

A NON-LOCAL MODEL OF A RANDOMLY FIBERED MEDIUM

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Summary The case study above, by the rather restrictive assumptions about the behaviors of the material thickness, the stiffness of the fibers and their section has not allowed us to highlight a non-local phenomenon. This is why we restrict ourselves in this chapter if the thickness h of the material does not depend on ε . Our attempt has so far failed. However we propose two deterministic and non-local energy merely the expected model.

We are interested in the determination of the macroscopic behavior of a randomly fibered mechanical structure whose reference configuration is the volume \mathcal{O} of \mathbb{R}^3 , with basis $\hat{\mathcal{O}}$. More precisely for ε we consider the union of fibers $T_\varepsilon(\omega)$ with basis $D(\omega)$ are disks distributed at random in \mathbb{R}^2 following a stochastic point process $\omega = (\omega_i)_{i \in \mathbb{N}}$ associated with a suitable probability space $(\Omega, \mathcal{A}, \mathbf{P})$. We aim to supply a deterministic equivalent variational limit when ε tends to zero, of the sequence of random integral functionals H_ε mapping $\Omega \times L^p(\mathcal{O}, \mathbb{R}^3)$ into $\mathbb{R}^+ \cup \{+\infty\}$, defined for every ω in $(\Omega, \mathcal{A}, \mathbf{P})$ by

$$H_\varepsilon(\omega, u) = \varepsilon^p \int_{\mathcal{O} \setminus T_\varepsilon(\omega)} f(\nabla u) dx + \int_{\mathcal{O} \cap T_\varepsilon(\omega)} g(\nabla u) dx$$

Where $u = 0$ on $\Gamma_0 := \hat{\mathcal{O}} \times \{0\}$. For more precision on the stochastic point process $(\omega_i)_{i \in \mathbb{N}}$ and for all question of measurability relating to the considered random maps we refer the reader to the next section. For short we sometimes write T_ε instead of $T_\varepsilon(\omega)$.

The functional H_ε models the internal energy of a mechanical structure made up of the union T_ε of thin parallel cylinders which represent the rigid fibers and a soft elastic material matrix occupying $\mathcal{O} \setminus T_\varepsilon$. We only have a statistical knowledge of the cross sections of the fibers in the sense that their positions are statistically homogeneous. From the mathematical point of view, this means that they are placed at random according to a stationary point process. The stiffness of the elastic material occupying $\mathcal{O} \setminus T_\varepsilon$ is of order ε^p . The functions u represent the displacements of the mechanical structure subjected to a given load L and clamped on the plane Γ_0 . We assume large deformations in the matrix and the fibers so that the strong and soft materials are hyperelastic.

From the mathematical point of view we reexamine the work of [3, 2, 1] in a stochastic setting and in the scope of nonlinear elasticity. We establish the almost sure convergence of $(\mathcal{P}_{H_\varepsilon})$ when $\varepsilon \rightarrow 0$ to the deterministic and homogeneous problem

$$(\mathcal{P}_H) \quad \min \left\{ H(u) - \int_{\mathcal{O}} L \cdot u dx : u \in L^p(\mathcal{O}, \mathbb{R}^3) \right\}$$

where the energy functional H is of non local nature. More precisely we establish the almost sure Γ -convergence of the sequence $(H_\varepsilon)_{\varepsilon > 0}$ to the infimum convolution $F_0 \nabla G_0$ defined for every $u \in L^p(\mathcal{O}, \mathbb{R}^3)$ by

$$H(u) := \inf_{v \in L^p(\mathcal{O}, \mathbb{R}^3)} \left(F_0(u - v) + G_0(\theta v) \right)$$

where F_0 and G_0 are the functionals energy Γ -limits of this two materials (and $\theta \in (0, 1)$ is the asymptotic volume fraction). The non-locality is describe in the energy $F_0(u - v)$, indeed, v is in some way a virtual displacements of fibers.

References

- [1] C. Licht, G. Michaille. *A nonlocal energy functional in pseudo-plasticity*. Asymptotic Analysis, **45**, (2005), 313-339.
- [2] G. Michaille, A. Nait Ali, S. Pagano. *Macroscopic behavior of a randomly fibered medium*. Journal de Mathématiques Pures et Appliquées, vol(42), **3** (2011) 230-252.
- [3] G. Michaille, A. Nait Ali, S. Pagano. *Two-dimensional deterministic model of a thin body with randomly distributed high-conductivity fibers*. Applied Mathematics Research eXpress, **3** (2014) 122-156.

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