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A macroeconomic model with structural change and disequilibrium : a study of the economic consequences of the Portuguese revolution of 1974

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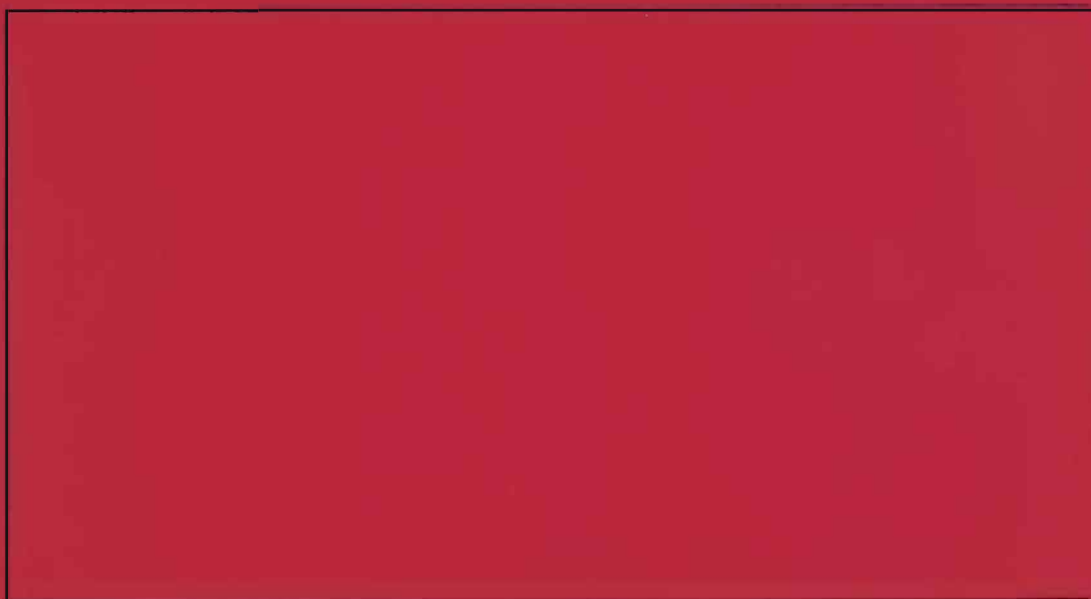
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A MACROECONOMIC MODEL WITH STRUCTURAL CHANGE
AND DISEQUILIBRIUM
A STUDY OF THE ECONOMIC CONSEQUENCES OF THE
PORTUGUESE REVOLUTION OF 1974

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A B S T R A C T

This paper presents and estimates a macroeconomic rationing model, using Portuguese data for the period 1955-1979.

Contrary to similar models found in the literature, the theoretical approach adopted in this study does not require the assumption of fully discrete regime nor that of a continuous framework. Instead, an intermediate specification is proposed which conserves the presence of regimes but involves no discrete jumps from one regime to another.

The presence of the min operator and the specification of a "clay-clay" production function raises a number of estimation problems. These are overcome by the use of some appropriate transformations and the adoption of a nonlinear two-stage least-squares procedure.

The results shed some light on the performances of the Portuguese economy during the period and afford a neat interpretation of the structural changes induced by the revolution of april 25th 1974.

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O. INTRODUCTION

Based on the principle that markets do not necessarily clear because prices adjust too sluggishly or in the wrong direction, in the past ten years there have been important developments in macroeconomics.

The fundamental assumption of this approach is that quantities adjust faster than prices. It might happen that agents sometimes are unable to exchange on one market all their goods at the prevailing price. If so, the transactions on the other markets will also be affected. For example, if firms can not sell all their production, they will reduce their demand for labour.

One implication of this kind of theory is the possibility of formalizing a type of structural change, i.e. whole economies or parts of economies passing through distinct regimes, each of them being ruled by different but stable behaviour relationships.

According to the above theory, there is a mis-specification in most of the actual macroeconomic models because their structure is always constant both in the estimation and in the predictive period.

Following the typology in Malinvaud (1977), most of these models represent economies which are constantly in a Keynesian regime, i.e. in a situation characterised by an excess supply of goods and labour.

Surely, attempts have been made to amend this. For instance there have been considered some indicators of disequilibrium like the degree of underutilisation of production capacities and the rate of unemployment. According to Muellbauer (1978), these indicators remain in the periphery of the model, despite the fact that they have improved the explanatory capacity of the model. They are fundamentally determined in an exogenous way and are not at ease with the theoretical framework of the rest of the model.

The models issued from the recent developments in macro-economics admit explicitly the possibility of excess demand (vs. supply) in the goods and the labour market. For this reason, they are generically known as "disequilibrium models".

Econometricians have developed techniques for modeling and estimating these models. Most of the work was done in the domain of a single market (see in particular Fair and Jaffe (1972), the pioneers ; Maddala and Nelson (1974), for a survey ; Quandt and Rosen (1978), for an application). More recently Ito (1980) and Gourieroux et al (1980) have extended these techniques to two markets, assuming discrete switching of regimes.

However, the number of empirical studies of complete macro-economic models is still very limited. In our opinion there are three kinds of reasons : first the still persisting insufficiency of theoretical developments in disequilibrium macroeconomics, specially in the field of dynamics. Some recent works (Green and Laffont (1981), Malinvaud (1980), Michel and Fourgeaud (1981) and Picard (1979)) deal with this problem. Second the econometric difficulties which are considerably increased if they are compared with traditional models. Last but not least, these models incorporate variables which are not statistically observed by actual accounting systems.

Though this work does not resolve all the above mentioned problems, it however shows :

(i) That, based on the framework of disequilibrium macroeconomics, it is possible to estimate a model which takes into account the possibility of structural change in the economy ;

(ii) That the econometric techniques necessary for the estimation of such a model are not too complicated and can be extended to more complete models.

As we did not intend to add to the theoretical developments of disequilibrium macroeconomics, we felt free on one hand to select features of this theory that seemed to be the empirically most promising ones, and on the other hand to suppress others, being of theoretical interest, but practically irrelevant.

The fundamental idea in the model is, that, at each period, one of the three following insufficiencies, namely goods demand, production capacity and labour supply, constitutes a bottleneck on the production of firms and obstructs economic activity. This gives three different possible approaches for determining production and employment.

However and contrarily to the common assumption of a discrete switching from one bottleneck to another, we will consider only one process under three different states of the economy.

More concretely the main features of the model are the following :

- (1) it is not assumed a complete jump from one regime to another.
The different states of the economy are characterised by different exact parts of the equations of the model. The stochastic specification is the same. We consider only one process and in this sense the economy will be on "average" in one single regime.
- (2) it is supposed that in an open economy the disequilibria verified in the production sphere are reflected in external trade.
- (3) households are not rationed in the goods market (the elasticity of imports at the world price is assumed to be infinite) and labour supply doesn't depend on transactions in this market.
- (4) a technology of a clay-clay production function is considered.
- (5) finally a nonlinear two-stage least-squares method is used for the estimation of the model.

The outline of the paper is as follows :

After having done a brief survey of some macroeconomic disequilibrium models already estimated (section 1) the presentation of the model is given. section 2.1 states the main characteristics and hypothesis and section 2.2 shows the adopted specification. The method of estimation is presented in section 2.3. section 2.4 deals with the way the model studies the structural changes in a given economy. The results of the estimation with Portuguese data are shown in section 3. The period of estimation is 1955-1979 but a particular attention is conferred to the structural changes which followed the Revolution of 25 April 1974. Some final remarks are given in section 4.

1. A BRIEF SURVEY OF SOME MACROECONOMIC DISEQUILIBRIUM MODELS
ALREADY ESTIMATED

We present below a brief characterisation of four studies dealing with the estimation of macroeconomic disequilibrium models. Each one corresponds to a special approach and undoubtedly they have inspired much of our work.

Kooiman and Kloek (1980) have estimated the Barro-Grossman model using the same concept of effective demand as Ito (1980). Two kinds of shortcomings characterise this approach. The first one is its complete static character. It is assumed that when an agent formalizes an effective demand in one market, he already knows the quantities he can exchange in the other markets. No reference is made to time. The constraints perceived by an agent about his demand or supply do not play any dynamic role, because they are straight derived from the actual exchanges (see Benassy (1977), for the concept of perceived constraints and Sneessens (1979), for a discussion about the implications of this approach).

The second shortcoming is the consideration at macroeconomic level of an efficient rationing criterion which implies the assumption that the transactions in all markets are exactly equal to the minimum of the demand and supply.

This assumption is in contradiction with empirical evidence, e.g. the simultaneous observation of unemployment and vacancies and the increase in labour productivity in periods of stimulation in economic activity.

The observation of these phenomena shows that, even if the firms are not rationed on the labour market, the transactions on this market (employment) will never be exactly equal to the labour demand (defined as the inverse of the production function). In order to understand the reason underlying such phenomena, one only needs to consider that, because of transaction costs, firms do not adjust, at each moment, labour supply to their needs. Under these conditions the rationing scheme will not be efficient.

Following Muellbauer (1978) one can say that, even if we assume that there is no inefficiency on the rationing at the microeconomic level, it will always be inefficient at the macroeconomic level. The justification is given through the aggregation : if some markets are in excess demand and other markets in excess supply, the total of the exchanges will not be equal to the minimum of the sum of total demands and of total supplies. Malinvaud (1980) states in the same direction that at a macroeconomic level the fluctuations of economic activity are smoother than at the microeconomic level and that the economy as a whole can hardly be in one single regime entirely.

So, in conclusion, the assumption of completely discrete regimes is not the appropriate one for estimating a complete macroeconomic model. This is reinforced by the fact that the econometric techniques, necessary for the estimation of the model under such an assumption, are heavy and hardly could be used for the estimation of a more disaggregate model (see in Ito (1980) and Gourieroux et al (1980), the methods of estimation of models with complete switching from one regime to another).

Sneessens (1979) proposes a solution for the two points mentioned above (efficiency of the rationing scheme and static character of the model).

After having noted that a rationing scheme will be only efficient if agents have a clear information about the market, the author adopts a rationing scheme that is only efficient in the expected values which each agent hopes to exchange. Moreover Sneessens admits that this specification is still insufficient for explaining the inefficiency empirically observed of the rationing scheme in the labour market. He proposes explicitly a formulization of the type

$$N = \min (E(\text{ND}), E(\text{NS})) - \frac{a_1}{1+b_1x^2} + \epsilon_t$$

where ϵ_t are residuals which represent the stochastic character of the rationing scheme ($E(\epsilon_t) = 0$), $x = E(ND) - E(NS)$ and where a_1 and b_1 are parameters that characterise the inefficiency of the allocation procedure in the labour market. This leads to the same type of criterion as the approach of Muellbauer, with another interpretation. However the author, in the end does not estimate this equation because he assumes that $b_1 x^2 = 0$ and that the constant term a_1 is incorporated in $E(ND)$ and in $E(NS)$. Consequently he has not answered the problem of inefficiency stated above.

About the second aspect, static character of the model, Sneessens introduces a certain dynamics in his model through the formalization of the perceived constraints by an agent.

For a better understanding of the difference of these two approaches, call YKA_t the constraints perceived by firms for their production, that is the maximum of production that firms think that they will be able to sell in period t ; $YC_t(.)$ the Walrasian production which means the level of production that maximizes their profits for a certain level of wages, prices and stocks; and call $NC_t(.)$ the level of employment corresponding to $YC_t(.)$.

The effective demand of labour ND_t will be

According to Sneessens :

$$(1) \quad ND_t = ND_t(.) + \alpha_1 (\bar{Y}_t - YC_t(.)) \quad (\alpha_1 < 0)$$

$$\bar{Y}_t = \min (YKA_t, YC_t(.))$$

$$YKA_t = \alpha E(YD_t) + (1 - \alpha) Y_t - 1$$

where α represents the "quantity of information" that firms have about future demand $Y D_t$. If they have an accurate information, α is equal to one, if they have no information α is zero. The agents will be wrong in their anticipations if $\alpha \neq 1$.

According to Kooiman and Kloek :

$$(2) \begin{aligned} ND_t &= NC_t(.) + \alpha_1 (Y_t - YC_t.(.)) && \text{if } Y_t = YD_t && (YD_t < YC_t) \\ &= NC_t(.) && \text{if } Y_t < YD_t \end{aligned}$$

Where Y is the effective production. Compared with the formalization (1), it is assumed that :

$$\begin{aligned} \bar{Y} &= Y && \text{if } Y = YD_t && (YD_t < YC_t) \\ \bar{Y} &= YC && \text{if } Y < YD_t \end{aligned}$$

We think that the approach of Sneessens is an ingenious way of introducing some dynamics in the model : if in a certain period t agents are wrong in their anticipations, they will change them for the next period. However the model that is specified must be coherent with this formalization. It must be able to answer what happens if in a certain period firms are wrong in their anticipations and if one assumes, like Sneessens does, that households are not rationed in the goods market. The model that was estimated by Sneessens with Belgium data is in our opinion too simple and can not answer this question.

Broer and Siebrand (1979) and Orsi (1980) follow approaches that lead to the abolition of regimes.

Before presenting each one of these, we discuss briefly the implications of such approaches. We think that one of the most important contributions of disequilibrium macroeconomics is to clarify the discussion about the possible theoretical states in which an economy can be found. In particular it has been shown that the same income policy can have different effects according to the state or regime in which an economy finds it self. Surely, it deals with theoretical states and we have already remarked that it is an important shortcoming to assume that

the economy will jump completely from one regime to another. However an approach which leads to the abolition of regimes gives a mixed regime which has a more difficult interpretation. One loses sight of alternative global regimes that may occur during the development of the trade cycle which can become a serious drawback if one is specially interested in global transitions of the economy between these regimes.

Broer and Siebrand assume that the level of transactions, defined by a transaction function, corresponds to a compromise between the supply and demand. Their main arguments are based on uncertainty that induces agents to compromise in favour of stable trade relation over time.

More concretely they consider a CES function.⁽¹⁾

$$(3) \quad Y_t = [a(YD_t)^p + (1-a)(YS_t)^p]^{-\frac{1}{p}}$$

where YD is the demand, YS is the supply, Y_t is the level of transaction, a is the weight of demand and p is a coefficient of nonlinearity.

The disequilibria are analysed through the parameters of the transaction function. However these parameters are considered constant all over the estimation period, which withdraws much of the disequilibrium character of the model.

Finally Orsi (1980) proposes a disequilibrium specification which is substantially a simultaneous version of the partial - adjustment model.

He defines for the endogenous variables of the model a partial - adjustment relation between equilibrium values which are generally not observed and observed values of these same variable. The typology of regimes disappears like in the Broer - Siebrand model. The disequilibria are considered through the adjustments between the equilibrium and observed values of endogenous variables.

(1) It is easily verified that $\lim_{p \rightarrow \infty} Y_t = \min(YD_t, YS_t)$

For a better understanding of his approach, we consider the following model which concerns only one market.

$$(4) \quad \begin{aligned} x_t^d &= a_1 Z_{1t} + a_2 P_t + \epsilon_{1t} \\ x_t^s &= b_1 Z_{2t} + b_2 P_t + \epsilon_{2t} \end{aligned}$$

x_t^d : demand ; Z_{1t} , Z_{2t} : vector of exogenous variables ;

x_t^s : supply ; P_t : price of the good exchanged in this market.

We are interested in estimating the exchanged quantity x_t on this market.

The "walrasian approach" consists in considering the price P_t as equilibrium price ($P_t = P_t^*$). So, we will have :

$$x_t = x_t^d = x_t^s = x_t^*$$

which, in practical terms, leads to estimating a traditional simultaneous equations system under equilibrium conditions. It is assumed that the observed values (x_t, P_t) , that means the statistical data, correspond to x_t^* and P_t^* .

The disequilibrium approach based on an efficient rationing criterion associates $X_t = \min (X_t^d, X_t^s)$ at each period, either with the demand curve or with the supply curve.(1)

Orsi considers that both of these situations do not constitute the general rule. He defines a partial adjustment between the observed values $(X_t$ and $P_t)$ and the equilibrium values $(X_t^*$ and $P_t^*)$ of the type

(1) The models exactly estimated depend on the hypothesis admitted for price P_t (see for a survey of these models Laffont and Monfort (1976) and Maddala and Nelson (1974)).

$$(5) \begin{bmatrix} x_t - x_{t-1} \\ p_t - p_{t-1} \end{bmatrix} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \begin{bmatrix} x_t^* - x_{t-1} \\ p_t^* - p_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}$$

where the Λ are adjustment coefficients and measure the speed of adjustment of (x_t, p_t) values to (x_t^*, p_t^*) .

The model to be estimated is composed of equations (4) and (5).

In the framework of a model where quantities adjust faster than prices, we can consider $\Lambda_{12} = \Lambda_{21} = \Lambda_{22} = 0$ and $x_t^* = \min(x_t^d, x_t^s)$. The rationing scheme defined by (5) can then be understood as follows ; agents do not exchange the quantities (x_t^*) that would maximise their utility, taking account only of the price and quantity constraints. There are other constraints that must be taken in consideration. For instance, at each period, firms keep a level of employment which is different from the optimum⁽¹⁾. There are other costs like training and hiring costs, which are not included in the production function but play an important role in decisions of firms about employment.

Interpreted in this way, the rationing criterion defined by Orsi can be used for explaining the transactions in the labour market, which will be done later.

However, this approach has two shortcomings when used systematically in a complete macroeconomic model. First it leads to the abolition of regimes (see above). Second the number of adjustment coefficients, which play a fundamental role, would become very big and would render the estimation very difficult. Orsi has estimated a model with only two equations (supply and demand for Italian exports) and even in

(1) This optimum is obviously defined as the level of employment that would maximize the profits of firms for a certain level of wages and prices, taking into account : (i) the technical constraints, given by the production function, (ii) the constraints perceived in goods market and (iii) the labour supply.

this simplified case he has fixed a priori the value for two (that means one half) of the adjustment coefficients. If we add that these coefficients must not be considered as constant for translating different speed of adjustment, we can imagine that the estimation of a macroeconomic model would become very hard.

As a way of conclusion of this brief survey of disequilibrium macroeconomic models, we note three points, which we specially have taken into account :

1) A complete macroeconomic model based on the assumption that the economy switches entirely from one regime to another doesn't find neither theoretical support nor empirical evidence. However a completely smoothing approach is also not advisable. First the parameters which measure the weight of each side of the market are considered like constants, which removes a lot of the disequilibrium character of the model. The consideration of these coefficients as variables would make the estimation practically impossible. Second the disappearing of the regimes drains from the model much of the contribution of recent developments in macroeconomics.

An intermediate approach is proposed in this work in which the existence of regimes is preserved but the evolution of the economy through different regimes is not discrete. It represents the same process under different states.

2) The formalization of perceived constraints by agents (specially firms), as in Sneessens (1979), constitutes an ingenious way of giving some dynamics to quantity rationing models which are fundamentally static. We will adopt this idea, but we shall formalize it in a way more compatible with our model.

3) A special importance must be accorded to the formalization of the external trade because most of the disequilibria that appear in a closed economy are "solved" through external trade in an open economy. For instance imports constitute an important element of supply in an open economy and they may avoid any rationing in the goods market. This will be done in Section 2.2.1.

2. THE MODEL

2.1. General structure, hypothesis and overview

The main characteristics of the model have already been noted. In this section we will state them in a more formal way and we will present our hypothesis about agents' behaviour.

There are three representative economic agents : households, firms and government and two goods : output and labour. Government action is considered exogenous and we admit the traditional specialisation of agents : firms supply output and demand labour, households demand output and supply labour.

An important characteristic of the model is the fundamental role played by firms in the determination of the production and employment levels.

More concretely we assume the following hypothesis about households behaviour :

(i) They are not constrained in the goods market, which means that their demand of consumption CD_t is equal to their ex post consumption (C_t).

Two kinds of reasons may justify this assumption. First we work at a macroeconomical level, so CD_t includes many substitutes : if an agent is rationed in one commodity, he can buy a substitute of this commodity. Second in the framework of an open economy, which is our case (see section 2.2.1), the rationing in the goods market will be hardly effective because of the possibility of imports when domestic production is insufficient.

(ii) The consumption function is the same for all the regimes.

This assumption states that households' consumption doesn't know important oscillations because of the existence of maintenance programs (like unemployment compensations, consumption habits and accumulated saving. This assumption is often used in the empirical work (eg. in Kooiman and Kloek (1980 and in Broer and Siebeand (1979)) because of the difficulties in the estimation. In the present work different specifications will be checked for the consumption function (see section 3.3.).

(iii) As a consequence of (i) we will assume that there is no spillover from the goods market to the labour supply (NS), which corresponds to walrasian supply NS (.).

About the behaviour of firms we consider that at the end of each period :

(i) They anticipate, for the next period t, a constraint on demand addressed to their production (YKA_t).

We will adopt a formalization of the type :

$$(6) \quad YKA_t = \gamma E(YD_t) + (1 - \gamma) Y_{t-1} + \gamma_1$$

where YD_t is the effective demand addressed to their products ⁽¹⁾.

Following Sneessens (1979) we consider that γ measures the information that producers have about future demand. For $\gamma = 1$ the producers know accurately the average level of demand ($\gamma_1 = 0$). For ($\gamma = 0$) they have no information about future demand. However in this case and contrarily to this author, producers will anticipate the value of the past year (Y_{t-1}) corrected by a coefficient γ_1 ($\gamma_1 \neq 0$) which may represent the average growth of Y_t during past periods. The inclusion of a constant term avoids that in periods of growth the anticipations were systematically pessimistic ($YKA_t < YD_t$) and in periods of depression systematically optimistic ($YKA_t > YD_t$).

(1) It includes, besides CD_t , the government and external demand which will be specified in the next section.

(ii) Taking in consideration YKA_t and the profitable production capacity $(YC_t(.))$ that corresponds to the level of production which would maximize its profits for the expected level of wages (\bar{w}_t) and prices (\bar{P}_t) , firms define the target level of production \bar{Y}_t as

$$(7) \quad \bar{Y}_t = \min (Y C_t(.), YKA_t)$$

where $Y C_t(.)$ is formally defined as :

$$YC_t(.) = F \left(F'^{-1} \left(\frac{\bar{w}_t}{P_t} \right) \right), F \text{ is the production function satisfying the usual conditions : } F'(N) > 0, F''(N) < 0 ; N \text{ being the labour factor.}$$

fyng the usual conditions : $F'(N) > 0, F''(N) < 0 ; N$ being the labour factor.

(iii) If they are not constrained with respect to labour, that means if $F^{-1}(\bar{Y}_t) < NS_t(.)$, firms will not change the programme of production. If they are constrained, $F^{-1}(\bar{Y}_t) > NS_t(.)$, they will define a new program according to availabilities of labour $F(NS(.))$.

Formally, if we call Y_t^* the target program of production, we will have :

$$(8) \quad Y_t^* = \begin{cases} \bar{Y}_t & \text{if } F^{-1}(\bar{Y}_t) \leq NS_t(.) \\ F(NS_t(.)) & \text{if } F^{-1}(\bar{Y}_t) > NS_t(.) \end{cases}$$

The determination of production and employment will be as follows :

The effective production (Y_t) will correspond on average to the target level of production Y_t^* .

$$(9) \quad E(Y_t) = Y_t^* \\ \text{or } Y_t = Y_t^* + \varepsilon_t$$

with $E(\varepsilon_t) = 0$

We note that Y_t^* is defined at the beginning of the period t . It corresponds to the level of production that maximizes the profits of firms, for the period t , taking into account their anticipations about : (i) the real wages which define YC_t ; (ii) the labour supply which gives $F(NS)$; (iii) the demand YKA_t . Hence, Equation (9) states that one (or more) of these three kinds of anticipations can be wrong. As a consequence some adjustments will necessarily take place and Y_t will not be exactly equal to Y_t^* .

As it was noted in Section 1 about ORSI'S model, the effective employment is determined according to an adjustment process.

$$(10) N_t - N_{t-1} = A_t \left[\Lambda F^{-1}(\bar{Y}_t) + (1-\Lambda) N_{t-1} + \Lambda_1 \right] + (1-A_t) NS_t(.) + \varepsilon_t$$

where A_t takes the value 1 if $F^{-1}(\bar{Y}_t) < NS_t$ and takes the value 0, otherwise.

The justification of Equation (10) can be given as follows.

For $A_t = 1$, it gives

$$(11) N_t - N_{t-1} = \Lambda (N_t^* - N_{t-1}) + \Lambda_1 + \varepsilon_t$$

where $N_t^* = F^{-1}(Y_t) = \min(F^{-1}(\bar{Y}_t), NS_t(.))$

This variable (N_t^*) is the level of employment that would maximize firms' profits under all constraints, i.e. YKA_t , YC_t and $NS_t(.)$. Because of the existence of hiring and firing costs the effective employment will be in general different from N_t^* . The parameter Λ represents the speed of adjustment between these two values. As Λ is lower, it is more expensive to adjust employment (Smith(1981)). The parameter Λ_1 avoids that we have systematically $N_t < N_t^*$ during the periods of growth of N_t and the contrary during the periods of decrease.

Equation (10) incorporates these aspects. In addition it has the advantage of considering that the speed of adjustment Λ is not constant and

depends on the situation in the labour market. It states that $\Lambda = 1$ and $\Lambda_1 = 0$ if $F^{-1}(\bar{Y}_t) > NS_t(\cdot)$, which means that in case of insufficiency of labour the firms would hire as much as possible.

As a final remark about this specification we note that it is implicitly assumed that the firms have information about $NS(\cdot)$ (more exactly they know its expected value) when their target level of production is defined.

The stochastic specification coherent with the model is :

$$(12) \quad YKA_t = \gamma E(YD_t) + (1 - \gamma) Y_{t-1} + \gamma_1$$

$$(13) \quad CD_t = CD_t(\cdot) + \varepsilon_{1t}$$

$$(14) \quad YD_t = CD_t + EXO_t + XD_t - MD_t$$

$$(15) \quad NS_t = NS(\cdot) + \varepsilon_{2t}$$

$$(16) \quad \bar{Y}_t = \min \left(F \left(F^{-1} \left(\frac{\bar{W}_t}{P_t} \right) \right), YKA_t \right)$$

$$(17) \quad YC_t = F \left(F^{-1} \left(\frac{\bar{W}_t}{P_t} \right) \right) + \varepsilon_{3t}$$

$$(18) \quad Y_t^* = \min (\bar{Y}_t, F(NS_t(\cdot)))$$

$$(19) \quad Y_t = Y_t^* + \varepsilon_{4t}$$

$$(20) \quad N_t - N_{t-1} = A_t \left[\Lambda F^{-1}(\bar{Y}_t + (1-\Lambda) N_{t-1} + \lambda_1) \right] + (1-A_t) NS_t(\cdot) + \varepsilon_{5t}$$

$$(21) \quad N_t^* = \min (NS_t(\cdot), F^{-1}(\bar{Y}_t))$$

where EXO_t is the exogenous part of YD_t , XD_t and MD_t are respectively the export and import demands for period t . Their formalization will be given in the next section.

It is easily shown that :

$$(22) \quad Y_t = F(N_t^*) = \min \left\{ YKA_t, F \left(F^{-1} \left(\frac{\bar{W}_t}{P_t} \right) \right), F(NS_t(\cdot)) \right\} + \varepsilon_{4t}$$

Scheme 1 illustrates the determination of each regime, called, according to the usual terminology : Keynesian unemployment, classical unemployment and repressed inflation. **Regime (4) is different from (3) because it corresponds to different anticipations.** Sneessens (1979) has called it underconsumption regime. However as both situations correspond to the same **observed values** we prefer to give them the same name : repressed inflation.

One can easily check that the model is coherent : the three regimes exhaust all the possibilities and at each time only one regime is observed. The model has only one reduced form.

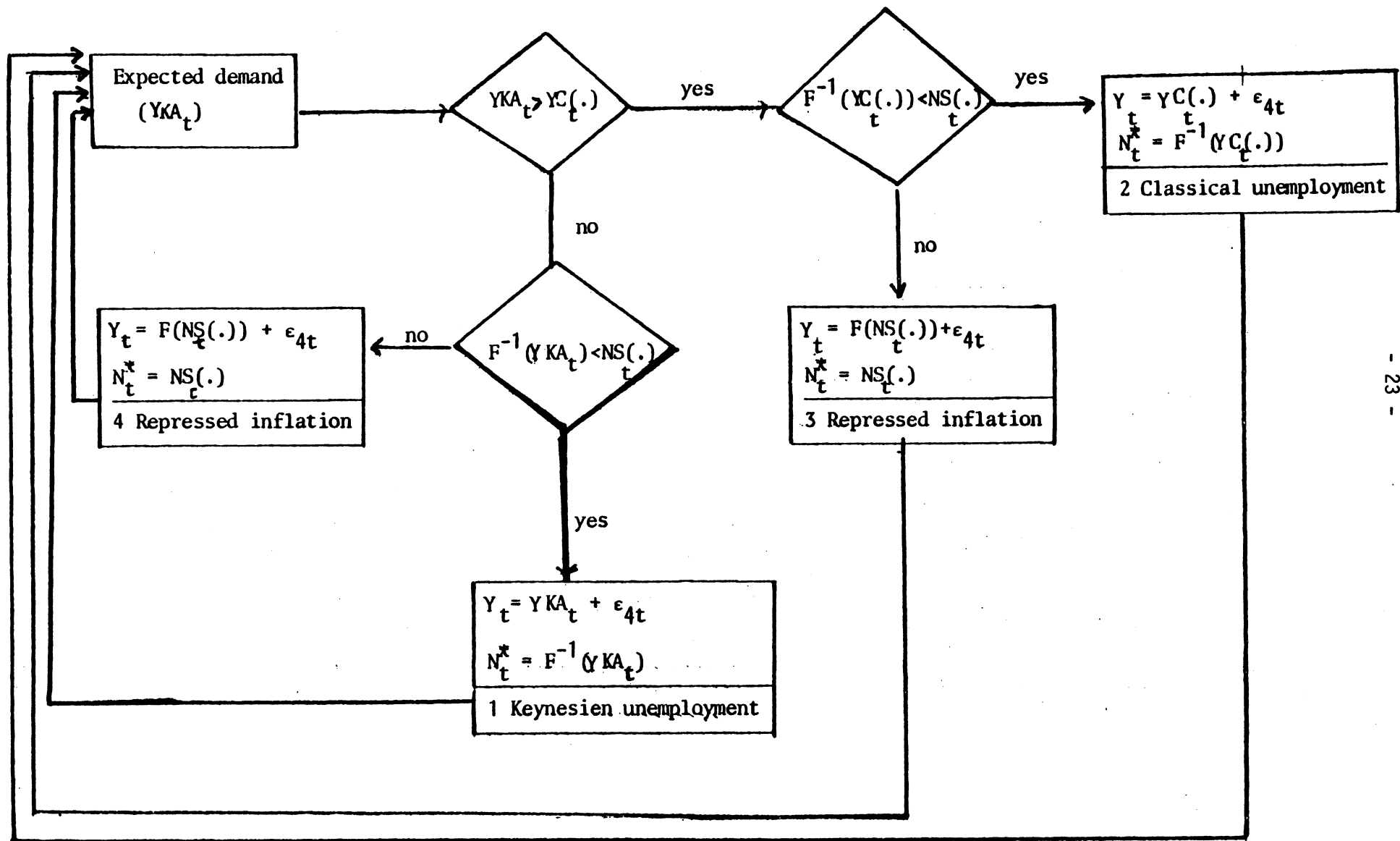
Before passing to the specification of the model we note once again that there is not a discrete switching from one regime to another : the regime in which the economy is at a certain period t depends on the bottleneck that defines the basic (or exact) structure of the model. There are three kinds of bottlenecks (1) :

1) In the Keynesian regime, firms anticipate to constrained in the demand for period t, which doesn't allow them to use all their production capacity, we remark producers are allowed to be wrong in their anticipations. However as we work with annual data these errors in the anticipations will not be important ($\gamma \approx 1$) because the firms have time to adapt their programs of production to demand during the period t. In any way the formulation of the block of the demand must account for such a possibility which we will do (section 2.2.1.).

2) In the classical regime the bottleneck is the profitable production capacity, originated by a too high level of real wages. We don't take endogenous investments which would increase the number of regimes. At each period of time and for a certain level of prices and wages, production capacity is fixed but is unknown.

(1) Fourgeaud and Mitchel (1981) consider a fourth possible bottleneck which corresponds to the physical capacity of production. We assume that in vue of maintenance costs, the profitable and physical capacity will be pratically the same.

Scheme 1 : THE DETERMINATION OF THE REGIME



3) Finally in regime of repressed inflation there is an insufficiency of labour. The employment is not derived from the usual "min condition" because this implies that firms go, each period t , to an "abstract market" and demand exactly the labour they need. We consider that firms do not adjust, at each period t , employment exactly to the needs of production because the training and hiring costs are not considered in the production function (1). Surely the speed of adjustment is not constant and depends on the situation in the labour market. This explains the productivity cycle which has been empirically established.

2.2. Specification

In this section we will present the specification used in the estimation and we will make some additional remarks about the equations of the model.

There are four blocks : demand, production capacity, labour supply and determination of production and employment. The meaning of the variables is given in Appendix A.

2.2.1. Block 1 : Demand

The following equations are considered in this block :

$$(23) \quad \ln YKA_t = \gamma E(\ln YK_t) + (1-\gamma) \ln Y_{t-1} + \gamma_1$$

$$(24) \quad YK_t = EXO_t + CD_t + XD_t - MD_t$$

$$(25) \quad \Delta \ln CD_t = g_1 + g_2 \Delta \ln DI_t + g_3 \ln \left(\frac{DI_t}{CD_t} \right)_{-1} + g_4 \ln \left(\frac{DI_t}{RNS_t} \right)_{-1} + g_5 C_t^{+0.1t}$$

$$(26) \quad \ln XD_t = \ln X_{t-1} + c_1 \Delta \ln DW_t + c_2 \Delta \ln \left(\frac{PEX_t}{PET_t} \right) + \epsilon_{2t}$$

$$(27) \quad \ln MD_t = \ln M_{t-1} + d_1 \Delta \ln CD_t + d_2 \Delta \ln (EXO_t) + d_3 \Delta \ln \left(\frac{PIM_t}{PY_t} \right) + \epsilon_{3t}$$

$$(28) \quad \ln X_t = \ln XD_t + c_3 \ln \left(\frac{YC_t}{Y_t} \right) + c_4 \ln \left(\frac{YK_t}{YKA_t} \right) + c_5 + \epsilon_{4t}$$

$$(29) \quad M_t = CD_t + EXO_t + X_t - Y_t$$

(1) This formally means that $h(Y) \neq F^{-1}(Y)$, where $h(Y)$ is the demand of labour for producing Y .

The first equation formalizes producers' expectations according to what was reported in Section 2.1.

Equation (24) is an identity and gives total demand for domestic goods and services YK_t .

Equation (25) is the consumption function and is basically the one derived by Hendry and Von Ungem-Stenberg (1979) and adopted in Kooiman and Klock (1980) (1).

Equations (26)-(29) formalize external trade. The first two give respectively exports (XD_t) and imports (MD_t) demand i.e. those exports and imports which would be realized if disequilibria were not verified in the supply side of the economy. Equations (28) and (29) translate the repercussions in the external trade of these disequilibria. The behaviour of imports is different from that of exports.

It is assumed that imports (Equation (29)) have an infinite elasticity. At the world price it is possible to import all the quantities desired. This assumption is coherent with the hypothesis of non rationing of households in goods market stated in Equation (25) (2).

The consideration of the variable YC_t/Y_t in Equation (28) can be explained in two ways.

First, if firms work nearly at full capacity (the economy is in a classical regime or in its neighbourhood), a rise in real wages will imply a fall of exports ($c_3 \geq 0$, $c_5 \leq 0$). Dreze and Modigliani (1981) point out that the amplitude of this deterioration will be different if firms accept some loss of sales and/or of profits or scrap.

(1) The accumulated real saving was replaced by the nonlabour real income because of the availability of data. The justification of the dummy variable C_t is given in Section 3., where alternative specifications for this equation were checked.

(2) we note that we can write $YK_t - Y_t = M_t - MD_t - (X_t - XD_t)$.

Second, if there is underutilisation of production capacities, firms are more competitive and they can "export" one part of this underutilisation. However, contrarily to Dixit (1978), we don't admit that this "export of underutilisation" is total. Firms can not sell abroad everything they want. According to Dixit's hypothesis an open economy could experience only classical and not Keynesian unemployment. Or as Malinvaud (1981) points out this argument is misleading because it relies on two usually unwarranted hypotheses :

- (i) that the foreign market is one of excess demand for goods,
- (ii) that this excess demand is instantaneously transmitted to the domestic market.

The presence of YKA_t/YK_t in this same Equation (28) has a similar interpretation. It states that exports will be affected if domestic producers have been wrong in their anticipations ($\gamma \neq 1$). In case of an overvaluation ($YKA_t > YK_t$) they will try to export a part of the surplus ($X \geq XD$). In the opposite case ($YKA_t < YK_t$) domestic producers may not be able to meet all their engagements ($X < XD$). Hence the parameter c_4 must be nonnegative ($c_4 \geq 0$).

2.2.2. Block 2 : Production function and the determination of production capacity

We have used a Clay-Clay technology with the following equations :

$$(30) \quad mc_t = \frac{1}{\ln(1+b')} (\ln \alpha' + \ln(1+a')t - \ln L_t) + t$$

$$(31) \quad YC_t = \alpha(1+a)^t \frac{\sum_{t-mc_t}^{t-1} E(v) e^{\epsilon 5t}}{t-mc_t}$$

$$(32) \quad NC_t = \frac{\alpha}{\alpha'} \left(\frac{1+a}{1+a'} \right)^t \frac{\sum_{t-mc_t}^{t-1} E(v)}{(1+b')^t} v$$

The same equations are presented in Vilares (1980) so we will only present some particular characteristics related to the present model. The first relation is derived from the well known neoclassical equality between the marginal productivity of labour ($u(t, t - mc_t)$) and the real wage (W_t/P_t). As the verification of this equality imposes restrictive assumptions we have changed it, replacing W_t/P_t by :

$$L_t = \sum_{i=0}^s \theta_i \frac{W_{t-i}}{P_{t-i}}, \quad \sum \theta_i = 1$$

which represents the real wage anticipated by firms. It determines, for each period, the expected production capacity (exact part of equation (31)). It can also be interpreted as follows : if in a certain period t an equipment is not profitable, firms will not scrap it if they anticipate a fall in the real wage that could make this equipment profitable again.

Formally, if the relevant markets are in perfect competition, we will have the following relation :

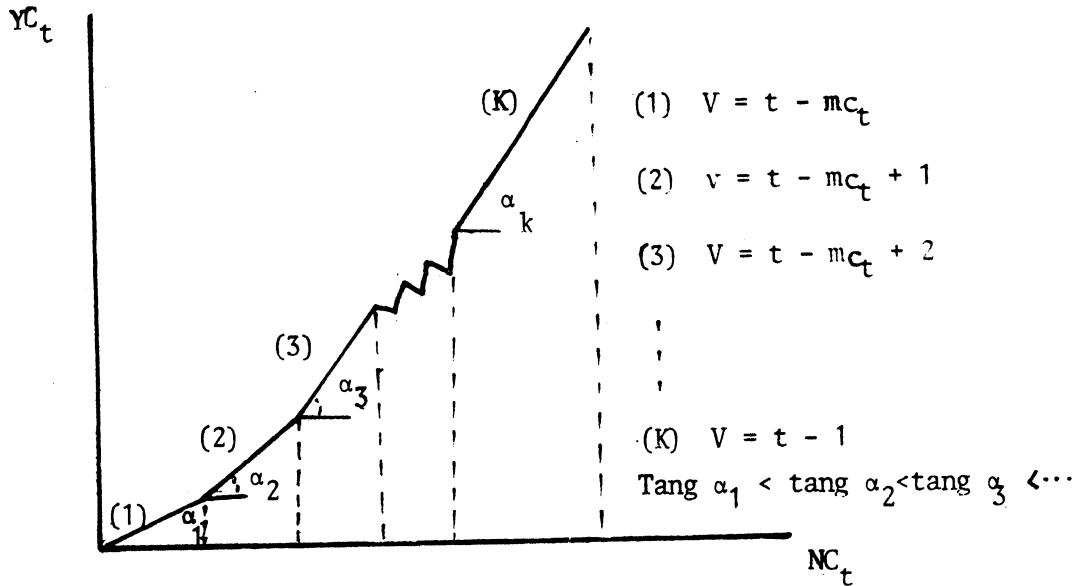
$$u(t, t - mc_t) = \alpha'(1 + a')^t (1 + b')^{t - mc_t} = L_t$$

from where Equation (30) is derived.

The parameters \underline{b}' and \underline{a}' ($b', a' \geq 0$) represent the growth rates of labour productivity. The first parameter (b') refers to embodied technical progress and the second one (a') represents not embodied technical progress. The value of mc_t defines the age of the oldest profitable vintage in the stock of capital and enters in the computation of YC_t (Equation (31)).

One of the main characteristics of a Clay-Clay technology is that labour productivity over an equipment depends on the date of the installation of this equipment. The stock of capital correspond to the sum of heterogenous vintages with different labour productivities. We represent in a generical way the production function by $YC = F(NC)$. We remark that $F(NC)$ is not differentiable and that the calculation of $YC = F(NC)$ or $NC = F^{-1}(YC)$ is much more complicated than in the case of Putty-Putty production functions. This is illustrated in Figure 1, below :

Figure 1 : THE NONDERIVABILITY OF THE PRODUCTION FUNCTION



The problem of the nonderivability of the production function will be solved by computing the derivatives of a smooth approximation of this function that is continuous and derivable (section 2.3.2.).

Equation (32) determines the maximum profit level of employment (NC_t). The parameter a represents the rate of growth of capital productivity and incorporates the obsolescence ($a \leq 0$).

2.2.3. Block 3 : Labour supply

The equations in this block are :

$$(33) \quad \ln NR_t = e_1 + e_2 \ln POF_t + e_3 \ln (RW_t)$$

$$(34) \quad \ln NRE_t = \ln NR_t + e_4 B_t \ln \left(\frac{NRE_t}{N_t} \right)_{-1} + e_5 (1-B_t) \ln \left(\frac{EM_t}{\overline{EM}_t} \right) + e_6 t$$

$$(B_t = 1 \text{ if } t \geq 74 ; B_t = 0 \text{ if } t < 74)$$

$$(35) \quad YR_t = \alpha(1+a)^t \sum_{t-mr_t}^{t-1} \frac{E(v)}{(1+b')^v} e^{\varepsilon_7 t}$$

$$(36) \quad NR_t = \frac{\alpha}{\alpha'} \left(\frac{1+a}{1+a'} \right)^t \sum_{t-mr_t}^{t-1} \frac{E(v)}{(1+b')^v}$$

As we have admitted that households are not constrained in the goods market, there is no spillover from the goods market and one can estimate labour supply in a separate block. This assumption is not restrictive because even in its absence one can argue that households supply of labour will still almost be unchanged provided they believe that the rationing will not persist in the future. Hence, the problem is to evaluate the labour supply, i.e. the labour availabilities in the economy NR. The direct replacement of NR by the statistical supply of labour NRE_t , that is by the sum of the employment and of the unemployment, is not possible because the two concepts are different (1). A stochastic relation was defined between these two variables (cf. below).

Equation (33) is taken from Rosen and Quandt (1978) and one can consult their work for its derivation (2)

Equation (34) gives in a very simple way the relation between NR_t and NRE_t for the Portuguese case. For the years preceding 1974 the ratio NRE/NR depends on the evolution of the Portuguese emigrants (EM_t) divided by its trend (\overline{EM}_t). For the other years such ratio (NRE/NR) is a nonlinear function of the rate of unemployment η_t (3).

This specification accounts for the important role played by the emigration in the labour market. Particularly, it considers that in periods of important emigration, a part of the emigrants are discouraged workers i.e. they belong to NR_t but not to NRE_t ($e_5 > 0$).

(1) Theoretically two aspects must be considered. On one hand NR_t must be bigger than NRE_t because of the discouraging effect: NR_t must include the number of people actually willing to work at prevailing conditions but not necessarily listed as either employed or looking for a job. On the other hand one part of NRE_t is not available because of the frictional unemployment.

(2) We have not considered the nonlabour income as explanatory variable, because in our estimations its coefficient was not significant. A new estimation of the Rosen-Quandt model with american data (Romer (1981)) has shown the same results.

(3) For these years equation(34) can be written as : $NRE_t/N_t = (1/1-\eta_{t-1})^4 e^{\varepsilon t}$
 $\eta_t = (NRE_t - N_t)/NRE_t$

After 1974, following the halt in immigration decided by European Governments, the Portuguese emigration has practically stopped and it doesn't reflect any longer the situation in the labour market. For this period (1974-1979) it is supposed that the growing level of unemployment (see why in section 3.) has constituted a discouraging factor and as a consequence e_5 must be positive.

In Section 3.3. the sensibility of the results to an alteration in the specification of this equation (34) is studied.

Equation (35) defines the level of production corresponding to an efficient utilisation of NR_t and Equation (36) is used to compute mr_t which is an intermediate variable for the evaluation of YR_t .

2.2.4. Block 4 : the determination of production and employment

The specification of the last six equations is easily derived from what we have said in section 2.1. :

$$(37) \quad \ln Y_t = \min (\ln YKA_t), E (\ln YC_t), E (\ln YR_t)) + \varepsilon_{8t}$$

$$(38) \quad YKA_t = \alpha(1 + a)^t \sum_{t-mka_t}^{t-1} E(v)$$

$$(39) \quad NKA_t = \frac{\alpha}{\alpha'} \frac{(1+a)^t}{1+a} \sum_{t-mka_t}^{t-1} \frac{E(v)}{(1+b')^v}$$

$$(40) \quad \ln N_t = A_t \left[\Lambda \min (\ln NKA_t, \ln NC_t) + (1-\Lambda) \ln N_{t-1} + \Lambda_1 \right] \\ + (1-A_t) \ln NR_t + \varepsilon_{9t}$$

Equation (37) has already been presented in section 2.1. with the only difference that it is now expressed in logarithms because of the nonlinearity of the adopted production function.

Equation (38) is used to determine the intermediate variable mka_t which is the inferior limit of the sum in NKA_t (Equation (39)).

Finally Equation (40) gives the level of employment according to what was reported in section 2.1.

2.3. The method of estimation

2.3.1. The complete model

The model to be estimated has 18 equations, (23) - (40). They are four blocks. As they are interdependent we can not estimate individually each block. The existence of the operator "expected value" leads to a model where only five equations (25), (28), (34), (37) and (40) are estimated as behaviour equations. In the other endogenous variables which are not observed only its exact part (its expected value) is used in the process of estimation.

Let T be the number of observations and m the number of endogenous variables, we assume the following general hypothesis about the residuals :

$$(41) \quad \begin{aligned} (1) \quad E(\epsilon_{ij}) &= 0 \quad i = 1, m \quad j = 1, T \\ (2) \quad E(\epsilon_{ij}\epsilon_{ks}) &\begin{cases} \sigma_{ik} & \text{if } j = s \quad i, k = 1, m \\ 0 & \text{if } j \neq s \quad j, s = 1, T \end{cases} \end{aligned}$$

2.3.2. The difficulties of estimation

As we have said in the introduction we will propose a nonlinear two-stage least-squares (NL2SLS) method.

However, before presenting it, three problems muste be solved :

- (i) Some of the endogenous variables are statistically not observed :

$$YKA_t, E(\ln YC_t), E(\ln YR_t), \ln XD_t, \ln MD_t, \ln NKA_t, \ln NC_t, \ln NR_t$$

(ii) The existence of a "min operator" in Equations (37) and (40) generates a discontinuous non differentiable model (derivatives can be computed but first derivatives are discontinuous). We note that the points of discontinuity are not known, since the concerned variables are not observed.

(iii) The non-differentiation of the production function (see section 2.2.2.).

2.3.2.1. The problems of the endogenous variables statistically not observed

This problem was systematically solved, replacing the not observed variables by their expressions given by the model. The case of $\ln YK_t$, $E(\ln YC_t)$, $E(\ln YR_t)$ raises some problems of a practical nature and we present below how we have solved them.

For the computation of YK_t we have proceeded as follows. First, we subtract equation(29) for $t = t-1$ from Equation (24), and we get :

$$YK_t - Y_{t-1} = EXO_t - EXO_{t-1} + CD_t + XD_t - X_{t-1} - (MD_t - M_{t-1}) - CD_{t-1}$$

or

$$(42) \quad \frac{YK_t - Y_{t-1}}{Y_{t-1}} = \frac{EXO_t - EXO_{t-1}}{Y_{t-1}} + \frac{CD_t - CD_{t-1}}{CD_{t-1}} \frac{CD_{t-1}}{Y_{t-1}} + \frac{XD_t - X_{t-1}}{X_{t-1}} \frac{X_{t-1}}{Y_{t-1}} - \frac{MD_t - M_{t-1}}{M_{t-1}} \frac{M_{t-1}}{Y_{t-1}}$$

Using the following approximations (1).

$$(YK_t - Y_{t-1}) / Y_{t-1} = \ln YK_t - \ln Y_{t-1}$$

$$(XD_t - X_{t-1}) / X_{t-1} = \ln XD_t - \ln X_{t-1}$$

$$(43) \quad (CD_t - CD_{t-1}) / CD_{t-1} = \ln CD_t - \ln CD_{t-1}$$

$$(MD_t - M_{t-1}) / M_{t-1} = \ln MD_t - \ln M_{t-1}$$

We get :

$$(44) \quad \ln YK_t = \ln Y_{t-1} + DA_t (X_{t-1}/Y_{t-1}) \ln(XD_t/X_{t-1}) - (M_{t-1}/Y_{t-1}) \ln(MD_t/M_{t-1})$$

where $DA_t = (EXO_t - EXO_{t-1})/Y_{t-1} + CD_{t-1}/Y_{t-1} \ln(CD_t/CD_{t-1})$

(1) They result form the first order Taylor expansions of $\text{EXP}(\ln YK_t)$, $\text{EXP}(\ln XD_t)$, $\text{EXP}(\ln MD_t)$, $\text{EXP}(\ln CD_t)$ around respectively $\text{EXP}(\ln Y_{t-1})$, $\text{EXP}(\ln X_{t-1})$, $\text{EXP}(\ln M_{t-1})$, $\text{EXP}(\ln CD_{t-1})$.

Whose expectation is (1) ;

$$(45) \quad E(\ln YK_t) = DA_t + (X_{t-1}/Y_{t-1}) E(\ln XD_t/XD_{t-1}) - (M_{t-1}/Y_{t-1}) E(\ln MD_t/M_{t-1}) \\ + \ln Y_{t-1}$$

Finally, we put :

$$(47) \quad \begin{aligned} DDW_t &= (X_{t-1}/Y_{t-1}) \Delta \ln DW_t \\ CEX_t &= (X_{t-1}/Y_{t-1}) \Delta \ln (PEX_t/PET_t) \\ CCD_t &= (M_{t-1}/Y_{t-1}) \Delta \ln CD_t \\ EEXO_t &= (M_{t-1}/Y_{t-1}) \Delta \ln EXO_t \\ CEM_t &= (M_{t-1}/Y_{t-1}) \Delta \ln (PIM_t/PY_t) \end{aligned}$$

This gives :

$$(48) \quad E(\ln YK_t) = DA_t + c_1 DDW_t + c_2 CEX_t - d_1 CCD_t - d_2 EEXO_t \\ - d_3 CEM_t + \ln Y_{t-1}$$

which allows to derive, from (23), $\ln YKA_t$

$$(49) \quad \ln YKA_t = \gamma DA_t + \ln Y_{t-1} + \gamma c_1 DDW_t + \gamma c_2 CEX_t - \gamma d_1 CCD_t \\ - \gamma d_2 EEXO_t - \gamma d_3 CEM_t + \gamma_1$$

The calculation of $E(\ln YC_t)$ is easier, because one only needs to take logarithms in (31). We will get in this case :

$$(50) \quad E(\ln YC_t) = \ln a + \ln(1+a)t + \ln \left(\sum_{t-mc_t}^{t-1} E(v) \right)$$

mc_t being computed by Equation (30).

(1) The expectations ("EXP") are always computed, conditionnal to the past values of the endogenous variables.

In an analogous way we get $E(\ln YR_t)$. From (35) we write :

$$(51) \quad E(\ln YR_t) = \ln a + \ln(1+a)t + \ln\left(\sum_{t-mr_t}^{t-1} E(v)\right)$$

mr_t being derived from Equation (36).

2.3.2.2. The existence of a min operator

This problem has been studied in the framework of the switching regression models when the points of structural change are not known (1). However until now, only linear models with two possible regimes have been estimated (see Goldfeld and Quandt (1972), Tishler and Zang (1979) and Ginsburgh, Tishler and Zang (1980)). Clearly, the theoretical extension to a nonlinear model with three regimes gives no problems, but the estimation becomes much more complicated because of the simultaneous character of our problem.

We start by formalizing the problem and then we will present the adopted solution.

Let YL_t , Y_{1t} , Y_{2t} and Y_{3t} represent respectively $\ln Y_t$, $\ln YKA_t$, $E(\ln YC_t)$, $E(\ln YR_t)$.

The problem is to estimate :

$$(52) \quad YL_t = \min (Y_{1t}, Y_{2t}, Y_{3t}) + \varepsilon_t$$

$$(\varepsilon_t = \varepsilon_{8t})$$

(1) If these points were known, the estimation would be much easier. One could use the technique of least square cubic splines presented in Poirier (1976) or simply the restricted least squares (RLS). Buse and Lim (1977) have shown that the standard cubic splines can be in most cases considered as a special case of RLS. After having estimated NKA_t , NC_t and NR_t one can use any one of these techniques for estimating Equation (40). (see Section 2.3.3.) so, its estimation will not be explicated here.

where Y_{1t} is given by (49), Y_{2t} by (30) and (50) and Y_{3t} by (36) and (51).

Assuming, for actual purposes, that our model has only these equations $\{(49), (30), (50), (36), (51)\}$ and that the ϵ_t is $NID(0, \sigma^2)$, the maximum likelihood for (52) can be obtained as the minimum of :

$$(53) \quad \Psi = \sum \epsilon_t^2$$

The minimisation is carried out with respect to the parameters of Y_{1t} , Y_{2t} and Y_{3t} : $\gamma, \gamma c_1, \gamma c_2, -\gamma d_1, -\gamma d_2, -\gamma d_3, \gamma_1, \ln(1+a), \ln \alpha, \ln(1+a')$ and $\ln \alpha'$ ⁽¹⁾. However as we have already said, the first derivatives of Ψ are discontinuous and its minimum can not be obtained via efficient gradient techniques.

The basic idea of the approach, generally used to solve this problem, is to replace the operator "min" by some smooth approximation generating a function Ψ which is continuous and derivable.

We have used the following approximation :

$$(54) \quad YL_t = \left[(Y_{1t})^P + (Y_{2t})^P + (Y_{3t})^P \right]^{\frac{1}{P}} + \epsilon_t$$

where P is a negatif integer ($P < 0$).

One can easily show that :

$$\lim_{P \rightarrow -\infty} YL_t = \min (Y_{1t}, Y_{2t}, Y_{3t}) + \epsilon_t$$

$$P \longrightarrow \dots \infty$$

$$(Y_{it} > 0, i = 1, 2, 3)$$

(1) the parameter b' is constant for each optimisation of Ψ (see below).

We transform equation (54) as :

$$(55) \quad Y_{L_t} = Y_{ot} \left[\left(\frac{Y_{1t}}{Y_{ot}} \right)^P + \left(\frac{Y_{2t}}{Y_{ot}} \right)^P + \left(\frac{Y_{3t}}{Y_{ot}} \right)^P \right]^{\frac{1}{P}}$$

where $Y_{ot} = \min_j | Y_{jt} | \quad j = 1, 2, 3$

It is clear that :

$$\frac{Y_{jt}}{Y_{ot}} \geq 1, \quad j = 1, 3$$

As $P < 0$

$$\lim_{P \rightarrow -\infty} A_t = \lim \left[\left(\frac{Y_{1t}}{Y_{ot}} \right)^P + \left(\frac{Y_{2t}}{Y_{ot}} \right)^P + \left(\frac{Y_{3t}}{Y_{ot}} \right)^P \right]^{\frac{1}{P}} = 1$$

$P \longrightarrow -\infty$

and the proof is complete.

This approximation implies that p must be large enough in order to get $A_t = 1$, otherwise we will have systematically $Y_{L_t} > Y_{ot} + \epsilon_t$. The residuals presented in equations (52) and (55) will not be the same and as a consequence the estimation will be biased.

The evaluation of Ψ and of its derivatives Ψ' can introduce underflow errors for large values of P (1). By this reason we have used the transformation noted in specification (55). Surely this transformation doesn't change the values of Ψ and of its derivatives Ψ' and at the same time it avoids most of the underflow errors (2).

(1) They are provoked by the presence in Ψ and in Ψ' (see Appendix B) of : $(Y_{jt})^P$, $j = 1, 3$. The computer gives an underflow error when it evaluates this term notwithstanding the fact that the value of Ψ and Ψ' is finite.

(2) Even in this case one may pay attention to these errors by introducing appropriate tests.

2.3.2.3. The nonderivability of the production function

Notwithstanding the fact that we have replaced the operator "min" by a smooth approximation, the function Ψ_t of Equation (55) is not yet differentiable with respect to $\ln(1+a')$, $\ln a'$, $\ln(1+a)$, $\ln a$. The reasons for this are the nonderivability of Y_{2t} with respect to the first two parameters and the nonderivability of Y_{3t} with respect to the four parameters (see Figure 1, section 2.2.2.).

On computing the derivatives of Ψ_t we have used the following smooth approximation

$$\begin{aligned}
 SC_t &= \sum_{t-mc_t}^{t-1} E(v) \approx \int_{t-mc_t}^{t-1} E(v) dv \\
 (56) \quad SR_t &= \sum_{t-mr_t}^{t-1} E(v) \approx \int_{t-mr_t}^{t-1} E(v) dv \\
 RR_t &= \sum_{t-mr_t}^{t-1} \frac{E(v)}{(1+b')^v} \approx \int_{t-mr_t}^{t-1} \frac{E(v)}{(1+b')^v} dv
 \end{aligned}$$

In all the other cases we have used the exact expressions.

The first partial derivatives of Ψ (Equation 53) are presented in Appendix B. They were computed taking into account the simultaneous character of the model.

2.3.3. The procedure of Estimation

By simple application of logarithms we are now in position to write the model in the following form which is the one used for the estimation :

$$(57) \quad \ln NR_t = e_1 + e_2 \ln POP_t + e_3 \ln RW_t$$

$$(58) \quad \ln NRE_t = \ln NR_t + e_4 B_t \ln(NRE_t/N_{t-1}) + e_5 (1-B_t) \ln(Em_t/\overline{EM}_t) + \epsilon'_{1t}$$

$$(59) \quad \Delta \ln CD_t = g_1 + g_2 \Delta \ln DI_t + g_3 \ln (DI_t/CD_{t-1}) + g_4 \ln (DI_t/EPMC_t) + \varepsilon'_{2t}$$

$$(60) \quad \ln YKA_t = \ln Y_{t-1} + \gamma DA_t + \gamma c_1 DDW_t + \gamma c_2 CEX_t - \gamma d_1 CCD_t - \gamma d_2 EEXO_t \\ - \gamma d_3 CEM_t + \gamma_1$$

$$(61) \quad E(\ln YK_t) = \ln Y_{t-1} + DA_t + c_1 DDW_t + c_2 CEX_t - d_1 CCD_t - d_2 EEXO_t \\ - d_3 CEM_t$$

$$(62) \quad mc_t = \frac{1}{\ln(1+b')} (\ln \alpha' + \ln(1+a') t - \ln L_t) + t$$

$$(63) \quad E(\ln YC_t) = \ln \alpha + \ln(1+a) t + \ln \sum_{t-mc_t}^{t-1} E(v)$$

$$(64) \quad \ln NR_t = \ln \frac{\alpha}{\alpha'} + \ln \left(\frac{1+a}{1+a'} \right) t + \ln \sum_{t-mr_t}^{t-1} \frac{E(v)}{(1+b')^v}$$

$$(65) \quad E(\ln YR_t) = \ln \alpha + \ln(1+a) t + \ln \sum_{t-mr_t}^{t-1} E(v)$$

$$(66) \quad \ln Y_t = \min (\ln (YKA_t), E(\ln YR_t), E(\ln YC_t)) + \varepsilon'_{3t}$$

$$(67) \quad \ln XD_t = \ln X_{t-1} + c_1 \Delta \ln DW_t + c_2 \Delta \ln (PEX_t/PET_t)$$

$$(68) \quad \ln MD_t = \ln M_{t-1} + d_1 \Delta \ln CD_t + d_2 \Delta \ln EXO_t + d_3 \Delta \ln (PIM_t/PY_t)$$

$$(69) \quad \ln X_t = \ln XD_t + c_3 \ln (YC_t/Y_t) + c_4 \ln (YK_t/YKA_t) + c_5 + \varepsilon'_{4t}$$

$$(70) \quad M_t = CD_t + EXO_t + X_t - Y_t$$

$$(71) \quad \ln YKA_t = \ln \alpha + \ln(1+a) t + \ln \sum_{t-mka_t}^{t-1} E(v)$$

$$(72) \quad \ln NC_t = \ln \frac{\alpha}{\alpha'} + \ln \left(\frac{1+a}{1+a'} \right)^t + \ln \frac{\sum_{t-mc_t}^{t-1} E(v)}{(1+b')^v}$$

$$(73) \quad \ln NKA_t = \ln \frac{\alpha}{\alpha'} + \ln \left(\frac{1+a}{1+a'} \right)^t + \ln \frac{\sum_{t-mka_t}^{t-1} E(v)}{(1+b')^v}$$

$$(74) \quad \ln N_t = A_t \Lambda \min (\ln NKA_t, \ln NC_t) + (1-\Lambda) \ln N_{t-1} + \Lambda_1 \\ + (1-A_t) \ln NR_t + \varepsilon'_5$$

where ε'_{1t} , ε'_{2t} , ε'_{3t} , ε'_{4t} and ε'_{5t} correspond respectively to ε_{6t} , ε_{1t} , ε_{8t} , ε_{4t} and ε_{9t} .

The procedure of estimation is as follows :

(i) We start by estimating labour supply replacing $\ln NR_t$, given by (57), in Equation (58). We get $\ln \hat{NR}_t$ which we replace in (64).

(ii) We estimate Equation (59) obtaining $\ln \hat{CD}_t$ and we replace in (60) and (68) $\ln CD_t$ by $\ln \hat{CD}_t$.

(iii) After having replaced $\ln YKA_t$, $E(\ln YC_t)$ and $E(\ln YR_t)$ in Equation (66) we estimate this equation using the procedure described in section 2.3.2.2.. We get estimations for parameters γ , γ_1 , c_1 , c_2 , d_1 , d_2 , d_3 , $\ln(1+a)$, $\ln \alpha$, $\ln(1+a')$, $\ln \alpha'$ which gives a simultaneous estimation of Equations : (60), (61), (62), (63), (65), (66), (67), (68), (71), (72) and (73).

(iv) We replace in Equation (69) $\ln Y_t$, $\ln YC_t$, $\ln YK_t$ and $\ln YKA_t$ by their estimations, computed in (iii), getting $\ln \hat{X}_t$. Imports (Equation (70)) are then estimated.

(v) Finally we estimate Equation (74) after having replaced $\ln NC_t$, $\ln NKA_t$ and $\ln NR_t$ by their estimation.

The parameter b' is estimated by an iteration method, which means : the complete model is estimated for each value of b' and we will choose the value of the parameter b' which gives the minimum of the product :

$$(75) \quad P = \prod_{j=3}^5 SS_j$$

with

$$SS_j = \sum_1^T \varepsilon_{jt}''^2$$

where ε_{3t}'' , ε_{4t}'' and ε_{5t}'' mean respectively the relative residuals of Equations (66), (69) and (74) (1).

The reasons of such a procedure are found in the work of Smallwood (1972). The author shows that for small values of the parameter b' there is indeterminacy of the parameters (in particular of $\ln a'$ and $\ln (1+a')$) of the model. So it would be necessary to consider constraints in the model which would complicate the estimation, specially the computation of derivatives, which is already very tedious (see Appendix B). In Appendix C we justify the choice of the minimum for P (Relation (75)) as an optimization criterion.

2.3.4. The properties of the estimators

The procedure that was presented is a typical two-stages procedure. First we estimate, in the reduced form, the endogenous variables which appear as explanatory variables in other equations. Then, we estimate in the structural form, the other variables, after having replaced the endogenous variables by their estimations.

The special feature of this estimation procedure is in Equation (66). It includes endogenous variables in the second member and it is nonlinear both in variables and in parameters.

(1) They are obviously derived from the absolute residues through the formule $\varepsilon'_t = 1 - \exp(-\varepsilon_t)$, where ε'_t is the relative residual and ε_t is the absolute residue).

The question is to know which properties the estimators have in such case.

Amemiya (1974) has studied this problem, under the form

$$Y_t = f(Z_t, B) + U_t$$

where U_t has standard properties, Z_t is an M -component vector consisting partly of endogenous variables and partly of exogenous variables and B is a G -component vector of unknown parameters.

He has presented a general nonlinear two stage least squares (NS2SLS) estimator \hat{B} , that is the value of B that minimizes

$\Psi(B) = (Y - f)' X(X'X)^{-1} X'(Y - f)$, where X is a $T \times K$ matrix of constants with rang K . (1).

His main results (2), under some general assumptions, which are not presented here, are :

- (i) B converges in probability to the true value B_0
- (ii) $\sqrt{T}(\hat{B} - B_0)$ converges in probability to

$$N \left\{ 0, \sigma^2 \left[\text{plim} \frac{1}{T} \frac{\partial f'}{\partial B} \Big|_{B_0} X(X'X)^{-1} \frac{\partial f}{\partial B'} \Big|_{B_0} \right]^{-1} \right\}$$

More explicitly, Bowden (1978) has pointed out the following result : if we replace the endogenous variables in Equation (66) by their estimations got by ordinary least squares, we will obtain, under very general assumptions, consistent estimators.

So, in conclusion : our estimator is consistent. It is less efficient than the nonlinear limited information maximum likelihood estimator (see Amemiya (1975)) but this latter is computationally much more difficult.

-
- (1) Following Amemiya (1975) we will call our estimator standard non linear two stage least squares in view of the particular choice of X .
 - (2) He has also shown that the well known properties of the two-stage least-squares are verified if f is not linear in B , but linear in Z .

2.4. Structural change and disequilibrium

As it was observed in the introduction the main goal of the empirical implementation of the model with Portuguese data is to study the structural changes occurred with the revolution of April 1974.

To this purpose and before presenting the results we evaluate in this section the special aptitude of this disequilibrium model to show structural changes. In particular we analyse the role played by the bottlenecks and define the balanced growth path.

We start by noting that each bottleneck corresponds to a different approach for determining the production and the employment level. They are represented in summary by Schemes 2, 3 and 4. These schemes show, in particular, that the disequilibria in the external trade are a function of the bottleneck. Most of the usual macroeconomic models consider only one of these approaches (in general the one corresponding to Schema 2). In this sense they can be considered as particular cases of the present model in so far as they consider only the situation in which the bottleneck is always caused by demand ⁽¹⁾. In the present model it is not assumed a priori which is the bottleneck that obstructs economic activity. This will be given by the data. In particular the estimations obtained for the production function (see below) may show that the economy finds it self in one of the situation represented by Schemas 3 and 4. The results can also show that during the period of estimation the economy has jumped from one regime (or bottleneck) to another. We will call this jump a structural change in the economy ⁽²⁾.

In order to fully understand these aspects we use approximations (56) and we compute :

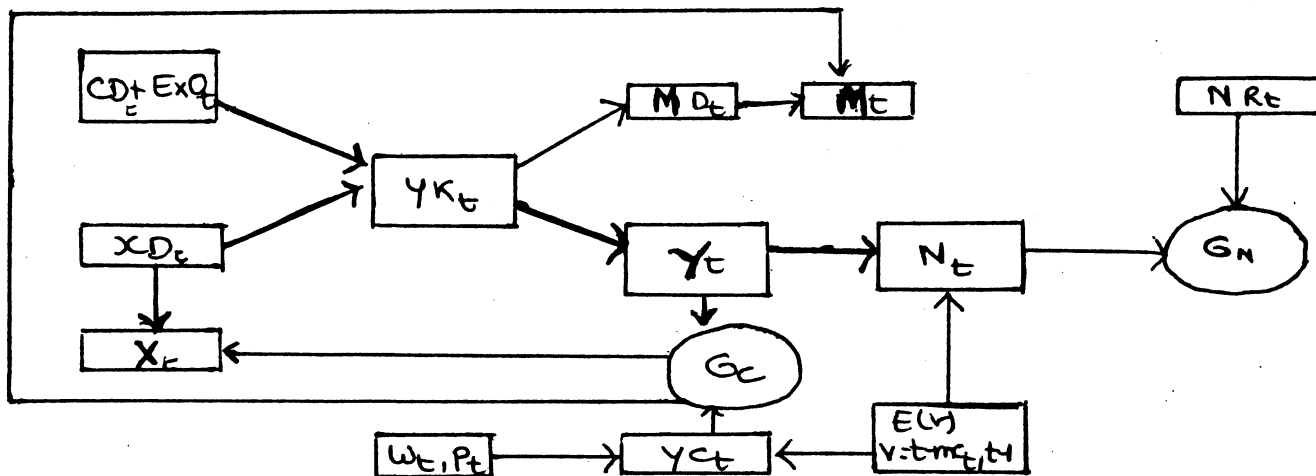
from (62) and (63) ;

$$(76) \quad mc'_t = 1 + (a' - l_t) / b'$$

(1) In a formal way this approach corresponds to $\gamma=1$ ($YKA_t = YK_t$) ;
 $YKA_t < YC_t$ and $YKA_t < YR_t$

(2) In a strictly econometric sense this definition is only correct in the framework of the usual macroeconomic models, the structure of which contemplates only a regime. The structure of the present model is general enough for allowing such a jump even if none of its component changes.

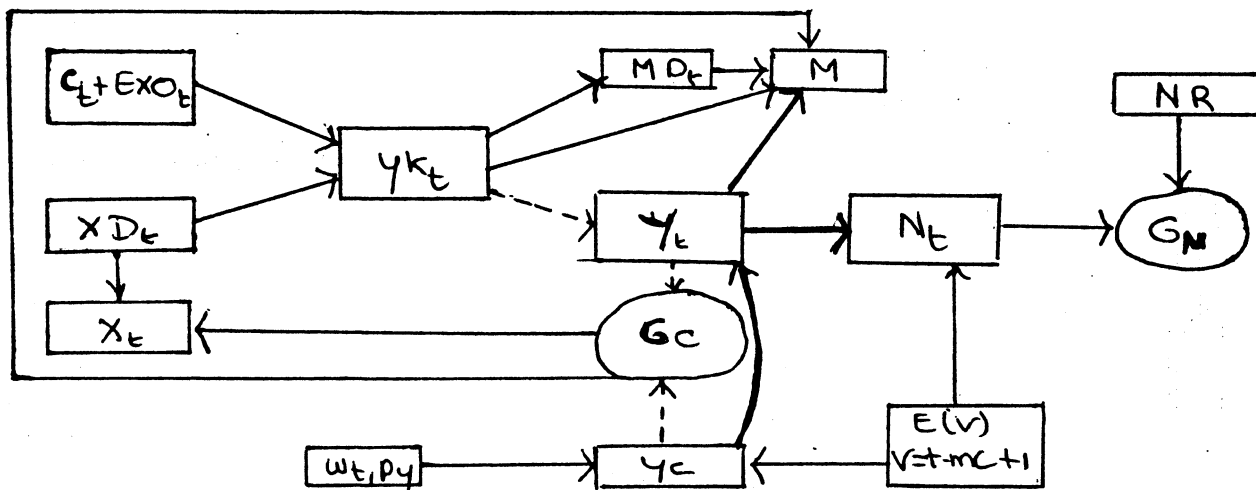
Schema 2 : KEYNESIAN UNEMPLOYMENT (bottleneck : demand)



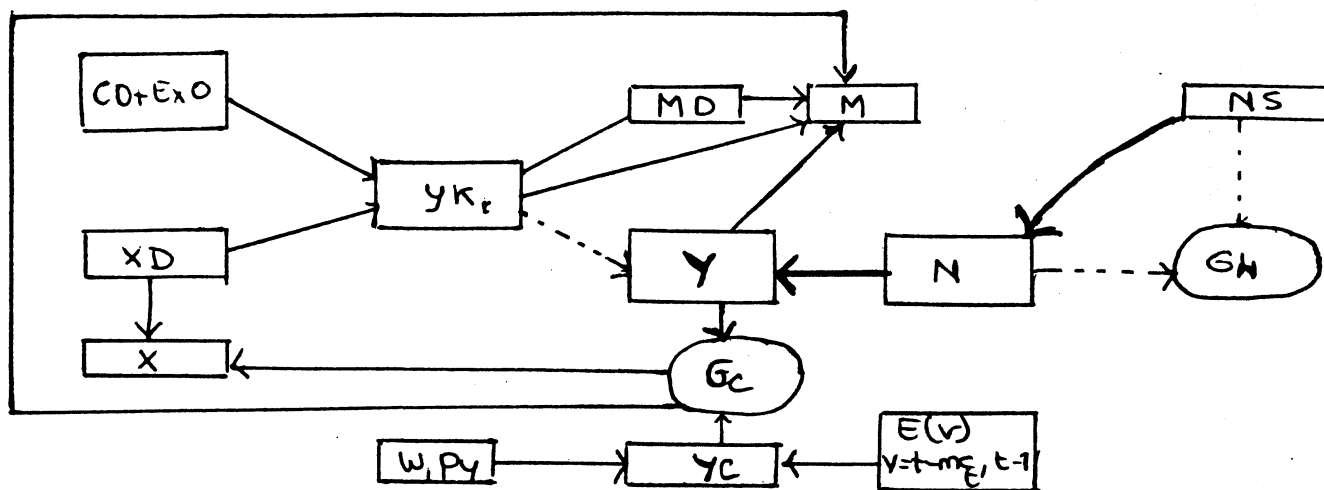
$$G_N = (NR - N) / N$$

$$G_C = (Y_C - Y) / Y$$

Schema 3 : CLASSICAL UNEMPLOYMENT (bottleneck : production capacity)



Schema 4 : REPRESSED INFLATION (bottleneck : labour supply)



$$(77) \quad y_{c_t} = a + (E(t-1) - E(t-mc'_t) (1-mc'_t)) / SC_t$$

and from (64) and (65) :

$$(78) \quad mr'_t = (nr_t + a'-a) \frac{RR_t (1+b')^{t-mr_t}}{E(t-mr_t)} - \frac{E(t-1)}{E(t-mr_t)} (1+b')^{t-mr_t}$$

$$(79) \quad y_{r_t} = a + (E(t-1) - E(t-mr'_t) (1-mr'_t)) / SR_t$$

In these relations ((76)-(79)) the meaning of the symbols is given according to : $x_t = d \ln X_t / d_t$; $X'_t = d X_t / d_t$; where X_t is the variable with respect to the rate of growth (x_t) and the derivative (X'_t) an computed (1).

For a constant rate of investment growth r , one can deduce the following results.

from (76) and (77)

1) if $l_t = a' + b'$

this implies that $mc'_t = 0$ and $yc_t = a + r$; the interpretation of which is classical : in case that the real wages increase at the same rate as the labour marginal productivity the value of mc_t doesn't change (2) . If, in such a situation, investment grows at a constant rate than YC_t grows also at a constant rate called by Solow et al (1966) natural rate of growth in a neoclassical framework with Harrod neutral technical progress ($a = 0$).

2) A situation characterized by $l_t < a + b'$ will give a speed of scrapping lower than in the case 1) ($mc'_t > 0$) and $yc_t > a + r$.

3) In a similar way $mc'_t < 0$ and $yc_t < a + r$ will be verified if $l_t > a' + b'$.

(1) One can easily check that the following approximations have also been used : $\ln (1+a') \approx a'$, $\ln (1+b') \approx b'$ in Relation (76); $\ln (1+a) \approx a$ in relations (77) and (79) and $\ln (1+a') - \ln (1+a) \approx a' - a$ in Relation (78)

(2) The results concerning the evolution of mc_t do not depend on investments growth. They are also exactly verified even in the discrete cas (Vilares (1980)).

from (78) and (79)

(i) for $nr_t = a + r - (a'+b')$; this gives $mr'_t = 0$ and $yr_t = a + r$ which has the following interpretation. If, at each periode t , the increases in labour supply are equal to the labour required for branding the new capital, the full employment can be assumed with a constant age for the oldest vintage in the stock of the capital.

(ii) For $nr_t > a + r - (a'+b')$; the model gives $mr'_t > 0$ and $yr_t > a + r$.

In this case the full employment will only be assured through a reduction of the scrapes.

(iii) In a similar way one can interpret the case where $nr_t < a + r - (a'+b')$, which corresponds to $mr'_t < 0$ and $yr_t < a + r$.

The developments which were reported allow us to conclude, that the model will portray :

a) constantly the Keynesian regime if the rate of growth of the anticipated demand (yka_t) is systematically inferior to yc_t , given by (76) and yr_t by (78).

b) a situation in which the capacity of production (YC_t) and the production at full employment (YR_t) grow at the same and constant rate ($yc_t = yr_t = a + r$) if the two following conditions are satisfied :

$$b1 : nr_t = r + a' - l_t$$

$$b2 : l_t = a' + b'$$

c) a balanced growth path, that is a situation where the bottlenecks play no role ($YKA_t = YC_t = YR_t = Y_t$) if, besides b), the firms anticipate will the demand ($\gamma = 1$), if $yk_t = a + r$ and if equilibrium ($YK = YC = YR$) is verified at the beginning of the growth path.

Clearly these situations are generally not observed since the variations of l_t , nr , r and y_k will assure very hardly the conditions stated above. The general case will be the one where the economic activity is obstructed in each period.

The type of bottleneck can change as a consequence of an important shock in one (or more) of the growth rates noted above. The final effect will depend on one hand on the amplitude of the variation and on the other hand on the situation preceding those shocks. All these aspects are illustrated by the model.

It seems so that the present model is specially equipped for analysing the structural changes in a given economy which was submitted to some important shocks. This statement will be empirically tested in the next section.

3. STRUCTURAL CHANGES IN THE PORTUGUESE ECONOMY OCCURRED
WITH THE APRIL 25TH REVOLUTION

The economic consequences of the Portuguese revolution of 25 april 1974 have given rise to an abundant litterature (see, in particular, Barbosa and Beleza (1979), Krugman and Macedo (1979) and OECD (1976)). So it is important to be very precise about the purpose of this empirical implementation, which is to illustrate the capacity of the present model to analyse structural changes in a given Economy that has been submitted to some important shocks.

We start with a brief summary of the main shocks. Then we comment the results given by the model and finally we study the robustness of the estimations.

3.1. A combination of shocks

The Portuguese Economy was in the space of two years (1974-75) exposed to an impressive number of shocks. We can divide them into two groups (see data and more details in OECD (1976)).

On one hand we have the shocks produced by the revolution, particularly :

1) The important increase in real wages, specially the lower wages. As prices were submitted to a political control, the real wages have jumped and a redistribution of income has taken place in favour of labour.

2) The loss of the colonies and as a consequence :

2.1) the loss of markets and of a source of raw materials ;

2.2) the arrival of settlers ;

2.3) the reduction of military effectives.

3) The extension of the public sector.

On the other hand we have the exogenous or external shocks, that is those which would affect the Economy under any circumstances, even if no revolution has taken place, like :

- i) the halt on emigration as a consequence of the decision taken by the governments from the states members of the European Communities ;
- ii) the rise in the price of oil and the world recession.

One can easily check (see the data in Appendix E) that these shocks have directly provoked a rise in the real wages of 17.3 % in 1974 and 12.6 % in 1975 and a rise in registration of the unemployment of 41.7 % in 1974 and 120 % in 1975. They are also indirectly responsible for the rupture in investments (-12.2 % in 1974 and -19.3 % in 1975).

3.2. The results

The model has been estimated for the period 1955-1979. The data are in Appendix E and the meaning of the variables, according to the Portuguese accounting system, is given in Appendix A. In this section we present the main results and some remarks about them. These remarks concern the specific contribution of the present model to the problem we analyse. The other results are shown in Appendix D.

In order to fully understand the results, they are presented in two groups. First we analyse the bottlenecks of the economic activity given by the evolution of YC_t , YR_t and YKA_t . Then we examine the repercussions on external trade of the disequilibria verified in the interior of the Economy.

3.2.1. The bottlenecks of the economic activity

The evolution of YC_t , YR_t and YKA_t given in Table 4 of Appendix D, suggests the following remarks :

- 1) The estimation of YKA_t and YK_t are very close ($\gamma \approx 1$, see Table 1), which can be justified by the fact that we use annual data. Under these conditions the bottlenecks which have obstructed the economic activity

(see below) were effective. They were not the result of wrong expectations of firms.

2) Figures 2 and 3 show that one can consider five phases in the process of production growth during the period of estimation.

The first one (phase A) is characterized by a considerable underutilization of production factors, specially labour. Figure 3 shows that there is a growing difference (except the year 1960) between GR and GC and between GC and GKA (see, in Figure 3, the definitions of these gaps). The beginning of the colonial war (1960-61) has not had immediate effects (this confirms the view expressed in ILO (1979)).

After 1963 this growing tendency of GR and GC is reversed, particularly the one of GR. The needs of the militar contingent and specially the pressure of the emigration (see this serie in Appendix E) start to take effect. Because these effects are still weak, this period (phase B) is characterized by an important growth of demand production and investments.

The situation starts to be critical after 1969 when a new phase begins (phase C). The strong emigration seconded by the colonial war had two effects. On the one hand they have provoked the scarcity of labour supply (direct effect). On the other hand the shortage on labour supply has led to real wage increases. As these increases were not followed by a sufficient renovation of the stock of capital in order to increase labour productivity (see, in Table 1, the quite modest values of a' and b'), they had, as a consequence a reduction of YC_t (indirect effect). Hence the results show that the Portuguese Economy was at the beginning of 1974 in a situation of insufficiency of labour (called repressed inflation) which provoked as an indirect effect a pressure on production capacities.

This situation could not last long. The april 25th revolution joined with external factors (see above) have forced a new direction and the Economy enters in the classical regime (phase D). Such result doesn't surprise if one considers on the one hand the developments of section 2.4. and on the other hand the situation of the Economy and the type of shocks : strong increase in labour supply and in real wages and a fall in investments.

After 1976 the remarkable decline in the real wage and the restrictions imposed to households consumption (OECD (1976)) have reversed the situation and a new phase starts (phase E).

The abrupt augmentation of the profitable production capacities (YC_t) for this period (1977-1979) deserves a particular remark. The model gives it because of the strong decline in real wages. If one supposes that $mc_{t+1} \leq mc_t + 1$, i.e., if a scrapped vintage doesn't return to the stock of capital, then the values of mc_t would be for these years 12.93, 13.93 and 14.93 (see Table 6 in Appendix D) and the values of GC_t would be those plotted by the dotted line (...) in Figures 2 and 3. This assumption which imposes $l_t \geq a'$ (Vilares (1980)) is only logical in the framework of a "normal situation". In an exceptional case like a revolution, firms can close temporarily or keep their equipments for some years even if they are not profitable. This can explain the behaviour of mc given in Table 6. Hence, the evolution of GC_t must be between the two bounds given in Figure 3.

3.2.2. External trade

The specification of the external raises a particular problem. Before 1974, about 22 % of the exports and 12 % of the imports came from the Previous Escudo Area (see, in Appendix E, the evolution of these values respectively represented by EXC and EXM). The formulation of XD and MD in terms of the usual variables -competivity, world and interior demand- is not appropriate for the part of the external trade coming from this area (1). This problem was solved by assuming that imports and exports demand of the Previous Escudo Area are exogenous, i.e. equal to their exchanged values.

We will make two remarks about the results obtained for external trade :

1) A general remark concerns the interest in endogenising the indicators of disequilibrium. Such an interest is illustrated in two ways :

(1) In Appendix E, it is shown how the index of price of competitors on foreign market (PET) and the weighted world trade index (DW) are computed.

i) by the important differences between X_t and XD_t and between M_t and MD_t ;

ii) by the estimations and standard errors obtained for the parameters c_3 , c_4 and c_5 which translate these disequilibria.

2) A more particular remark concerns the interpretation of external disequilibrium after 1974. It is coherent with what we have noted in section 2.2.1. Since the Economy was at the beginning of 1974 near the classical regime a rise in real wages has provoked in respect to the potential values XD and MD a fall in exports ($X < XD$) and a rise in imports ($MD > M$). After 1976 the abrupt augmentation in Yc_t has reversed the situation. It is useful to note that the only formalization of the external trade in terms of the usual variables of competitiveness : PEX/PET for XD_t ; and PIM/PY for MD_t ; explains a quite modest part of the evolution of external trade in this period. These aspects are shown in Table 5. Figure 4 illustrates the case of the exports.

3.3. The robustness of the results

We will do four remarks concerning the quality of the results.

1) Table 1 shows that practically all the parameters are significant (at the 5 % level) and they have a sign consistent with the interpretation given for them in the model.

2) The relative residuals obtained in the estimations are given in Table 2. They are quite modest if one thinks the important ruptures which have taken place in the Portuguese economy during the periode of estimation this is illustrated in Figure 5, where the residuals concerning N_t and Y_t are plotted.

3) As the labour supply and the consumption function are quite peripheral to the model. We have tested the robustness of the results using different specifications for these functions ⁽¹⁾. Table 3 shows the main

(1) It would be also interesting to compare the evolution of GC with an indicator of underutilization of the production capacities as it was done in the estimation of the present model with French data (Vilares (1981)). Unhappily there are no available data for such a confrontation.

results obtained for each one of these specifications. One must note the remarkable stability of the results. They don't question the validity of the comments made in section 3.2. the selected results are these of Model A. The presence of the dummy variable C_t in the consumption function is easily justified if one thinks that during the period 1974-76 the rise of real wages has exceptionally encouraged consumption. However if one doesn't consider this variable (models A_1 , A_3 and A_4) the main results rest fundamentally unchanged this confirms what was noted in section 2.4. : the model is general enough for portraying different situations without the help of the usual dummy variables.

4) Finally a general remark with a double sense.

First it is clear that all the results are conditioned by the quality of the data which in the Portuguese case is constantly called in question (Appendix E). However as we work at a very aggregate level (all the economy except the Public Administration) we believe that the possible imperfection in the data will only affect the intensity but not the nature of the disequilibria :

Second the quite good performances of the model must not hide the fact that the Portuguese revolution of 25 April 1974, like other social movements of this type, induces in the economic activity a lot of aspects of a qualitative nature that no model can contemplate.

Table 1 - PARAMETERS

e1	0,972	(0,012)	c1	1,93	(0,91) (*)	ln(1+a')	0,0047	(0,0023)
e2	0,766	(0,213)	c2	0,495	(0,231) (*)	lnα'	0,963	(0,431)
e3	0,077	(0,012)	d1	0,99	(0,48) (*)	c3	1,16	(0,41)
e4	0,276		d2	0,371	(0,108) (*)	c4	27,6	(12,6)
e5	0,009	(0,004)	d3	-0,214	(0,113) (*)	c5	-0,169	(0,08)
g1	-0,004	(0,003)	γ	1,03	(0,452)	Λ	0,085	(0,023)
g2	0,737	(0,019)	γ ₁	0,0024	(0,001)	Λ ₁	0,915	(0,42)
g3	0,287	(0,13)	ln(1+a)	-0,0305	(0,0108)	b'	0,041	(* *)
g4	-0,07	(0,03)	lnα	2,42	(0,89)			
g5	0,054	(0,02)						

Notes *) These standard errors are of the parameters Yc1, Yc2, -Yd1, -Yd2 and -Yc3, respectively (see page 35).

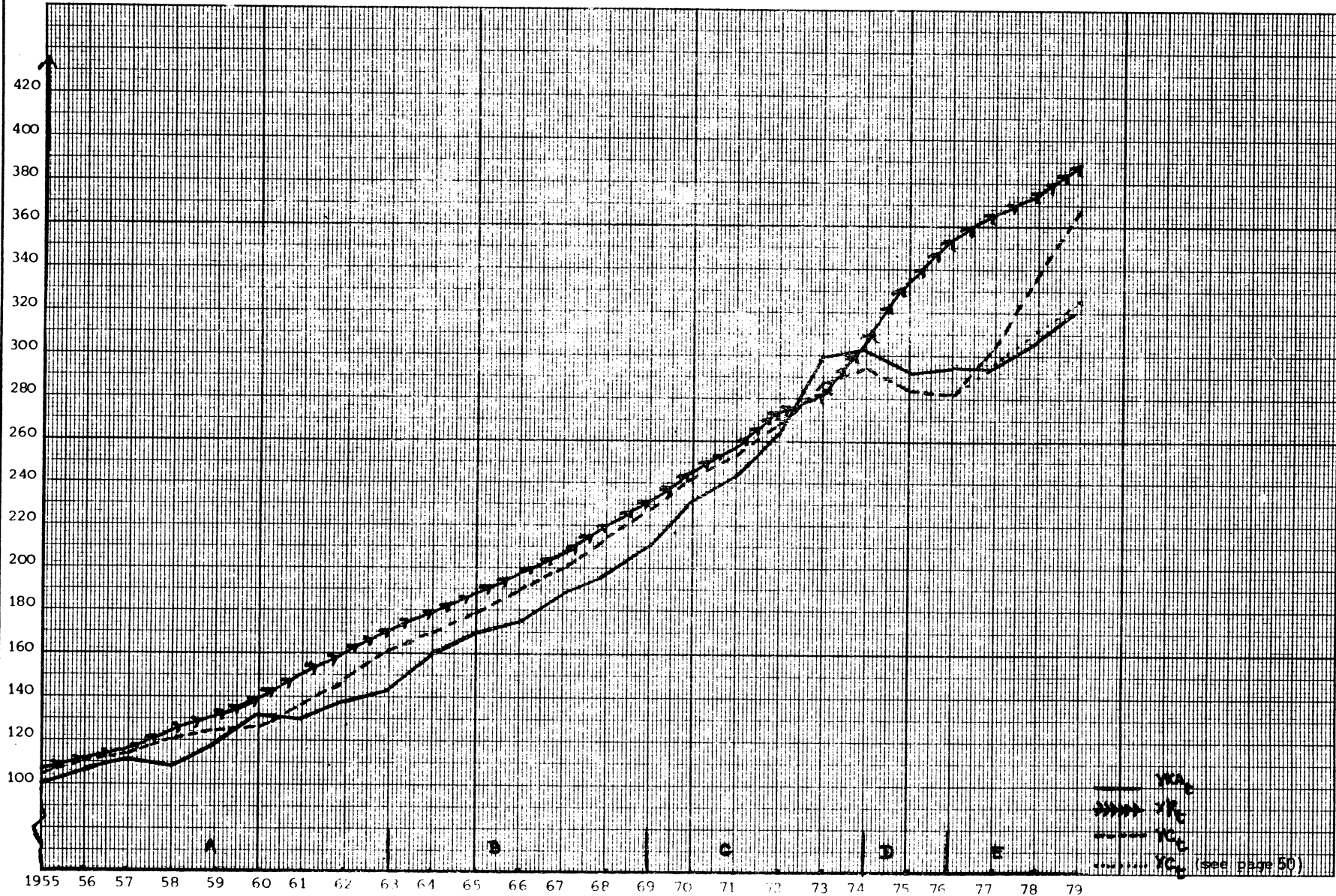
***) This parameter was estimated by iteration (see page 40).

Table 2 - RELATIVE RESIDUALS RESI_t (*)

T	RES1	RES2	RES3	RES4	RES5
55	-0.03555	0.02647	0.00686	0.02963	-0.00619
56	-0.02256	0.01436	-0.00697	0.00863	0.00163
57	-0.01102	-0.00102	-0.02319	-0.03145	-0.00125
58	0.01452	0.02532	0.03098	0.01812	0.02448
59	0.03150	0.01648	0.00100	-0.15484	0.02661
60	0.01171	-0.00573	0.01030	0.02868	-0.00560
61	0.01229	0.06891	0.00096	-0.00333	0.00851
62	0.00781	-0.06056	-0.00488	0.02649	0.00881
63	0.00998	0.03544	0.02619	-0.05190	0.01159
64	-0.00587	-0.03945	-0.02310	0.17232	0.00369
65	-0.00529	0.00277	-0.00737	0.00605	0.00286
66	0.00191	-0.00390	0.00166	0.04499	0.00525
67	-0.00150	-0.06402	-0.01242	-0.03922	-0.00969
68	-0.01075	0.08190	0.02429	-0.07367	-0.00764
69	-0.01430	-0.02449	-0.02170	-0.06789	-0.02180
70	0.00571	-0.06120	-0.03000	-0.07755	-0.00122
71	0.01152	0.01394	-0.00070	0.06198	-0.01345
72	0.01318	-0.07580	-0.00358	0.15625	-0.02109
73	0.00180	-0.00445	0.02150	-0.05506	-0.03968
74	-0.01295	0.00672	0.00241	-0.00391	-0.02360
75	-0.00453	0.00432	-0.04130	-0.01462	-0.01132
76	-0.01794	-0.01113	0.01367	-0.11488	0.00456
77	-0.00025	0.04058	0.02430	-0.00702	-0.00585
78	0.01149	0.01367	0.00863	0.00916	-0.00572
79	0.00661	-0.01737	-0.00836	0.06187	-0.00584

(*) RESI_t = (Z_t - ZE_t) / Z_t ; Z_t : observed value, ZE_t estimated value. The concerned variable is given by the procedure of estimation (page 39).

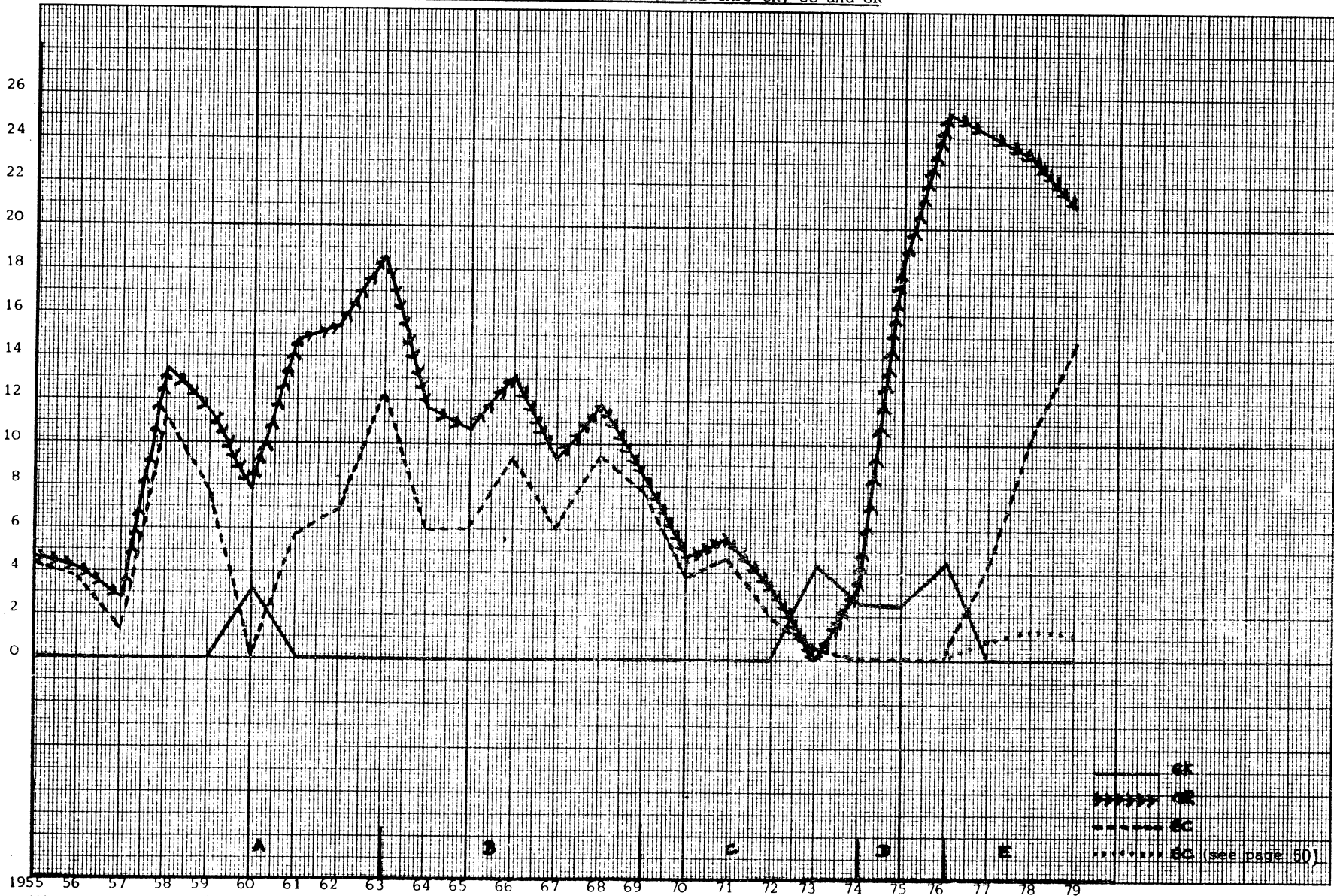
FIGURE 2 - THE BOTTLENECKS OF THE ECONOMIC ACTIVITY (1)



(1) The plotted values are those of Table 4 (Appendix D) divided by $0.01YE_{t_0}$ ($t_0 = 1952$).

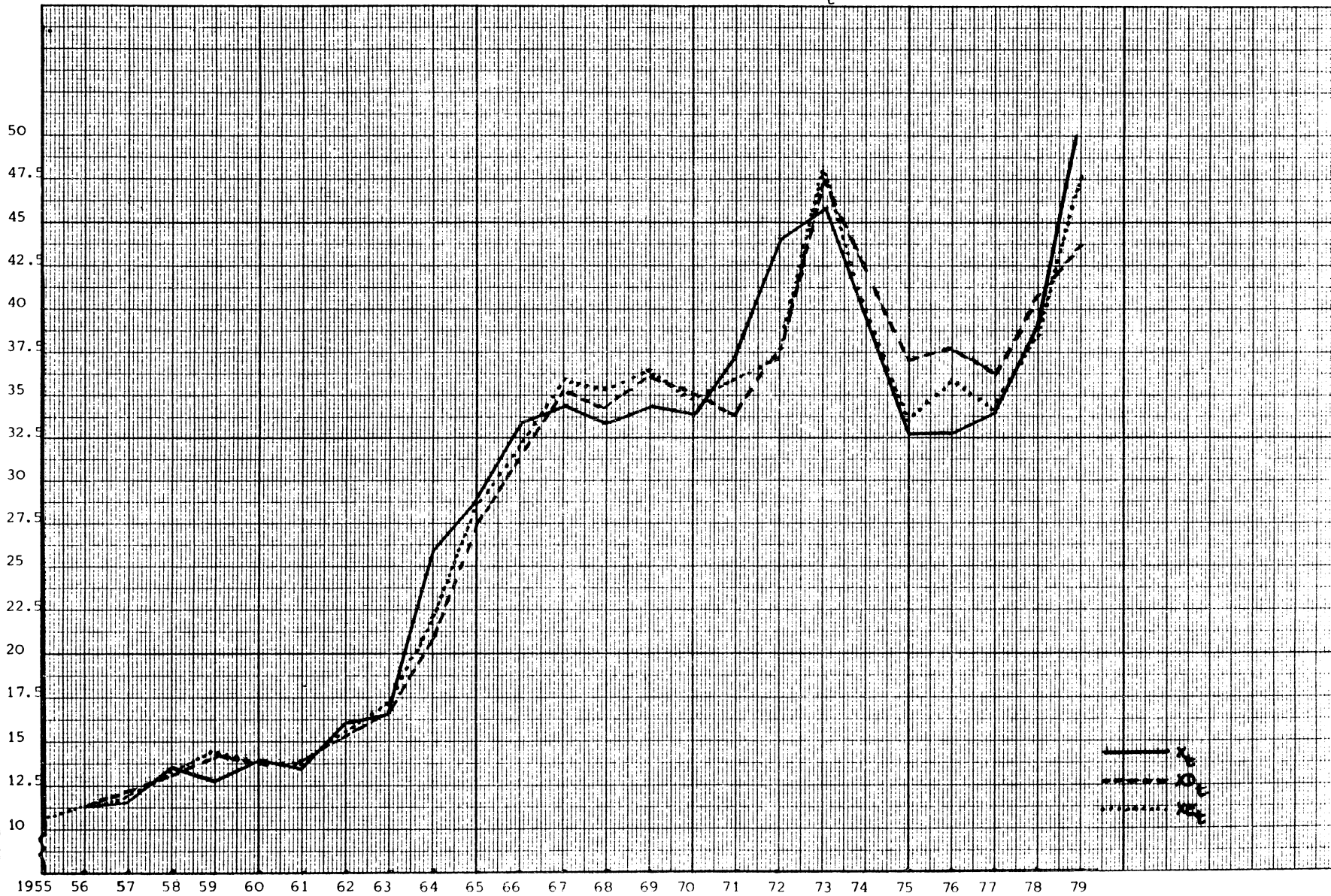
YWA
 YK
 YC
 YC_1
 YC_2 (see page 50)

FIGURE 3 - INTERNAL DISEQUILIBRIA. THE EVOLUTION OF THE GAPS GK, GC and GR ⁽¹⁾



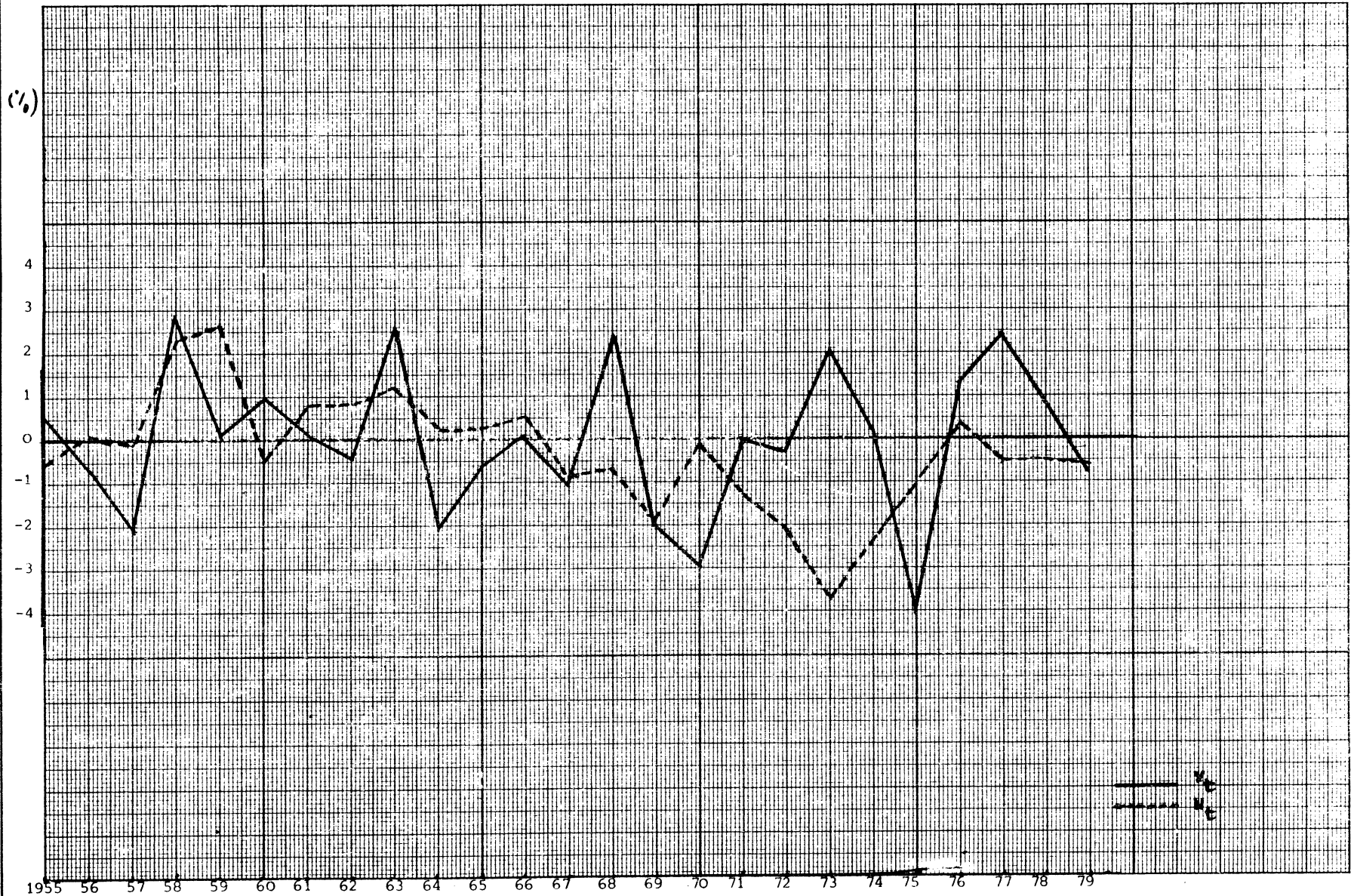
(1) $GKA = (YKA - YE) / 0.01YE$, $GR = (YR - YE) / 0.01YE$, $GC = (YC - YE) / 0.01YE$. (see Table 4 of Appendix D)

FIGURE 4 - EXTERNAL DISEQUILIBRIA. THE BEHAVIOUR OF X_t , XD_t and XE_t (1)



(1) cf. Appendix D (Table 5).

FIGURE 5 - RELATIVE RESIDUALS OBTAINED IN THE ESTIMATIONS FOR Y and N (1)



Y
N

(1) The plotted values are those of RES3 and RES5, given in Table 2, multiplied by 100.

Table 3 - ALTERNATIVE SPECIFICATIONS FOR CONSUMPTION AND LABOUR SUPPLY FUNCTIONS (Equations (58) and (59))

MODEL (※)	Parameters						Sum of the squares of the residuals SS_j					Regimes (years) ※※	
	a (%)	a' (%)	b' (%)	A	γ	c_3	SS1 (EQ. (58))	SS2 (EQ. (59))	SS3 (EQ. (66))	SS4 (EQ. (69))	SS5 (EQ. (74))	R	C
A1	-2,8 (1,1)	0,5 (0.2)	4	0,079 (0.029)	1 (0.44)	1,03 (0.54)	0,0049	0,0407	0,0107	0,1282	0,005	73	57, 60 74-76
A2	-2,9 (1.12)	0,6 (0.2)	4.1	0,08 (0.031)	1,038 (0.44)	1,48 (0.58)	0,0049	—	0,0105	0,1169	0,0044	73	55, 57, 60-61 74-76
A	-3 (1.1)	0,5 (0.2)	4.1	0,0849 (0.023)	1,03 (0.45)	1,16 (0.41)	0,0049	0,0365	0,0082	0,1284	0,0053	73	60, 74-76
A3	-2,8 (1.1)	0,5 (0.21)	4	0,081 (0.028)	1 (0.43)	1,056 (0.54)	0,0053	0,0407	0,0102	0,127	0,0064	73	57, 60 74-76
A4	-2,8 (1.12)	0,5 (0.22)	4	0,081 (0.028)	1 (0.43)	1,057 (0.54)	0,0051	0,0407	0,0102	0,127	0,0065	73	57, 60 74-76

Notes : ※ The results given in the text are those of model A. The other models are derived from this : model A1 : $C_t = 0, \forall t$ (Equation (59)) ; Model A2 : consumption is exogenous ; Model A3 : $C_t = 0$ and $B_t = 1, \forall t$ (without the number of emigrants as explanatory variable) ; Model A4 : $C_t = 0, \forall t$ and the exclusion of B_t , i.e. Equation (58) is estimated with both variables for all the period.

※※ The years which are not reported correspond to the keynesian regime (K).

4. CONCLUSIONS

When we started this research we intended to develop a model which was able to account for important structural changes in a given economy. In fact the usual macroeconomic models are not equipped for incorporation such structural changes because they admit explicitly a constant structure.

We have taken as framework the recent developments in disequilibrium macroeconomics the present model, contrarily to the usual macroeconomic models, incorporates in an endogenous way three different possible approaches for the determination of production and employment. We have showed that this aspect gives to the model a special aptitude for analysing structural changes in a given economy that has been submitted to some important shocks. The estimation method is not complicated and it can be extended to models of bigger dimension. No parameter is fixed a priori and only one (b') is estimated by an iteration procedure.

The results obtained in the estimation of the model with Portuguese data are satisfying and they shed light on three points. First they illustrate the interest of estimating some not observed, variables, like the full employment production capacity the profitable production capacity and exports and imports demand. These variables influence the evolution of production and employment. Second, they show that the present model gives a specific interpretation of the structural changes in the Portuguese Economy occurred with the April 25th revolution. Finally, the results elucidate the advantage of checking, on a model with reduced dimension, the restrictive hypothesis of a constant structure generally assumed in operational macroeconomic models.

The results would be improved if some developments had been carried out like the consideration of different consumption function according to each regime, the consideration in an endogenous way of investment and (or) inventories. However, under such conditions, the multiplication of the regimes risks to make the model lose its capacity to characterize, in a simple way, the evolution of the economy. The statistical difficulties, specially concerning involuntary inventories, also constitute an obstacle to such developments. In the Portuguese case these difficulties are considerably increased because of the particular insufficiency of data.

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APPENDIXES

- A - List of variables
- B - Computation of first partial derivatives of ψ
- C - The choice of the minimum for P as optimization criterion
- D - Results
- E - The data

APPENDIX A

LIST OF VARIABLES

All the values are at 1963 prices and in millions of escudos.

Endogenous variables

- YKA_t : producers' expectations for period t
- YK_t : total demand for domestic goods and services
- MD_t : domestic demand for foreign goods and services
- XD_t : external demand for domestic goods and services
- CD_t : households' consumption of goods and services
- X_t : exports of goods and services
- M_t : imports of goods and services
- NKA_t : efficient employment for producing YKA_t
- mka_t : age of the oldest vintage in use if YKA_t is produced (intermediate computable variable)
- YC_t : profitable production capacity
- mc_t : age of the oldest profitable vintage in the stock of capital
- NC_t : efficient employment for producing YC_t
- NR_t : labour disponibilities in the economy
- NRE_t : registered labour supply (employment + unemployment)
- YR_t : full emplyment production
- mr_t : age of the oldest vintage in use if YR_t is produced (intermediate computable variable)
- Y_t : gross national product (GNP)
- N_t : total of the employment, except in Public administration (in thousands).

Exogenous variables

- EXO_t : exogenous part in domestic demand (Public consumption + investment + change in stocks)
- PIM_t : import price index (PIM = 1, t = 1963)
- PEX_t : export price index (idem)
- PY_t : production price index (idem)
- PET_t : index of price of competitors on foreign market (see, in Appendix E, the way as it was computed)
- DW_t : weighted world trade (idem)
- DI_t : households real disposable income (the values of disposable income are divided by the consumption price index PC)
- L_t : $0.5 RW_t + 0.3 RW_{t-1} + 0.2 RW_{t-2}$
 $RW_t = W_t/PY_t$
 W_t = average wages for year in contos (thousands of escudos)
- POP_t : working age population (aged more than fourteen and less than fifteen years)
- $E(v)$: total of gross fixed capital formation in transports, machinery and equipment (see Appendix E, p. XIV)
- : year (1900, t = 0)
- RNS : non labour real income (the values of the nonlabour income are divided by PC)
- EM_t : emigrants (in thousands)

Dummy variables

- $A_t = \begin{cases} 1 & \text{if } \min(NKA_t, NC_t) < NR_t \\ 0 & \text{otherwise} \end{cases}$
- $B_t = \begin{cases} 1 & \text{if } t \geq 74 \\ 0 & \text{otherwise} \end{cases}$
- $C_t = \begin{cases} 1 & \text{if } 74 \leq t \leq 76 \\ 0 & \text{otherwise} \end{cases}$

APPENDIX B

COMPUTATION OF FIRST PARTIAL DERIVATIVES OF Ψ

In this appendix we compute the derivatives of the functions Ψ given in Equation (53). As in the optimisation the DFP algorithm⁽¹⁾ was employed the calculation of the first partial derivatives is sufficient.

If we call :

$$(1) F_{1t} = (Y_{1t})^P + (Y_{2t})^P + (Y_{3t})^P$$

where Y_{1t} is given by (60), Y_{2t} by (62)-(63), Y_{3t} by (64)-(65) and P is a negatif integer.

We can write :

$$(2) \Psi = \sum_1^T (Y_{L_t} - F_t)^2$$

where $F_t = (F_{1t})^{\frac{1}{P}}$ and $Y_{L_t} = \ln Y_t$

The vector of the parameters of optimisation x is considered in the following order :

$$(3) \begin{array}{lll} x(1) = \gamma & x(4) = -\gamma d_1 & x(8) = \ln (1+a) \\ x(2) = \gamma c_1 & x(5) = -\gamma d_2 & x(9) = \ln \alpha \\ x(3) = \gamma c_2 & x(6) = -\gamma d_3 & x(10) = \ln (1+a') \\ & x(7) = \gamma_1 & x(11) = \ln \alpha' \end{array}$$

Under these circumstances, we have :

$$(4) \begin{array}{l} Y_{1t} = \psi_1(x(i)) \quad i = 1,7 \\ Y_{2t} = \psi_2(x(i)) \quad i = 8,11 \\ Y_{3t} = \psi_3(x(i)) \quad i = 8,11 \end{array}$$

(1) We have used the optimisation programme GQOPT from the University of Princeton (U.S.A.).

and the computation of the derivatives can be done as follows :

$$(5) \quad \frac{\partial \psi}{\partial x(i)} = -2 \frac{T}{I} \sum (Y_{1t} - F_t) \frac{\partial F_t}{\partial x(i)}$$

$$i = 1, 11$$

$$(6) \quad \frac{\partial F_t}{\partial x(i)} = \frac{1}{P} (F_{1t})^{\frac{1}{P}-1} \frac{\partial F_{1t}}{\partial x(i)}$$

$$i = 1, 11$$

$$(7) \quad \frac{\partial F_{1t}}{\partial x(i)} = P(Y_{1t})^{P-1} \frac{\partial Y_{1t}}{\partial x(i)}$$

$$i = 1, 7$$

$$(8) \quad \frac{\partial F_{1t}}{\partial x(i)} = P(Y_{2t})^{P-1} \frac{\partial Y_{2t}}{\partial x(i)} + P(Y_{3t})^{P-1} \frac{\partial Y_{3t}}{\partial x(i)}$$

$$i = 8, 11$$

The computation of (7) is easy.

$$(9) \quad \frac{\partial Y_{1t}}{\partial x(i)} = Z(i, t)$$

$$i = 1, 7$$

Where $Z(i, t)$ represents the observation t of the explanatory variable i that is the variable corresponding to the parametre $x(i)$.

So, we will have :

$$(10) \quad \frac{\partial F_t}{\partial x(i)} = (F_{1t})^{\frac{1}{P}-1} (Y_{1t})^{P-1} Z(i, t)$$

$$i = 1, 7$$

For the computation of (8) we have used the approximation (56).
After some tedious calculations we can write :

$$(11) \quad \frac{\partial F_t}{\partial x(8)} = (F_{1t})^{\frac{1}{p}-1} \left[(Y_{2t})^{p-1} + (Y_{3t})^{p-1} \left(1 - \frac{RR_t}{SR_t}\right) (1+b')^{t-mr_t} \right] t$$

$$(12) \quad \frac{\partial F_t}{\partial x(9)} = \frac{1}{t} \frac{\partial F_t}{\partial x(8)}$$

$$(13) \quad \frac{\partial F_t}{\partial x(10)} = (F_{1t})^{\frac{1}{p}-1} \left[(Y_{2t})^{p-1} \frac{E(t-mc_t)}{SC_t \ln(1+b')} + (Y_{3t})^{p-1} \frac{RR_t}{SR_t} (1+b')^{t-mr_t} \right] t$$

$$(14) \quad \frac{\partial F_t}{\partial x(11)} = \frac{1}{t} \frac{\partial F_t}{\partial x(10)}$$

where RR_t , SR_t , SC_t represent the sums given in (56) and $E(t-mc_t)$ represents the investments in the period v ($v = t - mc_t$).

If we proceed now to the transformation noted in the text, we will get finally :

$$(15) \quad \frac{\partial F_t}{\partial x(i)} = (A_t)^{\frac{1}{p}-1} \left[\frac{Y_{1t}}{Y_{ot}} \right]^{p-1} Z(i,t)$$

$$i = 1,7$$

$$(16) \quad \frac{\partial F_t}{\partial x(8)} = (A_t)^{\frac{1}{p}-1} \left[\left(\frac{Y_{2t}}{Y_{ot}} \right)^{p-1} + \left(\frac{Y_{3t}}{Y_{ot}} \right)^{p-1} (1+b')^{t-mr_t} \left(1 - \frac{RR_t}{SR_t}\right) \right] t$$

$$(17) \quad \frac{\partial F_t}{\partial x(9)} = \frac{1}{t} \frac{\partial F_t}{\partial x(8)}$$

$$(18) \quad \frac{\partial F_t}{\partial x(10)} = (A_t)^{\frac{1}{p}-1} \left[\left(\frac{Y_{2t}}{Y_{ot}} \right)^{p-1} \frac{E(t-mc_t)}{SC_t \ln(1+b')} + \left(\frac{Y_{3t}}{Y_{ot}} \right)^{p-1} (1+b')^{t-mr_t} \frac{RR_t}{SR_t} \right] t$$

$$(19) \quad \frac{\partial F_t}{\partial x(11)} = \frac{1}{t} \frac{\partial F_t}{\partial x(10)}$$

where :

$$A_t = Y_{ot}^{-p} \quad F_{1t} = \left(\frac{Y_{1t}}{Y_{ot}} \right)^p + \left(\frac{Y_{2t}}{Y_{ot}} \right)^p + \left(\frac{Y_{3t}}{Y_{ot}} \right)^p$$

and

$$Y_{ot} = \min_j / Y_{jt} / (j= 1,3)$$

The computation of first partial derivatives of ψ is achieved, replacing in (5), $\partial F_t / \partial x(i)$ by the expressions (15)-(19).

APPENDIX C

THE CHOICE OF THE MINIMUM FOR P AS OPTIMIZATION CRITERION

As it is noted in the text the parameter b' is estimated by an iteration method. In this appendix we justify the choice of the minimum for P (Equation (75)) as an optimization criterion.

The difficulties in the estimation of the parameter b' are related to the presence of the parameter b' in three behaviour equations : (66), (69) and (74).

So, the present problem, under a general formalization, is to maximize the likelihood function of

$$(1) \quad \begin{aligned} Y_1 &= f_1(\theta, \theta_1) + \varepsilon_1 \\ Y_2 &= f_2(\theta, \theta_2) + \varepsilon_2 \end{aligned}$$

where f_1 and f_2 are nonlinear functions of the parameters θ , θ_1 and θ_2 ; Y_1 , Y_2 , ε_1 and ε_2 are T-component vectors with the usual meaning.

Assuming that the ε_{jt} are NID $(0, \sigma_j^2)$ ($j = 1, 2$) ; $t = 1, T$), the logarithme of the likelihood function L is easily computed.

$$(2) \quad L = \text{constant} - \frac{T}{2} \ln \sigma_1^2 - \frac{T}{2} \ln \sigma_2^2 - \frac{1}{2\sigma_1^2} \sum_1^T (Y_t - f_1)^2 - \frac{1}{2\sigma_2^2} \sum_1^T (Y_2 - f_2)^2$$

where $f_1 = f_1(\theta, \theta_1)$ and $f_2 = f_2(\theta, \theta_2)$

The function L is maximized with respect to σ_1^2 , σ_2^2 , θ , θ_1 and θ_2 .

If we compute $\partial L / \partial \sigma_1^2 = 0$ and $\partial L / \partial \sigma_2^2 = 0$, we will get the estimators for σ_1^2 and for σ_2^2 :

$$\hat{\sigma}_1^2 = \frac{\frac{1}{T} \sum_1^T (Y_1 - f_1)^2}{1} = \frac{\frac{1}{T} \sum_1^T \hat{\varepsilon}_{1t}^2}{1}$$

$$\hat{\sigma}_2^2 = \frac{\frac{1}{T} \sum_1^T (Y_2 - f_2)^2}{1} = \frac{\frac{1}{T} \sum_1^T \hat{\varepsilon}_{2t}^2}{1}$$

We compute now the concentrated likelihood function (L^*), replacing in (2), σ_1^2 and σ_2^2 by their estimators $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$.

$$(4) \quad L^* = \text{constant} - \frac{T}{2} \ln \frac{SS_1}{T} - \frac{T}{2} \ln \frac{SS_2}{T} - T$$

where

$$SS_j = \sum_1^T (Y_j - f_j)^2 = T \sigma_j^2 (\theta, \theta_1, \theta_2)$$

Hence, the minimum for L^* can be obtained, if one computes the minimum for :

$$(5) \quad P_1 = \frac{T}{2} \left(\ln \frac{SS_1}{T} + \ln \frac{SS_2}{T} \right)$$

or for

$$(6) \quad P = SS_1 \quad SS_2$$

and the proof is complete.

One can easily show that, in case of $\sigma_1 = \sigma_2 = \sigma$ the minimum for (6) corresponds to the minimum for

$$(7) \quad S = SS_1 + SS_2$$

The minimum for S was used in the estimation of the present model with French data (Vilares (1981)) and in Smallwood (1972) (1).

(1) The method we use here (minimum for P) was suggested by P. Balestra.

APPENDIX D - RESULTS (1)

Table 4 - BLOCK PRODUCTION

T	YKA	YK	YC	YR	YE
55	50172.3	50004.9	52421.3	52436.2	50172.3
56	53297.9	53083.4	55325.4	55524.5	53297.9
57	56585.3	56334.3	57226.7	58167.1	56585.3
58	54435.8	54334.0	60530.4	61691.2	54435.8
59	59196.5	58960.4	63866.9	65924.9	59196.5
60	66600.1	66201.6	64626.5	69694.4	64626.5
61	65561.9	65398.9	69424.3	75225.6	65561.9
62	69807.1	69508.4	74572.0	80559.7	69807.1
63	71847.1	71602.3	81024.0	85160.0	71847.1
64	80307.0	79906.1	85060.3	89599.6	80307.0
65	85085.5	84671.7	90123.3	94205.5	85085.5
66	87298.6	87002.9	95419.7	98768.1	87298.6
67	94563.1	94110.1	100224.2	103202.0	94563.1
68	98079.3	97698.9	107699.9	109638.7	98079.3
69	105621.7	105210.0	113989.0	114348.3	105621.7
70	116944.7	116221.5	121362.4	121750.0	116944.7
71	121828.2	121274.1	127479.6	129130.5	121828.2
72	131843.3	131205.9	134405.8	136805.1	131843.3
73	150486.2	149497.3	144011.6	143956.5	143956.5
74	152001.2	151488.0	148087.4	153336.6	148087.4
75	145915.2	145648.5	142479.6	168859.9	142479.6
76	147843.5	147139.2	141353.6	177535.1	141353.6
77	147304.0	146830.4	153388.8	183058.3	147304.0
78	153506.8	153064.8	169326.7	189123.1	153506.8
79	160857.2	160287.0	184533.1	194669.5	160857.2

Table 5 - EXTERNAL TRADE

T	XD	XE	X	MD	ME	M
55	10333.3	10133.5	10443.0	9853.9	12685.5	11624.8
56	11294.9	11173.7	11271.0	11810.4	14619.9	13512.5
57	12278.4	11976.2	11611.0	13961.4	16655.0	15774.7
58	13467.4	13526.4	13776.0	12915.5	14913.0	15135.4
59	14447.3	14770.4	12790.0	13777.9	14641.3	15881.5
60	13571.4	13538.3	13938.0	15320.4	17124.6	17746.8
61	13974.9	13615.2	13570.0	14114.2	20046.3	16742.7
62	15419.6	15732.0	16160.0	20153.6	19206.9	22571.6
63	16319.4	17155.4	16309.0	17567.8	20169.3	20584.8
64	20897.6	21427.0	25888.0	20134.5	27454.1	24126.2
65	27899.2	28583.2	28757.0	25238.0	30243.6	29259.7
66	31278.8	31945.2	33450.0	25952.8	31248.9	30217.6
67	35162.1	36007.9	34649.0	30930.2	29632.0	35207.6
68	34671.2	35940.0	33474.0	28461.1	36957.6	34777.8
69	36013.9	36779.4	34441.0	37852.4	41801.6	44275.8
70	35255.8	36508.4	33881.0	44859.2	45257.0	51252.5
71	34665.9	34922.4	37230.0	39623.7	49782.5	46153.3
72	37638.7	37239.7	44136.0	48805.7	53972.9	54886.6
73	47953.4	48511.6	45980.0	58007.5	57877.6	64313.2
74	41635.4	38908.4	38757.0	57803.1	65079.8	64670.5
75	37033.8	33193.1	32715.0	43457.8	51418.6	46001.1
76	37796.6	36473.3	32715.0	52297.3	45846.2	53561.6
77	35987.0	34888.3	34645.0	56681.3	59281.9	57417.7
78	38434.7	39339.2	39703.0	53657.9	54992.8	54047.3
79	43640.1	47452.3	50582.0	53423.5	55826.5	54012.0

(1) The estimated value of a given variable Z is represented by ZE (see, in Table 2, the relative residuals).

Table 6 - THE BEHAVIOUR OF mka_t, mk_t, ma_t, mc_t and m_t
 $m_t = \min (mka_t, mc_t, mr_t)$

T	MKA	MK	MR	MC	M
55	22.88	22.72	24.57	24.56	22.88
56	23.35	23.13	25.21	25.08	23.35
57	24.65	24.39	26.01	25.24	24.65
58	21.42	21.35	26.99	26.02	21.42
59	22.63	22.46	27.97	25.99	22.63
60	25.15	24.87	27.94	23.86	23.86
61	19.48	19.31	27.15	22.89	19.48
62	16.66	16.36	26.83	22.33	16.66
63	15.00	14.90	26.84	23.57	15.00
64	17.06	16.85	26.61	22.34	17.06
65	17.20	16.99	26.33	21.75	17.20
66	16.20	16.06	25.76	21.12	16.20
67	17.57	17.35	24.51	20.59	17.57
68	15.58	15.47	20.99	19.93	15.58
69	16.16	16.04	19.88	19.69	16.16
70	17.33	17.12	19.25	19.05	17.33
71	16.19	16.01	18.58	18.08	16.19
72	16.33	16.13	17.96	17.15	16.33
73	18.53	18.18	16.31	16.33	16.31
74	15.41	15.27	15.75	14.48	14.48
75	13.12	13.07	18.02	12.54	12.54
76	13.03	12.91	19.50	11.93	11.93
77	12.73	12.65	20.28	13.78	12.73
78	13.57	13.49	21.26	16.42	13.57
79	14.65	14.55	22.14	19.04	14.65

Table 7 - LABOUR SUPPLY (NR), OBSERVED (N) AND ESTIMATED (NE) EMPLOYMENT

T	NR	NE	N
55	2824.5	2747.6	2730.7
56	2859.1	2761.5	2766.0
57	2886.4	2801.8	2798.3
58	2953.4	2795.6	2865.8
59	3035.9	2870.0	2948.4
60	3028.9	2959.1	2942.6
61	3039.8	2927.2	2952.3
62	3049.6	2935.6	2961.7
63	3053.9	2936.5	2970.9
64	3050.0	2968.6	2979.6
65	3044.6	2980.8	2989.3
66	3036.9	2982.4	2998.1
67	3025.5	3004.4	2975.6
68	3004.9	2974.8	2952.2
69	2958.9	2962.2	2899.0
70	2990.4	2930.7	2927.1
71	3006.6	2951.4	2912.2
72	3010.0	2946.2	2885.3
73	2964.2	2964.2	2851.1
74	3003.9	2890.5	2823.9
75	3258.8	2838.6	2806.8
76	3361.7	2810.5	2823.4
77	3383.8	2830.1	2813.6
78	3416.2	2825.3	2809.2
79	3432.9	2827.4	2811.0

APPENDIX E

THE DATA

The structure of this appendix is as follows. First there are made some remarks about the accounting framework and there it is showed the data used in the estimations.

As there were, often, different and partial contradictory sources, the adopted principle was to select the source with the most data. For the missing years another source was chosen. If the two sources were not comparable, we have used the rates of growth of this second source. Where such has happend the respective source is preceded by the word "from".

The sources which do not belong to the references given in the text are shown at the end of this appendix. They are quoted by a number enclosed into a right parenthesis ([]).

The accounting framework

The Portuguese accounting system is the "former system of national accounts of OECD countries" : the insufficiencies in official data are related in most of the studies about the Portuguese economy (see, in particular, OECD (1976)). So, they only will be refered in the case of a complete lack of data.

We do three particular remarks about the accounting framework.

The first one concerns the block of employment. In order to account for the fact that the discouraging factor plays essentially for the wage earners, Equation (58) was estimated under the form.

$$\ln NRE_t = \ln NR1_t + e_4 B_t \ln \left(\frac{NRE_t}{N_t} \right)_{-1} + e_5 (1-B_t) \ln \left(\frac{EM_t}{EM_t} \right) + \varepsilon'_t$$

where $NRE = POPAC - EI$

$POPAC = N + EHBR + PDRE$

and

EI : individual producers

EHBR : employment in the exogenous sectors (Public Administration)

PDRE : unemployment (1)

NR1_t : total wage earners labour supply.

The labour supply for the endogenous sectors NR is computed afterwards.

$$NR = NR1 + EI - EHBR$$

The second remark respects E(v), i.e. the gross fixed capital formation in transports, machinery and equipment. The data concern the total of the economy. They include also the Public Administration E₁(v). We have preceed in this way because E₁(v) is only available for the period 1958-1975 and, in this period E₁(v) is a modest part of E(v).

Finally the last remark concerns the indicators of the international environment. They were defined in the following way.

$$PET = \pi_{i=1}^5 PEX_{it} \left(\frac{s_{it}}{s_{ito}} \right)^{xi}$$

$$DW = \pi_{i=1}^5 (Q_{it})^{xi}$$

(1) Official data in employment are "very heterogenous and partially contradictory" (OCDE (1977), p. 11) concerning unemployment the problem is still more complicated since there are no available data (at least as much as we know) for the years preceding 1974, except for the census (1950, 1960 and 1970). The problem was solved by using indirect information. For 1953-59 and 1961-69 it was used the indicator of the number of emigrants without economic activity and older than ten years given in [4]. For the period there is in [8] an indicator about the demand of employment, and it was used.

where $x_i = x_i^i / \sum x_i^i$, x_i^i : percentage of the exports to country i
 s_{it} : exchange rate of the money of country i in terms of the escudo
(to = 1963)
 PEX_{it} : exports price index of the country i (1963 prices)
 Q_{it} : index of the evolution of the GNP of the country i (1963 prices)

and the countries are :

united kingdom, west Germany, USA, France, and Netherlands (1).

As s_{it} was not available for all i, it was replaced by

$$s_{it} = sF_{it} \frac{s_{3t}}{sF_{3t}}$$

where sF_{it} is the exchange rate of the money of the country i in terms of the French franc and the country number 3 is USA (see above).

After having defined these variables (PET and DW) it becomes easy to justify why the exports and imports demand from the Previous Escudo Area are considered as exogenous (see page 50). First the rates of change do not play any competitive role. Second the only available data concerns x_i^i called EXC in this Appendix (2).

(1) They were chosen because their importance for the Portuguese external trade and the availability of data. Most of these data (Q_{it} , PEX_i and sF_i) were supplied by P. VILLA from INSEE (Paris).

(2) The part of the imports proceeding from the Previous Escudo Area is called EXM.

Table 8 - THE DATA (1)

YEAR	Y	EXO	CONSU	X	M	EQ	
1953	46434.	7252.	41277.	8106.	10201.	2247.	
1954	48620.	8871.	42257.	9398.	11906.	2349.	1927 609.
1955	50519.	7033.	45964.	10443.	12926.	2528.	1928 651.
1956	52929.	6779.	49182.	11271.	14303.	2319.	1929 654.
1957	55303.	8031.	51159.	11611.	15498.	2752.	1930 698.
1958	56176.	7397.	50104.	13776.	15101.	3272.	1931 621.
1959	59256.	7686.	53594.	12790.	14814.	3814.	1932 489.
1960	65299.	12744.	55467.	13938.	16850.	5328.	1933 617.
1961	65625.	11300.	61783.	13570.	21028.	5448.	1934 745.
1962	69468.	12595.	59904.	16160.	19191.	5097.	1935 877.
1963	73779.	12185.	66383.	16309.	21098.	5286.	1936 740.
1964	78494.	14035.	65361.	25888.	26790.	5717.	1937 760.
1965	84463.	16181.	68880.	28757.	29355.	5979.	1938 832.
1966	87444.	15716.	70105.	33450.	31827.	6242.	1939 688.
1967	93403.	17784.	70882.	34649.	29912.	9329.	1940 483.
1968	100521.	16509.	90517.	33474.	39979.	8701.	1941 558.
1969	103378.	17893.	93867.	34441.	42823.	10417.	1942 514.
1970	113539.	27568.	95288.	33881.	43198.	11490.	1943 454.
1971	121743.	26608.	107372.	37230.	49467.	13034.	1944 372.
1972	131373.	30962.	108696.	44136.	52421.	15620.	1945 569.
1973	147116.	38443.	125125.	45980.	62432.	15895.	1946 931.
1974	148445.	37781.	137311.	38757.	65404.	13953.	1947 1288.
1975	136829.	16927.	136096.	32715.	48909.	11262.	1948 1441.
1976	143312.	20310.	140859.	32715.	50572.	10586.	1949 1395.
1977	150972.	31264.	141704.	34645.	56641.	11105.	1950 1437.
1978	154843.	28491.	142271.	39703.	55622.	11338.	1951 1526.
1979	159524.	24666.	143124.	50582.	58848.	11259.	1952 2560.

Source : 1927/52 from [13] ; 1953/57 from [6] ; 1958/69 [4] ; 1970/75 [7] ;
1976/79 [2] and from [2] for EQ.

(1) The meaning of the variables is given in Appendix A.

Table 9 - THE DATA (1)

YEAR	PEX	PIM	PY	PC	PET	DW	EXC	EMC
1953	0.944	0.974	0.867	0.870	0.895	0.694	0.268	0.157
1954	0.931	0.879	0.853	0.868	0.889	0.709	0.250	0.170
1955	0.934	0.938	0.866	0.872	0.909	0.747	0.238	0.137
1956	0.954	0.927	0.896	0.885	0.958	0.769	0.246	0.119
1957	0.926	0.968	0.902	0.893	0.954	0.797	0.262	0.117
1958	0.806	0.962	0.937	0.983	0.976	0.830	0.274	0.147
1959	0.834	0.972	0.937	0.969	1.003	0.844	0.289	0.142
1960	0.863	0.983	0.929	0.984	0.967	0.895	0.257	0.144
1961	0.894	0.992	0.979	0.994	0.985	0.926	0.232	0.125
1962	0.913	0.980	0.974	0.988	0.989	0.991	0.225	0.126
1963	1.000	1.000	1.000	1.000	1.000	1.000	0.238	0.143
1964	0.917	1.053	1.018	1.080	1.002	1.079	0.250	0.149
1965	0.964	1.128	1.062	1.146	1.044	1.104	0.250	0.137
1966	0.918	1.125	1.118	1.239	1.069	1.142	0.236	0.134
1967	0.996	1.272	1.152	1.278	1.081	1.168	0.244	0.143
1968	1.046	1.064	1.163	1.167	1.050	1.193	0.250	0.158
1969	1.091	1.046	1.237	1.214	1.070	1.266	0.248	0.150
1970	1.231	1.246	1.260	1.284	1.153	1.304	0.245	0.148
1971	1.291	1.265	1.328	1.322	1.209	1.341	0.214	0.132
1972	1.377	1.383	1.430	1.429	1.241	1.393	0.146	0.116
1973	1.579	1.493	1.563	1.527	1.308	1.481	0.148	0.101
1974	2.264	2.143	1.884	1.875	1.639	1.487	0.110	0.105
1975	2.264	2.475	2.204	2.232	1.887	1.482	0.083	0.052
1976	2.404	2.801	2.585	2.607	2.237	1.581	0.049	0.025
1977	3.203	3.621	3.304	3.318	3.112	1.616	0.065	0.013
1978	3.919	4.467	4.050	4.052	4.130	1.686	0.055	0.007
1979	5.086	5.941	4.998	5.032	5.630	1.737	0.051	0.010

Source : PEX, PIM, PY, PC (those of table 8) ; EXC, EXM, DW, PET [12]
 (see note (1) on page XV).

(1) The meaning of the variables is given in Appendix A and in this Appendix
 (for EXC and EMC).

Table 10 - THE DATA

YEAR	POPAC	POP	EI	N	PDRE	EHBR
1953	2912.	5427.	784.	2696.	83.	133.
1954	2946.	5441.	784.	2720.	83.	143.
1955	2968.	5457.	784.	2731.	83.	152.
1956	3004.	5464.	789.	2766.	82.	156.
1957	3044.	5458.	791.	2798.	85.	164.
1958	3136.	5474.	823.	2866.	80.	190.
1959	3194.	5506.	810.	2948.	79.	167.
1960	3203.	5573.	814.	2943.	79.	183.
1961	3222.	5608.	816.	2952.	79.	191.
1962	3240.	5665.	816.	2962.	81.	197.
1963	3258.	5710.	815.	2971.	83.	204.
1964	3271.	5743.	811.	2980.	80.	211.
1965	3278.	5733.	806.	2989.	71.	218.
1966	3280.	5699.	790.	2998.	57.	225.
1967	3274.	5685.	786.	2976.	63.	235.
1968	3254.	5691.	774.	2952.	65.	237.
1969	3225.	5644.	755.	2899.	80.	246.
1970	3266.	5578.	749.	2927.	86.	253.
1971	3285.	5532.	754.	2912.	114.	259.
1972	3287.	5535.	745.	2885.	136.	266.
1973	3252.	5538.	721.	2851.	127.	274.
1974	3280.	5711.	709.	2824.	180.	276.
1975	3479.	5991.	710.	2807.	396.	276.
1976	3518.	6029.	707.	2823.	416.	279.
1977	3543.	6078.	685.	2814.	441.	288.
1978	3583.	6107.	688.	2809.	470.	304.
1979	3601.	6192.	690.	2811.	476.	314.

POP [1] ; 1953/59 from [3] ; 1960/77 [1] ; 1978/79 from [2] (see, about PDRE, page 13 of this appendix).

The meaning of the variables is given in this Appendix and in Appendix A (POP).

Table 11 - THE DATA (1)

YEAR	DI	W	RNS	EM	RDM
1953	50392.	7.	31881.	40.	18511.
1954	51646.	7.	32431.	41.	19215.
1955	54529.	7.	34298.	30.	20231.
1956	57900.	8.	36292.	27.	21608.
1957	60632.	8.	38059.	35.	22573.
1958	56272.	8.	34396.	34.	21876.
1959	61025.	9.	36639.	33.	24386.
1960	65004.	11.	38753.	33.	26251.
1961	68184.	12.	39914.	35.	28270.
1962	72511.	12.	41962.	38.	30549.
1963	78493.	12.	44824.	54.	33669.
1964	80952.	15.	45409.	86.	35543.
1965	85156.	16.	46495.	117.	38661.
1966	86528.	18.	47124.	133.	39403.
1967	94382.	20.	51107.	106.	43275.
1968	112393.	22.	63499.	104.	48894.
1969	119777.	24.	68307.	154.	51470.
1970	128573.	27.	66813.	173.	61760.
1971	142228.	31.	72847.	151.	69382.
1972	154454.	36.	80087.	105.	74367.
1973	176792.	43.	94778.	120.	82014.
1974	173292.	60.	83227.	70.	90065.
1975	156914.	79.	56448.	45.	100466.
1976	164331.	94.	59277.	33.	105055.
1977	165179.	107.	69232.	29.	95946.
1978	169263.	124.	75660.	24.	93603.
1979	178727.	147.	87538.	24.	91189.

Source : EM : 1953/73 [14] , 1974/79 [2] ;
 DI : 1953/75 [10] , 1976/79 [2] ;
 RNS : 1953/64 from [3] , 1965/75 [10] , 1976/79 [2] ;
 W : 1953/59 from [3] , 1960/69 [5] , 1970/76 [8] , 1977/79 from [2] .

(1) The meaning of the variables is given in Appendix A (RDM = DI-RNS).

THE SOURCES OF THE DATA

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