



**HAL**  
open science

# Job seeker's spatial equilibrium in a fuzzy mathematical framework

Catherine Baumont

► **To cite this version:**

Catherine Baumont. Job seeker's spatial equilibrium in a fuzzy mathematical framework. [Research Report] Laboratoire d'analyse et de techniques économiques(LATEC). 1990, 22 p., ref. bib.: 2 p. hal-01545755

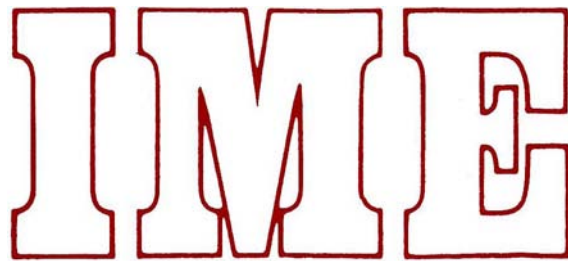
**HAL Id: hal-01545755**

**<https://hal.science/hal-01545755>**

Submitted on 23 Jun 2017

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



# INSTITUT DE MATHÉMATIQUES ÉCONOMIQUES

LATEC C.N.R.S. URA 342

**DOCUMENT de TRAVAIL**



UNIVERSITE DE BOURGOGNE

FACULTE DE SCIENCE ECONOMIQUE ET DE GESTION

4, boulevard Gabriel - 21000 DIJON - Tél. 80 39 54 30 - Fax 80 39 56 48

ISSN : 0292-2002

9003

**JOB SEEKER'S SPATIAL EQUILIBRIUM  
IN A FUZZY MATHEMATICAL FRAMEWORK**

**Catherine BAUMONT**



# JOB SEEKER'S SPATIAL EQUILIBRIUM IN A FUZZY MATHEMATICAL FRAMEWORK

## **Abstract**

This paper incorporates complex behavior in a job seeking strategy model. Since the state of information is imperfect, a job seeker faces an imprecise labor market. His behavior is fuzzy and variable. It is shown how this complex behavior can be described by fuzzy subsets theory and bifurcation analysis. The latter examines the discontinuous phenomena. Then it is proved that a job seeker's fuzzy equilibrium with a spatial constraint exists.

## **Keywords**

Fuzzy Utility - Fuzzy Behavior - Spatial Equilibrium - Discontinuous Behavior - Labor Market Area.

## 1. INTRODUCTION.

The empirical literature on job seek theory is often focused on the studies of the linkage between two elements of a job seek strategy : migration and job seek efficiency [d'Avirsenet (1979), Goss E.P. and Schoening N.V. (1984), Greenwood M.J., Hunt G.L. and Mc Dowell J.M. (1986), Lesourne J. (1986), Renault E. and Lesourne J. (1986)] ; wage requirement and job seek efficiency [Barron J.M. and Mellow W. (1979), Kahn L.M. (1978)] ; imperfect information and job seeking strategy [Laffond G. and Lesourne J. (1986), Rea S.A. Jr (1981)] and job seeker's qualifications and job choices [Lösch A. (1954)].

The aim of this paper is to combine these different elements i.e. to introduce the complex behavior of a job seeker in a job seek strategy. More precisely, we consider that the labor market is an imprecise space where job seekers have a fuzzy and spatial behavior. The job seeker's fuzzy behavior results from many influences of the different characteristics of the jobs : wage, qualifications, job location and of the job seeker's ones : qualifications, socio-demographic characteristics, residential location and wished wage.

Since informations on vacancies, wages and job locations are imperfect, job seekers cannot have precise and constant behaviors. So they are not able to say whether a job belongs to their sets of preferences or not (without any other choice) but they are able to say that a job imperfectly belongs to them. That is designated by "fuzzy behavior".

The job seeker's spatial behavior is analyzed as a choice between migration and geographical immobility. First the analysis is job oriented (given a residence look for jobs). Then, if the job seeker does not choose a job in his residential labor market, he decides to migrate.

## 2. THE ANALYTICAL FRAMEWORK

A particular economic framework is given.

The job seek strategy depends on four principal factors : wage, information, space and qualification -which are interdependant-. For example, the level of qualification determines the level of wage, or a man who does not want to migrate will accept a lower wage and if the information is costly the job seeker will accept more easily a lower wage. Finally, we can set down that the wage absorbs most of the influences of

the other factors. A job seek strategy will be mainly articulated around the wages wished by job seekers and offered by employers.

The job seeker fuzzy behavior will consist in obtaining the maximum wage which also depends on qualifications, space and information. We set down that the wage utility measures the job utility because of the following comments : first the wage levels take into account some characteristics linked to the practise of a job : hard job, tiresome job, unhealthy job. Second, other characteristics are not systematically taken into account by the wage because their "monetary" integrations depend on the work contract negotiation between a job seeker and an employer. For example, a job seeker who lives far from his future job location will try to obtain an higher wage to compensate the distance costs. Finally, wage utility is a good approximation of job utility if most of the influence of these factors : space, information and qualification is taken into account.

However, we suppose that these last factors constrain the job seek strategy. More precisely, we set down that a job seeker faces only with the spatial constraint. In fact, the state of information determines the set of jobs known by a job seeker and his qualification determines the jobs he can obtain. So these two factors influence the job seek strategy before it begins. On the contrary, the influence of the factor "space" appears during its progress.

The spatial behavior of a job seeker can be resumed as follows : a job seeker tries to make his residential location and his job location coincide. So, either he does not want to migrate and he prefers to move daily from home to job location or he accepts to migrate when these daily moves are too costly and tiresome.

This spatial behavior is a passage from a refusal to migrate to an acceptance to migrate, i.e. is a spatial behavior breaking off. It can be analyzed by the bifurcation theory.

### 3. THE MODEL

A job seeker follows a strategy of a fuzzy wage utility maximization under a fuzzy spatial constraint. We study this behavior with the theory of fuzzy subsets and with the bifurcation theory. We will obey the following plan : first we will define the jobs sets and second we

will describe the objective and constraint.

We draw the attention on the following point : only ordinary (non fuzzy) mathematical concepts are underlined. For example  $A \subset \underline{X}$  is a fuzzy subset of the  $\underline{X}$  referential.

### 3.1. Jobs sets

A job seeker prospects on a specific labor market which is the set of a priori possible jobs that we note  $\underline{\mathcal{E}}$ .  $\underline{\mathcal{E}}$  is characterized by the state of the labor market and by its capacity to deliver information.

The state of the labor market depends on economics circumstances and we will suppose that the unemployment rate is rather high. Jobs are perfectly localized and types of jobs are known through available information. So for each job man asks some qualifications and man offers a certain wage.

We define  $\underline{\mathcal{E}}$  as the set of jobs offered in a given location for a certain level of qualification and for a given wage.

We set down that :

$$\underline{\mathcal{E}} = \left\{ \left[ \begin{array}{c} 1 \\ e_s^q \end{array} \right] \right\}$$

where  $l$  represents the job locations  $l = 0 \dots m$

$q$  represents the qualifications asked for the jobs  $q = 1 \dots n$

and  $s$  represents the offered wages  $s = 1 \dots r$ .

For example, the job  ${}^1 e_4^3$  is a job offered in Lille (a town whose code is 1) for a professional whose qualification level is 3 and for a wage of  $x$  FF (code 4 in the wages scale).

However, the space of offered jobs is different from the space of accessible jobs. The latter depends on the job seeker's qualifications. We will note  $\underline{\mathcal{E}}$  the set of technical possible jobs.

$$\text{Then } \underline{\mathcal{E}} \subset \underline{\mathcal{E}} \text{ and } \underline{\mathcal{E}} = \left\{ \left[ \begin{array}{c} 1 \\ e_s^{q'} \end{array} \right] \right\} \text{ where } q' = 1 \dots t ; t \leq n.$$

$q'$  are the job seeker's qualifications.

Finally, we set down that  $\underline{\mathcal{E}} = \left\{ e^h \right\}$  where  $e^h = \left[ \begin{array}{c} 1 \\ e_s^1 \end{array} \right]$  with  $q' = h$ .



The job seeker's decision belongs to  $\underline{E}$ .

### 3.2. The objective

A job seeker tries to maximize a fuzzy wage utility which is imprecise because it depends on the different effects of the different characteristics of the jobs : the wage offered by the employer, the job location, the qualifications required for the job, and it depends on the job seeker's requirements : the wished minimum wage (the reservation wage) and the wished job locations.

#### Remarks

1) The maximization wage utility objective is compatible with the reservation wage concept. So, for constant wage requirements, the objective is to obtain a wage equal to or higher than the reservation wage. If wage requirements can be reviewed falling, the objective is to obtain the nearest wage from the reservation wage. If wage requirements can be reviewed rising, the objective is to obtain the highest possible wage.

2) The state of information is imperfect, so the reservation wage is not exactly known. This also justifies the use of a fuzzy wage utility. Finally, a job seeker will prefer more or less a job following its offered wage actually observed for this type of jobs on the labor market.

Let  $F$  be the fuzzy objective of a job seeker and  $\mu_F$  be the membership function which designates the fuzzy wage utility.

We define  $F$  and  $\mu_F$  as follows :

$$F \subset \underline{E} \quad F = \left\{ e_s, \mu_F ; \forall e_s \in \underline{E} ; \mu_F(e_s) \in [0, 1] \right\}$$

where  $e_s = [ \overset{l}{e}_s \overset{h}{e}_s ]$  and :  $\mu_F : \underline{E} \longmapsto [0, 1]$

$$e_s \longmapsto \mu_F(e_s) \in [0, 1].$$

Let  $s^*$  be the reservation wage and  $s_m$  be the average wage really observed for the job  $\overset{l}{e}_s \overset{h}{e}_s$  which is paid for a wage  $s$ .

We will note  $\varepsilon = s^* - s_m$  the estimation error of the reservation wage.

- If  $s^*$  is overestimated,  $\varepsilon$  is positive and  $\max(s^*, s_m) = s^*$ .

- If  $s^*$  is underestimated,  $\varepsilon$  is negative and  $\max(s^*, s_m) = s_m$ .

So  $\mu_F$  is defined as follows :

$$\left\{ \begin{array}{ll} \mu_F(e_s) = 0 & \text{if } s < \min(s_m, s^*) \\ \mu_F(e_s) = 1 & \text{if } s \geq \max(s_m, s^*) \\ \mu_F(e_s) \in ]0, 1[ & \text{if } \min(s_m, s^*) \leq s < \max(s_m, s^*) \end{array} \right.$$

If the reservation wage is exactly estimated, we find again the traditional case, i.e. a job seeker will accept the job  $e_s^{lh}$  if  $s \geq s^* = s_m$ , then  $\mu_F(e_s) = 1$  and he will refuse it if  $s < s^* = s_m$ , then  $\mu_F(e_s) = 0$ .

### 3.3. The constraint

The job seeker's spatial behavior depends on the labor market area he prospects. If we define this area as a space centred on the job seeker's residential location, we define a linkage between the job seeker's mobility and the labor market area he prospects. The latter is limited by the maximum distance between the residential location and the job location he will accept. We note  $R^*$  this maximum distance.

Since the distances are costly,  $R^*$  depends on the job seeker's earnings. Since job seeker wage requirements are imprecise and since a job seeker can have other earnings over wage (financial yields, spouse wage, game incomes, work on the side), the whole job seeker's earnings is imprecise, so  $R^*$  is not perfectly determined and the spatial constraint is fuzzy.

We can define the spatial fuzzy constraint  $C$  as follows :

$$C = \{e^1, \mu_C, \forall e^1 \in E, \mu_C(e^1) \in [0, 1]\}$$

where  $e^1 = [e_s^{lh}]$  and  $\mu_C$  is the membership function of a job to the  $C$  fuzzy subset.

We set down :  $E \longmapsto [0, 1]$

$$\mu_C : e^1 \longmapsto \mu_C(e^1) \in [0, 1]$$

Let  $l_0$  be the residential location and  $l$  be the job places with  $l = 0 \dots m$ .

$$\begin{cases} \mu_C(e^l) = 0 & \text{if } d(l_0, l) \geq R^* \\ \mu_C(e^l) = 1 & \text{if } d(l_0, l) = 0 \\ \mu_C(e^l) \in ]0, 1[ & \text{if } 0 < d(l_0, l) < R^* \end{cases}$$

where  $d(l_0, l)$  is the distance between  $l_0$  et  $l$ .

We show that  $\mu_C$  decreases when  $d(l_0, l)$  increases

$$\forall \left( \begin{bmatrix} l \\ e_s^h \end{bmatrix}, \begin{bmatrix} l' \\ e_{s'}^{h'} \end{bmatrix} \right) \in \underline{E} \times E \quad \text{where } l' \geq l ; h' \neq h ; s' \approx s$$

$$\text{if } d(l_0, l) \geq d(l_0, l') \text{ then } \mu_C(e^l) \leq \mu_C(e^{l'})$$

Now we can define the job seeker's fuzzy and spatial equilibrium.

#### 4. JOB SEEKER'S FUZZY AND SPATIAL EQUILIBRIUM

A job seeker must find a compromise between the wages he can obtain and the location of the jobs. In order to solve this problem, we must define the spatial behavior of a job seeker before we shall determine the job seeker's fuzzy and spatial equilibrium.

##### 4.1. The spatial behavior

A job seeker prospects on a more or less large labor market area because of his mobility constraint. Since daily moves are costly and tiresome, he will measure the profitability of the offered jobs. For a given wage, the labor market area is a disk whose center is  $l_0$  and whose radius is  $R^*$ . We can set down the following assumptions :

H1 : The residential utility at  $l_0$  is noted  $U_0$  and it measures the profitability of the job offered at  $l$ . For every wage  $s$ , the residential utility is maximum when  $l = l_0$ .

H2 : For a given wage  $s$ , the residential utility decreases when the distance  $d(l_0, l)$  increases.

H3 : For a given wage  $s$ , the residential utility equals 0 when  
 $d(l_0, l) = R^*$ .

H4 : If  $d(l_0, l) = R^*$ , a job seeker will prefer to migrate and go to live in  $l$ , where the new residential utility will become maximum.

Under these different assumptions the spatial behavior of a job seeker can be interpreted as follows : for a given wage, the residential utility equals 0 when it is more interesting to migrate than to move daily from home to job location. So, a job seeker will change his behavior, i.e. he will accept to migrate after he has refused to do it.

Finally, the problem of migration is to determine a maximum distance  $R^* = d(l_0, l)$  for which we observe a spatial behavior break off.

This problem can be solved by the bifurcation theory since a bifurcation point is a point for which the type of the set of solutions changes. Here, for a distance  $R^*$ , the residential utility  $U_0$  equals 0 but it becomes maximum in  $l$ . We actually have a bifurcation point. Now we are going to define it.

The residential utility is a function of the time man can pass there and of the money man can spend there.

Therefore, we set down that  $U_0 = \mathcal{T}_{l_0} \times \mathcal{P}_0$ ,

where  $U_0$  = residential utility in  $l_0$ ,

$\mathcal{T}_{l_0}$  = total leisure time passed in  $l_0$ ,

$\mathcal{P}_0$  = wage left over.

This definition allows us to count all type of leisure either by their costs or their time if they are free.

#### Remark

The introduction of time supposes that we actualize the different costs compared with the job seeker's age. But we set down that the reference period is smaller or equal to one year. This condition

avoids the actualization and is compatible with information on jobs which are quite precise for a year. More precisely we have :

$$U_0 = (\mathcal{T} - tw_1 - n R_1 V_1) (tw_1 S - n C_1 R_1 - \bar{S}_0 A_0)$$

where  $T$  = reference period (week, month, year...),

$tw_1$  = work time on  $T$  for the job  $l$ ,

$n$  = number of moves from  $l_0$  to  $l$  and from  $l$  to  $l_0$  during  $T$ ,

$V_1$  = move time from  $l_0$  to  $l$ ,

$R_1$  =  $d(l_0, l)$  (km),

$S$  = wage rate per work time,

We note  $tw_1 S = s$  = weekly wage or monthly wage or yearly wage,

$C_1$  = move cost per km for  $d(l_0, l)$ ,

$S_0$  = cost of living at  $l_0$ ,

$A_0$  = Consumption at  $l_0$ ,

We note  $\bar{S}_0 A_0 = \bar{s}_0$  : consumption spent during  $T$ .

We set down  $\bar{s}_0 = a s$  (consumption spent is a constant part of the wage  $s$ ).

$$\text{Then } U_0 = (\mathcal{T} - tw_1 - nR_1V_1) ((1 - a) s - nC_1 R_1)$$

Remark

1) When  $d(l_0, l) = 0$  then  $U_0 = U \max = U^*$

$$U^* = (\mathcal{T} - tw_1) (1 - a) s_0$$

2) Migration costs (psychological and monetary costs) are not counted because they are negligible compared with  $n \times C_1 \times R_1$  and  $n \times V_1 \times R_1$  because they represent only one move from  $l_0$  to  $l$ .

A bifurcation point  $R^*$  is stated when  $U_0 = 0$

$$\text{i.e. } U_0 = \mathcal{T}_{l_0} \times \mathcal{J}_0 = 0$$

$$\text{then } \mathcal{T}_{l_0} = 0 \text{ or } \mathcal{J}_0 = 0$$

$$\text{We obtain } T - tw_1 - n V_1 R_1 = 0 \text{ or } s(1 - a) - n C_1 R_1 = 0$$

$$\text{and } R_1^1 = \frac{T - tw_1}{n V_1} \text{ or } R_1^2 = \frac{s(1 - a)}{n C_1}$$

Finally we have  $R^* = \min (R_1^1, R_1^2)$ .

Remarks

1) The residential utility can be minimum for a distance  $R^{**}$ , solution of the equation :

$$\frac{\delta U_0}{\delta R} = 0$$

i.e.  $R^{**} = \frac{V_1(s(1-a)) + C_1(T - tw_1)}{2n V_1 C_1}$

$$R^{**} = \frac{1}{2} (R_1^1 + R_1^2)$$

Then if  $U_0$  is minimum, its value is negative and we cannot accept it in economics.

2) We choose  $R^* = \min(R_1^1, R_1^2)$  in order to respect the traditional behavior of leisure preference.

3)  $R^*$  is a maximum radius of a labor-market area centred on the residential location of a job seeker who refuses to migrate.

All the points of the frontier of the disk  $(l_0, R^*)$  are bifurcation points, i.e.  $U_0$  equals 0 but it becomes maximum when a job seeker migrates from  $l_0$  to  $l$ . So, if a job seeker changes his behavior, we have a discontinuous phenomena that we can represent as follows :

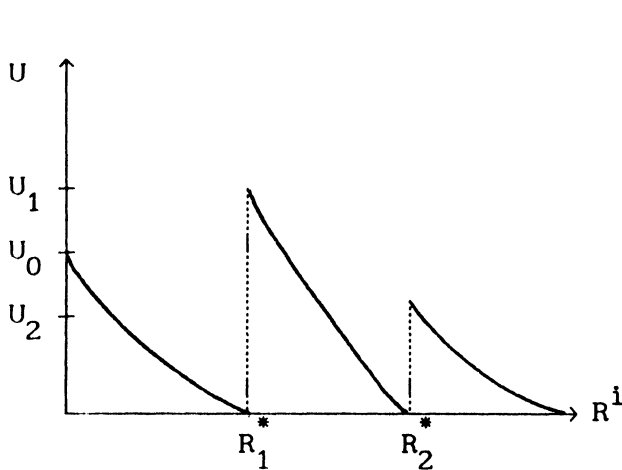


Figure 1

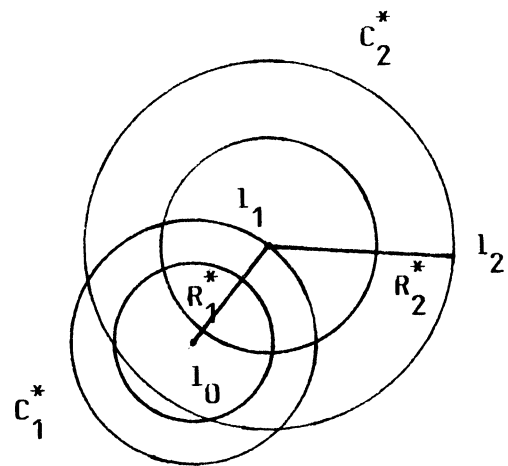


Figure 2

Remarks

The new residential utility value  $U^*$  depends on the cost of life at  $l$  so it can be lower, higher or equal to  $U_0$  (figure 1).

The value of  $R^*$  mainly depends on offered wages. So for each wage level, a specific set of solutions is symbolized by a circle (figure 2) and the higher the wage is, the more slowly the residential utility

decreases (figure 1).

Bifurcation phenomenas are symbolized by the dotted lines (figure 1) which represent the discontinuity of the residential utility function. A numerical application using French data illustrates our purpose.

For example :  $T = 4 \text{ weeks} \approx \text{one month} = 672 \text{ h}$

$$tw_1 = 4 \times 39 = 156 \text{ h}$$

$$n = 4 \times 5 \times 2 = 40 \text{ moves for five work days per week}$$

$$C_1 = 1,6 \text{ FF (fiscal tariff for an average powerful car)}$$

$$V_1 = 1/80 \text{ h (for an average speed of 80 km/h)}$$

$$s = 5 \text{ 000 FF}$$

$$\bar{s}_0 = 0,7 \text{ s (we suppose that 70 \% of earnings is spent for}$$

the consumption of food, clothes, health and home).

We obtain :  $R_1 = 1032 \text{ km}$  or  $R_2 = 23,4 \text{ km}$

So :  $R^* = \min (1032, 23,4) = 23,4 \text{ km}$

Then a man who earns 5 000 FF for 4 weeks will have to migrate to live at his job location if it is 23,4 km or more far from his residential location. The value  $R_1 = 1032 \text{ km}$  is explained because total leisure time includes total sleeping time and this result proves the importance of the time over earning one.

#### 4.2. Job seeker's fuzzy and spatial equilibrium

The optimum job  $e^*$  is the best for the fuzzy objective and for the fuzzy constraint. This type of job is a fuzzy optimal decision which does not allow improving the objective without deteriorating the constraint and vice-versa.

Therefore  $e^* \in D$  with  $D = F \cap C$  where  $D \subset \underline{E}$  is defined as follows :

$$D = \left\{ e_s^h ; \mu_D, \forall e_s^h \in \underline{E}, \mu_D(e_s^h) = F(e_s) \wedge \mu_C(e^1) \right\}$$

where  $\mu_D$  is the membership function of a job to the D fuzzy subset.

To solve a problem of maximization of non fuzzy objective under non fuzzy constraint we need a convexity assumption for the C subset. In case of fuzzy subsets, convexity can be required for F or C. We choose to put convexity on F.

The solution  $e^*$  is determined in the following manner :

$$\begin{aligned}\mu_D(e^*) &= \sup_{\underline{E}} \mu_D \left( \begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \right) = \sup_{\begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \in \underline{A}} \mu_F \left( \begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \right) = \\ &= \sup_{\alpha \in ]0, 1[} \left[ \alpha \wedge \sup_{\begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \in \underline{C}_\alpha} \mu_F \left( \begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \right) \right]\end{aligned}$$

where  $\underline{A} = \left\{ \begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix}, \in \underline{E} / \mu_C(e^1) \geq \mu_F(e_s) \right\}$

and  $\underline{C}_\alpha$  is the  $\alpha$ -cut

$$\underline{C}_\alpha = \left\{ \begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \in \underline{E} / \mu_C \left( \begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \right) \geq \alpha \right\}$$

Finally, we must find  $\alpha^*$  so that  $\sup_{\begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \in \underline{C}_{\alpha^*}} \mu_F(e_s) = \alpha^* = \sup_{\underline{E}} \mu_D(e^*)$

where  $\alpha^*$  is a fixed point for the function  $\sup_{\begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \in \underline{C}_\alpha} \mu_F(e_s) = \varphi(\alpha)$

A solution exists if and only if the function  $\varphi(\alpha)$  has a fixed point, i.e. if and only if  $\varphi(\alpha)$  is continuous and decreasing over the interval  $[0, 1]$  due to the following theorem.

Theorem

If the function  $\sup_{\begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \in \underline{C}_\alpha} \mu_F \left( \begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \right)$  is continuous and decreasing over the

interval  $[0, 1]$  then :

- 1)  $\sup_{\begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \in \underline{C}_\alpha} \mu_F \left( \begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \right)$  has a fixed point.  $\exists \bar{\alpha} [0, 1] / \bar{\alpha} = \sup_{\begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \in \underline{C}_{\bar{\alpha}}} \mu_F \left( \begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \right)$
- 2)  $\sup_{\begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \in \underline{E}} \mu_D \left( \begin{smallmatrix} 1 \\ e_s^h \end{smallmatrix} \right) = \bar{\alpha}$



Therefore we shall prove that the function  $\varphi(\alpha)$  is continuous and decreasing over  $[0, 1]$ .

$$[0, 1] \longmapsto [0, 1]$$

We set down :  $\varphi(\alpha)$  :

$$\alpha \longmapsto \varphi(\alpha) = \text{Sup}_{\substack{1 e_s^h \\ \in \underline{C}_\alpha}} \mu_F \left( \begin{matrix} 1 e_s^h \\ \end{matrix} \right)$$

First we prove that  $\varphi(\alpha)$  is decreasing.

With the property of decreasing of the sequence  $\{\underline{C}_\alpha\}$  we have :

$$\forall (\alpha_1, \alpha_2) \in [0, 1] \times [0, 1] \text{ if } \alpha_1 \leq \alpha_2 \text{ then } \underline{C}_{\alpha_2} \subseteq \underline{C}_{\alpha_1}$$

$$\text{and } \text{Sup}_{\substack{1 e_s^h \\ \in \alpha_2}} \mu_F(e_s) \leq \text{Sup}_{\substack{1 e_s^h \\ \in \underline{C}_{\alpha_1}}} \mu_F(e_s) \Rightarrow \varphi(\alpha_2) \leq \varphi(\alpha_1)$$

Then, we will prove that  $\varphi(\alpha)$  is continuous over  $[0, 1]$  with the theorem due to Tanaka-Okuda-Asai [21].

#### Theorem

If the fuzzy subset  $F$  is strictly convex, then the function  $\text{Sup}_{\substack{1 e_s^h \\ \in \underline{C}_\alpha}} \mu_F(e_s)$  is continuous.

$$1 e_s^h \in \underline{C}_\alpha$$

We know that a fuzzy subset is strictly convex if and only if its membership function is strictly quasi-concave. We are going to prove that  $\mu_F$  is strictly quasi-concave.

$\mu_F$  is strictly quasi-concave if and only if the following condition is true.

$$\forall e_s \in \left[ \begin{matrix} 1 e_s^h \\ \end{matrix} \right] \in F \quad \forall e_{s'} \in \left[ \begin{matrix} 1 e_{s'}^h \\ \end{matrix} \right] \in F \text{ with } e_s \neq e_{s'}$$

$$3) \text{ if } \mu_F(e_s) \geq \mu_F(e_{s'}) \Rightarrow \forall \lambda \in ]0, 1[ \mu_F(\lambda e_s + (1 - \lambda)e_{s'}) \geq \mu_F(e_{s'})$$

Then  $\mu_F$  is strictly quasi concave and  $F$  is strictly convex. So we have to prove proposition (3).

If  $e_s$  is different from  $e_{s'}$ , these two jobs are different either by the

location or by the wage or by the qualification or by any combination of two or three of these elements. We have to examine seven different cases.

First case

$$s \neq s' ; h = h' \text{ and } l = l'$$

The two jobs are only different by the wage.

So in terms of wage  $(\lambda s + (1 - \lambda) s') > \min (s, s')$

Then the job  $(\lambda e_s + (1 - \lambda) e_{s'})$  is preferred to the job  $e_s$ , and  $\mu_F(\lambda e_s + (1 - \lambda) e_{s'}) > \mu_F(e_s)$ .

Second case

$$s = s' ; h \neq h' \text{ and } l = l'$$

The two jobs are only different by the qualification. If  $e_s$  is preferred to  $e_{s'}$ , because of the qualification, the job seeker will prefer again a job where he can use this qualification  $h$  to a job  $e_s$ , where he cannot use it. Then  $\mu_F(\lambda e_s + (1 - \lambda) e_{s'}) > \mu_F(e_s)$ .

Third case

$$s = s' ; h = h' \text{ and } l \neq l'$$

If  $\mu_F(e_s) > \mu_F(e_{s'})$  then  $d(l_0, l) < d(l_0, l')$ .

$\forall e_{s''} = \begin{bmatrix} l'' \\ e_s \end{bmatrix}$  where  $l'' = \lambda l + (1 - \lambda)l'$  we will have

$\mu_F(e_{s''}) > \mu_F(e_s)$ , then  $\mu_F(\lambda e_s + (1 - \lambda) e_{s'}) > \mu_F(e_s)$ .

Fourth case

$$s \neq s' ; h \neq h' \text{ and } l = l'$$

If  $e_s$  is preferred to  $e_{s'}$ , because of the combination  $(e, h)$  then, according to the individual division of labor theory (Lösch [13]),  $s$  is higher than  $s'$ . We find again the first case and

$\mu_F(\lambda e_s + (1 - \lambda) e_{s'}) > \mu_F(e_s)$ .

Fifth case

$$s \neq s' ; h = h' \text{ and } l \neq l'$$

If  $e_s$  is preferred to  $e_{s'}$ , then the residential utility for the job  $e_s$  is higher than the residential utility for the job  $e_{s'}$ .

Then the value of the residential utility for the job  $(\lambda e_s + (1 - \lambda)e_{s'})$  will be higher than the residential utility for the job  $e_{s'}$ .

Therefore  $\mu_F(\lambda e_s + (1 - \lambda) e_{s'}) > \mu_F(e_{s'})$ .

#### Sixth case

$$s = s' ; h \neq h' \text{ and } l \neq l'$$

If  $e_s$  is preferred to  $e_{s'}$ , because of an optimum combination of  $l$  and  $h$  ; then all the compromise will be still preferred to  $e_{s'}$ , which is the wished situation. Then  $\mu_F(\lambda e_s + (1 - \lambda) e_{s'}) > \mu_F(e_{s'})$ .

#### Seventh case

$$s \neq s' ; h \neq h' \text{ and } l \neq l'$$

The jobs  $e_s$  and  $e_{s'}$  are completely different. We can find again every previous case by determining what is or what are the different elements that make  $e_s$  preferred to  $e_{s'}$ .

Therefore,  $\forall (e_s ; e_{s'}) \in F^2$  if  $\mu_F(e_s) > \mu_F(e_{s'})$   
then  $\forall \lambda \in ]0, 1[ \mu_F(\lambda e_s + (1 - \lambda) e_{s'}) > \mu_F(e_{s'})$

However, a job seeker can be indifferent for  $e_s$  and  $e_{s'}$ , or he cannot compare the two jobs.

If the job seeker is indifferent for  $e_s$  and for  $e_{s'}$ , the preference for a combination of this two jobs is evident.

So we obtain (4)  $\mu_F(e_s) \geq \mu_F(e_{s'}) \Rightarrow \mu_F(\lambda e_s + (1 - \lambda) e_{s'}) > \mu_F(e_{s'})$ .

If the job seeker cannot compare the two jobs we have the axiome of the fuzzy subsets :  $\mu_F(\emptyset) = 0$ .

So  $\mu_F(e_s) = \mu_F(e_{s'}) = 0$  and the proposition (4) is confirmed.

This proposition (4) proves that  $\mu_F$  is strongly quasi concave, then  $\mu_F$  is strictly quasi-concave,  $\forall (e_s , e_{s'}) \in F^2$ .

Then  $F$  is strictly convex and  $\varphi(\alpha)$  is continuous over  $[0, 1]$ .

Moreover,  $\varphi(\alpha)$  is decreasing over  $[0, 1]$ . Then  $\varphi(\alpha)$  has a fixed point  $\alpha^*$  so that

$$\alpha^* = \sup_{\substack{1e^h \in \underline{C}^* \\ 1e^h \in \underline{E}}} \mu_F(e_s) = \sup_{1e^h \in \underline{E}} \mu_D(1e_s^h)$$

Now we must examine if this solution is unique, i.e. if the two following conditions are confirmed, first the function  $\mu_F$  must be strictly quasi concave (that is true) and second  $\forall 1e_s^h \in (\underline{C}_\alpha - \underline{C}_1)$  we must have  $\mu_F(1e_s^h) \neq 1$  and we cannot confirm this condition because we can find a job  $e_s$  so that  $\mu_F(1e_s^h) = 1$ .

Finally, a solution of the problem of the job seeker's fuzzy and spatial equilibrium exists but it is not unique.

This conclusion does not prevent our model to be relevant to describe a job seek strategy when the labor market is considered to be an imprecise space. And a numerical example will illustrate the model while it will be explaining how the solution can be found.

## 5. NUMERICAL APPLICATION

The solution  $e^*$  belongs to  $C \in \underline{E}$ , i.e. it belongs to the open disk  $(l_0, R)$  and it requires the qualifications  $h$  of the job seeker.

The job seeker has for each of his qualifications a reservation wage  $s^{*h}$ , a fuzzy utility  $F(s^{*h})$  and a seek labor market area determined by  $U_0(s^{*h})$ . A job seeker follows an optimum strategy if he begins to seek the jobs whose combinations wage qualification are the best.

If he does not find a job, he will have to change his spatial behavior and move to go to live at the job location.

Before we realize a numerical application, we are going to define numerically the membership function  $\mu_F$  and  $\mu_C$ .

$\mu_F$  is defined as follows :

$$\begin{aligned} \underline{E} &\longmapsto [0, 1] \\ \mu_F : e_s &\longmapsto \mu_F(e_s) \in [0, 1] \end{aligned}$$

$$\text{where } \begin{cases} \mu_F(e_s) = 0 & \text{if } s < \min(s^*, s_m) \\ \mu_F(e_s) = 1 & \text{if } s \geq \max(s^*, s_m) \\ \mu_F(e_s) = \frac{\min(s, s_m) - s}{|\epsilon|} & \text{if } \min(s^*, s_m) \geq \max(s^*, s_m) \end{cases}$$

$s^*$  is the reservation wage.

$s_m$  is the average wage actually observed.

$s^* - s_m = \epsilon$  is the estimation error of  $s^*$ .

$\mu_C$  is defined as follows :

$$\mu_C : \begin{array}{l} E \longrightarrow [0, 1] \\ e^1 \longrightarrow \mu_C(e^1) \in [0, 1] \end{array}$$

$$\text{where } \begin{cases} \mu_C(e^1) = 0 & \text{if } U_0 \leq 0 \iff d(l_0, 1) \geq R^* \\ \mu_C(e^1) = U_0 / U^* & \text{if } U_0 > 0 \iff 0 \leq d(l_0, 1) < R^* \end{cases}$$

$$U^* = s_0(1 - a)(T - tw_0).$$

We know that  $\mu_D(e^*) = \alpha^*$  where

$$\alpha^* = \sup_{\substack{l \\ e_s^h \in A}} \mu_F(e_s) = \sup_{\alpha \in [0, 1]} [\alpha \wedge \sup_{\substack{l \\ e_s^h \in \underline{C}_\alpha}} \mu_F(e_s)]$$

$$\text{with } \underline{A} = \left\{ \substack{l \\ e_s^h} \in \underline{E} / \mu_C \left( \substack{l \\ e_s^h} \right) \geq \mu_F \left( \substack{l \\ e_s^h} \right) \right\}$$

$$\text{and } \underline{C}_\alpha = \left\{ \substack{l \\ e_s^h} \in \underline{E} / \mu_C \left( \substack{l \\ e_s^h} \right) \geq \alpha \right\}$$

So we can calculate  $\alpha^*$  from  $\underline{A}$  or from  $\alpha$ .

If we choose the first mathematical programming we determine  $\underline{A}$ . The optimal job  $e^*$  belongs to  $\underline{A}$  and it has the highest value  $\mu_F(e_s)$ .

If we choose the second mathematical programming we determine for each value of  $\alpha \in [0, 1]$  the corresponding fuzzy subset  $\underline{C}_\alpha$ . The optimum job  $e^*$  belongs to  $\underline{C}_\alpha^*$  and it has the highest value  $\mu_F(e_s)$ .

If we find some jobs equally placed, the job seeker will choose among them the one which satisfies other wishes.

Now we state a numerical example.

The job seeker residential location is  $l_0$ . He has three qualifications  $h = \{1, 2, 3\}$  and he can choose among ten jobs.

$$\underline{E} = \left\{ e_2^{11}, e_3^{21}, e_4^{23}, e_5^{32}, e_5^{42}, e_5^{33}, e_5^{41}, e_6^{52}, e_8^{62}, e_8^{53} \right\}$$

We know the following values :

$$\begin{aligned} d(l_0, l_1) &= 5 \text{ (km)} & s_1 &= 4\ 000 \text{ (FF/month } \approx 4 \text{ weeks)} \\ d(l_0, l_2) &= 10 & s_2 &= 4\ 500 \\ d(l_0, l_3) &= 17 & s_3 &= 5\ 000 \\ d(l_0, l_4) &= 22 & s_4 &= 5\ 500 \\ d(l_0, l_5) &= 30 & s_5 &= 6\ 000 \\ d(l_0, l_6) &= 40 & s_6 &= 6\ 500 \\ & & s_7 &= 7\ 000 \\ & & s_8 &= 7\ 500 \end{aligned}$$

$$s_{m1} = 4\ 900 \text{ (FF/month } \approx 4 \text{ weeks)}$$

$$s_{m2} = 6\ 600$$

$$s_{m3} = 6\ 500$$

$$s_1^* = 5\ 300 \text{ (FF/month } \approx 4 \text{ weeks)}$$

$$s_2^* = 5\ 900$$

$$s_3^* = 5\ 800$$

We have obtained the following values for the membership functions  $\mu_F$  and  $\mu_C$ .

$l_h$ $e_s$	$e_2^{11}$	$e_3^{21}$	$e_4^{23}$	$e_5^{32}$	$e_5^{42}$	$e_5^{33}$	$e_6^{41}$	$e_8^{52}$	$e_8^{62}$	$e_8^{53}$
$\mu_C$	0,76	0,57	0,61	0,39	0,21	0,39	0,21	0,015	0	0,14
$\mu_F$	0	0,25	0	0,14	0,14	0,28	1	0,85	1	1

With the first mathematical programming we obtain :

$$\underline{A} = \left\{ e_2^{11}, e_3^{21}, e_4^{23}, e_5^{32}, e_5^{33}, e_5^{42} \right\}$$

$$\text{Sup}_{e_s \in \underline{A}} \mu_F(e_s) = \text{Sup} \{ 0 ; 0,25 ; 0 ; 0,14 ; 0,28 ; 0,14 \} = 0,28$$

$$\text{Hence } \alpha^* = 0,28 = \mu_F(e_5^{33}) \text{ and } e^* = e_5^{33}$$

With the second mathematical programming we obtain :

$\alpha$	$\underline{C}_\alpha$	$\text{Sup}_{e_s \in \underline{C}_\alpha} \mu_F(e_s) = \alpha'$	$\alpha' \wedge \alpha$
0	$\underline{E}$	1	0
0,1	$\left( e_2^{11}, e_3^{21}, e_4^{23}, e_5^{32}, e_5^{42}, e_5^{33}, e_5^{41}, e_8^{53} \right)$	1	0,1
0,2	$\left( e_2^{11}, e_3^{21}, e_4^{23}, e_5^{33}, e_5^{42}, e_5^{32}, e_5^{41} \right)$	1	0,2
0,3	$\left( e_2^{11}, e_3^{21}, e_4^{23}, e_5^{32}, e_5^{33} \right)$	0,28	0,28
0,4	$\left( e_2^{11}, e_3^{21}, e_4^{23} \right)$	0,25	0,25
0,5	$\left( e_2^{11}, e_3^{21}, e_4^{23} \right)$	0,25	0,25
0,6	$\left( e_2^{11}, e_4^{23} \right)$	0	0
0,7	$\left( e_2^{11} \right)$	0	0
0,8	$\emptyset$	-	-
0,9	$\emptyset$	-	-
1	$\emptyset$	-	-

Then  $\text{Sup} (\alpha \wedge \alpha') = 0,28$

$$\text{So } \alpha^* = 0,28 = \mu_F \left( e_5^{33} \right) \quad e^* = e_5^{33}$$

$$e_s \in \underline{C}_{0,28}$$

A job seeker will choose the job  $e_5^{33}$  which is located 17 km for from his residential location and whose wage is 6 000 FF per month. This wage is

higher than his reservation wage.

## CONCLUSION

The following conclusions can be drawn.

1. The theory of fuzzy subsets allows to describe less strictly the job seek strategy than with the reservation wage theory since a job seeker wishes to maximize the fuzzy wage utility.

2. The bifurcation theory allows to analyse the job seeker's spatial behavior as a variable behavior that refuses the traditional constant behavior assumption.

3. Finally, the problem of the job seeker's fuzzy and spatial equilibrium admits solutions. The solution is not often unique but this result does not penalize the model because a job seeker can yet choose among the different optimal solutions.

4. The theory of fuzzy subsets allows the complex behavior analysis and their applications to performant models for decisions making.



## REFERENCES

- BARRON J.M. and MELLOW W. "Search Effort in The Labor-Market", The Journal of Human Ressources, (1979), 389-404.
- BEN AKIVA M. and DE PALMA A. "Analysis of a Dynamical Residential Location Choice Model With Transaction Costs", Journal of Regional Science, 2, (1986), 321-343.
- D'ARVISENET P. "Prospection sur le marché du travail et mobilité géographique", Revue Consommation, 2, (1979), 4-32.
- DE PALMA A. "Bifurcation and Choice Behavior in Complex System", in Bifurcation Analysis, M. Hazewinkel and al Eds., (1985), 31-43.
- DUBOIS D. and PRADE H. "Fuzzy Sets and Systems : Theory and Applications", Academic Press, (1980).
- GOSS E.P. and SCHÖENING N.C. "Search Time, Unemployment and the Migration Decision", The Journal of Human Ressources, (1984), 570-579.
- GREENWOOD M.J., HUNT G.L. and McDOWELL J.M. " Migration and Employment Changes : Empirical Evidence on the Spatial and Temporal Dimension of the Linkage", Journal of Regional Science, 2, (1986), 223-234.
- HAZEWINKEL M. "Bifurcation Phenomena : A Short Introductory Tutorial With Examples", in Bifurcation Analysis, M. Hazewinkel and al Edition, (1985), 13-30.
- KAHN L.M. "The Returns to Job Search : a Test of Two Models", The Review of Economics and Statistics, (1978), 496-503.
- LAFFOND G. and LESOURNE J. "La dynamique des marchés avec processus de recherche et coût d'information", Economie Appliquée, Archives de l'ISMEA, 3/4, (1985), 703-767.
- LESOURNE J. "La science économique et l'autoorganisation", Document de Travail de l'I.M.E., n° 68, (1984), Dijon, France.

LÖSCH A. The Economics of Location, translated from the second revised edition by William H. Woglom with the assistance of Wolfgang F. Stolper, New Haven, Yale University Press, (1954).

NEGREPONTI-DELIVANI "Le comportement nouveau en matière d'emploi de la part des ménages", Revue d'Economie Politique, 1, 1979, 42-56.

PONSARD C. "On the Imprecision of Consumer's Spatial Preferences", Papers of the Regional Science Association, 42, 1979, 59-71.

PONSARD C. "L'équilibre spatial du consommateur dans un contexte imprécis", Sistemi Urbani, 3, 1981, 107-133.

PONSARD C. "Producer's Spatial Equilibrium with a Fuzzy Constraint", European Journal of Operational Research, 10, 1982, 302-313.

PREVOT M. "Bifurcation Sets. An Application to Urban Economics", in Bifurcation Analysis, Hazewinkel and al Eidtion, 1985, 101-114.

REA S.A. Jr "Workmen's Compensation and Occupational Safety Under Imperfect Information", The American Economic Review, 1981, 80-93.

RENAULT E. et LESOURNE J. "Autoorganisation et dispersion géographique des marchés", Economie Appliquée, Archives de l'ISMEA, 314, 1985, 637-663.

TANAKA H., OKUDA T. and ASAI K. "On Fuzzy Mathematical Programming", Journal of Cybernetics, 3, 1974, 37-46.

VISCUSI W.K. "Imperfect Job Risk Information and Optimal Workmen's Compension Benefits", Journal of Public Economics, 14, 1980, 319-337.

