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## A Quadratic Model and A Heuristic for Sizing an Hybrid Renewable Energy System.

Serigne Gueye · Rachid Belfkira ·

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December 29, 2009

**Abstract** An Hybrid Renewable Energy System (HRES) may be defined as a system in which various renewable energy components (solar panels, wind turbines,...,batteries) are interconnected in such a way to satisfy, at any time, a demand of electrical energy. Since electrical power supplied by each component (taking independently) depends on different environment conditions (sun, wind), and since the demand fluctuates, the objective of such system is to be able to produce energy at any time by optimally exploiting favourable weather

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conditions for each component, and to stock available energy (for latter use in non favourable conditions) of surproduction periods. The induced problem is to compute the optimal number of each component, minimizing installation and maintenance costs. We propose an integer quadratic programming model to solve this problem. The model is linearized and solved with a heuristic scheme. Numerical results, based on data obtained on a site in Dakar (Senegal) in which the system has to be installed, are provided.

**Keywords** 0-1 Quadratic Programming, Heuristic, Linearization, Renewable Energy system.

## 1 Introduction

Renewable energy is energy generated from natural resources such as sunlight, wind, rain, tides, which are naturally replenished. The environmental problems, high oil prices and increasing government support, are driving increasing renewable-energy commercialization. The industry is currently growing more and more. From the end of 2004 to the end of 2008, solar photovoltaic (PV) capacity increased sixfold to more than 16 gigawatts (GW), wind power capacity increased 250 percent to 121 GW, and total power capacity from new renewables increased 75 percent to 280 GW. The renewable energy technologies are non-polluting but in the same time suffer to at least two "critics".

It is well known that the electrical power generation from the renewable energy technology is site-dependent and intermittent. The power generated depends

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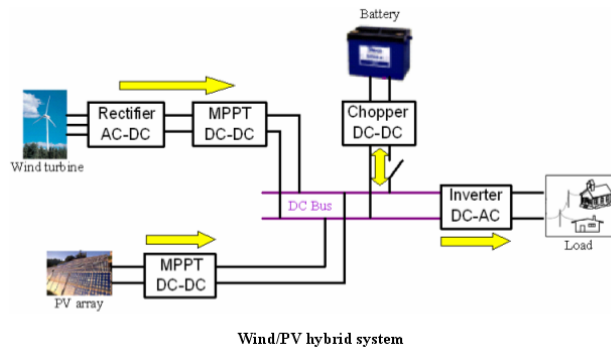
on the environment conditions, which fluctuate randomly and impact considerably the power generation. However, some of them have complementary profiles. This is specially the case for solar and wind energy. For instance, in summer significant solar radiations may be expected whereas in winter wind speed is higher. In a geographical point of view, the locations of a country (or a region) may also have complementary potential in the sense that some locations may be more suitable for a kind of renewable energy than some others. For example, the north-west european coast benefit to high wind exposition while in the south solar radiations are higher. Hence, one may imagine systems exploiting as best as possible weather conditions of any seasons, or potential of regions (or locations), in such a way to deliver at any time required electrical power. A part of this kind of system limited to individual uses (not at a scale of regional or country production) have been imagined by GREAH Researchers (Groupe de Recherche en Electronique et Automatique du Havre).

The expensive installation and maintenance costs of renewable energy systems, in comparison to other energy sources (gas, oil,...), is also a critical point. Wind and solar power costs are currently higher than gas or oil energy sources but the trend is going down with economies of scale and better materials. Thus, when implementing any renewable energy technology, one has to analyze carefully the problem of optimizing induced costs.

The hybrid system proposed by Belfkira et al [1] may be summarized as in the

figure 1. Very schematically, it may be viewed as a network where the sources are the wind turbines and solar panels and the destination is the customers. The energy flow fulfilled by the sources is summed up on a so called "bus". When the customer demands are lower to the energy supplied the surplus are stored in batteries. In the case of bad weather conditions, the demand may exceed the offer of the sources, hence the batteries are used if possible. Many additional power equipments (rectifier, chopper,...) are necessary to implement in practice this system. The explanations of such equipments is out of the scope of this paper. Shortly, these equipments are used to convert the electrical power from a given form to another (i.e Alternative Current (AC) to Direct Current (DC) and vice versa).

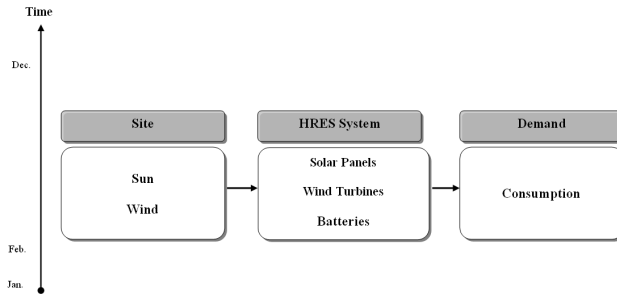
Each material of this system is expensive in terms of installation and maintenance costs. Thus, we seek to find the optimal numbers of solar panels, wind turbines and batteries needed to satisfy the demand of electrical energy at any time of the year. These optimal numbers correspond to the minimal acquisition, installation and maintenance costs of the system. The demand of electrical energy and the amount of energy that may be produced by the different sources are in practice difficult to know accurately since it depends on customer consumption profiles, meteorologic conditions during a year, etc. To evaluate this demand two approaches may be used : stochastic and deterministic. In the stochastic approach the demand and power of the renewable energy sources over the time are modeled as stochastic variables, then a stochastic



**Fig. 1** HRES

programming model has to be solved. In the deterministic case, an observation period of length  $T$  is considered, for instance 6 significant months of the year. This period is discretized (i.e  $t = 0, \dots, T$ ). Then at any time  $t$ , the demand is evaluated by a constant giving the average consumption. Observations are also done in this period on solar radiations, temperature, wind speed. These data are then used to evaluate at each time  $t$  the electrical power generated by solar panels and wind turbines of different types. This paper is concerned with the deterministic approach.

In section 2, we introduce data notations. In section 3, we detail the quadratic model proposed to find the optimal size of the renewable energy system taking into account deterministic informations. The model has been applied to size a renewable energy system in a site located at the Cheikh Anta Diop University at Dakar (Senegal). The observation period considered in this site is 6 months with a discretization of 1 hour. This small step of discretization induced a very large scale mathematical program, impossible to solve optimally in our



**Fig. 2** Supply Chain

computing environment. A heuristic is then proposed in section 4. The numerical results are provided in section 5. Finally conclusion and perspectives are drawn in section 6.

## 2 Renewable Energy Supply Chain

Providing electrical energy corresponding to customer demands may be represented as a supply chain starting from the renewable energy sources and ending at the final consumers as shown in figure 2. At any time of the year (from January to December) and in any weather conditions we want to implement an Hybrid Renewable Energy System (HRES for short) giving the required quantity of energy. The sizing decision problem is located precisely in the link "HRES System" of this logistic chain. For this decision, informations about the links at the left (Site) and at the right (Demand) of the HRES system is necessary at any time. These informations are obtained by observations in a given period of the year. A period of size  $T$  (6 to 12 months) is chosen and discretized, i.e  $t = 0, 1, 2, \dots, T$ . At any time  $t$ , some measures

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are performed related to site and demand properties : solar radiations, temperature and wind speed. The informations about the customers consist on an evaluation of the demand at time  $t$ . All of these data are supposed to be fixed scalar. With these data and additional informations about the physical properties of the renewable energy components, it is possible to compute at any time  $t$  the power fulfilled by each component and the customer demand.

We notice  $d_t$  the demand at time  $t$ . Let us consider that different types  $i = 1, \dots, I$  (resp.  $j = 1, \dots, J$ ) of solar panels (resp. wind turbines) exist. The (known) electrical power provided by a solar panel (resp. wind turbine of type  $j$ ) of type  $i$  at time  $t$  is noticed  $P_{it}$  (resp.  $W_{jt}$ ).  $P_{it}$  and  $W_{jt}$  values are given by equations depending on material and site properties.

Given the demand  $d_t$  and powers  $P_{it}$  and  $W_{jt}$ , our purpose is to size the HRES System in such a way to minimize the installation and maintenance costs with respect to the demand and the batteries life cycle. The next section deals with the associated optimization model.

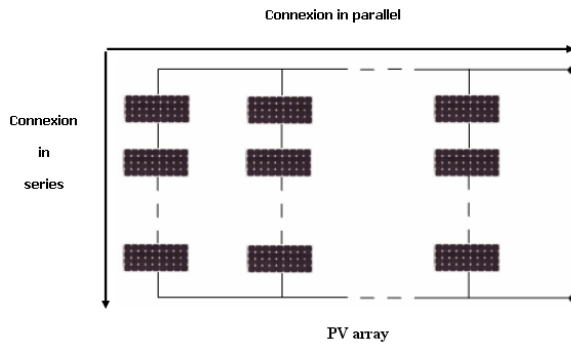
### 3 Model

We subdivide the model presentation in subsections corresponding to the different parts of any mathematical program : Decision variables, Objective Function, Constraints.



### 3.1 Decision Variables

Decisions that we have to take deal with the optimal numbers of solar panels, wind turbines and batteries. In order to understand the meaning of the variables, we briefly show below how the renewable energy components of the system are organized. The solar panels, of type  $i$ , are interconnected in series and in parallel in a matrix structure called "PV array" where PV stands for photovoltaic (see figure 3 below). Vertical connexions are called "strings".



**Fig. 3** PV array

Strings are connected in parallel, providing a PV Array. Let us notice  $v_i^s$  (resp.  $v_i^p$ ) the number of solar panels connected in series (resp. parallel).  $v_i^s$  do not depends, in practice, on site or demand properties but only on system properties. Using electrical engineering theory,  $v_i^s$  may be computed accurately. This is a constant. At the opposed, the suitable value of  $v_i^p$  depends on the environment and on the demand. Hence,  $v_i^p$  is a decision variable. Let us remark that each type corresponds to a PV array. Since the number ( $I$ ) of solar types is fixed, the number of PV array is upper bounded by  $I$ . Our decision is thus

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to find how many strings to install for each PV array type.  $v_i^p = 0$  means that the PV array  $i$  will not be implemented.

Batteries are organized as solar panels. Similar picture as above may be also used for battery array by replacing the solar panels by batteries. We notice  $y_k^s$  (resp.  $y_k^p$ ) the number of batteries of type  $k$  connected in series (resp. in parallel). As for solar panels,  $y_k^s$  is known and  $y_k^p$  is a decision variable. The HRES system is supposed to use battery panels when the renewable energy quantity is insufficient, and to store electrical energy in batteries when the offer exceeds the demand. The set of all batteries of any type is called *the battery bank*, and  $Y_t$  stands for the total amount of electrical power available in this bank. Since this bank may be used to store or to keep energy, the values of  $Y_t$  should reflect these two alternatives. By convention, a positive value of  $Y_t$  means that the bank is in charging process (energy storage) while a negative value stands for discharging process.

A wind turbine is composed of two parts : a tower and the turbine composed of rotor shaft, and electrical generator at the top of the tower. The number of wind turbines of type  $j$  to install will be noticed  $t_j$ .

The variables  $v_i^p$ ,  $t_j$ ,  $y_k^p$  are integers while  $Y_t$  is continuous. These integer variables will be bounded respectively by maximal number of solar strings of type  $i$  in parallel ( $\overline{v_i^p}$ ), maximal number of wind turbines of type  $j$  ( $\overline{t_j}$ ) and

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maximal number of battery strings of type  $k$  in parallel ( $\overline{y_k^p}$ ).

$$\text{i.e. } 0 \leq v_i^p \leq \overline{v_i^p} \quad i = 1, \dots, I$$

$$0 \leq y_k^p \leq \overline{y_k^p} \quad k = 1, \dots, K$$

$$0 \leq t_j \leq \overline{t_j} \quad j = 1, \dots, J$$

### 3.2 Objective Function

For each component (solar panels, batteries and wind turbines), three costs have been considered : the acquisition cost (noticed 1), the installation cost (noticed 2) and the annual maintenance cost (noticed 3). We notice  $c_i^1$  (resp.  $c_i^2, c_i^3$ ) the acquisition (resp. installation, maintenance) cost of a solar panel of type  $i$ . We seek to optimize maintenance cost over a period of 20 years generally admitted as the usual lifespan of solar panels and batteries. Hence, the total cost of a solar panel of type  $i$  is

$$c_i = c_i^1 + c_i^2 + 20c_i^3, \quad i = 1, 2, \dots, I.$$

Let  $b_k^1$  (resp.  $b_k^2, b_k^3$ ) be the acquisition (resp. installation, maintenance) cost of one battery of type  $k$ . At the opposed of solar panels, over the period of 20 years, a battery will be changed  $r_k$  times while solar panels are only maintained.  $r_k$  is fixed. The total cost of a battery of type  $k$  is then :

$$b_k = b_k^1 + b_k^2 + (20 - r_k)b_k^3 + r_k(b_k^1 + b_k^2), \quad k = 1, 2, \dots, K.$$

For explanation, battery is bought ( $b_k^1$ ) and installed ( $b_k^2$ ). When a battery is

changed no maintenance cost have to be considered but acquisition and installation ( $b_k^1 + b_k^2$ ). Thus the number of time that maintenance is performed corresponds to  $20 - r_k$  and the corresponding maintenance cost is  $(20 - r_k)b_k^3$ .

Some costs are associated to each part of a wind turbine. We notice  $w_j^1$ ,  $w_j^2$ ,  $w_j^3$  the acquisition, installation and maintenance costs of a turbine of type  $j$ , and  $h_j^1$ ,  $h_j^2$ ,  $h_j^3$  the acquisition, installation and maintenance costs of a tower of type  $j$ . The resulting total cost is :

$$w_j = w_j^1 + w_j^2 + 20w_j^3 + h_j^1 + h_j^2 + 20h_j^3.$$

With these notations, the total cost of the renewable energy components, plus the batteries cost is

$$\sum_{i=1}^I c_i v_i^s v_i^p + \sum_{k=1}^K b_k y_k^s y_k^p + \sum_{j=1}^J w_j t_j.$$

This is the objective function that we have to minimize. Notice that the function is linear since  $v_i^s$  and  $y_k^s$  are known.  $v_i^s v_i^p$  is the total number of solar panels of type  $i$  for the PV array of type  $i$ . Since each one cost  $c_i$ , the total cost of a panel of type  $i$  is  $c_i v_i^s v_i^p$ . The expression of the batteries cost is obtained similarly.

### 3.3 Constraints

Two kind of constraints have been formulated : the demand constraints and material constraints on batteries.

### 3.3.1 Demand constraints

At any time  $t$ , the total amount of energy produced must be greater or equal to the demand  $d_t$ . Since  $P_{it}$  is the power of a solar panel of type  $i$  at time  $t$ , and  $W_{jt}$  the power of a wind turbine of type  $j$  at time  $t$ , then the total power of solar panels and wind turbines is :

$$E_t = \sum_{i=1}^I P_{it} v_i^s v_i^p + \sum_{j=1}^J W_{jt} t_j, \quad t = 0, 1, \dots, T.$$

It follows that  $E_t - Y_t$  is the total energy fulfilled by renewable energy sources and battery panels. This total energy must always fit the demand. The demand constraints are then

$$E_t - Y_t \geq d_t, \quad t = 0, 1, \dots, T. \quad (1)$$

### 3.3.2 Battery constraints

The total amount of energy in the battery bank is called the "state of charge" of the battery bank at time  $t$  and is noticed  $u_t$ . For safety reasons and maximal capacity of batteries,  $u_t$  is bounded by known minimal and maximal values  $\underline{u}$  and  $\bar{u}$ .

$$\underline{u} \leq u_t \leq \bar{u}, \quad t = 0, 1, \dots, T. \quad (2)$$

If  $u_k^s$  is the maximal capacity of a battery string of known size  $y_k^s$  then the maximal capacity of a battery array with  $y_k^p$  strings of size  $y_k^s$  is  $y_k^p u_k^s$ . It follows that

$$\bar{u} = \sum_{k=1}^K y_k^p u_k^s. \quad (3)$$

$\underline{u}$  is a minimal value under which the battery must not be discharged to avoid serious damages. Its value is obtained as follows :

$$\underline{u} = (1 - DoD)\bar{u}, \quad (4)$$

where  $DoD$  is calling *the Depth of Discharge*.  $DoD$  is a material property provided by the battery manufacturer and giving the authorized discharging percentage from the maximal capacity.

$u_t$  ( $t = 0, 1, \dots, T$ ) is a serie expressed as follows :

$$\begin{cases} u_0 = \bar{u} \\ u_{t+1} = u_t + \frac{Y_t}{\beta}\eta, \text{ if } Y_t \geq 0 \text{ (charging process)} \\ u_{t+1} = u_t + \frac{Y_t}{\beta}, \text{ if } Y_t < 0 \text{ (discharging process)} \end{cases}$$

where  $\eta \in [0, 1]$  and  $\beta \geq 0$  are two physical properties expressing the performance of the battery, and the HRES bus voltage.

To modelize the two possible values for  $u_{t+1}$ , binary variables  $\lambda_t$  have been introduced. Let

$$\lambda_t = \begin{cases} 1 \text{ in the charging process} \\ 0 \text{ in the discharging process} \end{cases}$$

With these variables, we have :

$$u_{t+1} = u_t + \frac{Y_t}{\beta}[\eta\lambda_t + (1 - \lambda_t)]$$

It follows (by recurrence) that

$$u_{t+1} = \bar{u} + \sum_{l=1}^t \frac{Y_l}{\beta} [\eta\lambda_l + (1 - \lambda_l)], \quad t = 0, 1, \dots, T. \quad (5)$$

Thus, from inequations ( 2), and equations ( 3), ( 4) and ( 5), we deduce that

$$-DoD \sum_{k=1}^K y_k^p u_k^s \leq \sum_{l=1}^{t-1} \frac{Y_l}{\beta} [\eta\lambda_l + (1 - \lambda_l)] \leq 0, \quad t = 1, 2, \dots, T. \quad (6)$$

In addition, we also have to link variables  $\lambda_t$  with  $Y_t$  signs. When  $\lambda_t = 1$  (resp.  $\lambda_t = 0$ ) we are in charging (resp. discharging) process, so  $Y_t$  must be non negative (non positive). The resulting inequations are :

$$(1 - \lambda_t)\underline{Y} \leq Y_t \leq \lambda_t \bar{Y}, \quad 0 \leq t \leq T, \quad (7)$$

where  $\bar{Y}$  (resp.  $\underline{Y}$ ) is the maximal (resp. minimal) possible total capacity in the battery bank.  $\bar{Y}$  and  $\underline{Y}$  are computed using the upper bounds on variables  $y_k^p$ .  $\bar{Y}$  is the maximal capacity of the battery bank obtained by taking for each type  $k$  the maximal value  $\bar{y}_k^p$ ,

$$i.e. \bar{Y} = \sum_{k=1}^K \bar{y}_k^p u_k^s \beta,$$

and  $\underline{Y} = -\bar{Y}$ .

### 3.4 Final Formulation

With the subsections above the complete formulation for the optimal sizing may be written. We have :

$$MODEL(T) : Min \quad \sum_{i=1}^I c_i v_i^s v_i^p + \sum_{k=1}^K b_k y_k^s y_k^p + \sum_{j=1}^J w_j t_j$$

$$\text{s-t} \quad (1) \quad \sum_{i=1}^I P_{it} v_i^s v_i^p + \sum_{j=1}^J W_{jt} t_j - Y_t \geq d_t, \quad 0 \leq t \leq T$$

$$(6) \quad -DoD \sum_{k=1}^K y_k^p u_k^s \leq \sum_{l=1}^{t-1} \frac{Y_l}{\beta} [\eta \lambda_l + (1 - \lambda_l)] \leq 0, \quad 1 \leq t \leq T$$

$$(7) \quad (1 - \lambda_t) \underline{Y} \leq Y_t \leq \lambda_t \bar{Y}, \quad 0 \leq t \leq T$$

$$0 \leq v_i^p \leq \bar{v}_i^p \quad i = 1, \dots, I$$

$$0 \leq y_k^p \leq \bar{y}_k^p \quad k = 1, \dots, K$$

$$0 \leq t_j \leq \bar{t}_j \quad j = 1, \dots, J$$

#### 4 Resolution

Following the formulation sections, we are interested here in the resolution scheme of  $MODEL(T)$ . As a preliminary, let us first introduce some general remarks on the problem. The objective function of the problem is linear, and except the constraints (6) all others constraints are also linear. Depending on the value of  $T$  the resulting formulation may be very large. In our numerical experimentations, in section 5,  $T$  is equal to 4343 (6 months) and there are two types of each renewable energy components. As a consequence, there are

:



- $T + 1$  binary variables  $\lambda_t = 4344$
- $T + 1$  continuous variables  $Y_t = 4344$
- $I = 2$  : 2 types of solar panels.
- $J = 2$  : 2 types fo wind turbines.
- $K = 2$  : 2 types of batteries.

Hence, the total number of variables is 8694 variables in which 4350 are integers. We also have :

- $T + 1$  constraints (1) = 4344.
- $2 * (T + 1)$  constraints (6) = 8686.
- $2 * (T + 1)$  constraints (7) = 8688.

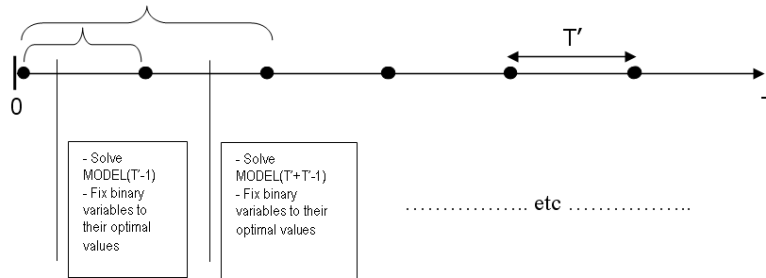
The total number of constraints is 21718. It follows, from this value of  $T$ , a very large scale integer non linear mathematical program. Both for this huge size and limited machine capacity, it has been chosen to solve this problem by a heuristic. The resolution method is detailed in the sequel.

#### 4.1 Heuristic

Our heuristic may be classified in referent domain optimization methods (see Glover and Laguna [4]). The idea is quite simple. Solving exactly  $MODEL(T)$  for little value of  $T$  is possible since the resulting formulation is also of little

size. Thus, even if we cannot solve  $MODEL(T)$  for big value of  $T$  (such as 4343), one may choose a lower value (says  $T' = \frac{4344}{2} = 2172$ ), then solve  $MODEL(T' - 1)$ , fixed all variables  $\lambda_t, Y_t$  for  $t = 0, 1, \dots, T' - 1$  to their optimal values in  $MODEL(T' - 1)$ , and finally solved  $MODEL(2 * T' - 1)$ . In doing so, at each of the two iterations, the number of variables  $\lambda_t$  and  $Y_t$  of the resulting problems are always divided by two.

Our heuristic is a sophistication of this idea in which any value for  $T'$  (not necessarily  $\frac{T}{2}$ ) may be chosen and where the variables fixed at each iteration are only binary variables  $\lambda_t$ . We choose to fix only variables  $\lambda_t$  because at each iteration the optimal solution is obtained using a standard linearization of the quadratically constrained problem  $MODEL(T)$ . In the resulting linear formulation, the "difficult" variables are not the continuous one's ( $Y_t$ ) but the binary one's. The general scheme of our heuristic is summarized in figure 4. Let  $T$  be the size of the observation period. Since  $T$  is currently too large, we



**Fig. 4** Heuristic Scheme

take a much more lower period  $T'$ .  $T$  is then subdivided in  $\lfloor \frac{T}{T'} \rfloor$  subperiods

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of size  $T'$  as shown in the figure. At the first iteration,  $MODEL(T' - 1)$  is solved optimally. Then binary variables,  $\{\lambda_t\}_{t=0,1,\dots,T'-1}$  are fixed (definitely) to their optimal values. At the second iteration  $MODEL(2T' - 1)$  is solved and  $\{\lambda_t\}_{t=T'+1,1,\dots,2T'+1}$  are fixed, etc. The corresponding algorithm is the following :

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### HEURISTIC

- $\mathbf{T} = 6$  months of 1 hour step i.e 4343
- $\mathbf{T}' =$  value to be fixed ( $\mathbf{T}' \ll \mathbf{T}$ ).
- **newStart** = 0
- **newT** =  $\mathbf{T}'$

While  $\text{newT} \leq T$

- **Solve optimally**  $MODEL(\text{newT}-1)$ ,
- Let  $\lambda_B^*$  be optimal values of variables  $\lambda_B$
- for  $t = \text{newStart}$  to **newT-1**
- $\lambda_B(t) = \lambda_B^*(t)$
- $\text{newStart} = \text{newStart} + T'$
- $\text{newT} = \text{newT} + T'$

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*end While*

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Let  $T_k = kT'$  ( $0 \leq k \leq q$ ) where  $q$  is the ratio of the division of  $T$  by  $T'$

$$i.e T = T'q + r \text{ with } 0 \leq r < q$$

Let also  $T_{q+1} = T'q + r = T$ .  $T_1, T_2, \dots, T_{q+1}$  are the discretization of  $[0, T]$ .

Let us consider two indices,  $i < j$ , of the set  $\{1, 2, \dots, q + 1\}$ . We notice  $(v^p, y^p, t, \lambda, Y)$  a solution of  $MODEL(T_j)$  where :  $v^p = (v_1^p, v_2^p, \dots, v_I^p) \in \mathbb{N}^I$ ,  $y^p = (y_1^p, v_2^p, \dots, v_K^p) \in \mathbb{N}^K$ ,  $t = (t_1, t_2, \dots, t_J) \in \mathbb{N}^J$ ,  $\lambda = (\lambda_0, \lambda_2, \dots, \lambda_{T_j}) \in \{0, 1\}^{T_j}$ , and  $Y = (\lambda_0, \lambda_2, \dots, \lambda_{T_i}) \in \mathbb{R}^{T_j}$ .

It can be seen that by projecting  $\lambda$  (resp.  $Y$ ) on  $\{0, 1\}^{T_i}$  (resp.  $\mathbb{R}^{T_i}$ ), the resulting point  $(v^p, y^p, t, \lambda^*, Y^*)$ , where  $\lambda^*$  (resp.  $Y^*$ ) are the restrictions of  $\lambda$  on the space  $\{0, 1\}^{T_i}$  (resp.  $\mathbb{R}^{T_i}$ ), remains feasible for  $MODEL(T_i)$ . Thus we have the following property.

*Property 1* For any mathematical program ( $P$ ) let  $V(P)$  denotes its optimal value. We have  $V(MODEL(T_i)) \leq V(MODEL(T_j)) \forall 1 \leq i \leq j \leq q + 1 \square$

It follows that since  $V(MODEL(T_1)) \leq V(MODEL(T_2)) \leq \dots \leq V(MODEL(T))$ ,

$V(MODEL(T_1))$  is a lower bound of the optimal value of  $MODEL(T)$  that may be used to evaluate the quality of the solution of the heuristic. It corresponds, in our heuristic, to the optimal value computed at the first iteration of the while loop. In the subsequent iterations, the objective values found cannot be used as lower bounds since some variables  $\lambda$  has been fixed. Obviously, notice that greater is the value  $T'$  and better is this lower bound. In our nu-

merical experiments (see section 5) this lower bound has been computed for increasing value of  $T'$ .

#### 4.2 Solving optimally **MODEL(newT-1)**

At each iteration of the algorithm, **MODEL(newT-1)** is supposed to be solved exactly. The optimal solution is found using a linearization of the quadratic constraints (6). As a result **MODEL(newT-1)** becomes an integer linear program that may be solved by any linear programming package (as CPLEX). The linearization is performed as follows.

Let us recall constraints (6) :

$$(6) \quad -DoD \sum_{k=1}^K y_k^p u_k^s \leq \sum_{l=1}^{t-1} \frac{Y_l}{\beta} [\eta \lambda_l + (1 - \lambda_l)] \leq 0, \quad 1 \leq t \leq T.$$

It is well-known (see Fortet [3], Billionnet [2]) that a standard linearization of the products  $Y_l \lambda_l$  is obtained by introducing new variables

$$x_l = Y_l \lambda_l,$$

and additional constraints :

$$\begin{aligned} \lambda_l \underline{Y} &\leq x_l \leq \lambda_l \bar{Y} \\ x_l &\leq \lambda_l \underline{Y} + Y_l - \underline{Y} \\ \lambda_t \bar{Y} + Y_l - \bar{Y} &\leq x_l \end{aligned}$$

It follows that a linearization applied to  $MODEL(newT - 1)$  introduces  $T'$  additional discrete variables  $x_t$ ,  $4T'$  additional linearization constraints. In

addition,  $2T'$  constraints of type (6) and variables  $\lambda_t$  and  $Y_t$  ( $t = \text{newStart}$  to  $\text{newT}-1$ ) are added at each iteration. Thus, even if  $T'$  is sufficiently small  $MODEL(\text{newT} - 1)$  may be difficult to solve because of new constraints and variables introduced at each iteration. So, it is important to analyze carefully the constraints and variables in such a way to introduce only necessary elements. This is the goal of the next subsection.

#### 4.3 Some implementation tricks

Iteration by iteration one may observe that  $2T'$  constraints of type (6) are introduced. For each of them new linearization variables  $x_t$  has to be considered and associated linearization constraints are also added in the model. Thus, the formulation grows significantly. To control this growth, it is important to add only necessary constraints taking into account variable fixations done previously. In practice, a naive approach consisting simply in adding all constraints and variables at each iteration leads, in our computing environment, to intractable problem, memory problem and very low processing time. We show, in this subsection, that it is not necessary to add all constraints (6) at each iteration taking into account informations about the previous binary variables fixing.

Let us consider constraint (6) for a fixed value of  $t$

$$-DoD \sum_{k=1}^K y_k^p u_k^s \leq \sum_{l=1}^{t-1} \frac{Y_l}{\beta} [\eta \lambda_l + (1 - \lambda_l)] \leq 0.$$

Let also consider constraint ( 7) for the value  $t$

$$(1 - \lambda_t)\underline{Y} \leq Y_t \leq \lambda_t\bar{Y}.$$

If  $\lambda_t = 1$  then, because of constraint ( 7), we have  $0 \leq Y_t$ . Thus, let go back to constraint ( 6) for  $t + 1$

$$i.e. \quad -DoD \sum_{k=1}^K y_k^p u_k^s \leq \sum_{l=1}^t \frac{Y_l}{\beta} [\eta\lambda_l + (1 - \lambda_l)] \leq 0.$$

Since  $\lambda_t = 1$  and  $\beta, \eta \geq 0$ , we necessarily have

$$-DoD \sum_{k=1}^K y_k^p u_k^s \leq \sum_{l=1}^t \frac{Y_l}{\beta} [\eta\lambda_l + (1 - \lambda_l)],$$

because

$$\sum_{l=1}^t \frac{Y_l}{\beta} [\eta\lambda_l + (1 - \lambda_l)] = \sum_{l=1}^{t-1} \frac{Y_l}{\beta} [\eta\lambda_l + (1 - \lambda_l)] + \frac{Y_t}{\beta} \eta,$$

$$\frac{Y_t}{\beta} \eta \geq 0 \text{ and } -DoD \sum_{k=1}^K y_k^p u_k^s \leq \sum_{l=1}^{t-1} \frac{Y_l}{\beta} [\eta\lambda_l + (1 - \lambda_l)].$$

Thus, only the right-hand-side inequality has to be added when  $\lambda_t = 1$ , that is to say

$$\sum_{l=1}^t \frac{Y_l}{\beta} [\eta\lambda_l + (1 - \lambda_l)] \leq 0.$$

For similar reason, when  $\lambda_t = 0$ , only the left-hand-side inequality is necessary

$$i.e. \quad -DoD \sum_{k=1}^K y_k^p u_k^s \leq \sum_{l=1}^t \frac{Y_l}{\beta} [\eta\lambda_l + (1 - \lambda_l)].$$

The same observations hold for  $\lambda_{t+1}$ ,  $\lambda_{t+2}$ , and so on.

The implementation trick consists after optimally solving **MODEL(newT-1)**

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to discard all unnecessary inequalities in constraints ( 6), and associated linearization variables in such a way to reduce the problem number of constraints.

One may observed also that fixing a binary variable  $\lambda_t$  is equivalent to fix also the expression of the associated linearization variable  $x_t$ . As a consequence the problem size may be also reduced in terms of variables since all variables  $x_t$  may be replaced by their corresponding values :  $Y_t$  (when  $\lambda_t = 1$ ) or 0 (when  $\lambda_t = 0$ ).

Without these simple tricks it was not possible, for certain value of  $T'$ , in our computing environment, to get enough memory to perform all iterations of the algorithm.

## 5 Numerical results

The heuristic algorithm has been implemented for a real world case in a site at Dakar (Senegal). The exact location of the site may be viewed in the figure above (indication C : University Cheikh Anta Diop). The site is at  $14^{\circ}41$  *N* Longitude,  $17^{\circ}27$  *W* Latitutde and 24 *m* sea level. Given some observations, the practical objective of the experimentation is to size as better as possible an Hybrid Renewable Energy System. Data related on environmental observations have been collected by colleagues of Ecole Supérieure Polytechnique de Dakar (ESP Dakar). Two of these important informations are the solar radiations and wind speed over the observation period. Figure 6 show these



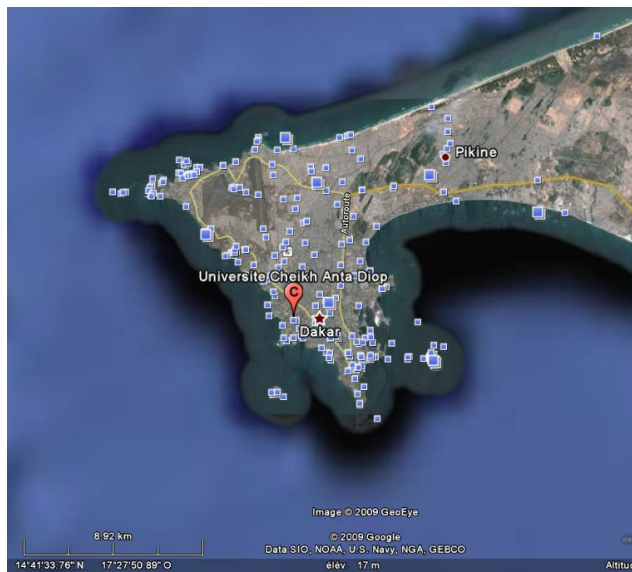


Fig. 5 Geographical Location in Dakar (Senegal)

informations. The wind speed is in average  $4.3 \text{ m/s}$  and the solar radiation is in average (approximately)  $200 \text{ W/m}^2$ .

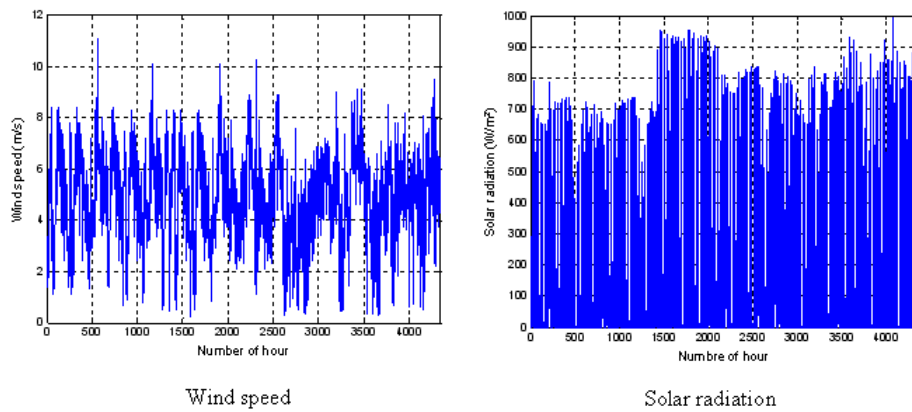


Fig. 6 Observations

The number of types and the period length is given in the table 1.

$T$	$I$	$J$	$K$
4343	2	2	2

**Table 1** Period Length (T), Types (I,J,K)

Because of the huge value  $T$ , describing entirely the matrices  $P = \{P_{it}\}_{\substack{1 \leq i \leq I \\ 0 \leq t \leq T}}$  and  $W = \{W_{jt}\}_{\substack{1 \leq j \leq J \\ 0 \leq t \leq T}}$ , as well as the demand vector  $d = \{d_t\}_{0 \leq t \leq T}$  in this paper is of no interest.

The costs are in table 2. The maximum number of each renewable energy components (i.e. values  $\overline{v_i^p}, \overline{y_k^p}, \overline{t_j}$ ) has been fixed to 50. We also have  $DoD = 0.8$ ,  $\beta = 48 \text{ Volt}$  and  $\eta = 0.8$ . The experiments have been performed on a PC Dell with 2 G0 RAM, Intel Core Processor of 1.86 GHz and Linux Suze Operating System. The linear programming model of each iteration has been solved with Ilog Cplex 10.1.

In practice, the sizing process is done once. As a corollary, there is no "strong constraint" on the processing time. We are not necessarily searching for a

$c_1$	$c_2$	$b_1$	$b_2$	$w_1$	$w_2$
783.9 Eur.	675.1 Eur.	986.58 Eur.	2067.12 Eur.	27849.9 Eur.	23034.7 Eur.

**Table 2** Costs

fast algorithm giving good solutions in few times but for an algorithm giving "good" feasible solutions in a "reasonable" amount of time. We have considered that more than 1000sec. (maximum fixed time limit at each iteration) for solving exactly  $\mathbf{MODEL}(\mathbf{newT} - 1)$  at a iteration of the algorithm is unreasonable. When Cplex branch-and-bound iterates more that this time limit, the process is stop and we consider that our algorithm fails to find a solution since  $\mathbf{MODEL}(\mathbf{newT} - 1)$  is supposed to be solved optimally.

A more flexible version of our heuristic may be to return in this case the best integer integer solution found by Cplex and then to continue the heuristic loop using this solution. This version was not explored in this paper. It will be the subject of other computational experiments.

At the initial point of our heuristic (see subsection 4.1), a value for  $T'$  has to be chosen. The values  $T'$  start with 100 and are increased by 50. The results are reported in table 3. For each  $T'$  the value of the first optimization is stored in the column  $LB$  (Lower Bound). The objective function values at the end of all iterations are in the column  $Cost$ , and the corresponding solutions in the remaining columns. As explained in section 4.1 each value of the column  $LB$  value is a lower bound of the optimal value of the problem. It follows that the maximum value, noticed  $MaxLB$  of this column corresponds to the best lower bound found. At the opposed, the minimal value of the column  $cost$ , noticed  $MinCost$ , is the best solution found by the heuristic. Thus comparing

$T'$	LB	Cost	$v_1^p$	$v_2^p$	$t_1$	$t_2$	$y_1^p$	$y_2^p$	time (sec.)
100	69542.6	88791.1	40	1	0	0	4	1	1693.4
150	71491.6	80952.1	35	1	0	0	4	1	1309.92
200	71491.6	83652.8	39	1	0	0	1	2	1137.5
250	72678.2	88791.1	40	1	0	0	4	1	991.6
300	73870.1	82895.7	36	1	0	0	2	2	1063.7
350	73870.1	95486.3	36	1	0	0	1	4	1013
400	73870.1	89596.4	35	1	0	0	2	3	947.4
450	73870.1	85220.6	40	1	0	0	1	2	920.8
500	73870.1	93489.1	40	1	0	0	1	3	1305
550	78949.4	95438.1	44	1	0	0	2	2	946.9
600	78949.4	87599.1	39	1	0	0	2	2	875.6
650	78949.4	85220.6	40	1	0	0	1	2	976.7
700	78949.4	83276.9	39	1	0	0	3	1	1179.7
<i>MinCost</i>		<b>80952.1</b>							
<i>MaxLB</i>	<b>78949.4</b>								

**Table 3** Numerical results

*MaxLB* and *MinCost* gives a idea of the quality of the heuristic.

In our experiments, the values of  $T'$  greater than 700 lead either to cplex memory problem (*CPLEX ERROR 1001 : Out of Memory*) or "unreasonable" processing time (more than 1000sec. in a iteration) to solve optimally **MODEL(newT - 1)**. The best heuristic solution (*MinCost*) is equal to **80952.1**.

For this solution the heuristic suggest to take no wind turbines, whatever is the type, a type 1 (resp. type 2) PV array of size  $v_1^p v_1^s = 35 * 2 = 70$  (resp.  $v_2^p v_2^s = 1 * 3 = 3$ ) and a type 1 (resp. type 2) battery array of size

$y_1^p y_1^s = 4 * 4 = 16$  (resp.  $y_2^p y_2^s = 1 * 4 = 4$ ). The relative deviation with the best lower bound, MaxLB, is equal to  $\frac{80952.1-78949.4}{78949.4} * 100 \approx 2.5\%$ . So, we are not so far from the optimal value but without guarantee that we have really reached it.

The results correspond to meteorologic conditions and geographic location of the site. Indeed, as attested by the wind speed graph, the wind exposure of the site is low. For the types of turbines of this experiment, it is estimated (by the specialists) that "good" electrical production may be expected for wind speed around 6 *m/s*. The observations show that, in average, the wind speed of the site is about 4.3 *m/s*. This is rather insufficient. And, taking into account the expensive costs (see table 2) of wind turbines, there is no evidence for investment on such technologies. This may explain why the optimization process suggests no wind turbines. At the opposed, because of significative solar radiations of this geographical area, as well as competitive costs of solar panels in comparison to wind turbines, the optimization suggests numerous solar panels of each type. Thus, all of these results are not surprisingly and show the practical validity of the model.

## 6 Conclusion and perspectives

In this paper, a quadratically constrained model for sizing an Hybrid Energy Renewable System (HRES) was proposed. The goal of this system is to take advantage of different weather conditions, geographic properties in such a way to produce required electrical energy using various renewable energy sources.

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Because of the very large size of the formulation, a heuristic has been proposed to solve the model. The method proposed may be classified in referent domain based optimization method [4]. The formulation has been used with data collected in a Dakar site. The results are consistent in the light of the meteorological informations on this site.

However, because of the heuristic approach, optimality of this solution is not guarantee. It remains to compute a lower bound in order to evaluate the quality of the solution. We have seen above that using simply a linear relaxation do not work in our computing environment because of memory problem resulting by the very large scale integer linear problem. In further researches, we seek to fix this point with decomposition methods exploiting the specific structure of the constraints, in particular constraints 6.

Another interesting perspective is to take into account the stochastic nature of the problem. All data used in the model have been supposed deterministic using some approximations. It is well known that meteorologic conditions are stochastic and since the energy produced by solar panels or wind turbines depends on these conditions then the power of a solar panel of type  $i$  at time  $t$  (i.e  $P_{it}$ ) and the power of a wind turbine of type  $j$  at time  $t$  (i.e  $W_{jt}$ ) should be stochastic variables. Our perspective in this field will deal to propose stochastic programming modeling and associated resolution scheme.

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