LÉVY NMF FOR ROBUST NONNEGATIVE SOURCE SEPARATION

Paul Magron,[1,2] *Roland Badeau,*[2] *Antoine Liutkus,*[3] *

[1] Signal Processing Laboratory, Tampere University of Technology (TUT), Finland, paul.magron@tut.fi
[2] LTCI, Télécom ParisTech, Université Paris-Saclay, Paris, France, roland.badeau@telecom-paristech.fr
[3] Inria, Nancy Grand-Est, Multispeech team, LORIA UMR 7503, France, antoine.liutkus@inria.fr

ABSTRACT

Source separation, which consists in decomposing data into meaningful structured components, is an active research topic in many fields including music signal processing. In this paper, we introduce the Positive α-stable (PαS) distributions to model the latent sources, which are a subclass of the stable distributions family. They notably permit us to model random variables that are both nonnegative and impulsive. Considering the Lévy distribution, the only PαS distribution whose density is tractable, we propose a mixture model called Lévy Nonnegative Matrix Factorization (Lévy NMF). This model accounts for low-rank structures in nonnegative data that possibly has high variability or is corrupted by very adverse noise. The model parameters are estimated in a maximum-likelihood sense. We also derive an estimator of the sources, which extends the validity of the Wiener filtering to the PαS case. Experiments on synthetic data and realistic music signals show that Lévy NMF compares favorably with state-of-the art techniques in terms of robustness to impulsive noise and highlight its potential for decomposing nonnegative data.

Index Terms— Lévy distribution, Positive alpha-stable distribution, nonnegative matrix factorization, audio source separation.

1. INTRODUCTION

Source separation consists in extracting underlying components called *sources* that add up to form an observable signal called *mixture*. It has applications in many fields, including data analysis or audio processing [1].

A groundbreaking idea presented in [2] is to exploit the fact that the observations are often nonnegative, and to decompose them as a sum of nonnegative terms only, permitting only constructive interactions of the latent factors. This Nonnegative Matrix Factorization (NMF) has shown successful in many fields of audio signal processing such as automatic transcription [3] and source separation [4]. NMF, originally introduced as a rank-reduction method, approximates a nonnegative data matrix X as a product of two low-rank nonnegative matrices W and H. The factorization can be obtained by optimizing a cost function measuring the error between X and WH, such as the Euclidean, Kullback-Leibler (KL [2]) and Itakura-Saito (IS [4]) cost functions. This may often be framed in a probabilistic framework, where the cost function appears as the negative log-likelihood of the data, e.g. Gaussian [4] or Poisson [5]. Addressing the estimation problem in a Maximum A Posteriori (MAP) sense makes it possible to incorporate some prior distribution over the parameters W and H in order to enforce desirable properties for the parameters such as harmonicity, temporal smoothness [6] or sparsity [7].

However, the above-mentioned distributions fail to provide good results when the data is very impulsive or contains outliers. This comes from their rapidly decaying tails, that cannot account for really unexpected observations. The family of heavy-tailed *stable* distributions [8] was thus found useful for robust signal processing [9]. A subclass of this family, called the Symmetric α-stable (SαS) distributions, has been used in audio [10, 11] for modeling complex-valued Short-Term Fourier Transform (STFT). An estimation framework based on Markov Chain Monte Carlo (MCMC) methods has been proposed [12] to perform the separation of SαS mixtures. The common ground of these methods is to assume all observations as independent but to impose low-rank constraints on the nonnegative *dispersion parameters* of the sources, and not on their actual outcomes.

In this paper, we address the problem of modeling and separating nonnegative sources from their mixture, while still constraining their dispersion to follow an NMF model. To do so, we consider another subclass of the stable family which models nonnegative random variables: the Positive α-stable (PαS) distributions. They also benefit from being heavy-tailed and are thus expected to yield robust estimates. Since the Probability Density Function (PDF) of those PαS distributions does not admit a closed-form expression in general, we study more specifically the Lévy case, which is a particular analytically tractable member of the family. We introduce the *Lévy NMF* model, where the dispersion parameters of the sources are structured through an NMF model and where realizations are necessarily nonnegative. The parameters are then estimated in a Maximum Likelihood (ML) sense by means of a Majorize-Minimization (MM) approach. We also derive an estimator of the sources which extends the validity of the generalized Wiener filtering to the PαS case. Several experiments conducted on synthetic and realistic music signals show the potential of this model for a nonnegative source separation task and highlight its robustness to impulsive noise.

This paper is structured as follows. Section 2 introduces the Lévy NMF mixture model. Section 3 details the parameters estimation and presents an estimator of the sources. Section 4 experimentally demonstrates the denoising ability of the model and its potential in terms of source separation for musical applications.

2. LÉVY NMF MODEL

2.1. Positive α-stable distributions

Stable distributions, denoted $\alpha\mathcal{S}(\mu, v = \sigma^\alpha, \beta)$, are heavy-tailed distributions parametrized by four parameters: a shape parameter $\alpha \in]0; 2]$ which determines the tails thickness of the distribution

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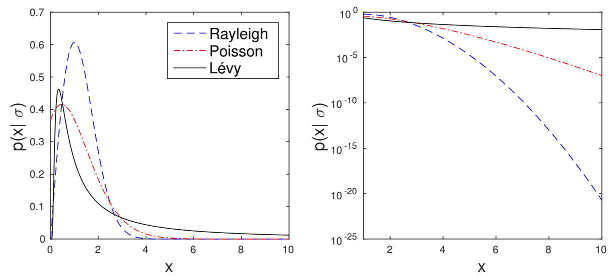

Figure 1: PDF of several distributions with a nonnegative support on a linear (left) and logarithmic (right) scale.

(the smaller α, the heavier the tail of the PDF), a location parameter $\mu \in \mathbb{R}$, a scale parameter $\sigma \in]0;+\infty[$ measuring the dispersion of the distribution around its mode, and a skewness parameter $\beta \in [-1;1]$. For convenience, we define everywhere in this study $v = \sigma^\alpha$. The SαS distributions, which are such that $\beta = 0$, are an important subclass of this family and a growing topic of interest in audio [10, 12].

Such distributions are said "stable" because of their additive property: a sum of K independent stable random variables $X_k \sim \alpha\mathcal{S}(\mu_k, v_k, \beta)$ is also stable: $X = \sum_k X_k \sim \alpha\mathcal{S}(\mu, v, \beta)$, with $\mu = \sum_k \mu_k$ and $v = \sum_k v_k$.

Stable distributions do not in general have a nonnegative support. However, it can be shown [13] that when $\beta = 1$ and $\alpha < 1$, the support of the distribution is $[\mu; +\infty[$. In this paper, we consider that $\mu = 0$ thus the support is $\mathbb{R}_+$. The PαS distributions are therefore such that $\mathcal{P}\alpha\mathcal{S}(v) = \alpha\mathcal{S}(0, v, 1)$ with $\alpha < 1$.

2.2. Lévy NMF mixture model

The only α for which the PDF of a PαS distribution can be expressed in closed form is the Lévy case $\mathcal{L}(v) = \mathcal{P}\frac{1}{2}\mathcal{S}(v)$:

$$p(x \mid v = \sqrt{\sigma}) = \begin{cases} \sqrt{\dfrac{\sigma}{2\pi}} \dfrac{1}{x^{3/2}} e^{-\frac{\sigma}{2x}} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

As it can be seen in Fig. 1, the Lévy distribution has a heavier tail than the most commonly used distributions for nonnegative data modeling (the Rayleigh distribution is non-other that the distribution of the modulus of a complex Gaussian variable).

Let us consider a nonnegative data matrix $X \in \mathbb{R}_+^{F\times T}$ (it can be, for instance, the modulus of the STFT of an audio signal). Each entry of X is modeled as the sum of K independent Lévy-distributed components $X_k(f,t) \sim \mathcal{L}(v_k(f,t))$. Note that all entries are independent, which allows us to extend the standard notation to matrices. Then, $X \sim \mathcal{L}(v)$ with $v = \sum_k v_k$.

The scale parameters are structured by means of an NMF [2], which preserves the additive property of the model as in [11]: $v = WH$,with $W \in \mathbb{R}_+^{F\times K}$ and $H \in \mathbb{R}_+^{K\times T}$. We then refer to this model as the *Lévy NMF* model.

3. PARAMETERS ESTIMATION

The parameters W and H are estimated in an ML sense, which is natural in a probabilistic framework. The log-likelihood of the data is given by:

$$\begin{aligned} L(W,H) &\overset{c}{=} \frac{1}{2}\sum_{f,t} \log([WH](f,t)^2) - \frac{[WH](f,t)^2}{X(f,t)} \\ &\overset{c}{=} -\frac{1}{2} d_{IS}([WH]^{\odot 2}, X), \end{aligned}$$

where $\overset{c}{=}$ denotes equality up to an additive constant which does not depend on the parameters, $\cdot^{\odot 2}$ is entry-wise exponentiation and d_{IS} denotes the IS divergence [4]. We thus remark that maximizing the log-likelihood of the data in the Lévy NMF model is equivalent to minimizing the IS divergence between $[WH]^{\odot 2}$ and X, which boils down to minimizing the following cost function:

$$\mathcal{C}(W,H) = \sum_{f,t} \frac{[WH](f,t)^2}{X(f,t)} - 2\log([WH](f,t)). \tag{2}$$

3.1. Naive multiplicative updates

The cost function (2) can be minimized with the same heuristic approach that has been pioneered in [2] and used in many NMF-related papers in the literature. The gradient of $\mathcal{C}$ with respect to a parameter θ (W or H) is expressed as the difference between two nonnegative terms: $\nabla_\theta \mathcal{C} = \nabla_\theta^+ - \nabla_\theta^-$, which leads to the multiplicative update rules:

$$\theta \leftarrow \theta \odot \frac{\nabla_\theta^-}{\nabla_\theta^+} = \theta \odot a_\theta, \tag{3}$$

where $\odot$ (resp. the fraction bar) denotes the element-wise matrix multiplication (resp. division). For Lévy NMF:

$$a_W = \frac{[WH]^{\odot -1} H^T}{([WH] \odot X^{\odot -1})H^T}, \tag{4}$$

and

$$a_H = \frac{W^T [WH]^{\odot -1}}{W^T([WH] \odot X^{\odot -1})}. \tag{5}$$

Provided W and H have been initialized as nonnegative, they remain so throughout iterations. Even though this methodology often leads to a non-increasing cost function, this monotonicity is not guaranteed in general, and does not hold in the particular case of Lvy NMF, which motivates the research for a novel optimization approach.

3.2. Majorize-Minimization updates

We then propose to adopt an MM approach [14] to derive novel update rules. The core idea of this strategy is to find an auxiliary function G which majorizes the cost function $\mathcal{C}$:

$$\forall(\theta, \overline{\theta}), \mathcal{C}(\theta) \leq G(\theta, \overline{\theta}), \text{ and } \mathcal{C}(\overline{\theta}) = G(\overline{\theta}, \overline{\theta}). \tag{6}$$

Then, given some current parameter $\overline{\theta}$, we aim at minimizing $G(\theta, \overline{\theta})$ in order to obtain a new parameter θ. This approach guarantees that the cost function $\mathcal{C}$ will be non-increasing over iterations. Such an auxiliary function is obtained in a similar fashion as in [15, 16]. We introduce the auxiliary parameter $\overline{W}$ and the weight:

$$\rho_k(f,t) = \frac{\overline{W}(f,k)H(k,t)}{\overline{V}(f,t)}, \text{ with } \overline{V} = \overline{W}H. \tag{7}$$

Since $\sum_k \rho_k(f,t) = 1$, we can apply the Jensen's inequality to the convex function $x \mapsto x^2$:

$$[WH](f,t)^2 = \left(\sum_k \rho_k(f,t) \frac{W(f,k)H(k,t)}{\rho_k(f,t)} \right)^2 \tag{8}$$

$$\leq \sum_k \rho_k(f,t) \left(\frac{W(f,k)H(k,t)}{\rho_k(f,t)} \right)^2, \tag{9}$$

which leads to the majorization of the first term of $\mathcal{C}$ in (2):

$$\sum_{f,t} \frac{[WH](f,t)^2}{X(f,t)} \leq \sum_{f,t} \frac{\overline{V}(f,t)}{X(f,t)} \sum_k \frac{W(f,k)^2 H(k,t)}{\overline{W}(f,k)}. \tag{10}$$

In a similar fashion, we majorize the second term in (2) ($x \mapsto -\log(x)$ is also a convex function) and therefore we obtain the following auxiliary function G:

$$G(W,H,\overline{W}) = \sum_{f,t,k} \frac{\overline{V}(f,t)H(k,t)}{X(f,t)\overline{W}(f,k)} W(f,k)^2 - 2\frac{\overline{W}(f,k)H(k,t)}{\overline{V}(f,t)} \log \left(W(f,k) \frac{\overline{V}(f,t)}{\overline{W}(f,k)} \right). \tag{11}$$

We then set the partial derivative of G with respect to W to zero, which leads to an update rule on W. The update rule on H is obtained in exactly the same way. Thus, the updates are:

$$\theta \leftarrow \theta \odot a_\theta^{\odot 1/2}, \tag{12}$$

where a_W and a_H are given by (4) and (5). A MATLAB implementation of this algorithm can be found at [17].

Remark: Let us assume that $K = 1$ and that $\forall f$, $W(f) = 1$. Then the Lévy NMF and ISNMF [4] updates on H rewrite:

$$H_{\text{Lévy}}(t) \leftarrow \sqrt{\frac{F}{\sum_f \frac{1}{X(f,t)}}} \text{ and } H_{\text{IS}}(t) \leftarrow \frac{1}{F} \sum_f X(f,t). \tag{13}$$

Therefore, the update on H in the Lévy NMF (resp. ISNMF) model boils down to an harmonic (resp. arithmetic) mean on X. Since the harmonic mean is known to attenuate the impact of strong outliers (compared to the arithmetic mean), the corresponding Lévy NMF procedure is expected to yield robust estimates.

3.3. Estimator of the components

For a source separation task, it can be useful, once the model parameters are estimated, to derive an estimator $\hat{X}_k$ of the isolated components X_k. In a probabilistic framework, a natural estimator is given by the posterior expectation of the source given the mixture $\mathbb{E}(X_k|X)$. For SαS random variables, such an estimator is provided by a generalized Wiener filtering [10]. One contribution of this paper is to prove that this result still holds for the PαS family. Derivations may be found in the companion technical report for this paper [18]. For Lévy-distributed random variables ($\alpha = 1/2$), we then have:

$$\hat{X}_k = \mathbb{E}(X_k|X) = \frac{v_k}{\sum_l v_l} \odot X = \frac{W_k H_k}{WH} \odot X. \tag{14}$$

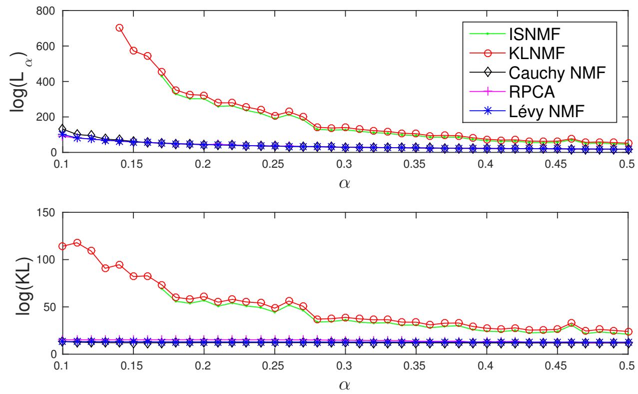

Figure 2: Fitting impulsive noise: estimation quality.

4. EXPERIMENTAL EVALUATION

4.1. Fitting impulsive noise

To test the ability of Lévy NMF to model impulsive noise, we have generated 5 components' pairs for W and H by taking the 4th power of random Gaussian noise, in order to obtain sparse components. The entries of the product $[WH]$, of dimensions 50×50, were then used as the scale parameters of independent PαS random observations, for various values of α in the range $0.1 - 0.5$: small values of α lead to very impulsive observations. We ran 200 iterations of the Lévy, IS [4], KL [2] and Cauchy [11] NMFs, and RPCA [19, 20] algorithms with rank $K = 5$. To measure the quality of the estimation, we computed the KL divergence and the α-dispersion, defined as a function of the data shape parameter α:

$$L_\alpha = \sum_{f,t} |\sigma(f,t) - \hat{\sigma}(f,t)|^{1/\alpha}, \tag{15}$$

where $\sigma^{\odot\alpha} = WH$ (resp. $\hat{\sigma}^{\odot\alpha} = \hat{W}\hat{H}$) contains the synthetic (resp. estimated) parameters. Results averaged over 100 synthetic data runs are presented in Fig. 2. The Lévy NMF algorithm shows very similar results to those obtained using RPCA or Cauchy NMF, with slightly better results than these methods for very small values of α. The reconstruction quality is considerably better than with ISNMF and KLNMF. Those results demonstrate the potential of the Lévy NMF model for fitting data which may be very impulsive.

4.2. Music spectrogram inpainting

We propose to test the denoising ability of the Lévy NMF model when the data is corrupted by very impulsive noise. When audio spectrograms are corrupted by such noise, the retrieval of the lost information is known as an *audio inpainting* task. We consider 6 guitar songs from the IDMT-SMT-GUITAR [21] database, sampled at 8000 Hz. The data X is obtained by taking the magnitude spectrogram of the STFT of the mixture signals, computed with a 125 ms-long Hann window and 75 % overlap. The spectrograms are then corrupted with synthetic impulsive noise that represents 10 % of the data. We run 200 iterations of the algorithms with rank $K = 30$ in order to estimate the clean spectrograms.

We present the obtained spectrograms in Fig. 3 (KLNMF and ISNMF lead to similar results). The traditional NMF techniques are not able to denoise the data: the estimation of the parameters is de-

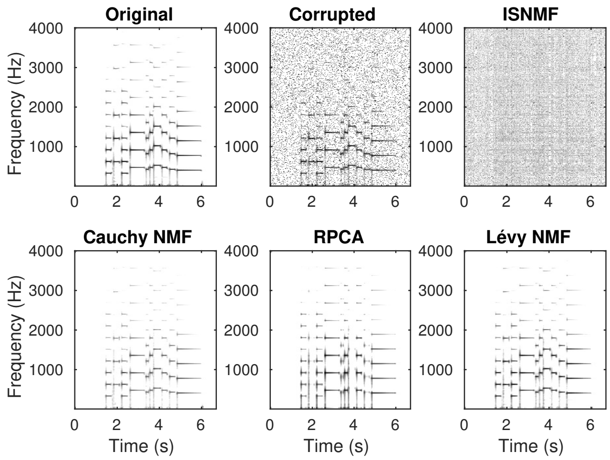

Figure 3: Music spectrogram restoration.

Table 1: Audio inpainting performance

	Log(KL)	SDR (dB)
ISNMF	9.0	−23.5
KLNMF	6.2	−8.9
Cauchy NMF	3.4	7.6
RPCA	3.6	7.4
Lévy NMF	**3.2**	**9.2**
Weighted ISNMF	3.8	4.5

teriorated by the presence of impulsive noise. Conversely, the noise has been entirely removed in the Lévy NMF estimate. This is confirmed in the second column of Table 1 which presents the quality of the estimation measured with the KL divergence between the original and estimated spectrograms averaged over the 6 audio excerpts. The best results are obtained with Lévy and Cauchy NMFs.

Remarkably, none of the algorithms used above is informed with any prior knowledge about the location of the noise. As a complementary experiment, we informed ISNMF with this knowledge by incorporating a mask containing the position of the noise into the NMF, resulting into a weighted ISNMF [22]. The blind Lévy NMF still leads to better results than the informed ISNMF.

Finally, the phase of the mixture is assigned to the estimated STFT in order to resynthesize time signals. Note that this is theoretically justified only for ISNMF and Cauchy NMF models: improved separation quality could therefore be reached by using an additional phase retrieval algorithm such as [23]. The quality of the estimation is measured by means of the Signal to Distortion Ratio (SDR) computed with the BSS Eval toolbox [24] and presented in the last column of Table 1. We remark that Lévy NMF outperforms all the other approaches in terms of SDR, but also in a subjective perceptual evaluation (the interested reader can find some sound excerpts at [25]). It is thus promising for such an audio inpainting application, since it does not require any additional noise modeling or detection technique.

4.3. Musical accompaniment enhancement

We propose to test the performance of the Lévy NMF model for enhancing the accompaniment in music signals. We consider 50 music

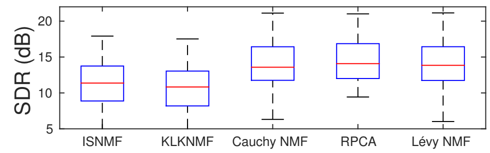

Figure 4: Musical accompaniment enhancement quality. Each boxplot is made up of a central line indicating the median of the data, upper and lower box edges indicating the 1^{st} and 3^{rd} quartiles, and whiskers indicating the minimum and maximum values.

excerpts of various genres from the DSD100 database (a remastered version of the database used for the SiSEC 2015 campaign [26]). Each track consists of a singing voice signal and a musical accompaniment signal. While musical accompaniment is assumed to be well represented by a low-rank NMF model, voice is assumed to be similar to impulsive noise. Thus, when fitting a model that is robust to such a noise on the data, we expect that the voice will be treated as noise, therefore leading to enhance the musical accompaniment.

The data X is obtained by taking the magnitude spectrogram of the STFT of the mixture signals, computed as in the previous experiment. We then run 200 iterations of the algorithms with a rank of factorization $K = 30$, and we estimate the sources by means of the generalized Wiener filtering (14). The phase of the mixture is then assigned to the estimated musical accompaniment source in order to resynthesize time signals. The quality of the enhancement is measured by means of the SDR between the original and estimated accompaniment tracks and presented in Fig. 4.

Musical accompaniment enhancement performed with Lévy NMF leads to better results than traditional NMF techniques. An informal subjective evaluation of the synthesized signals (sounds examples available at [25]) shows that ISNMF and KLNMF methods do not lead to a significant attenuation of the voice, while with the other techniques, the voice is hardly perceptible, at the cost of very few artifacts. Lévy NMF competes with other robust methods such as Cauchy NMF and RPCA, and thus appears to be a good candidate for robust musical applications.

5. CONCLUSION

In this paper, we introduced the Lévy NMF model, which structures the dispersion parameters of PαS distributed sources when $\alpha = 1/2$. Experiments have shown the potential of this model for robustly decomposing realistic nonnegative data.

Such a model could be useful in many other fields where the source separation issue frequently occurs and where the Lévy distribution finds applications, such as optics [27]. Future work could focus on novel estimation techniques for the Lévy NMF model, using for instance a MAP estimator, which would permit us to incorporate some prior knowledge about the parameters [6]. Besides, drawing on [12], the family of techniques based on MCMC could be useful to estimate the parameters of any PαS distribution. Alternatively, one could extend the Lévy NMF model to the family of inverse gamma distributions it belongs to. Although this would result in losing the additivity property and thus the theoretical foundation for α-Wiener filtering, this would allow for convenient analytical derivations thanks to tractable likelihood functions. This strategy is reminiscent of recent work [28] where the tractable Student-t distribution is used instead of the SαS one.

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