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## A Benders' decomposition algorithm for the real-time Railway Traffic Management Problem

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### Abstract

In this paper, we propose a Benders' decomposition algorithm to tackle the real-time railway traffic management problem. Specifically, we decompose the mixed-integer linear programming formulation at the basis of RECIFE-MILP (Pellegrini et al., 2015), a state-of-the-art algorithm for this problem. In our Benders' decomposition, we minimize total delays by making train routing and scheduling decisions in the master problem. Given these decisions, we compute the trains arrival and passing times in the slave problem to deduce the total delay. By applying our Benders' decomposition algorithm to RECIFE-MILP, we tackle large instances representing traffic in the Rouen-Rive-Droite control area, in France.

### Keywords

real-time railway traffic management problem, MILP, Benders' decomposition.

## 1 Introduction

For many railway systems, during peak hours, the infrastructure capacity is completely exploited for ensuring the trains circulations. Many trains travel within short time through critical points. If a disturbance occurs, the traffic may be perturbed and, as a result, conflicts may emerge. In a conflict, multiple trains would claim the same track section concurrently if traveling at the planned speed. Hence, some trains must be stopped or decelerated for ensuring safety, and delays propagate. In locations such as junctions, which are areas where multiple lines cross, the emergence of conflicts is very frequent and effectively dealing with them may be particularly difficult.

Two kinds of delays occur in railway traffic: primary and secondary delay. The unexpected events which disturb the traffic cause the primary delay. The secondary delay is the result of delay propagation due to the interactions between trains. We do not have means to avoid unexpected events; hence the primary delay cannot be prevented. Instead, the secondary delay may be avoided or reduced by suitably managing traffic. The real-time Railway Traffic Management Problem (rtRTMP) consists in modifying trains route and schedule to limit the delay propagation. This problem is tackled by dispatchers. They do it manually, so the result of their choices is generally suboptimal. The use of an efficient algorithm to help dispatchers decision making is crucial to ensure an effective traffic management when disruptions occur. The design of this efficient algorithm is typically based

on two main models used to represent the railway infrastructure: macroscopic and microscopic. In the macroscopic model, the infrastructure is seen as a set of nodes (junctions or stations) connected by lines, and the separation between trains is imposed through the control of generic headway times. In the microscopic one, the infrastructure is seen as a set of track sections where the train separation is imposed according to the signaling system, as it happens in reality. With the former, we can solve very large instances, but with low precision at the local level; hence, the optimization choices may have unexpected effects on the traffic when they are actually implemented, due for example to the specific signaling system in place. With the latter, these unexpected effects are avoided by considering in the optimization all necessary details. However, solving large instances may become very difficult due to memory or computational time limits.

In the operations research literature, many algorithms have been proposed to address the rtRTMP. In the following, we report the most relevant contributions. We group the contributions in two categories: those that propose algorithms without focusing on the size of the instances, and those that explicitly tackle large instances through decomposition. For what concerns the first category, several algorithms have been proposed (for recent surveys, see Cacchiani et al. (2014); Corman and Meng (2015)). In some papers, the authors propose heuristic solution approaches in which train routes are fixed and the optimization concerns the scheduling decisions. An algorithm which determines trains optimal scheduling on a single track is proposed by Higgins et al. (1996). This algorithm can both give support to trains dispatchers to tackle the rtRTMP and evaluate the impact of timetable changes. A heuristic algorithm is designed by Sahin (1999) to tackle train rescheduling problem in disrupted situations. In Dessouky et al. (2006), the authors propose a branch-and-bound algorithm for rescheduling trains using fixed routes. Some authors propose heuristic solution approaches that consider a limited number of alternative routes selected during the solution process (D'Ariano et al., 2008; Corman et al., 2010; Acuna-Agost et al., 2011); others start with a small number of alternative routes and increase it throughout the solution process (Caimi et al., 2011, 2012). While several algorithms exist to tackle the rtRTMP considering a limited number of alternative routes, few algorithms have been proposed to address the rtRTMP considering all alternative routes. An effort in this direction is the work of Rodriguez (2007); Törnquist Krasemann (2012); Pellegrini et al. (2014, 2015). An algorithm based on a constraint programming approach is proposed by Rodriguez (2007). Törnquist Krasemann (2012) proposes a greedy heuristic for the rtRTMP, modeling track segments. The impact of the microscopic representation of the infrastructure on the optimal solution is assessed by Pellegrini et al. (2014). In the paper, the authors show that the consideration of a fine granularity of the infrastructure representation, up to the track-circuit level, may allow the detection of better solutions than the use of the rough granularity, which is limited to the block section level. The RECIFE-MILP algorithm is proposed by Pellegrini et al. (2015). All these algorithms do not explicitly focus on the problem represented by the size of the instances to be tackled. However, a few papers exist dealing with this problem. For example, D'Ariano and Pranzo (2009); Corman et al. (2012, 2014) propose heuristic approaches to decompose the instances to be solved. The authors decompose large instances of the rtRTMP by dispatching areas and apply a bi-level optimization approach to solve the problem. At the lower level, the system manages traffic in their control areas without any knowledge of the traffic flow elsewhere. At the higher level, a coordinator module is responsible for the traffic management over a railway network including several areas with a global vision of the traffic flows. Others, as Lusby et al. (2012), propose classic mathemat-

ical decomposition approaches as column generation. This column generation approach is applied to a set packing model to tackle real-time train routing at junctions. The set packing model of the problem is formulated as an integer linear program with a resource based constraint system. Meng and Zhou (2014) propose a Lagrangian relaxation approach to tackle the simultaneous train rerouting and rescheduling on N-track railway network. A logic Benders' decomposition approach is proposed by Lamorgese and Mannino (2015); Lamorgese et al. (2016) to solve the rtRTMP. In the paper, the authors combine a macroscopic and a microscopic model to perform a decomposition of the initial problem into two sub-problems (master and slave problem). This approach allows dealing with large instances but it does not consider many details of the microscopic infrastructure characteristics. In a recent seminar, Lamorgese and Mannino (2016) present a standard Benders' decomposition approach to tackle the train's rescheduling problem formulated with an alternative graph model. In their approach, the authors replace the standard Benders' feasibility and optimality cuts with strong cuts obtained by strengthening and lifting the standards ones.

In this work, we consider RECIFE-MILP, which allows solving some rtRTMP instances to optimality considering all details in the infrastructure. However, it has been shown (Pellegrini et al., 2015) that its performance may strongly worsen when tackling very large instances in the short time allowed by the real-time nature of the problem. For this formulation, we propose a Benders' decomposition (Benders, 1962) to increase the size of the instances which can be effectively tackled. In our Benders' decomposition algorithm, we separate the rtRTMP into two problems: the master and slave. The master problem is the real-time train routing and scheduling problem, it contains the routing and scheduling variables (binary variables) and one dummy variable representing the contribution of continuous variables to the master problem objective function. The schedule here is intended as the order in which trains cross common tracks, and do not include time information. Once an incumbent solution is found in the master problem, it is sent to the dual of the slave problem. This solution is a specific route and schedule assignment for each train in the instance. The slave problem contains the continuous variables which determine the track sections occupation and reservation times, and deduces the delay suffered by each train. Based on the result of the dual problem, we add an optimality or feasibility cut to the master problem to cut off the current solution, if suboptimal or infeasible for the overall problem. This process is repeated until the algorithm converges to an optimal solution or the maximum computational time imposed by the real-time nature of the problem has elapsed. Starting from this standard Benders' reformulation, we propose a reduced one to improve the computational performance. Solving these reformulations with an iterative and an integrated Benders' algorithm, we tackle instances representing traffic in the Rouen-Rive-Droite control area, in France.

The remainder of this paper is organized as follows. Section 2 presents the RECIFE-MILP formulation of the rtRTMP. The RECIFE-MILP Benders' reformulation and the Reduced RECIFE-MILP Benders' reformulation proposed are presented in Section 3 and Section 4, respectively. Computational experiments are reported in Section 5. Conclusions and directions for future work are discussed in Section 6.

## 2 RECIFE-MILP formulation

In this section, we present the RECIFE-MILP formulation of the rtRTMP proposed by Pellegrini et al. (2015). RECIFE-MILP is a heuristic algorithm based on the truncated solution

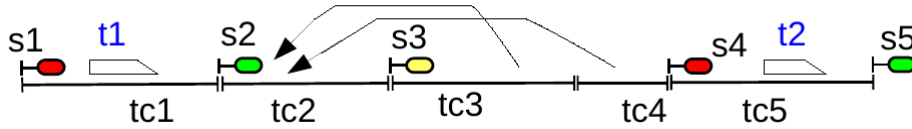


Figure 1: Example of the track-circuit reservation with the 3-aspect signaling system. The arrows represent the reference track-circuits for  $tc_3$  and  $tc_4$ , i.e., the first track-circuit of the  $n$ -2nd preceding block section, with  $n$  equal to 3.

of mixed-integer linear programming (MILP) formulation. This formulation is based on a microscopic representation of the infrastructure, capable to consider all the characteristics of the infrastructure in deep details.

Before presenting the RECIFE-MILP formulation of the rtRTMP, we define some technical expressions used to indicate some relevant elements of the railway system.

**Track-circuit:** a detection device on which the presence of a train is automatically detected.

**Running time:** the time spent by the head of a train on a track-circuit when traveling at the planned speed.

**Clearing time:** the time elapsed between the moment when the train's head leaves a track-circuit and the moment when its tail leaves it.

**Occupation time:** the time interval during which a train physically occupies a track-circuit.

**Block section:** a block section is a sequence of track-circuits whose access is controlled by a signal.

**Reservation time:** in the blocking time theory (Pachl, 2009), a train is allowed to enter a block section opened by an  $n$ -aspect signal only if the following  $n - 1$  block sections are available. Before entering, the train reserves these  $n - 1$  block sections, and in particular it reserves all their track-circuits. Hence, the reservation time of the track-circuit starts before a train enters the reference track-circuit (the first track-circuit of the  $n - 2$ nd preceding block section, e.g., in the example of Figure 1, the reference track-circuit of  $tc_3$  is  $tc_2$  in case of a 3-aspect signaling system) and ends when it enters the track-circuit itself.

**Formation time:** in addition to the reservation time, a supplementary time is needed before a train starts the occupation of a sequence of block sections. This supplementary time is the formation time. It is used for example to take into account the signal visibility distance.

**Release time:** the release time corresponds to the lapse of time in which the reservation of a track-circuit is still active after a train exits it.

**Utilization:** we name utilization the sum of reservation and occupation time.

**Route:** the complete sequence of track-circuits traversed by a train during its trip is named route.

If a train starts its trip at null speed, in the model its occupation of the first track-circuit is accounted only from the time at which it starts moving. Its staying still on the track-circuit before that time is represented through the reservation time.

In RECIFE-MILP, in addition to the actual track-circuits, two dummy ones are considered,  $tc_0$  and  $tc_\infty$ . They represent the entry and the exit locations of the infrastructure, respectively. Each track-circuit has a running time and a clearing time which depend on the type of train traversing them and on the route traveled. For the dummy track-circuits, the

running time and clearing times are null.

In the RECIFE-MILP formulation, the following notation is used:

$T$   $\equiv$  set of trains;

$w_t$   $\equiv$  weight associated to train  $t$ 's delay;

$ty_t$   $\equiv$  type corresponding to train  $t$  (train characteristics);

$init_t, exit_t$   $\equiv$  earliest time at which train  $t$  can be operated and earliest time at which it can reach its destination given  $init_t$  and the route assigned in the timetable;

$R_t, TC_t$   $\equiv$  set of routes and track-circuits available for train  $t$ ;

$TC^r$   $\equiv$  set of track-circuits composing route  $r$ ;

$p_{r,tc}, s_{r,tc}$   $\equiv$  track-circuits preceding and following  $tc$  along  $r$ ;

$rt_{ty,r,tc}, ct_{ty,r,tc}$   $\equiv$  running and clearing time of  $tc$  along  $r$  for a train of type  $ty$ ;

$ref_{r,tc}$   $\equiv$  reference track-circuit of  $tc$  along route  $r$ ;

$bs_{r,tc}$   $\equiv$  block section including track-circuit  $tc$  along route  $r$ ;

$for_{bs}, rel_{bs}$   $\equiv$  formation and release time for block section  $bs$ ;

$M$   $\equiv$  large constant.

The formulation contains non-negative **continuous variables**:

for all triplets of  $t \in T, r \in R_t$  and  $tc \in TC^r$ :

$o_{t,r,tc}$  : time at which  $t$  starts the occupation of  $tc$  along  $r$ ,

$l_{t,r,tc}$  : longer stay of  $t$ 's head on  $tc$  along  $r$ , due to dwell time  
and scheduling decisions (delay);

for all pairs of  $t \in T$  and  $tc \in TC_t$ :

$sU_{t,tc}$  : time at which  $t$  starts  $tc$  utilization;

$eU_{t,tc}$  : time at which  $t$  ends  $tc$  utilization;

for all  $t \in T$ :

$D_t$  : delay suffered by train  $t$  when exiting the infrastructure.

In addition it includes **binary variables**:

for all pairs of  $t \in T$  and  $r \in R_t$ :

$$x_{t,r} = \begin{cases} 1 & \text{if } t \text{ uses } r, \\ 0 & \text{otherwise,} \end{cases}$$

for all triplets of  $t, t' \in T$  such that the index  $t$  is smaller than the index  $t'$ , and  $tc \in TC_t \cap TC_{t'}$ :

$$y_{t,t',tc} = \begin{cases} 1 & \text{if } t \text{ utilizes } tc \text{ before } t' (t \prec t'), \\ 0 & \text{otherwise } (t \succ t'). \end{cases}$$

In the formulation, the objective (1) is the minimization of the total weighted delays suffered by trains at their exit from the infrastructure. This weighting of the objective function allows taking into account different train priorities. These priorities may be linked to the type of circulation (e.g., freight or passenger) or other aspects as the number of passengers traveling on each train. A train  $t$  cannot be operated earlier than  $init_t$  (Constraints (2)). Constraints (3) indicate that the start time of track-circuit occupation along a route is zero

if the route itself is not used. A train starts occupying track-circuit  $tc$  along a route after spending in the preceding track-circuit its longer stay and its running time, if the route is used (Constraints (4)). Constraints (5) impose that a train must use exactly one route. The value of delay  $D_t$  at least equals the difference between the actual and the scheduled arrival times at the exit of the infrastructure (Constraints (6)). A train's utilization of a track-circuit starts as soon as the train starts occupying the track-circuit  $ref_{r,tc}$  along one of the routes including it, minus the formation time (Constraints (7)). The utilization of a track-circuit lasts till the train utilizes it along any route, plus the formation and the release time (Constraints (8)). The track-circuit utilizations by two trains must not overlap (Constraints (9) and (10)).

$$\min \sum_{t \in T} w_t D_t. \quad (1)$$

$$o_{t,r,tc} \geq init_t x_{t,r} \quad \forall t \in T, r \in R_t, tc \in TC^r : p_{r,tc} = tc_0, \quad (2)$$

$$o_{t,r,tc} \leq M x_{t,r} \quad \forall t \in T, r \in R_t, tc \in TC^r, \quad (3)$$

$$o_{t,r,tc} = o_{t,r,p_{r,tc}} + l_{t,r,p_{r,tc}} + r_{t,r,ty_t,p_{r,tc}} x_{t,r} \quad \forall t \in T, r \in R_t, tc \in TC^r : p_{r,tc} \neq tc_0, \quad (4)$$

$$\sum_{r \in R_t} x_{t,r} = 1 \quad \forall t \in T, \quad (5)$$

$$D_t \geq \sum_{r \in R_t} o_{t,r,tc_\infty} + \sum_{r \in R_t} r_{t,r,ty_t,tc_\infty} x_{t,r} + c_{t,r,ty_t,tc_\infty} x_{t,r} - exit_t \quad \forall t \in T, \quad (6)$$

$$sU_{t,tc} \leq \sum_{\substack{r \in R_t: \\ tc \in TC^r}} \left( o_{t,r,ref_{r,tc}} - for_{bs_r,tc} x_{t,r} \right) \quad \forall t \in T, tc \in TC_t, \quad (7)$$

$$eU_{t,tc} \geq \sum_{\substack{r \in R_t: \\ tc \in TC^r}} o_{t,r,ref_{r,tc}} + rel_{bs_r,tc} x_{t,r} + c_{t,r,ty_t,tc} x_{t,r} + r_{t,r,ty_t,tc} x_{t,r} + l_{t,r,tc} \quad \forall t \in T, tc \in TC_t, \quad (8)$$

$$eU_{t,tc} - M(1 - y_{t,t',tc}) \leq sU_{t',tc} \quad \forall t, t' \in T, t < t', tc \in TC_t \cap TC_{t'}, \quad (9)$$

$$eU_{t',tc} - M y_{t,t',tc} \leq sU_{t,tc} \quad \forall t, t' \in T, t < t', tc \in TC_t \cap TC_{t'}. \quad (10)$$

For a more detailed discussion of this formulation we refer the interested reader to Pellegrini et al. (2014, 2015).

### 3 RECIFE-MILP Benders' reformulation

By fixing routing and scheduling variables in the RECIFE-MILP formulation (1)-(10) (let them be  $\bar{x}_{t,r}$  and  $\bar{y}_{t,t',tc}$ ) we get the following Benders' Sub-Problem (BSP). In our Benders' decomposition algorithm, we do not solve the BSP (11)-(19): its dual is solved to generate Benders' cuts. Hence, we present also the formulation of the Dual of the Benders' Sub-Problem (DBSP). Based on the DBSP (20)-(33) formulation, we present the Restricted Benders' Master Problem (RBMP) that is the reformulation of RECIFE-MILP (1)-(10).

#### Benders' Sub-Problem (BSP)

$$\min \sum_{t \in T} w_t D_t. \quad (11)$$

$$o_{t,r,tc} \geq \text{init}_t \bar{x}_{t,r} \quad \forall t \in T, r \in R_t, tc \in TC^r : p_{r,tc} = tc_0, \quad (12)$$

$$o_{t,r,tc} \leq M \bar{x}_{t,r} \quad \forall t \in T, r \in R_t, tc \in TC^r, \quad (13)$$

$$-o_{t,r,tc} + o_{t,r,p_{r,tc}} + l_{t,r,p_{r,tc}} = -rt_{r,ty_t,p_{r,tc}} \bar{x}_{t,r} \quad \forall t \in T, r \in R_t, tc \in TC^r : p_{r,tc} \neq tc_0, \quad (14)$$

$$D_t - \sum_{r \in R_t} o_{t,r,tc_\infty} \geq -(\text{exit}_t - \sum_{r \in R_t} rt_{r,ty_t,tc_\infty} \bar{x}_{t,r} - ct_{r,ty_t,tc_\infty} \bar{x}_{t,r}) \quad \forall t \in T, \quad (15)$$

$$sU_{t,tc} - \sum_{\substack{r \in R_t: \\ tc \in TC^r}} o_{t,r,ref_{r,tc}} \leq - \sum_{\substack{r \in R_t: \\ tc \in TC^r}} \text{for}_{bs_{r,tc}} \bar{x}_{t,r} \quad \forall t \in T, tc \in TC_t, \quad (16)$$

$$eU_{t,tc} - \sum_{\substack{r \in R_t: \\ tc \in TC^r}} o_{t,r,ref_{r,tc}} - l_{t,r,tc} \geq \sum_{\substack{r \in R_t: \\ tc \in TC^r}} \text{rel}_{bs_{r,tc}} \bar{x}_{t,r} + ct_{r,ty_t,tc} \bar{x}_{t,r} + rt_{r,ty_t,tc} \bar{x}_{t,r} \\ \forall t \in T, tc \in TC_t, \quad (17)$$

$$eU_{t,tc} - sU_{t',tc} \leq M - M \bar{y}_{t,t',tc} \quad \forall t, t' \in T, t < t', tc \in TC_t \cap TC_{t'}, \quad (18)$$

$$eU_{t',tc} - sU_{t,tc} \leq M \bar{y}_{t,t',tc} \quad \forall t, t' \in T, t < t', tc \in TC_t \cap TC_{t'}. \quad (19)$$

Let  $\alpha_{t,r,tc}$ ,  $\beta_{t,r,tc}$ ,  $\lambda_{t,r,tc}$ ,  $\theta_t$ ,  $\phi_{t,tc}$ ,  $\omega_{t,tc}$ ,  $\eta_{t,t',tc}$ ,  $\psi_{t,t',tc}$ , be the dual variables associated with Constraints (12)-(19) respectively. The Dual of the Benders' Sub-Problem (DBSP) is written as follows.



### Dual of Benders' Sub-Problem (DBSP)

$$\begin{aligned}
& \max \left( \sum_{t \in T} \sum_{r \in R_t} \sum_{tc \in TC^r} \bar{x}_{t,r} (\text{init}_t \alpha_{t,r,tc} + M \beta_{t,r,tc} - r t_{r,ty_t,pr,tc} \lambda_{t,r,tc}) \right. \\
& + \sum_{t \in T} \sum_{r \in R_t} \sum_{tc \in TC_t} \bar{x}_{t,r} ((\text{rel}_{bsr,tc} + c t_{r,ty_t,tc} + r t_{r,ty_t,tc'}) \omega_{t,tc} - \text{for}_{bsr,tc} \phi_{t,tc}) \\
& \quad - \sum_{t \in T} (\text{exit}_t - \sum_{r \in R_t} \bar{x}_{t,r} (r t_{r,ty_t,tc_\infty} + c t_{r,ty_t,tc_\infty})) \theta_t \\
& \left. + \sum_{t \in T} \sum_{t' \in T} \sum_{tc \in TC_t \cap TC_{t'}} M ((1 - \bar{y}_{t,t',tc}) \eta_{t,t',tc} + \bar{y}_{t,t',tc} \psi_{t,t',tc}) \right). \quad (20)
\end{aligned}$$

$$\alpha_{t,r,tc} + \beta_{t,r,tc} + \lambda_{t,r,tc} - \sum_{tc \in TC_t} \phi_{t,tc} - \omega_{t,tc} \leq 0 \quad \forall t \in T, r \in R_t, tc \in TC^r : p_{r,tc} = tc_0, \quad (21)$$

$$\lambda_{t,r,tc} - \lambda_{t,r,pr,tc} + \beta_{t,r,tc} - \sum_{tc \in TC_t} \phi_{t,tc} - \omega_{t,tc} \leq 0 \quad \forall t \in T, r \in R_t, tc \in TC^r, \quad (22)$$

$$\lambda_{t,r,tc} - \sum_{tc \in TC_t} \omega_{t,tc} \leq 0 \quad \forall t \in T, r \in R_t, tc \in TC^r, \quad (23)$$

$$\alpha_{t,r,tc_\infty} + \beta_{t,r,tc_\infty} - \lambda_{t,r,tc_\infty} - \theta_t \leq 0 \quad \forall t \in T, r \in R_t, \quad (24)$$

$$\theta_t \leq 1 \quad \forall t \in T, \quad (25)$$

$$\phi_{t,tc} - \eta_{t,t',tc} - \psi_{t,t',tc} \leq 0 \quad \forall t, t' \in T, t < t', tc \in TC_t \cap TC_{t'}, \quad (26)$$

$$\omega_{t,tc} + \eta_{t,t',tc} + \psi_{t,t',tc} \leq 0 \quad \forall t, t' \in T, t < t', tc \in TC_t \cap TC_{t'}, \quad (27)$$

$$\alpha_{t,r,tc} \geq 0 \quad \forall t \in T, r \in R_t, tc \in TC^r, \quad (28)$$

$$\beta_{t,r,tc} \leq 0 \quad \forall t \in T, r \in R_t, tc \in TC^r, \quad (29)$$

$$\lambda_{t,r,tc} \in \mathbb{R} \quad \forall t \in T, r \in R_t, tc \in TC^r, \quad (30)$$

$$\omega_{t,tc}, \theta_t \geq 0 \quad \forall t \in T, tc \in TC_t, \quad (31)$$

$$\phi_{t,tc} \leq 0 \quad \forall t \in T, tc \in TC_t, \quad (32)$$

$$\eta_{t,t',tc}, \psi_{t,t',tc} \leq 0 \quad \forall t, t' \in T, t < t', tc \in TC_t \cap TC_{t'}. \quad (33)$$

Given the DBSP (20)-(33), we can generate the cuts that are necessary to reformulate the RECIFE-MILP formulation (1)-(10) as the Restricted Benders' Master Problem (RBMP).

### Restricted Benders' Master Problem (RBMP)

$$\min z \quad (34)$$

$$\begin{aligned} z - & \left( \sum_{t \in T} \sum_{r \in R_t} \sum_{tc \in TC_r} x_{t,r} (\text{init}_t \alpha_{t,r,tc} + M \beta_{t,r,tc} - r t_{r,ty_t,pr,tc} \lambda_{t,r,tc}) \right. \\ & + \sum_{t \in T} \sum_{r \in R_t} \sum_{tc \in TC_t} x_{t,r} ((\text{rel}_{bs_r,tc} + ct_{r,ty_t,tc} + r t_{r,ty_t,tc'}) \omega_{t,tc} - \text{for}_{bs_r,tc} \phi_{t,tc}) \\ & \quad \left. - \sum_{t \in T} (\text{exit}_t - \sum_{r \in R_t} x_{t,r} (r t_{r,ty_t,tc_\infty} + ct_{r,ty_t,tc_\infty})) \theta_t \right) \\ & + \sum_{t \in T} \sum_{t' \in T} \sum_{tc \in TC_t \cap TC_{t'}} M ((1 - y_{t,t',tc}) \eta_{t,t',tc} + y_{t,t',tc} \psi_{t,t',tc}) \geq 0 \\ & ((\alpha, \beta, \lambda, \theta, \phi, \omega, \eta, \psi) \in P_S) \end{aligned} \quad (35)$$

$$\begin{aligned} & \sum_{t \in T} \sum_{r \in R_t} \sum_{tc \in TC_r} x_{t,r} (\text{init}_t \alpha_{t,r,tc} + M \beta_{t,r,tc} - r t_{r,ty_t,pr,tc} \lambda_{t,r,tc}) \\ & + \sum_{t \in T} \sum_{r \in R_t} \sum_{tc \in TC_t} x_{t,r} ((\text{rel}_{bs_r,tc} + ct_{r,ty_t,tc} + r t_{r,ty_t,tc'}) \omega_{t,tc} - \text{for}_{bs_r,tc} \phi_{t,tc}) \\ & \quad - \sum_{t \in T} (\text{exit}_t - \sum_{r \in R_t} x_{t,r} (r t_{r,ty_t,tc_\infty} + ct_{r,ty_t,tc_\infty})) \theta_t \\ & + \sum_{t \in T} \sum_{t' \in T} \sum_{tc \in TC_t \cap TC_{t'}} M ((1 - y_{t,t',tc}) \eta_{t,t',tc} + y_{t,t',tc} \psi_{t,t',tc}) \leq 0 \\ & ((\alpha, \beta, \lambda, \theta, \phi, \omega, \eta, \psi) \in R_S) \end{aligned} \quad (36)$$

$$\sum_{r \in R_t} x_{t,r} = 1 \quad \forall t \in T, \quad (37)$$

$$x_{t,r} \in \{0, 1\} \quad \forall t \in T, r \in R_t, \quad (38)$$

$$y_{t,t',tc} \in \{0, 1\} \quad \forall t, t' \in T, t < t', tc \in TC_t \cap TC_{t'}, \quad (39)$$

$$z \geq 0. \quad (40)$$

The additional dummy variable  $z$  represents the contribution of the RECIFE-MILP continuous variables to the RBMP (34)-(40) objective function, while sets  $P_s$  and  $R_s$  contain the extreme points and extreme rays, respectively, of the polyhedron  $S$  representing the feasible solution space of the DBSP (20)-(33). The RBMP (34)-(40) has only one dummy variable  $z$  and the binary variables  $x_{t,r}$  and  $y_{t,t',tc}$  of the original problem, RECIFE-MILP (1)-(10). Note that the values of  $(\alpha, \beta, \lambda, \theta, \phi, \omega, \eta, \psi)$  are known because they are either the extreme points  $P_s$  or the extreme rays  $R_s$  calculated in the DBSP (20)-(33). Constraints (35) and (36) represent respectively the classic Benders' optimality and feasibility cuts. Constraints (37) represent the routing constraints equivalent to (5) in the RECIFE-MILP formulation (1)-(10). We have thus reformulated the RECIFE-MILP formulation (1)-(10) as an equivalent problem (RBMP (34)-(40)) with binary variables and one continuous dummy variable.

At the beginning of our RECIFE-MILP Benders' decomposition algorithm, we seek a feasible solution to the RBMP (34)-(40) by fixing sets  $P_s$  and  $R_s$  to empty ( $P_s := \emptyset$ ,  $R_s := \emptyset$ ). The problem obtained, when the constraints (35) and (36) are removed, is called the Unrestricted Benders' Master Problem (UBMP) which is formulated as follows.

#### Unrestricted Benders' Master Problem (UBMP)

$$\min z \quad (41)$$

$$\sum_{r \in R_t} x_{t,r} = 1 \quad \forall t \in T, \quad (42)$$

$$x_{t,r} \in \{0, 1\} \quad \forall t \in T, r \in R_t, \quad (43)$$

$$y_{t,t',tc} \in \{0, 1\} \quad \forall t, t' \in T, t < t', tc \in TC_t \cap TC_{t'}, \quad (44)$$

$$z \geq 0. \quad (45)$$

We tackle our reformulation of the RECIFE-MILP formulation (1)-(10) with two Benders' decomposition algorithms: the iterative algorithm and the integrated one. The integrated algorithm uses the lazy constraint callback method. Here, we embed the DBSP (20)-(33) into the branch-and-bound procedure of the UBMP (41)-(45). In particular, at any node where an incumbent is found, we run a callback in which the DBSP (20)-(32) is solved. If the DBSP (20)-(33) is unbounded we add the appropriate feasibility cut (Constraints (35)) to the UBMP (41)-(45). If the DBSP (20)-(33) is bounded we seek if its objective function value is equal to the value of the UBMP (41)-(45) objective function. If so, a new integer incumbent solution is found and it is tested for the optimality. If it is not optimal, we add the appropriate optimality cut (Constraints (36)). Thus, in the integrated algorithm instead

of iteratively solving the UBMP (41)-(45) and the DBSP (20)-(33), the whole problem is solved in one single integrated procedure. The algorithm stops when the optimal solution is found or when the computational time limit is elapsed. A similar approach is followed by Wu (2013) and Feizollahi et al. (2015).

For comparison, we also tackle the problem with the iterative algorithm in which the UBMP (41)-(45) and the DBSP (20)-(33) are iteratively solved at the optimum. At each iteration, we add to the UBMP (41)-(45) either a Benders' optimality cut or a Benders' feasibility cut depending the solution of the DBSP (20)-(33) until the optimum is found (the objective function value is the same for the UBMP (41)-(45) and the DBSP (20)-(33)) or when the computational time limit is elapsed.

Note that for the rtRTMP, the UBMP (41)-(45) cannot be infeasible: any solution that fixes a routing decision for each train ( $x$  variables) is feasible because there are no constraints on the scheduling ( $y$  variables). As a result, the infeasibility case of the UBMP (41)-(45) is not considered in the algorithms.

#### 4 Reduced RECIFE-MILP Benders' reformulation

By studying the rtRTMP structure, we realize that most  $x$ -variables in an incumbent solution of the UBMP (41)-(45) are zero. Hence, many constraints in the sub-problem are negligible: for example when we know the routing decision for each train, Constraints (13) in the BSP (11)-(19) become unnecessary because they are trivially satisfied or their role is simply to set some variables equal to zero. Hence, many variables (as  $\beta_{t,r,tc}$ ) and many constraints (as some Constraints (21) and (22)) in the DBSP (20)-(33) are also unnecessary. For the same reason, we can discard all constraints in the BSP (11)-(19) associated with the unused routes ( $\bar{x}_{t,r} = 0$ ). Moreover, we observe that, in the BSP (11)-(19), the disjunctive Constraints (18) and (19) are imposed for all track-circuits which may possibly be used by two trains  $t$  and  $t'$ . However, if the routes chosen for the two trains imply that a possibly shared track-circuit is used by either only one or none of them, the scheduling decision implied by the corresponding  $y$ -variable is actually negligible from a practical point of view: RECIFE-MILP will fix this variable, but this information will not be used for the practical implementation of the solution since, in reality, no precedence needs to be set unless both trains are passing in the same location. Hence, the value of these practically negligible  $y$ -variable can be disregarded in the BSP (11)-(19), together with the corresponding constraints. Remark that, when only one train uses a possibly shared track-circuit, only one assignment of the  $y$ -variable is feasible: if  $t$  is using it, then the variable must be equal to 0 since the utilization variables of  $t'$  must be set to 0; if  $t'$  is using it, then the variable must be set to 1. This implication is not explicit in the UBMP (41)-(45): it can generate solutions which result infeasible for the BSP (11)-(19) due to the negligible  $y$ -variables. To avoid the generation of Benders' feasibility cuts due to these variables, we remove from the BSP (11)-(19) the Constraints (18) and (19) which correspond to the track-circuits not actually used by each pair of trains. The result of the constraint and variable removals, from the BSP (11)-(19) and hence from the DBSP (20)-(33), is the Reduced RECIFE-MILP Benders' reformulation, whose details are described in the rest of this section.

In the Reduced RECIFE-MILP Benders' reformulation, we apply the same Benders' algorithms, the iterative algorithm and the integrated one, replacing the BSP (11)-(19) and the DBSP (20)-(33) by the Red-BSP (46)-(53) and the Red-DBSP (54)-(66) described in the following.

### Reduced Benders' Sub-Problem (Red-BSP)

Let,  $\bar{r}$  be the route used by train  $t$  and  $\bar{r}'$  the route used by train  $t'$ , where  $t, t' \in T$ . Remark, the set of track-circuits available for train  $t$  ( $TC_t$ ) is reduced to  $TC^{\bar{r}}$  (set of track-circuits composing the route  $\bar{r}$ ) and the set of track-circuits available for train  $t'$  ( $TC_{t'}$ ) is also reduced to  $TC^{\bar{r}'}$ . The Reduced Benders' Sub-Problem (Red-BSP) can then be formulated as follows.

$$\min \sum_{t \in T} w_t D_t. \quad (46)$$

$$o_{t,\bar{r},tc} \geq \text{init}_t \quad \forall t \in T, \bar{r}, tc \in TC^{\bar{r}} : p_{\bar{r},tc} = tc_0, \quad (47)$$

$$-o_{t,\bar{r},tc} + o_{t,\bar{r},p_{\bar{r},tc}} + l_{t,\bar{r},p_{\bar{r},tc}} = -rt_{\bar{r},ty_t,p_{\bar{r},tc}} \quad \forall t \in T, \bar{r}, tc \in TC^{\bar{r}} : p_{\bar{r},tc} \neq tc_0, \quad (48)$$

$$D_t - o_{t,\bar{r},tc_\infty} \geq -(\text{exit}_t - rt_{\bar{r},ty_t,tc_\infty} - ct_{\bar{r},ty_t,tc_\infty}) \quad \forall t \in T, \quad (49)$$

$$sU_{t,tc} - o_{t,\bar{r},ref_{\bar{r},tc}} \leq -for_{bs_{\bar{r},tc}} \quad \forall t \in T, tc \in TC^{\bar{r}}, \quad (50)$$

$$eU_{t,tc} - o_{t,\bar{r},ref_{\bar{r},tc}} - l_{t,\bar{r},tc} \geq rel_{bs_{\bar{r},tc}} + ct_{\bar{r},ty_t,tc} + rt_{\bar{r},ty_t,tc} \quad \forall t \in T, tc \in TC^{\bar{r}}, \quad (51)$$

$$eU_{t,tc} - sU_{t',tc} \leq M - M\bar{y}_{t,t',tc} \quad \forall t, t' \in T, t < t', tc \in TC^{\bar{r}} \cap TC^{\bar{r}'}, \quad (52)$$

$$eU_{t',tc} - sU_{t,tc} \leq M\bar{y}_{t,t',tc} \quad \forall t, t' \in T, t < t', tc \in TC^{\bar{r}} \cap TC^{\bar{r}'}. \quad (53)$$

As a consequence, the Red-DBSP (54)-(66) is the following.

### Reduced Dual of Benders' Sub-Problem (Red-DBSP)

$$\begin{aligned} & \max \left( \sum_{t \in T} \sum_{tc \in TC^{\bar{r}}} \text{init}_t \alpha_{t,\bar{r},tc} - rt_{\bar{r},ty_t,p_{\bar{r},tc}} \lambda_{t,\bar{r},tc} \right. \\ & + \sum_{t \in T} \sum_{tc \in TC_t} (rel_{bs_{\bar{r},tc}} + ct_{\bar{r},ty_t,tc} + rt_{\bar{r},ty_t,tc}) \omega_{t,tc} - for_{bs_{\bar{r},tc}} \phi_{t,tc} \\ & \quad \left. - \sum_{t \in T} (\text{exit}_t - rt_{\bar{r},ty_t,tc_\infty} + ct_{\bar{r},ty_t,tc_\infty}) \theta_t \right) \\ & + \sum_{t \in T} \sum_{t' \in T} \sum_{tc \in TC^{\bar{r}} \cap TC^{\bar{r}'}} M((1 - \bar{y}_{t,t',tc}) \eta_{t,t',tc} + \bar{y}_{t,t',tc} \psi_{t,t',tc}). \end{aligned} \quad (54)$$

$$\alpha_{t,\bar{r},tc} + \lambda_{t,\bar{r},tc} - \phi_{t,tc} - \omega_{t,tc} \leq 0 \quad \forall t \in T, \bar{r}, tc \in TC^{\bar{r}} : p_{\bar{r},tc} = tc_0, \quad (55)$$

$$\lambda_{t,\bar{r},tc} - \lambda_{t,\bar{r},p_{\bar{r},tc}} - \phi_{t,tc} - \omega_{t,tc} \leq 0 \quad \forall t \in T, \bar{r}, tc \in TC^{\bar{r}}, \quad (56)$$

$$\lambda_{t,\bar{r},tc} - \omega_{t,tc} \leq 0 \quad \forall t \in T, \bar{r}, tc \in TC^{\bar{r}}, \quad (57)$$

$$\alpha_{t,\bar{r},tc_\infty} - \lambda_{t,\bar{r},tc_\infty} - \theta_t \leq 0 \quad \forall t \in T, \bar{r}, \quad (58)$$

$$\theta_t \leq 1 \quad \forall t \in T, \quad (59)$$

$$\phi_{t,tc} - \eta_{t,t',tc} - \psi_{t,t',tc} \leq 0 \quad \forall t, t' \in T, t < t', tc \in TC^{\bar{r}} \cap TC^{\bar{r}'}, \quad (60)$$

$$\omega_{t,tc} + \eta_{t,t',tc} + \psi_{t,t',tc} \leq 0 \quad \forall t, t' \in T, t < t', tc \in TC^{\bar{r}} \cap TC^{\bar{r}'}, \quad (61)$$

$$\alpha_{t,\bar{r},tc} \geq 0 \quad \forall t \in T, \bar{r}, tc \in TC^{\bar{r}}, \quad (62)$$

$$\lambda_{t,\bar{r},tc} \in R \quad \forall t \in T, \bar{r}, tc \in TC^{\bar{r}}, \quad (63)$$

$$\omega_{t,tc}, \theta_t \geq 0 \quad \forall t \in T, tc \in TC^{\bar{r}}, \quad (64)$$

$$\phi_{t,tc} \leq 0 \quad \forall t \in T, tc \in TC^{\bar{r}}, \quad (65)$$

$$\eta_{t,t',tc}, \psi_{t,t',tc} \leq 0 \quad \forall t, t' \in T, t < t', tc \in TC^{\bar{r}} \cap TC^{\bar{r}'}. \quad (66)$$

Finally, we need also to change the formulation of the RBMP (34)-(40) to take into account the changes in the Red-DBSP (54)-(66). In particular, Constraints (35) and (36) become Constraints (67) and (68). The cuts now include only the variables which we considered in the Red-DBSP (54)-(66).

$$\begin{aligned}
& z - \left( \sum_{t \in T} \sum_{tc \in TC^{\bar{r}}} x_{t,\bar{r}} (\text{init}_t \alpha_{t,\bar{r},tc} - r t_{\bar{r},ty_t,p_{\bar{r},tc}} \lambda_{t,\bar{r},tc}) \right. \\
& + \sum_{t \in T} \sum_{tc \in TC^{\bar{r}}} x_{t,\bar{r}} ((rel_{bs_{\bar{r},tc}} + ct_{\bar{r},ty_t,tc} + r t_{\bar{r},ty_t,tc}) \omega_{t,tc} - for_{bs_{\bar{r},tc}} \phi_{t,tc}) \\
& \quad \left. - \sum_{t \in T} (\text{exit}_t - x_{t,\bar{r}} (r t_{\bar{r},ty_t,tc_\infty} + ct_{\bar{r},ty_t,tc_\infty})) \theta_t \right) \\
& + \sum_{t \in T} \sum_{t' \in T} \sum_{tc \in TC^{\bar{r}} \cap TC^{\bar{r}'}} M((1 - y_{t,t',tc}) \eta_{t,t',tc} + y_{t,t',tc} \psi_{t,t',tc}) \geq 0 \\
& \quad ((\alpha, \lambda, \theta, \phi, \omega, \eta, \psi) \in P_S) \tag{67}
\end{aligned}$$

$$\begin{aligned}
& \sum_{t \in T} \sum_{tc \in TC^{\bar{r}}} x_{t,\bar{r}} (\text{init}_t \alpha_{t,\bar{r},tc} - r t_{\bar{r},ty_t,p_{\bar{r},tc}} \lambda_{t,\bar{r},tc}) \\
& + \sum_{t \in T} \sum_{tc \in TC^{\bar{r}}} x_{t,\bar{r}} ((rel_{bs_{\bar{r},tc}} + ct_{\bar{r},ty_t,tc} + r t_{\bar{r},ty_t,tc}) \omega_{t,tc} - for_{bs_{\bar{r},tc}} \phi_{t,tc}) \\
& \quad - \sum_{t \in T} (\text{exit}_t - x_{t,\bar{r}} (r t_{\bar{r},ty_t,tc_\infty} + ct_{\bar{r},ty_t,tc_\infty})) \theta_t \\
& + \sum_{t \in T} \sum_{t' \in T} \sum_{tc \in TC^{\bar{r}} \cap TC^{\bar{r}'}} M((1 - y_{t,t',tc}) \eta_{t,t',tc} + y_{t,t',tc} \psi_{t,t',tc}) \leq 0 \\
& \quad ((\alpha, \lambda, \theta, \phi, \omega, \eta, \psi) \in R_S) \tag{68}
\end{aligned}$$

## 5 Computational experiments

In this section we assess the quality of the RECIFE-MILP Benders' reformulation and the Reduced RECIFE-MILP Benders' reformulation. The two reformulations are solved using the iterative algorithm and the integrated one. We consider a case-study representing traffic in the Rouen-Rive-Droite control area to run our experiments. This case-study is recognized to be difficult to tackle in the time limit imposed for the rtRTMP (Pellegrini et al., 2015). We set the computational time available for the optimization to one minute, three minutes and five minutes. The implementation is done using IBM ILOG CPLEX Concert Technology for C++ (IBM ILOG CPLEX version 12.6 (IBM, 2012)). The weight associated to train  $t$ 's delay is set to 1 for all trains, since we have no information on different trains' priority. The route formation and release times are 15 and 5 seconds for all block sections, respectively.

The control area including Rouen-Rive-Droite comprises six stations, with two to six platforms, and one bifurcation. The presence of multiple stations with several possible platform assignments implies the availability of a very large number of alternative routes. The existence of these routes is the main source of complexity of traffic management in this location, together with the presence of mix traffic. The control area is depicted in Figure 2. The 190 track-circuits compose 189 block sections and 11 347 routes. The one-day timetable considered includes 186 trains: 2 high-speed and 107 conventional passenger trains, 33 freight trains and 44 local movements.

From this one-day timetable, we create 100 random scenarios: 20% of trains, randomly selected, suffer a random delay between 5 and 15 minutes at their entrance in the control

area. We generate one rtRTMP instance from each of these 100 scenarios by considering all the trains entering the control area within an hour horizon. We set the time horizon from 6:00 am to 07:00 am. This time horizon corresponds to the morning peak hour.

The so obtained one-hour instances include between 10 and 13 trains (mean 11). Each train can use between 1 and 192 routes (mean 45), which translates into a RECIFE-MILP formulation with about 117 000 continuous variables, 78 000 binary variables and 364 000 constraints, for an instance including 10 trains. We run the experiments on a computer with eight Intel Xeon 3.5 Ghz processors and 128 GB RAM.

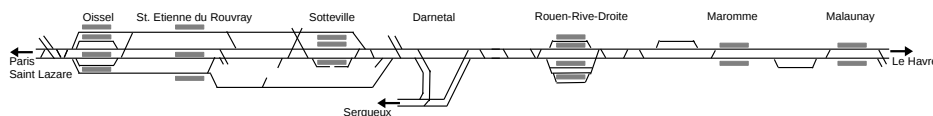


Figure 2: Rouen-Rive-Droite control area

In the next subsections, we compare first the two Benders' reformulations proposed: the RECIFE-MILP Benders' reformulation and the Reduced RECIFE-MILP Benders' reformulation. Second we compare the iterative algorithm to the integrated one. Finally we compare the best pair reformulation-algorithm to the RECIFE-MILP algorithm proposed by Pellegrini et al. (2015).

### 5.1 Comparison of the two Benders' reformulations

To compare the two Benders' reformulations proposed in this paper, we tackle the 100 instances described at the beginning of this section using the integrated algorithm. We stop the algorithm after 20 DBSP (20)-(33) solutions and we compare the computational time employed for the two reformulations. The results show that with the RECIFE-MILP Benders' reformulation, the computational time for solving the DBSP (20)-(33) is very long: 165 seconds in average. The corresponding quantity for the Reduce RECIFE-MILP Benders' reformulation is 0.31 seconds. Indeed, the former is not suitable for tackling the rtRTMP, where the available computational time is only of some minutes. The latter instead complies with the real-time nature of the problem. In the following, then, we will only consider the Reduced RECIFE-MILP Benders' reformulation for the experiments.

### 5.2 Comparison of the two Benders' algorithms: the iterative and the integrated ones

In this section, we compare the iterative algorithm to the integrated one to solve the Reduced RECIFE-MILP Benders' reformulation.

The computational analysis on the 100 instances previously presented, limiting the computation time to 5 minutes, shows that the integrated algorithm outperforms the iterative one. For each instance, the integrated algorithm finds better solutions than the iterative one. The integrated algorithm improves the quality of the objective function value of the iterative one by 77% in average.



Table 1: Performance of the Reduced RECIFE-MILP Benders' algorithm vs the RECIFE-MILP algorithm on 100 instances (time limit 60 seconds)

<b>Algorithms</b>	<b># optima</b>	<b># best solutions found</b>
RECIFE-MILP	1	96
Reduced RECIFE-MILP Benders'	0	4

Table 2: Performance of the Reduced RECIFE-MILP Benders' algorithm vs the RECIFE-MILP algorithm on 100 instances (time limit 180 seconds)

<b>Algorithms</b>	<b># optima</b>	<b># best solutions found</b>
RECIFE-MILP	9	97
Reduced RECIFE-MILP Benders'	0	3

### 5.3 Reduced RECIFE-MILP Benders' algorithm versus RECIFE-MILP algorithm

In this section, we compare the performance of our Reduced RECIFE-MILP Benders' algorithm to the RECIFE-MILP one. We choose the RECIFE-MILP algorithm as the benchmark because it is an efficient algorithm for the rtRTMP.

In Table 1, 2 and 3, we report the results achieved on the 100 instances considered, setting the computational time available to one, three and five minutes, respectively. The first column indicates the name of the algorithm. The two last columns contain the number of instances solved to the optimum within the available computational time and the number of instances in which each algorithm found the best solution, respectively. The results show that the RECIFE-MILP algorithm achieves the best performance for more cases than the Reduced RECIFE-MILP Benders' algorithm. For the instances on which the Reduced RECIFE-MILP Benders' algorithm is the best, we observe that the percentage of optimality cuts with respect to the total number of Benders' cuts is very high: 97% in average. Instead, when the RECIFE-MILP algorithm is the best, we remark that this percentage decreases: 85% in average. Hence, we conjecture that the performance of the Reduced RECIFE-MILP Benders' algorithm would improve if the percentage of the optimality cuts generated during the integrated branch-and-bound process increased. For the instances in which the algorithms get a feasible but sub-optimal solution, the gap is still high: 98% in average for the RECIFE-MILP algorithm and 100% in average for the Reduced RECIFE-MILP Benders' algorithm. This is due to the weakness of the linear relaxation of the formulation, which we will investigate deeply in future works.

Table 3: Performance of the Reduced RECIFE-MILP Benders' algorithm vs the RECIFE-MILP algorithm on 100 instances (time limit 300 seconds)

<b>Algorithms</b>	<b># optima</b>	<b># best solutions found</b>
RECIFE-MILP	10	96
Reduced RECIFE-MILP Benders'	0	4

## 6 Conclusion and future work

In this paper, we proposed two reformulations of RECIFE-MILP for the rtRTMP: the RECIFE-MILP Benders' reformulation and the Reduced RECIFE-MILP Benders' reformulation. We solve them through two Benders' decomposition algorithms, the iterative and the integrated one. The latter implements the lazy constraint callback that is the integration of the Benders algorithm into the branch-and-bound procedure. We tackle large and difficult instances representing traffic at the Rouen-Rive-Droite control area, in France. The results show that the RECIFE-MILP Benders' reformulation is not appropriate for this problem. Instead, the Reduced RECIFE-MILP Benders' reformulation can be appropriate thanks to its better computational performance. Moreover, the integrated algorithm outperforms the iterative one. Nevertheless, the classic RECIFE-MILP achieves the best result in the majority of the instances tackled. We conjecture that the Benders' algorithm performance would improve if the percentage of optimality cuts generated increased. In future works, we will focus on how to generate strong inequalities in the unrestricted Benders' master problem. These inequalities must make the reduced dual of Benders' sub-problem bounded and warrant the generation of optimality cuts.

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