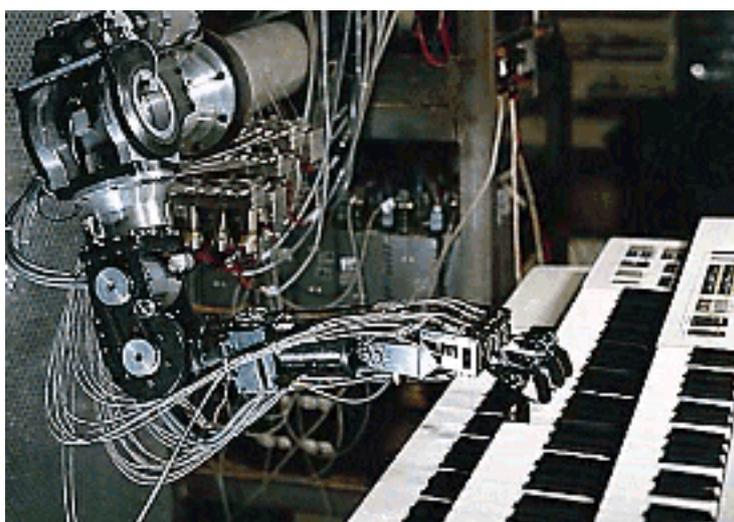
Figure 4

Precise research moments – practicals- are weekly organized ; pupils working in pairs have to solve « open » problems (see figure 4 above).

- The problem terms are choosed to enhance interaction between calculator and paper work (for example the research of the number of solutions of the equation $e^x = x^{2n}$) and to enhance a research process : the question has to be actualy problematic ;
- The pairs are choosed to constitute complementary research teams ;
- The students have to note successive stages of their work (their results as their dead-end findings) into their research notebooks ;
- The teacher lets time necessary to try various methods ; he gives partial correction to each pair after the first session in order to boost thinking again. Time in all the more necessary in such complex environments ;
- Great emphasis is laid upon the synthesis moments, both to take stock of the practicals' results and highlight the diversity of problem-solving ways discovered by students pairs ;
- This last stage ends with a period of presentation of a new mathematic notion by the teacher : clearness of the notion definition is the indispensable « counterpoint » to the diversity of points of view which have led to its introduction.

Thus the introduction of technology does not simplify by itself the teacher's work, nor the pupils. Controlling complex calculator environment needs in fact to build complex teaching and learning environment. We argue than, **under these conditions**, work in this kind of environment can lead to a transformation of student's relationship to mathematics (giving priority to the creative side of work). Moreover, one can observe a transformation of students' collective relationships to knowledge, giving greater

importance to social aspects. This transformation of the relationship with knowledge is a condition to give to the students the desire to learn, and to learn together, to understand the world, to distinguish what is true, what is false. It will be our temporary conclusion : a symbolic calculator doesn't create automatically new behaviour for mathematical research or study ! Calculator environment needs to be built by the teacher.



References

- Artigue M. (1995). Une approche didactique de l'intégration des E.I.A.O.. In Guin, Nicaud and Py (Eds). *Environnements Interactifs d'Apprentissage avec Ordinateurs* (pp. 17-19). Paris : Eyrolles.
- Dreyfus T. (1993). Didactic design of computer-based learning environments. In C. Keitel and K. Ruthven (Eds), *Learning from Computers : Mathematics Education and Technology*, Vol 121, Nato, Serie F (pp. 159-186). Springer-Verlag.
- Duval, R. (1995). *Semiosis et pensée humaine*. Peter Lang (Paris).
- Guin, D. and Trouche, L. (1999). The Complex Process of Converting Tools into Mathematical Instruments. The Case of Calculators. *International Journal of Computers for Mathematical Learning*, **3(3)**.

Noss, R. and Hoyles C. (1996). *Windows on Mathematical Meanings* (pp. 153-166). Dordrecht : Kluwer Academic Publishers.

Rabardel, P. and Verillon P. (1995). Cognition and artifacts : A contribution to the study of thought in relation to instrumented activity, *European Journal of Psychology in Education* **9(3)**.

Robert, A. and Tenaud, I. (1989). Une expérience d'enseignement de la géométrie en Terminale C, *Recherche en didactique des mathématiques*, **9(1)**.

Shoaf, M., (1997). Using the total power of the TI-92 ! From discovery explorations to compel lab reports, *International Journal of Computers for Mathematical Learning*, **4(3)**.

Trouche L. and Guin D. (1996). Seeing is reality : how graphic calculators may influence the conceptualisation of limits. *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education* (pp. 323-333), Valencia **4**.

Trouche, L. (1997). A propos de l'apprentissage des limites de fonctions dans un environnement calculatrice, étude des rapports entre processus d'instrumentation et processus de conceptualisation. *Ph. Doctoral Thesis* (Université de Montpellier II)

Vigotsky, L.S. (1962). *Thought and Language*. Cambridge, MA : MIT Press.

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