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► **To cite this version:**

E. Nefzaoui, Younes Ezzahri, Karl Joulain, Jérémie Drevillon. Tunable Radiative Thermal Rectifiers: Toward Thermal Logical Circuits. The 15th International Heat Transfer Conference (IHTC-15), Aug 2015, Kyoto, Japan. hal-01546293

HAL Id: hal-01546293

<https://hal.science/hal-01546293>

Submitted on 23 Jun 2017

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TUNABLE RADIATIVE THERMAL RECTIFIERS : TOWARD THERMAL LOGICAL CIRCUITS

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ABSTRACT

Thermal rectification can be defined as an asymmetry in the heat flux when the temperature difference between two interacting thermal reservoirs is reversed. We present a thermal rectifier concept based on far-field radiative heat transfer. The device is composed of two opaque thermal baths 1 and 2 at temperatures T_1 and T_2 respectively and exchanging heat through thermal radiation. The two interacting bodies are made of spectrally selective photonic structures with different thermo-optical properties. First, we show that the mismatch between the two structures optical properties temperature dependence is at the origin of thermal rectification. Then, we show that rectification ratio of a given device can be simply tuned with a few parameters such as operating temperature, the emission peak position and width. Presented results pave the way to the use of widely studied photonic structures in thermal management application and thermal logical circuits development.

KEY WORDS: Thermal management, Thermal insulation, Thermal storage, Energy conversion and storage, Energy efficiency Thermophysical properties, Nano/Micro scale measurement and simulation, Photon, phonon and electron transport, Radiation, Cryogenics, Heat transfer enhancement.

1. INTRODUCTION

Thermal rectification can be defined as an asymmetry in the heat flux when the temperature difference between two interacting thermal reservoirs is reversed. A non zero rectification means that a reversal of the thermal gradient induces, in addition to the reversal of the heat flux direction, a variation of its magnitude. The realization of a device exhibiting such an uncommon behavior, a thermal rectifier for instance, would pave the way to the development of thermal circuits in the manner non-linear electronic devices marked the genesis of modern electronics[31]. Indeed, as in electronics, thermal logical circuits need a thermal diode which can be defined as an ideal rectifier, *i.e.* a one-way heat transmitter. Consequently, an increasing interest has been given to thermal rectifiers during the past decade and a few models of radiative thermal rectifiers have been proposed during the last few years. In this paper, we generalize the concept of the first far field radiative thermal rectifier we recently proposed [15] to various spectrally selective photonic structures. We also show that the performances of the proposed rectifiers can be tuned through several parameters : the operating temperature, the emission peak position and width, essentially. A parametric optimization of this device shall then lead to higher rectification ratios and, consequently, provide the elementary component of complex thermal logical circuits. The present paper is organized as follows : In Sec. 2 we propose an unequivocal definition of the rectification ratio and relate it to definitions proposed by other authors for comparison sake. In Sec. 3, we draw up a map of

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previously proposed radiative thermal rectifiers through an extensive review of photon mediated thermal rectification literature in order to position the presented devices and provide a road map for future works. In Sec. 4, we present a general model for photonic structures based rectifiers. Finally, we propose in Sec. 5 several implementations based on resonant Fabry-Prot cavities which illustrate the dependence of the proposed device on different parameters.

2. ON RECTIFICATION DEFINITION

Consider a system of two thermal baths 1 and 2 at two different temperatures T_1 and T_2 , respectively. The rectification ratio can be defined by :

$$R = \frac{|q_{FB} - q_{RB}|}{\max(q_{FB}, q_{RB})} \quad (1)$$

where q_{FB} is the heat flux in the initial state and q_{RB} the heat flux when the thermal gradient between the two thermal reservoirs is reversed, *i.e.* $T_1 \rightarrow T_2$ and $T_2 \rightarrow T_1$. Other authors, while choosing FB as the configuration maximizing exchanged RHF, proposed :

$$R_1 = \frac{q_{FB} - q_{RB}}{q_{RB}} \quad (2)$$

and

$$R_2 = \frac{q_{FB}}{q_{RB}} \quad (3)$$

These various definitions do not allow a immediate comparison of literature results. However, they can be simply related by :

$$R = \frac{R_1}{R_1 + 1} = \frac{R_2 - 1}{R_2} \quad (4)$$

In this paper, we will prefer Def. 1 since it is unequivocal. Indeed, it does not depend on the choice of the initial forward and reverse bias configurations. Besides, and contrary to Defs. 2 and 3 which may lead to infinite rectifications ratios, it bounds rectification ratio in the $[0, 1]$ range. No rectification, *i.e.* no variation in the exchanged flux magnitude between FB and RB is observed for $R = 0$ while $R = 1$ corresponds to a perfect rectifier, *i.e.* a perfect one way heat transmitter. Finally, shall we here point the issue of the rectification ratio dependence on the temperature difference $\Delta T = |T_2 - T_1|$ which is not addressed by any of the proposed definitions. Indeed, devices reported in literature operate at different temperature ranges and with different thermal gradients. Moreover, we generally observe an increasing rectification ratio when ΔT increases which makes this latter a key parameter to describe radiative thermal rectifiers. Defining a rectification ratio per temperature unit is thus necessary in order to make the different devices performances comparable. Such a definition is straightforward when used materials exhibit regular thermo-optical properties (TOP), *i.e.* small variations of their optical properties with small temperature variations. However, it is not applicable anymore for PCM where TOP are strongly non linear around the critical temperature. Consequently, since the temperature difference is not accounted for in the definition of the rectification ratio, $(R, \Delta T)$ couples shall be systematically compared instead of mere R values.

3. A REVIEW

First works focused on heat conduction devices and have led to rectification models based on non-linear lattices [4, 10, 11, 23], graphene nano-ribbons [5, 34] and several other interesting mechanisms [22, 33]. Some authors

went beyond the thermal rectification issue and proposed theoretical models of thermal logical gates [30] and a thermal transistor [13]. Experimental implementations of rectifiers based on carbon and boron nitride nanotubes [3], semiconductor quantum dots [21] and bulk cobalt oxides [8] have also been realized. Extensive reviews treating thermal rectification in solids can be found in Refs. [12, 19].

During the last four years, a few authors tackled the question of radiation based thermal rectification. A theoretical study and an experimental suggestion of a radiative thermal rectifier based on non-linear solid-state quantum circuits operating at very low temperatures (a few mK) have been first presented [20]. A rectification ratio up to 10% is predicted. Moreover, three theoretical schemes of radiative thermal rectification based on near-field thermal radiation control have lately been proposed [1, 17, 32]. A rectification ratio up to 30% (according to the present paper rectification definition and using the references data) is theoretically predicted for temperature differences ranging between 100 K and 300 K. Very recently, comparable rectification ratios have also been reached, for the same temperature differences, by the first reported far-field radiative thermal rectifier made of Fabry-Prot cavities [15]. We reported a maximal rectification ratio of 19%. Conceiving devices based on far-field thermal radiation shall pave the way to fast experimental implementations since efficient observation of near-field effects on thermal radiation in simple plane-plane geometries is still out of reach of current experimental setups [9, 18].

Shall we note here that the interacting bodies in all these early radiative rectifiers are discrete modes resonators: bulk materials supporting surface resonances, surface phonon-polaritons [17] or surface plasmon-polaritons [1] for instance, or the both of them [32] (**explain how it works and cite [16]**), nanostructured materials with cavity modes [15] or nonlinear quantum resonators [20]. Rectification is therefore achieved by controlling the coupling between the two bodies modes. Switching from a state where the two thermal reservoirs modes are strongly coupled, forward bias (FB), to a state of weak coupling, reverse bias (RB), leads to a decrease in the exchanged radiative heat flux, thus to a thermal rectification.

More recently, the idea of broadband radiative thermal rectification (BRTR) emerged with the outstanding experimental and theoretical studies of radiative heat flux modulation with a phase change material (PCM), VO_2 for instance [26–29]. Heat flux contrasts up to 80% and 90% have been experimentally proven in the far-field [28] and the near-field [29], respectively. Theoretically, a contrast of near-field radiative heat flux of almost 100% is predicted for a bulk plane-plane glass/ VO_2 configuration [27]. Shall we here emphasize that these contrasts are only observed around VO_2 metal-insulator transition temperature ($T_c = 67^\circ\text{C}$) which strongly restricts their potential practical scope. With a much simpler experiment analyzing a heated VO_2 film thermal radiation with an infrared camera and FTIR spectrometer, similar results confirmed the non-linear thermal emission in the far-field with respect to small temperature variations around T_c [7]. Exploiting these contrasts to design a VO_2 -based thermal rectifiers has then been proposed in both the far-field [6, 35] and the near-field [2]. High rectification ratios up to 70% and 90% for small and large temperature differences are predicted [2].

Consequently, BRTR seems to be a promising path for efficient thermal rectification. In principle, a BRTR can be simply achieved with two planes of different random materials separated by a vacuum gap. Indeed, since non-identical materials are not likely to have identical thermo-optical properties (TOP), *i.e.* the same temperature dependence of their optical properties, inverting the temperature gradient between the two planes would modify the spectral distribution of the exchanged radiation, and very probably the net heat flux. However, randomly chosen materials wouldn't lead to good rectifiers since TOP are generally very small [24, 25], *i.e.* optical properties vary slowly with temperature. Hence, PCM would be a relevant alternative since their optical properties exhibit strong variations with respect to temperature around the transition temperature T_c .

4. MODEL

Figure 1 presents a schematic of the proposed device composed of two parallel planar bodies 1 and 2 separated by a gap of thickness d and characterized by their dielectric functions and temperatures ($\epsilon_1(T_1)$, T_1) and

$(\epsilon_2(T_2), T_2)$, respectively. The two bodies are assumed to be in vacuum. Thus, they only exchange radiative

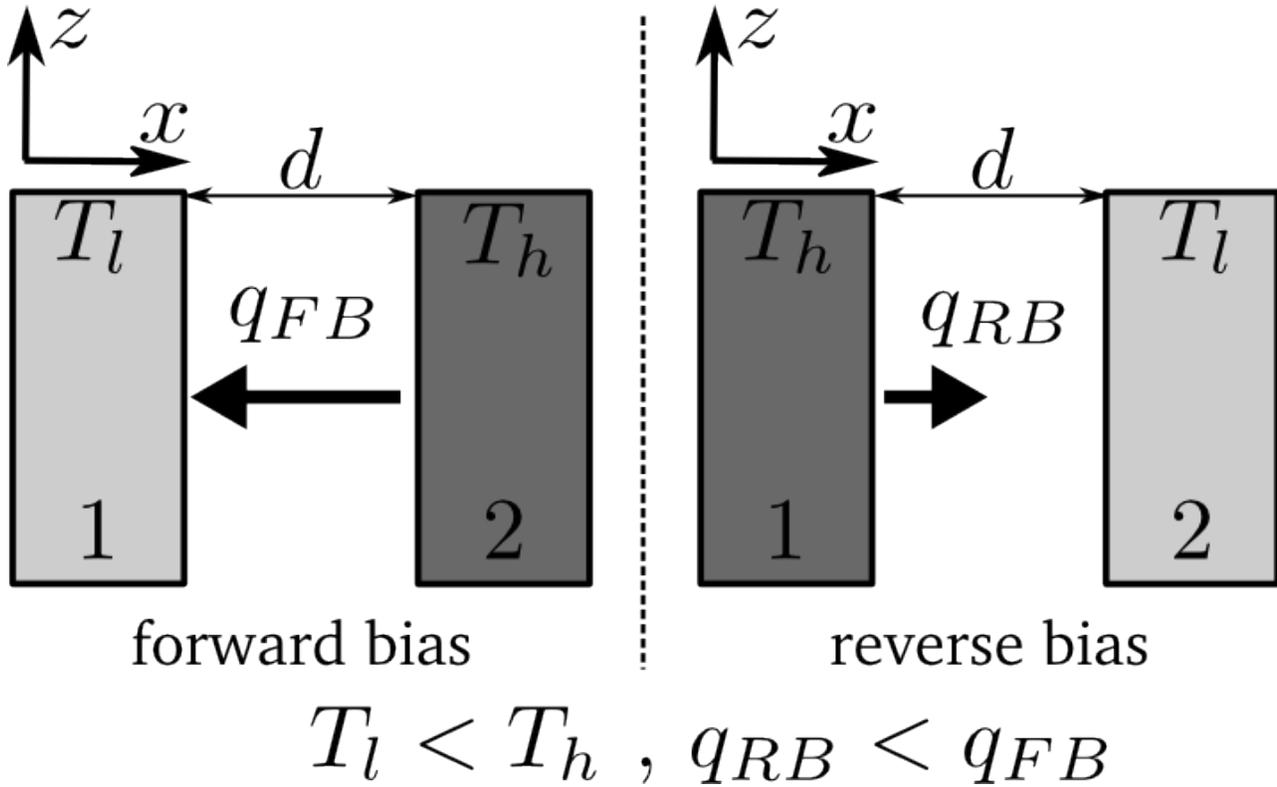


Fig. 1 Two parallel planar bodies separated by a distance d . In forward bias configuration, $T_1 = T_h$ and $T_2 = T_c$. In reverse bias configuration, $T_1 = T_c$ and $T_2 = T_h$.

heat flux (RHF) density. The considered bodies radiative properties, in particular their emissivities and reflectivities (ϵ and ρ respectively), are completely governed by their dielectric functions and geometries. In the case of opaque bodies, energy conservation and Kirchhoff's laws combination leads to the following relation between the monochromatic emissivity and reflectivity at a given temperature:

$$\epsilon(T, \lambda) = 1 - \rho(T, \lambda) \quad (5)$$

We also assume the two bodies are lambertian sources, thus ϵ and ρ direction-independent.

The gap width d is assumed to be much larger than the dominant thermal radiation wavelength (Wien wavelength) $\lambda_W(T) = hc/k_B T$ where h , c , k_B and T are Planck constant, the speed of light in vacuum, Boltzmann constant and absolute temperature, respectively. The net RHF density exchanged by the two media resumes then to the far field contribution which can be written [14]:

$$\dot{q}(T_1, T_2) = \pi \int_{\lambda=0}^{\infty} [I^0(\lambda, T_1) - I^0(\lambda, T_2)] \tau(\lambda, T_1, T_2) d\lambda \quad (6)$$

where

$$I^0(\lambda, T) = \frac{2hc}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} \quad (7)$$

is the black body intensity at a temperature T and

$$\tau(\lambda, T_1, T_2) = \frac{\epsilon_1(\lambda, T_1) \epsilon_2(\lambda, T_2)}{1 - \rho_1(\lambda, T_1) \rho_2(\lambda, T_2)} \quad (8)$$

is the monochromatic RHF density transmission coefficient between 1 and 2. Now, let us assume the two bodies are selective emitters so that they behave as quasi-monochromatic spectral emitters, *i.e.* they present reflectivity valleys (or emissivity peaks) of the same finite width $\Delta\lambda$ at given wavelengths, $\lambda_{p,i}$, $i \in \{1, 2\}$ for instance. Outside the emission peaks, they can be seen as piecewise gray bodies and their reflectivities can be written as follows:

$$\rho_i(\lambda, T) = \begin{cases} \rho_{min}(T) \simeq 0 & \text{if } \lambda \in [\lambda_{p,i}(T) - \Delta\lambda/2, \lambda_{p,i}(T) + \Delta\lambda/2] \\ \rho_{iI}(T) \simeq 1 & \text{if } \lambda < \lambda_{p,i}(T) - \Delta\lambda/2 \\ \rho_{iII}(T) \simeq 1 & \text{if } \lambda > \lambda_{p,i}(T) + \Delta\lambda/2 \end{cases} \quad (9)$$

These reflectivities are illustrated in Fig. 2 insets for a given set of parameters : $\Delta\lambda = 1 \mu\text{m}$, $\rho_{min} = 10^{-2}$, $\rho_{iI} = \rho_{iII} = 0.99$ and $\lambda_{p,1}(T) = \lambda_W(T_h = 500K)$. Body 1 emissivity is temperature independent. Body 2 emissivity is initially identical to that of body 1 in forward bias ($\lambda_{p,2}(T_c) = \lambda_{p,1}$) while its peak shifts to the red by $\delta_{rev}\lambda_p$ when the temperatures of the two bodies are reversed *i.e.* $\lambda_{p,2}(T_h) = \lambda_{p,1} + \delta_{rev}\lambda_p$. This shift with the temperature of one of the selective emitters emission peak is the key to achieve thermal rectification. In practice, this shift is completely governed by the used materials thermo-optical properties (TOP), *i.e.* $\partial\epsilon'/\partial T$ and $\partial\epsilon''/\partial T$ where ϵ' and ϵ'' are the real and imaginary part of the material dielectric permittivity respectively. Consider for now a small shift verifying $\delta_{rev}\lambda_p < \Delta\lambda$ thus the two peaks still partially overlap in reverse biased configuration. The transmission coefficient resulting from this emissivities choice for both configurations is plotted in Fig. 2. We can then note a one- μm wide spectral transmission window of RHF in forward bias and a narrower window in reverse bias. This asymmetry when the temperatures of the two bodies are inverted would lead to a thermal rectification. In fact, the exchanged RHF in forward bias is given by :

$$\begin{aligned} \dot{q}_{for} &\simeq \sigma T_h^4 [F_{\lambda_{p,1}+\Delta\lambda/2}(T_h) - F_{\lambda_{p,1}-\Delta\lambda/2}(T_h)] \\ &- \sigma T_c^4 [F_{\lambda_{p,1}+\Delta\lambda/2}(T_c) - F_{\lambda_{p,1}-\Delta\lambda/2}(T_c)] \\ &+ \pi \int_{\lambda \in \mathcal{L}_{for}} \frac{\epsilon_1(\lambda, T_h)\epsilon_2(\lambda, T_c)}{1 - \rho_1(\lambda, T_h)\rho_2(\lambda, T_c)} [I^0(\lambda, T_h) - I^0(\lambda, T_c)] \end{aligned} \quad (10)$$

where σ is Stefan-Boltzmann constant and

$$\sigma T^4 = \pi \int_0^\infty I^0(\lambda', T) d\lambda' \quad (11)$$

and the spectral fraction of black body radiation emitted in the spectral range $\lambda' \leq \lambda$ and $\mathcal{L}_{for} = [0, \lambda_{p,1} - \Delta\lambda/2] \cup [\lambda_{p,1} + \Delta\lambda/2, \infty]$ is

$$F_\lambda(T) = \pi \frac{\int_0^\lambda I^0(\lambda', T) d\lambda'}{\sigma T^4} \quad (12)$$

On the other hand, the exchanged RHF in reverse bias is given by :

$$\begin{aligned} \dot{q}_{rev} &\simeq \sigma T_h^4 [F_{\lambda_{p,1}+\Delta\lambda/2}(T_h) - F_{\lambda_{p,1}+\delta_{rev}\lambda_p-\Delta\lambda/2}(T_h)] \\ &- \sigma T_c^4 [F_{\lambda_{p,1}+\Delta\lambda/2}(T_c) - F_{\lambda_{p,1}+\delta_{rev}\lambda_p-\Delta\lambda/2}(T_c)] \\ &+ \pi \int_{\lambda \in \mathcal{L}_{rev}} \frac{\epsilon_1(\lambda, T_c)\epsilon_2(\lambda, T_h)}{1 - \rho_1(\lambda, T_c)\rho_2(\lambda, T_h)} [I^0(\lambda, T_h) - I^0(\lambda, T_c)] \end{aligned} \quad (13)$$

where $\mathcal{L}_{rev} = [0, \lambda_{p,1} + \delta_{rev}\lambda_p - \Delta\lambda/2] \cup [\lambda_{p,1} + \Delta\lambda/2, \infty]$.

The piecewise decompositions of the exchanged RHF in Eqs. 10 and 13 take into account the fact that $\epsilon_i \simeq 1$ within the structure i peak. Since $\epsilon_i \simeq 0$ elsewhere else, the terms with the pre-factor $\epsilon_1\epsilon_2/(1 - \rho_1\rho_2)$ in

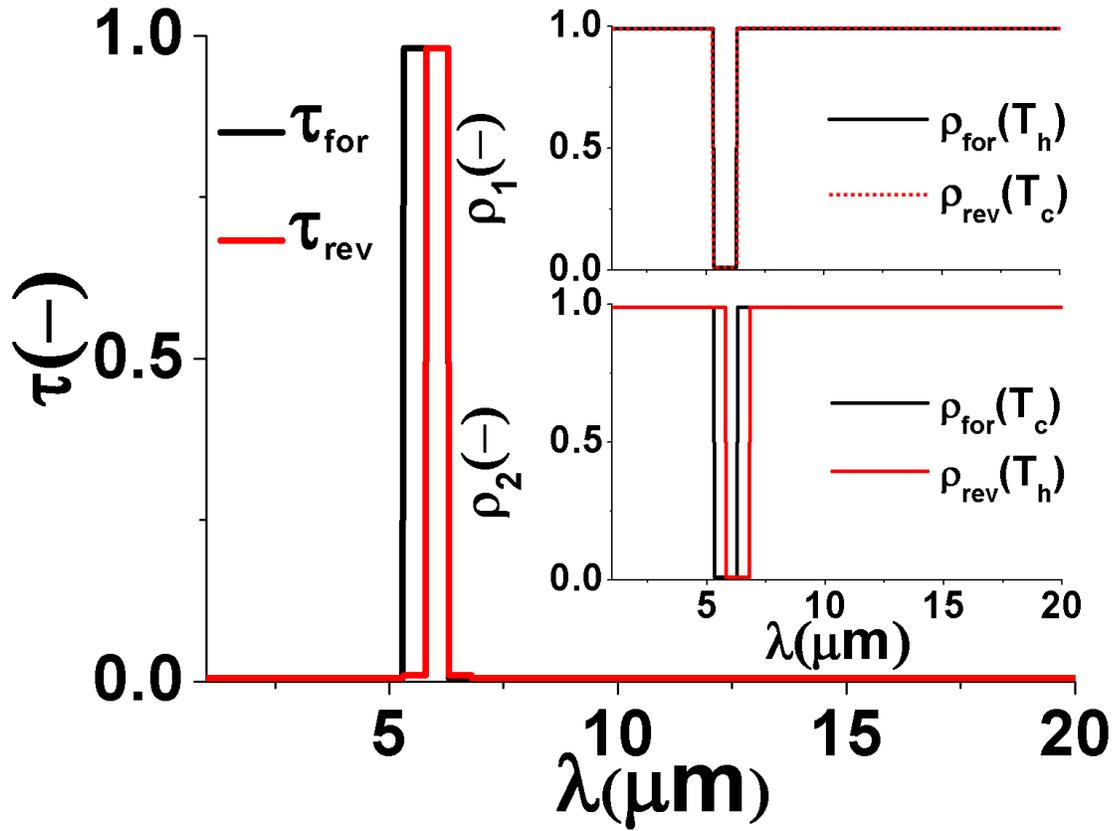


Fig. 2 The transmission coefficient in forward and reverse biased configurations and bodies 1 and 2 reflectivities in forward (top inset) and reverse bias (bottom inset) configurations. In both configurations, $\lambda_{p,1}(T) = \lambda_W(T_h) = 5.8 \mu\text{m}$ while $\lambda_{p,2}(T_c) = \lambda_{p,1}$ in forward bias and $\lambda_{p,2}(T_h) = \lambda_{p,1} + \delta_{rev}\lambda_p$ in reverse bias where $\delta_{rev}\lambda_p = \Delta\lambda/2$.

the integrand are of the first order. In reverse biased configuration, we can note that the spectral domains of the order zero and the first order terms are reduced and extended respectively due to the shift between the two peaks. The reduction of the dominating term (order 0) due to its spectral domain diminution, if it is different from the higher order term (1st order) increase, would lead to an asymmetry of the RHF and therefore a rectification phenomenon. For the above presented situation, calculations lead to: $\dot{q}_{for} = 122.67 \text{ W}\cdot\text{m}^{-2}$, $\dot{q}_{rev} = 63.97 \text{ W}\cdot\text{m}^{-2}$ and $R = 0.48$. This rectification value is of the order of the largest reported values for radiative thermal rectifiers.

According to the previous assumptions, this rectification ratio obviously depends on the chosen parameters, in particular the temperatures T_h and T_c , the peaks position λ_p compared to $\lambda_W(T_h)$, their width $\Delta\lambda$ compared to the black body spectrum useful width at the largest temperature, the shift between the two peaks in reverse bias $\delta_{rev}\lambda_p$ compared to $\Delta\lambda$ and the reflectors and emitters quality characterized by ρ_{ij} and ρ_{min} , respectively. It is clear that the maximal rectification ratio is reached when the peaks shift in reverse bias is larger than the peaks width and when the bodies are perfect reflectors outside the peaks, *i.e.* when $\delta_{rev}\lambda_p/\Delta\lambda \geq 1$ and $\rho_{ij} = 1$ respectively : reverse bias RHF vanishes and rectification reaches 1. The first condition on $\delta_{rev}\lambda_p$ is more easily realized with narrow emissivity peaks. However, narrower emissivity peaks would decrease forward bias RHF density which makes the rectification ratio more sensitive to the RHF density noise (high order terms), *i.e.* to heat transmitted in a spectral range outside the peaks thus to ρ_{ij} . Besides, since ρ_{ij} and ρ_{min} are temperature

dependent, other behaviors may be observed if these parameters actually vary when the temperature gradient is reversed. An increase of ρ_{ij} in reverse bias for instance, even small, would induce a supplementary exchanged RHF. This supplementary RHF may counterbalance or overcome the flux decrease due to the peaks shift. The final rectification is then a balance between the two phenomena.

5. IMPLEMENTATIONS

6. CONCLUSIONS

ACKNOWLEDGMENTS

Authors gratefully acknowledge the support of the Agence Nationale de la Recherche through the Source-TPV Project No. ANR 2010 BLAN 0928 01. This work pertains to the French Government program “Investissements d’Avenir” (LABEX INTERACTIFS, reference ANR-11-LABX-0017-01).

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