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# Existence and uniqueness of periodic solutions of integro-differential equations with two variable in vector valued spaces

Bahloul Rachid<sup>1\*</sup>

## 1 Introduction

Motivated by the fact that neutral functional integro-differential equations (abbreviated, NFDE) with finite delay arise in many areas of applied mathematics, this type of equations has received much attention in recent years. In particular, the problem of existence of periodic solutions, has been considered by several authors.

In this poster, we study the existence of periodic solutions for the following integro-differential equation with two variable of the following form

$$\frac{\partial}{\partial \mathbf{x}} \frac{\partial}{\partial \mathbf{y}} (\mathbf{u}(\mathbf{x}, \mathbf{y}) - \mathbf{L}(\mathbf{u}_{\mathbf{x}, \mathbf{y}})) + \mathbf{A}\mathbf{u}(\mathbf{x}, \mathbf{t}) = \mathbf{G}(\mathbf{u}_{\mathbf{x}, \mathbf{y}}) + \int_{-\infty}^{\mathbf{x}} \int_{-\infty}^{\mathbf{y}} \mathbf{a}(\mathbf{x} - \mathbf{s}, \mathbf{y} - \xi) \mathbf{u}(\mathbf{s}, \xi) d\xi d\mathbf{s} + \mathbf{f}(\mathbf{x}, \mathbf{y}) \quad (1.1)$$

Where  $(A, D(A))$  is a closed linear operator on a Banach space  $X$ ,  $L$  and  $G$  are linear bounded operators,  $a \in L^1(\mathbb{R}_+ \times \mathbb{R}_+)$  and  $f : \mathbb{R} \times \mathbb{R} \rightarrow X$  is a locally  $p$ -integrable and  $2\pi$ -periodic function for  $1 \leq p < \infty$ .

$u_{x,y}(\theta, \xi) = u(x + \theta, y + \xi)$  for all  $\theta, \xi \in [-r, 0], x, y \geq 0$ .

## 2 A criterion for periodic solutions

Let  $X$  be a Banach Space. Firstly, we denote by  $\mathbb{T}$  the group defined as the quotient  $\mathbb{R}/2\pi\mathbb{Z}$ . There is an identification between functions on  $\mathbb{T}$  and  $2\pi$ -periodic functions on  $\mathbb{R}$ . We consider the interval  $[0; 2\pi)$  as a model for  $\mathbb{T}$ . Given  $1 \leq p < \infty$ , we denote by  $L^p(\mathbb{T} \times \mathbb{T}; X)$  the space of  $2\pi$ -periodic locally  $p$ -integrable functions from  $\mathbb{R} \times \mathbb{R}$  into  $X$ ,

For  $f \in L^p(\mathbb{T} \times \mathbb{T}; X)$ , we denote by  $\hat{f}(k, z)$ ,  $(k, z) \in \mathbb{Z}$  the  $(k, z)$ -th Fourier coefficient of  $f$  that is defined by:

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$$\widehat{f}(k, z) = \left(\frac{1}{2\pi}\right)^2 \int_0^{2\pi} \int_0^{2\pi} e^{-i(kx+zt)} f(x, t) dx dt.$$

for  $(k, z) \in \mathbb{Z} \times \mathbb{Z}$  and  $(x, t) \in \mathbb{R} \times \mathbb{R}$ .

**Notation 2.1.** : Let  $(k, z) \in \mathbb{Z} \times \mathbb{Z}$  and  $1 \leq p < \infty$ . Denote by  $H^{1,p}(\mathbb{T} \times \mathbb{T}; X) = \left\{ u \in L^p(\mathbb{T} \times \mathbb{T}; X) : \exists v \in L^p(\mathbb{T} \times \mathbb{T}; X), \widehat{v}(k, z) = -kz\widehat{u}(k, z) \right\}$ .  $\widetilde{\mathbf{a}}(\lambda, \mu) = \int_0^{+\infty} \int_0^{+\infty} e^{-(\lambda x + \mu y)} \mathbf{a}(\mathbf{x}, \mathbf{y}) d\mathbf{x} d\mathbf{y}$ ,  $\lambda, \mu \in \mathbb{C}$ , be the Laplace transform of  $a$

**Lemma 2.2.** For  $1 \leq p < \infty$ , a sequence  $\{M_{k,z}\}_{(k,z) \in \mathbb{Z} \times \mathbb{Z}}$  is said to be an  $L^p$ -multiplier (or  $(L^p, L^p)$ -multiplier) if for each  $f \in L^p(\mathbb{T} \times \mathbb{T}, X)$ , there exists  $u \in L^p(\mathbb{T} \times \mathbb{T}, Y)$  such that  $\widehat{g}(k, z) = M_{k,z}\widehat{f}(k, z)$  for all  $(k, z) \in \mathbb{Z} \times \mathbb{Z}$ .

**Definition 2.1.** For  $1 \leq p < \infty$ , a sequence  $\{M_{k,z}\}_{(k,z) \in \mathbb{Z} \times \mathbb{Z}}$  is said to be an  $L^p$ -multiplier (or  $(L^p, L^p)$ -multiplier) if for each  $f \in L^p(\mathbb{T} \times \mathbb{T}, X)$ , there exists  $u \in L^p(\mathbb{T} \times \mathbb{T}, Y)$  such that  $\widehat{g}(k, z) = M_{k,z}\widehat{f}(k, z)$  for all  $(k, z) \in \mathbb{Z} \times \mathbb{Z}$ .

**Definition 2.2.** : For  $1 \leq p < \infty$ , a sequence  $\{M_{k,z}\}_{(k,z) \in \mathbb{Z} \times \mathbb{Z}}$  is said to be an  $(L^p, H^{1,p})$ -multiplier if for each  $f \in L^p(\mathbb{T} \times \mathbb{T}, X)$ , there exists  $u \in H^{1,p}(\mathbb{T} \times \mathbb{T}, Y)$  such that  $\widehat{g}(k, z) = M_{k,z}\widehat{f}(k, z)$  for all  $(k, z) \in \mathbb{Z} \times \mathbb{Z}$ .

### 3 A characterization of strong $L^p$ -solution

**Definition 3.1.** : Let  $f \in L^p(\mathbb{T} \times \mathbb{T}; X)$ . A function  $u \in H^{1,p}(\mathbb{T} \times \mathbb{T}; X)$  is said to be a  $2\pi$ -periodic strong  $L^p$ -solution of Eq. ((1.1)) if  $u \in D(A)$ ,  $u(0, t) = u(2\pi, t)$ ,  $u(x, 0) = u(x, 2\pi)$  for all  $(x, t) \in \mathbb{T} \times \mathbb{T}$  and Eq.((1.1)) holds almost every where.

**Proposition 3.1.** Let  $X$  be a Banach space. Suppose that for every  $f \in L^p(\mathbb{T} \times \mathbb{T}; X)$  there exists a unique strong solution of Eq. ((1.1)) for  $1 \leq p < \infty$ . Then

- (1) for every  $(k, z) \in \mathbb{Z} \times \mathbb{Z}$  the operator  $(-kzI + A - \widetilde{\mathbf{a}}(ik, iz))$  has invertible.
- (2)  $\left\{ -kz(-kzI + A - \widetilde{\mathbf{a}}(ik, iz))^{-1} \right\}_{(k,z) \in \mathbb{Z} \times \mathbb{Z}}$  is an  $L^p$ -multiplier.

## 4 Existence of solution

**Lemma 4.1.** *Let  $1 \leq p < \infty$  and  $X$  be a Banach space. The following assertions are equivalent*

(1)  $\{M_{k,z}\}_{(k,z) \in \mathbb{Z} \times \mathbb{Z}}$  is an  $(L^p, H^p)$ -multiplier.

(2)  $\{-kzM_{k,z}\}_{(k,z) \in \mathbb{Z} \times \mathbb{Z}}$  is an  $L^p$ -multiplier.

Such that  $M_{k,z} = (-kzI + kzL_{kz} + A - G_{kz} - \tilde{a}(ik, iz))^{-1}$

**Theorem 4.2.** **Let  $A$  be a closed linear operator and  $1 \leq p < \infty$ . If**

**(1) for every  $(k, z) \in \mathbb{Z} \times \mathbb{Z}$  the operator  $(-kzI + kzL_{kz} + A - G_{kz} - \tilde{a}(ik, iz))$  has invertible.**

**(2)  $\{-kz(-kzI + kzL_{kz} + A - G_{kz} - \tilde{a}(ik, iz))^{-1}\}_{(k,z) \in \mathbb{Z} \times \mathbb{Z}}$  is an  $L^p$ -multiplier.**

**Then for every  $f \in L^p(\mathbb{T} \times \mathbb{T}; X)$  there exists a unique  $2\pi$ -periodic strong  $L^p$ -solution of (Eq 1).**

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