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**Periodic solutions of partial degenerate differential
equations with n delay**

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Introduction

Ce travail est motivé par l'étude des équations différentielles fonctionnelles à retard. Le **retard** est défini comme étant la propriété d'un système physique pour lequel la réponse à une action appliquée est retardée dans son effet par exemple le retard du **Tonner-eclair**(visuelle-sonore). Pour un observateur, le tonnerre ne se manifeste pas en même temps que l'éclair (retard d'un tonnerre). Si l'orage est trop loin, on n'entend pas le tonnerre (retard infini).

Les équations différentielles fonctionnelles à retard sont apparues dans différentes disciplines scientifiques (dynamique de population, trafic urbain, physique, biologie,..)

Modèle en Biologie

$$\frac{d}{dt}N(t) = -dN(t) + bN(t - r)$$

où d et b sont respectivement les taux de mortalité et de naissance de la population de cellule $N(t)$.

r est la durée de gestation moyenne de la population.

Problème

In this work, we study the existence of strong solutions for the following degenerate partial functional differential equation with n delay

$$\begin{cases} \frac{d}{dt} Mx(t) = Ax(t) + \sum_{j=1}^n Bx(t - r_j) + f(t) \\ x(0) = x(2\pi). \end{cases} \quad (1)$$

where $f \in L^p([-r_{2\pi}, 0], X)$ for some $1 \leq p < \infty$, $r_{2\pi} = 2\pi N$ ($N \in \mathbb{N}$) and we suppose A , B and M are a closed linears operators such that $D(A) \subset D(B) \cap D(M)$. u_t denotes the history function in $C([-r, 0]; X)$ defined by $x_t(\theta) = x(t + \theta)$ for $\theta \in [-r, 0]$.

Definition : R -bounded

Let X and Y be Banach spaces. A family of operators $T = (T_j)_{j \in \mathbb{N}^*} \subset B(X, Y)$ is called R -bounded, if there is a constant $C > 0$ and $p \in [1, \infty)$ such that for each $n \in \mathbb{N}$, $T_j \in T$, $x_j \in X$ and for all independent, symmetric, $\{-1, 1\}$ -valued random variables r_j on a probability space (Ω, M, μ) the inequality

$$\left\| \sum_{j=1}^n r_j T_j x_j \right\|_{L^p(0,1;Y)} \leq C \left\| \sum_{j=1}^n r_j x_j \right\|_{L^p(0,1;X)}$$

is valid. The smallest C is called R -bounded of $(T_j)_{j \in \mathbb{N}^*}$ and it is denoted by $R_p(T)$.

Example

Let $r > 0$ be fixed, $L \in B(L^p([-r, 0], X); X)$ and $k \in \mathbb{Z}$, we define the operator L_k on X by : $L_k x = L(e_k x)$ for all $x \in X$. We have $(L_k)_{k \in \mathbb{Z}}$ is R -bounded and $R_p(L_k) \leq r^{1/p}$.

Definition

Let $f \in L^p(\mathbb{T}; X)$. A function $x \in H^{1,p}(\mathbb{T}; X)$ is said to be a 2π -periodic strong L^p -solution of Eq. (1) if $x(t) \in D(A)$ for all $t \geq 0$ and Eq. (1) holds almost every where.

Necessary condition

Proposition

Let X be a **Banach space**. Suppose that for every $f \in L^p(\mathbb{T}; X)$ there exists a unique strong solution of Eq. (1) for $1 \leq p < \infty$. Then

- 1 for every $k \in \mathbb{Z}$ the operator $\Delta_k = (ikM - A - \sum_{j=1}^n B_{j,k})$ has bounded inverse
- 2 $\{ik\Delta_k^{-1}\}_{k \in \mathbb{Z}}$ is R -bounded.

Condition sufficient

Definition : UMD Space

A Banach space X is said to be UMD space if the Hilbert transform is bounded on $L^p(\mathbb{R}; X)$ for all $1 < p < \infty$.

Example

1. Any Hilbert space is an UMD space.
2. $L^p(0,1, X)$ are UMD spaces for every $1 < p < \infty$
3. $L^4(0,1, X)$ is UMD space, the bound $C_4 \leq 1 + \sqrt{2}$
4. $L^8(0,1, X)$ is UMD space, the bound $C_8 \leq 1 + \sqrt{2} + \sqrt{4 + 2\sqrt{2}}$

Theorem

Let X be an UMD space and $A : D(A) \subset X \rightarrow X$ be a closed linear operator. Then the following assertions are equivalent for $1 < p < \infty$.

- 1 for every $f \in L^p(\mathbb{T}, X)$ there exists a unique strong L^p -solution of Eq.(1).
- 2 $\sigma_{\mathbb{Z}}(\Delta) = \phi$ and $\{ik\Delta_k^{-1}\}_{k \in \mathbb{Z}}$ is R-bounded.
$$\Delta_k = (ikM - A - \sum_{j=1}^n B_{j,k})$$

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Merci pour votre attention