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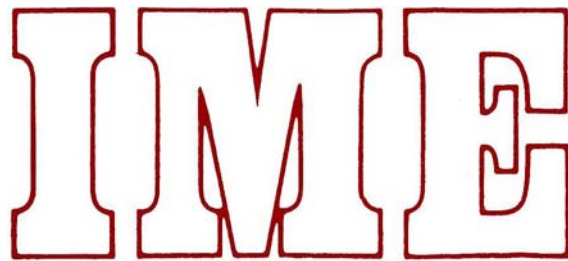
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RISK SHARING, THE MINIMUM WAGE, AND THE
BUSINESS CYCLE

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1. Introduction

This paper constructs a dynamic general equilibrium model in which labor incomes are influenced by risk sharing considerations and borrowing restrictions. We show that the dynamic properties of such an economy, in which the sharing of income and risk is effected solely via the labor market, are consistent with the principal stylized facts of the business cycle. We consider a situation in which workers are unable to borrow against their future income. This capital market imperfection is seen to alter the workings of the labor market whereby the latter substitutes as the vehicle for income and risk reallocation. The implications of this substitution for labor markets have been highlighted in the implicit contracts literature. Our objective here is to show how the introduction of such considerations affects the time series properties of a specific dynamic, multi-agent general equilibrium model.

Our dynamic context is a variant of the one good stochastic growth model. This identifies us as following in the tradition of the Real Business Cycle (RBC) literature (see e.g., Kydland and Prescott (1982) and Hansen (1985)). We do this not only because the enormous flexibility of the RBC methodology allows us to incorporate simple contracting phenomenon in a natural way but also because the RBC literature has a well developed sense of model evaluation: comparing the covariance/variance matrix of the model's artificial time series with that of the U.S. economy.

The second ingredient in what follows is a (static) model of labor contracting proposed by Drèze (1986 and 1989). In Drèze's model, ex ante optimal income and risk sharing arrangements may imply wage

discrimination between old and young workers, and the institution of a (state dependent) minimum wage resulting in unemployment. Socially desirable unemployment compensation is paid out of the proceeds of a tax on employer's profits.

The present paper can thus be more precisely viewed as an attempt to integrate simple non-Walrasian contracting theory into the RBC paradigm. While we believe such an exercise is of considerable interest on a purely theoretical and methodological level, this is not our only justification for undertaking such a study. Indeed, we believe non-Walrasian model features have considerable potential for explaining certain macroeconomic phenomena which have, to date, not yet been entirely successfully rationalized in the literature. The most important of these phenomena concerns the observed low variability of wages relative to employment found in U.S. and other countries' time series data.¹ Time series generated by the simplest RBC model -- the one good neoclassical stochastic growth model with a labor leisure choice -- have exactly the opposite property: wages vary much more than employment. This confounding of model prediction by the actual data is the so-called "wage-employment variability paradox."

Several resolutions have been proposed to this paradox. The first, and probably most controversial, was Kydland and Prescott's (1982) suggestion that the puzzle itself was evidence of a larger willingness on the part of economic agents to intertemporally substitute work and leisure across time than is implied by the standard time additive utility function. Their proposal -- to work with a modified utility function -- was met with much skepticism on the part of labor economists who declared themselves unable to find in the data traces of the

required level of intertemporal substitutability. In another paper, the same authors -- Kydland and Prescott (1988a) -- show that taking due account of the variable work week of capital would also go a long way towards resolving the paradox. Following yet another trail, Hansen (1985) explores, quite successfully, the implications of the non-convexity of the consumer-worker's choice set due to the impossibility of continuously adjusting the length of the work day; that is, he considers the implications of accepting as a fact the observation that most people have a limited menu of work schedules (full time or not at all).

In our first venture into non-Walrasian RBC modeling (Danthine and Donaldson (1987)) we have argued that the wage-employment variability paradox suggests a need to integrate non-Walrasian elements into the RBC paradigm. We first tested the potential of efficiency wage considerations to account for the low variability of wages relative to employment. We showed that efficiency wage features, while leading naturally to unemployment, do not necessarily produce relative wage rigidities in the business cycle sense and, for realistic levels of unemployment, do not appear necessarily to resolve the paradox. The present paper follows upon that first attempt. It is motivated by the observation that another major class of non-Walrasian labor market models -- those with labor contracting -- is "designed" to produce relative wage rigidities and, in that sense, is more likely to give rise to a satisfactory RBC model. (This class of models is known, however, to be less satisfactory in explaining unemployment). This hypothesis must be examined not only in light of its actual impact on the variability of employment and wages but also with respect to its

compatibility with the other stylized facts of the business cycle. Our first question can thus be cast as follows: Assuming any observed wage rigidity is the result of risk sharing considerations motivated by capital market imperfections, can we account for the relative variability of employment and wages while also retaining the desirable features of simpler Walrasian RBC models?

Now the contract literature to date is very large (see Rosen (1985) for a survey) and it is not possible to relate our work to all its many individual strands. Generally speaking, the economy's investment decision has not been modeled in this literature while it is an emphasis of our formulation. Difficulties in obtaining clear-cut comparative statics results in the context of a capital accumulation model with labor contracting (noted in Rosen (1985)) encourage the numerical approach we adopt. Those papers perhaps most directly related to our work are Wright (1988), Osano (1988), and McDonald and Solow (1981). Wright's (1988) set up (infinitely lived firms and finitely lived workers) is similar to our own although he does not consider capital accumulation; as a result, the optimality of contract equilibrium that he obtains does not carry over to our setting. Osano (1988) also considers a closely related model but with infinitely lived firms and workers; he proves that an appropriately defined contract equilibrium supports the social optimum. No attempt is made to characterize this equilibrium, however, which is another emphasis of this paper. Lastly, McDonald and Solow (1981) consider a model of a firm and union bargaining over a wage contract where the outcome is subject to certain equity conventions. Although they deal in a static partial equilibrium setting, their results support the possibility that short term contracts

could lead to fluctuations in employment with relatively stable wages.

An outline of the paper is as follows: Section 2 proposes a dynamic model into which optimal risk sharing can be explicitly incorporated; as outlined above, this involves merging a model proposed by Drèze (1989) into a basic RBC paradigm. Section 3 describes the algorithm by which we compute this equilibrium while Section 4 details a summary of our numerical results. Section 5 is reserved for concluding comments.

2. Model Formulation

Optimal risk sharing requires that the ratio of agents' marginal utilities of consumption be constant across all states of nature. Such an optimal allocation of risks will generally be prevented if some subset of agents is restricted from participating in the financial markets. Under our formulation workers are prohibited from borrowing and lending. This prohibition is in the spirit of the observation that a worker's main wealth is non-diversifiable human capital and that firm specific human capital (especially) does not collateralize consumption loans in modern economies. Such a constraint will have significant spill-over effects on the workings of other markets as agents attempt to reallocate risks in other ways. We focus on the institution of the labor market as a vehicle for reallocating risks and we hypothesize the existence of a social contract whereby extremes of income inequality are to be avoided.

2.1 Firms, Workers, and Equilibrium

There are J identical firms each producing the unique commodity with the same constant-returns-to-scale technology as described by a production function of the form $f(k_t, \ell_t^0, \ell_t) \Gamma_t$ where k_t denotes firm specific capital, Γ_t the economy wide shock to technology (common to all firms) and ℓ_t^0 and ℓ_t , respectively, denote firm levels of old and young labor employed (all period t levels). In general, a lower case variable will denote firm or individual specific levels of that variable while upper case variables denote economy wide aggregates.

Firms are owned by infinitely-lived dynasties of entrepreneurs (capitalists) who are entitled to the residual profits from production.² Capitalists' consumption and savings decisions solve the following

standard problem:

$$\begin{aligned}
 (1) \quad & \max_{\{(c_t), (z_t)\}} E\left(\sum_{t=0}^{\infty} \beta^t V(c_t)\right) \\
 & \text{s.t.} \quad c_t + z_t \leq \pi(k_t, K_t, \Gamma_t) \\
 & \quad k_{t+1} = (1-\Omega)k_t + z_t, \quad k_0 \text{ given,}
 \end{aligned}$$

where $V(\cdot)$ denotes the period utility function of a representative capitalist, c_t and z_t , respectively, his period t consumption and investment, β his period discount factor, Ω the period depreciation rate, and E the expectations operator. The expression $\pi(k_t, K_t, \Gamma_t)$ is the period profit function of a representative capitalist with individual capital stock k_t when the state of the economy is summarized by the aggregate capital stock K_t and shock Γ_t . The specifics of this function will be described in a moment.

Problem (1) contains the core of our model's dynamics and is similar to previous RBC models in that respect. Note, however, that under our interpretation, unlike more standard models, the production and investment decisions are made by the same economic agent. These investment decisions, in turn, are significantly affected by the existence of non-Walrasian labor market institutions described below.

The structure of the labor market is as follows: workers live T periods with $2L/T$ new workers being born and the same number of (old) workers, $2L/T$, dying at each date. There is thus a stationary population of $2L$ workers of which the fraction ξ are viewed as "young" unskilled apprentices and the fraction $(1-\xi)$ are viewed as "old" skilled workers. In what follows we will choose $\xi = \frac{1}{2}$ while noting that the implied "midlife discontinuity" -- graduating from unskilled to skilled

status -- can simply be viewed as marking the conclusion of an extensive training program. Alternatively, skill levels could be smoothed out by allowing several different skill designations and modifying the production technology accordingly. Every worker, young or old, is assumed to supply one unit of labor inelastically in each period of his life.

We assume that firms offer efficient labor contracts to old workers. Such contracts must clearly specify full employment since there is no disutility to work. Each of the J firms thus employs, in equilibrium, its share $(1/J)$ of the total supply of old workers. These contracts further imply optimal risk sharing between the risk averse old workers and the less risk averse capitalists; consequently, they must be of the form

$$(2) \quad \ell_t^0 = L_t^0/J = L^0/J, \text{ with}$$

$W^0(K_t, \Gamma_t)$, the period t wage paid by all firms to old workers, satisfying, for all t ,

$$(3) \quad u'(W^0(K_t, \Gamma_t)) = \theta V'(c(k_t, K_t, \Gamma_t))$$

where $c(k_t, K_t, \Gamma_t)$ solves problem (1). Here $u(\)$ denotes the period utility function of a representative worker (old or young) and

$$(4) \quad k_t = K_t/J.$$

Equation (4) anticipates the fact that in equilibrium each of the identical firms will hold the same amount of capital. As before, K_t and Γ_t are the economy wide state variables.

The parameter θ governs the relative income shares of capitalists

and old workers and may be viewed as reflecting, ex post, their relative bargaining strengths. In the spirit of the implicit contract literature, θ will be fixed at a level such that profit earners will voluntarily enter into such contracts. This means, specifically, that θ will be chosen so that the expected utility ($EV(\cdot)$) of the profit earners in the presence of contracting with old workers will be no less than what would be the case if the wages of the old were governed by Walrasian determination.

Following Drèze (1989) we next depart from the optimal contracting literature by postulating the impossibility of contractual relationships between firms and young workers. We thus assume, in effect, that for a portion of the labor force efficient risk sharing cannot be achieved privately. Firms decide, on a purely profit maximizing basis, how much young labor to hire for the current period given their current capital stock and ex post to the realization of the value of the technology shock. That is, the level of young employment, $L(K_t, \Gamma_t)$ in each state of the economy (K_t, Γ_t) is given by

$$(5) \quad L(K_t, \Gamma_t) = J\ell_t, \text{ where } \ell_t \text{ solves}$$

$$(6) \quad f_3(k_t, L^0/J, \ell_t)\Gamma_t = W(K_t, \Gamma_t)$$

with $k_t = \frac{K_t}{J}$. Here $W(K_t, \Gamma_t)$ denotes the state contingent wage of the young workers and it is to the origins of this wage that we now turn.

Given incomplete capital markets, Walrasian wage determination in the young "casual" labor market may entail considerable income variability and an inefficient allocation of income risk. Most real world economies have developed institutions designed to prevent extremes of income inequality. Accordingly, we postulate the existence, in our

artificial economy, of a system combining a minimum wage with unemployment compensation financed by a tax on firm profits. The state contingent minimum wage $W(K_t, \Gamma_t)$ and unemployment compensation $TX(K_t, \Gamma_t)$ maximize the following social welfare function:

For every (K, Γ) , $W(K, \Gamma)$ and $TX(K, \Gamma)$ solve:

$$(7) \quad \max_{\{W(K, \Gamma), TX(K, \Gamma)\}} \lambda JV(C(K, \Gamma)) + L^0 u(W^0(K, \Gamma)) +$$

$$L(K, \Gamma)u(W(K, \Gamma)) + [L - L(K, \Gamma)]u(TX(K, \Gamma))$$

subject to: (i) $W(K, \Gamma) \geq TX(K, \Gamma)$ multipliers
(μ)

(ii) $L \geq L(K, \Gamma)$ (Φ)

with $L(K, \Gamma)$ determined by equations (5) and (6) given $W(K, \Gamma)$, and $W^0(K, \Gamma)$ satisfying equation (3).

Here $C(K, \Gamma)$ is the consumption of the representative capitalist at equilibrium. The solution to (7) can be viewed as the social contract arrived at through a bargaining process between firm owners and young workers where the institutional arrangement is common knowledge, the constraints are agreed upon by all the participants, and the relative bargaining power is summarized by the parameter λ .

Notice that the solution to (7) is a schedule which defines the optimal wages to the young and old and unemployment benefits to the unemployed young in every state. We therefore write $W(K, \Gamma)$, $W^0(K, \Gamma)$ and $TX(K, \Gamma)$ to emphasize that these quantities need not be identical across states.

As our analysis to follow demonstrates, the solution to problem (7) will impose the condition that $W(K, \Gamma) = TX(K, \Gamma)$ whenever there is unemployment. If constraint 7(i) had alternatively been written as $W(K, \Gamma) \geq \epsilon TX(K, \Gamma)$, for $\epsilon > 1$, a constant, then unemployment would be ex

ante voluntary but ex post involuntary for the unemployed.

Before characterizing the solution to this problem we now conclude the above discussion by describing the profit and consumption functions of the representative profit earners and making precise the equilibrium concept appropriate to this model.

The profit function $\pi(k_t, K_t, \Gamma_t)$ can be written as:

$$(8) \quad \pi(k_t, K_t, \Gamma_t) = \max_{\ell_t} [f(k_t, \ell_t^0, \ell_t) \Gamma_t - \ell_t^0 W^0(K_t, \Gamma_t) - \ell_t W(K_t, \Gamma_t) - tx(K_t, \Gamma_t)].$$

where $tx(K_t, \Gamma_t)$ is the individual firm's share of the total tax burden levied to finance unemployment compensation. In equilibrium, one must have

$$(9) \quad Jtx(K, \Gamma) = (L - L(K, \Gamma))TX(K, \Gamma)$$

for every state (K, Γ) . It is important to note that for each individual firm, the tax tx is a lump sum amount that is unrelated to the firm's employment policy and is thus not experience related.

Problem (1) together with (8) is a standard dynamic programming formulation. Using Bellman's optimality principle, it can be shown to be equivalent to finding the value function $Q(k_t, K_t, \Gamma_t)$ where

$$(10) \quad Q(k_t, K_t, \Gamma_t) = \max_{0 \leq z_t \leq \pi(k_t, K_t, \Gamma_t)} (V(\pi(k_t, K_t, \Gamma_t) - z_t) + \beta \int Q((1-\Omega)k_t + z_t, K_{t+1}, \Gamma_{t+1}) dH(\Gamma_{t+1}; \Gamma_t)),$$

for appropriate laws of motion on the aggregate state variables which we assume are known to the profit earners (rational expectations):

(i) $K_{t+1} = (1-\Omega)K_t + Z(K_t, \Gamma_t)$ (law of motion on aggregate investment), and

(ii) the conditional distribution of Γ_{t+1} given Γ_t according to the known probability distribution $dH(\Gamma_{t+1}; \Gamma_t)$.

In terms of our previous discussion note also the identity
 $c(k, K, \Gamma) - \pi(k, K, \Gamma) - z = \pi(k, K, \Gamma) - z(k, K, \Gamma)$.

This seemingly simple formulation belies the complex interactions between the profit earners' and the workers' problems. Indeed, the optimal wage and employment schedules for workers of both generations depend upon the form of the aggregate investment function $Z(K_t, \Gamma_t)$ while, in turn, the form of the aggregate investment function -- being the sum of individual investment decisions -- depends upon the choice of factor wages in the next time period. A continuous function $Q(\cdot)$, as defined in (10), will exist under the customary conditions, including the continuity and concavity of $u(\cdot)$, $V(\cdot)$ and $f(\cdot)$ and the continuity of the $W(\cdot)$, $W^0(\cdot)$ and $tx(\cdot)$ functions. For a precise description of the sufficient conditions and the associated existence proof the reader is referred to Danthine and Donaldson (1990a).

Assuming differentiability of $Q(\cdot)$, the optimal $z(k_t, K_t, \Gamma_t)$ function which solves (10) is characterized via the standard first order condition:

$$(11) \quad V'(\pi(k_t, K_t, \Gamma_t) - z_t) = \beta \int Q_1((1-\Omega)k_t + z_t, K_{t+1}, \Gamma_{t+1}) dH(\Gamma_{t+1}; \Gamma_t)$$

By application of the envelope theorem,

$$(12) \quad \begin{aligned} Q_1(k_t, K_t, \Gamma_t) &= V'(\pi(k_t, K_t, \Gamma_t) - z_t) [\pi_1(k_t, K_t, \Gamma_t) + (1-\Omega)] \\ &= V'(\pi(k_t, K_t, \Gamma_t) - z_t) [f_1(k_t, \ell_t^0, \ell_t) \Gamma_t + (1-\Omega)]. \end{aligned}$$

Note that this formulation differs from those in which we can express the consumer-investor's (the profit earner's) income as the sum

of his wage income and his capital rental income, the latter expressed as the marginal product of capital (the competitive return on capital) multiplied by his capital holdings. This is for two reasons. First, the profit earners do not provide explicit labor services in this formulation. Second, and more significantly, optimal risk sharing between workers and profit earners may force the residual profit to differ from the return on capital even in the presence of constant returns. Thus we view profit earners as entrepreneurs who contribute whatever capital they have to the production process every period and who receive in return the residual profit after wages and unemployment taxes have been paid. We also note that this reverses the roles normally assigned to firms and workers in more conventional RBC paradigms.

We may now use the homogeneity of the J profit earners, together with the constant returns to scale assumption

$$(13) \quad Jf\left(\frac{K_t}{J}, \frac{L^0}{J}, \frac{L_t}{J}\right) = f(K_t, L^0, L_t)$$

to express aggregate profits as:

$$(14) \quad \Pi(K_t, \Gamma_t) = f(K_t, L^0, L(K_t, \Gamma_t))\Gamma_t - W^0(K_t, \Gamma_t)L^0 - W(K_t, \Gamma_t)L(K_t, \Gamma_t) - TX(K_t, \Gamma_t)(L - L(K_t, \Gamma_t)).$$

Aggregate consumption of the profit earners is then given by

$$(15) \quad C_t = \Pi(K_t, \Gamma_t) - Z(K_t, \Gamma_t).$$

The aggregate investment function $Z(K_t, \Gamma_t)$ is thus defined recursively by the equation:

$$\begin{aligned}
(16) \quad V'(C_t) &= V'(\Pi(K_t, \Gamma_t) - Z(K_t, \Gamma_t)) \\
&= \beta \int V'(\Pi(K_{t+1}, \Gamma_{t+1}) \\
&\quad - Z(K_{t+1}, \Gamma_{t+1})) [f_1(K_{t+1}, L^0, L(K_{t+1}, \Gamma_{t+1})) \Gamma_{t+1} + (1-\Omega)] dH(\Gamma_{t+1}; \Gamma_t)
\end{aligned}$$

in conjunction with the law of motion on aggregate capital (eqn. 10(i)), and equation (6) defining the aggregate demand for young labor.

Our notion of equilibrium can now be spelled out:

Definition: Equilibrium in this model is a quadruple of continuous functions $W(K_t, \Gamma_t)$, $W^0(K_t, \Gamma_t)$, $TX(K_t, \Gamma_t)$ and $Z(K_t, \Gamma_t)$ which simultaneously solve problem (7) and equation (16), for all values of (K_t, Γ_t) in the feasible range.

Now the existence of equilibrium can be guaranteed only provided the technology and preferences satisfy substantially restrictive assumptions. Since a detailed consideration of the existence issues is not the focus of this paper, the readers is once again referred to Danthine and Donaldson (1990a).

An issue still remaining is whether the equilibrium defined for this economy is optimal by some reasonable criterion. The answer to this question turns out, of course, to be negative. Indeed, the unemployment insurance scheme for young workers will force wages to the young above their Walrasian levels in certain states of nature giving rise to unemployment and lost output (income) in those states. This reduction in total income and the fact that the unemployment insurance tax is financed solely by profit earners together reduce the profit earner's income relative to what it would be in the corresponding constrained optimum (that for which there is risk sharing for older workers and young workers are paid their Walrasian wage in every state of nature). Since in this class of models investment is typically a normal good, we would expect investment to be thus reduced as a result (a fact

confirmed by our simulation results).

2.2 Equilibrium Characterization

Problem (7) together with (16) represents an adaptation of Drèze's contracting model (1989, Appendix 5) embedded in a dynamic context. Like Drèze (1989), the solution to (7) can take one of two forms. In 'good' states of the world (those with favorable shocks and a high level of capital), the demand for labor will be sufficient to ensure full employment of the young at a Walrasian (equilibrium) wage which is above the minimum wage acceptable given the equity considerations captured by the social welfare function in (7). In that case, constraint (i) is not binding ($\mu = 0$), while constraint (ii) is ($\Phi > 0$) with $L = L(K, \Gamma)$.

We will not explore the above possibility but rather concentrate on those sets of parameters for which, in all (stationary) states of the world, constraint (i) is binding ($\mu > 0$) and $W(K, \Gamma) = TX(K, \Gamma)$ and, consequently, constraint (ii) is not ($\Phi > 0$ and $L > L(K, \Gamma)$); i.e., unemployment is positive). It can be shown that the wages of the young will then fall short of the wages of the old ($W(K, \Gamma) \leq W^0(K, \Gamma)$) in these states. For this second case, the precise first order conditions for (7) are:

$$(17i) \quad -\lambda V'(\cdot) [(L(K, \Gamma) + TX(K, \Gamma) \frac{\partial L(K, \Gamma)}{\partial W(K, \Gamma)}] + u'(\cdot) L(K, \Gamma) + \mu = 0$$

$$(17ii) \quad -\lambda V'(\cdot) [L - L(K, \Gamma)] + u'(\cdot) [L - L(K, \Gamma)] - \mu = 0,$$

where we have omitted the obvious arguments for economy of presentation.

Rearranging (17ii) gives

$$(18) \quad [u'(\cdot) - \lambda V'(\cdot)] [L - L(K, \Gamma)] = \mu,$$

which shows μ to be positive (i.e., $W(K, \Gamma) = TX(K, \Gamma)$) only if there

cannot be full employment of workers at the efficient wage. Substituting (18) into (17i) one also obtains

$$(19) \quad [u'(\cdot) - \lambda V'(\cdot)]L(K, \Gamma) + \lambda V'(\cdot) \frac{\partial L(\cdot)}{\partial W(\cdot)} \tau X(K, \Gamma) = 0, \text{ or}$$

$$(20) \quad u'(\cdot) - \lambda V'(\cdot) \left[1 - \frac{W(K, \Gamma)}{L(K, \Gamma)} \frac{\partial L(K, \Gamma)}{\partial W(K, \Gamma)} \right],$$

$$(21) \quad = \lambda V'(\cdot) \left[1 - \frac{L(K, \Gamma)}{L} \cdot \eta_{LW} \right],$$

where η_{LW} is the wage elasticity of (young) labor demand. Equation (21) is Drèze's (1989) equation A.50 which, as he shows, results from equating the marginal gain for workers of an increase in the minimum wage, $u'(\cdot)$, to the marginal loss of profit earners, $\lambda V'(\cdot) \left[L - \frac{W(K, \Gamma) \partial L(K, \Gamma)}{\partial W(K, \Gamma)} \right]$, the latter being the loss of income due to the higher wage payment or the higher unemployment tax (L) plus the loss of output due to the reduced employment:

$$-f_3(K, L^0, L(K, \Gamma)) \Gamma \frac{\partial L(K, \Gamma)}{\partial W(K, \Gamma)} = -W(K, \Gamma) \frac{\partial L(K, \Gamma)}{\partial W(K, \Gamma)}.$$

This concludes our formal discussion of the model and our notion of equilibrium. We next proceed to an overview of the numerical algorithm by which equilibrium is calculated.

3. Numerical Procedure

The task before us is to solve problem (7) and equation (16) simultaneously for a given set of model parameters ($\alpha, \beta, \Omega, \rho, \bar{\Gamma}, \underline{\Gamma}, \gamma, \nu, \theta, \lambda$ -- to be defined shortly). This was accomplished via a recursive iteration procedure that acts on the form of the investment function thereby generating the equilibrium $Z(K, \Gamma)$ as the limit of a monotone increasing sequence of approximating functions. All calculations were performed on a (K, Γ) grid partition of a neighborhood surrounding the certainty capital stock steady state. The maximum distance between any two partition elements -- the "norm" of the partition -- ranged between .005 and .02. The procedure is as follows: First choose $Z_0(K, \Gamma) = 0$. Using this $Z_0(K, \Gamma)$, solve problem (6)-(10) to determine the corresponding wage and employment functions, $W_0(K, \Gamma)$, $W_0^0(K, \Gamma)$ and $L_0(K, \Gamma)$. This is a matter of solving at most a system of two non-linear equations. Standard sub-routines are available for this purpose.

Using these latter functions, next solve equation (16) -- which characterizes the equilibrium function -- to obtain a $Z_1(K, \Gamma)$. The $Z_1(K, \Gamma)$ is obtained in a somewhat novel way. Since our solution process for problem (7) requires an explicit functional form for the $Z(K, \Gamma)$ function and since the standard procedures for solving equation (16) provides an equilibrium $\tilde{Z}_1(K, \Gamma)$ defined only on a discrete set of capital stock \times shock pairs (a partition of the state space), we chose to approximate the true $Z_1(K, \Gamma)$ in the following manner. Given the $\tilde{Z}_1(K, \Gamma)$ we regressed investment as a function of the capital stock and shock to technology, for all (K, Γ) pairs in the same neighborhood of the steady state, to obtain an expression of the form: $Z_1(K, \Gamma) = \alpha + \beta K + \delta \Gamma$. Work by Larry Christiano (1988) as well as ourselves (Danthine, Donaldson, and Mehra (1989)) is persuasive that $Z_1(K, \Gamma)$ so obtained is a good approximation to $\tilde{Z}_1(K, \Gamma)$. Our procedures for obtaining $\tilde{Z}_1(K, \Gamma)$ as

the discrete solution to equation (16) follow along the lines of techniques described in Danthine and Donaldson (1987), and Coleman (1989).

Using the $Z_1(K, \Gamma)$, we next solve problem (7) again to secure a new set of wage and employment functions $W_1(K, \Gamma)$, $W_1^0(K, \Gamma)$ and $L_1(K, \Gamma)$. From these latter functions a new $Z_2(K, \Gamma)$ is obtained as a solution to eqn. (16) and the process repeats itself. We thereby construct a sequence of monotonic increasing functions which is bounded above and thus convergent. The corresponding sequences of wage functions $\{W_n^0(K, \Gamma)\}$ and $\{W_n(K, \Gamma)\}$ are each monotone decreasing and similarly converge; $\{L_n(K, \Gamma)\}$ is also monotone increasing (via equations (5)-(6)) and convergent.

Using the equilibrium investment function $Z(K, \Gamma)$ thus obtained, the time series of capital stock was generated as per 10(i) for a specified (see below) sequence of random technology shocks. With all wage and quantity expressions defined as functions of K and Γ , the corresponding time series for the variables followed accordingly.

Using the same set of parameters, this entire procedure was then followed by a nearly identical one yet for which $W^0(K, \Gamma)$ was set equal to the marginal product of old workers in every state while the wages of the young remained governed by the same insurance agreement otherwise specified in problem (7). If the $E(V(\cdot))$ so obtained in this second simulation equals or slightly falls short of the $E(C(\cdot))$ obtained in the first simulation above, the risk sharing contracts for old workers were deemed incentive compatible: firms would voluntarily enter into such contracts as they are welfare non inferior for their owners. If the reverse were true (i.e., the introduction of the contract reduced the welfare of the profit earners), θ was increased until the incentive compatibility relationship was restored.

As for functional forms, we chose $V(c) = \ln(c)$ and $u(c) = \frac{c}{\gamma}$, while $f(K, L^0, L)\Gamma$ was assumed to be of the form $MK^\alpha(L_0^{1-\nu}L^\nu)^{1-\alpha}\Gamma$. The shock to technology Γ was required to follow a two-state Markov process with transition probability matrix

$$\Gamma_{t+1} = \bar{\Gamma} \quad \underline{\Gamma}$$

$$\Gamma_t = \begin{matrix} \bar{\Gamma} \\ \underline{\Gamma} \end{matrix} \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$$

with p , $\bar{\Gamma}$, and $\underline{\Gamma}$ parameters of choice. The discipline imposed on the choice of this subset of parameters was twofold: (i) that p be chosen high enough that the pattern of autocorrelations of output lagged with itself over several periods resembles the analogous pattern for output of the U.S. economy and (ii) that $\underline{\Gamma}$, $\bar{\Gamma}$ be chosen so that the standard deviation of (detrended) output for the model economy again approximated its counterpart for the U.S. economy. These considerations (especially (ii)) led us to choose $\bar{\Gamma} = 1.009$ and $\underline{\Gamma} = .992$, with $p = .9$. By way of comparison, we note that in order for our shock process to match the first and second unconditional moments of Kydland and Prescott's (1982), we would have had to choose $\bar{\Gamma} = 1.02$, $\underline{\Gamma} = .98$ and $p = .97$. In particular, the shocks would have had to be much larger. This observation suggests that the 'transmission mechanism' in an economy with the types of restrictions we have imposed, is more powerful than its analogue for purely Walrasian models. We view this as especially significant in light of the fact that with proper accounting (in the Solow tradition) of the size of technology shocks, existing Walrasian paradigms are unable to account fully for the observed variability of output.

We next considered β , α and Ω ; following Kydland and Prescott (1982), these were fixed at, respectively, .99, .36 and .10 (see Kydland and Prescott

(1982) for the complete rationale). The choice of the β value to be .99, in particular, gives an average period return to physical capital of 1%, which is approximately what is observed quarterly for the U.S. economy. This implies that our model period corresponds to one quarter.

The remaining parameters were more difficult to fix unambiguously. To our knowledge, there is no convincing study that attempts to measure the coefficient of relative risk aversion (CRRA) of shareholders vis-a-vis nonshareholders. In conformity with previous work, we use a logarithmic utility function for the profit earners. We further adopt the intuitive assumption that those who choose not to be entrepreneurs are more risk averse than those who do. The results reported below correspond to a value of $\gamma = -6$, which we view as being in the admissible range $[-2, -10]$ in light of earlier microstudies, notably Drèze (1981). In the absence of hard empirical evidence on ν , the fraction of total labor income going to "casual" young workers as opposed to workers under contract, we simply fix ν at $\frac{1}{2}$. The parameter M is purely a scale parameter; it was chosen to fix the level of unemployment in the range of 4% to 10% which is reasonable for what has been observed for the U.S. economy. As noted earlier the parameter θ was determined entirely endogenously within the model such as to give the expected utility of profit earners with and without risk sharing labor contracts to the old as being the same. Lastly, the parameter λ determined the degree of income inequality between profit earners and young workers. It was similarly endogenously determined in such a way as to given reasonable relative income allocations.

4. Numerical Results

We first provide a statistical summary of the U.S. economy - our reference case - and then present a detailed comparative overview of the

model's performance with special attention to the "wage-employment variability paradox."

Aggregate time series for the U.S. economy display certain statistical regularities a replication of which constitutes the base test that a potentially valid model should pass. Table (1) below summarizes these statistics for a representative post-war time period (3rd quarter 1955 through 1st quarter 1984):

Table (1)
Statistical Properties -- U.S. Economy

(a) standard deviation percent, (b) correlation with output

<u>Series</u>	<u>(a)</u>	<u>(b)</u>
output	1.76	1.00
consumption	1.29	.85
investment	8.60	.92
capital stock	.63	.04
hours (employment)	1.66	.76
productivity (average)	1.18	.42

Source: Hansen (1985), Table 1; the above results are derived from quarterly data which have been detrended using the Hodrick and Prescott (1980) filter methodology.

As is evident from Table (1) column (a), investment varies proportionately much more than output while consumption and capital stock less so. Hours are substantially more variable than productivity (output/hours). Referring to column (b), consumption and investment are especially highly correlated with output; the same is true of hours and, to a somewhat lesser degree, productivity. Note also that capital stock is essentially uncorrelated with output. Although not reported in the Table, (detrended) output is highly intertemporally autocorrelated: its correlation with its one and two quarter lagged values is, respectively, .84 and .57 (c.f., Kydland and Prescott (1982)).

Table (2) presents the analogous contract model statistics for a representative set of parameters.

Table (2)
Model Statistics: Basic Aggregates

$$\begin{array}{cccc} M = .49, & \lambda = 7, & \gamma = -6.00, & \nu = .50 \\ \theta = 16, & p = .9, & \Gamma = .992 & \bar{\Gamma} = 1.009 \end{array}$$

(a) standard deviation in percent, (b) correlation with output

<u>series</u>	undetrended		detrended ⁽ⁱ⁾	
	<u>(a)</u>	<u>(b)</u>	<u>(a)</u> ⁽ⁱⁱ⁾	<u>(b)</u>
output	4.39	1.00	1.76	1.00
total consumption	2.72	.61	.80	-.74
(i) profit earner consumption	18.72	.92	2.76	.88
(ii) old worker consumption	3.71	.78	1.35	.37
(iii) young worker consumption	2.67	-.38	2.00	-.95
(iv) total worker consumption	2.18	.44	.90	-.80
investment	13.92	.89	8.90	.98
capital stock	6.97	.84	.65	.01
total hours	3.01	.91	1.78	.98
(i) young hours	6.02	.91	3.72	.98
productivity (average)	2.14	.83	.32	.05

Welfare Measures and Related Statistics

	<u>Mean Values</u>	<u>Welfare</u>
output	1.16	average utility ⁽ⁱⁱⁱ⁾
total consumption	.86	(i) profit earners -3.33
(i) prof. earn. consumption	.04	(ii) old workers -32.19
(ii) old worker consumption	.42	(iii) young workers -37.17
(iii) young worker consumption	.40	
investment	.30	
capital stock	11.60	
young hours	.90	
unemployment rate (young)	10%	

notes: (i) detrended using Hodrick and Prescott (1980) methodology
(ii) standard deviation of filtered series
(iii) average utility per time period

There are a number of aspects of this case that deserve comment.

1. First and foremost, the wage-employment variability paradox appears essentially solved. The puzzle can be summarized by first

noting that for U.S. data, (detrended) hours vary essentially as much as output and the ratio of the standard deviation of hours to the standard deviation of (average productivity) is about 1.3 (1.6/1.18). For the basic Walrasian stochastic growth model, however, hours and productivity each vary approximately half as much as output with the latter ratio being close to one. In our risk sharing model, hours vary slightly more than output and the hours - productivity relative variability ratio is 5.56 (1.78/.32). While this latter result is extreme and in a sense goes way overboard, we deem it acceptable given the strong assumptions we have adopted (no borrowing by workers yet full insurance via the labor market), assumptions that would have to be relaxed in a more realistic version of the model. The small variation in average productivity is also due in part to the extremely small productivity shocks required to produce output variation identical to that of the U.S. economy.

2. This resolution of the 'business cycle puzzle' does not affect the ability of the model to replicate the other major stylized facts. The relative variability of the aggregate output, consumption, and investment series thus corresponds generally to the observations made above for the U.S. economy. This is especially encouraging as it shows that the integration of 'real-world like' institutional arrangements designed to promote income and risk sharing into the standard paradigm can be accomplished without losing the latter's most attractive features.

3. The performance of the model in terms of the observed correlations is generally superior to what is obtained in pure Walrasian models, with one exception. On the positive side, investment is highly

correlated with output and capital stock is essentially uncorrelated with output, as per the data. Hours are more highly correlated with output than what is observed for the U.S. economy but in this regard our model performs no worse than the other Walrasian paradigms (e.g., Hansen's (1985) indivisible labor economy and Kydland and Prescott's (1982) time-to-build economy display similar correlations of .98 and .95 respectively). With regard to average productivity, its correlation with output is substantially lower than that of hours and in this sense the model is more respectful of the data than the Walrasian paradigms (Hansen's (1985) and Kydland and Prescott's models give nearly identical correlations of, respectively, .86 and .87 along this dimension). We note the substantial effect that detrending has on many of these correlations.

4. On the negative side, young worker consumption (wages) is negatively correlated with output, a byproduct of the minimum wage mechanism in conjunction with the no borrowing constraint. This effect is substantially strengthened by the detrending process. With young worker consumption more variable than old worker consumption (detrended series) and highly countercyclical, the stochastic pattern of young worker consumption relative to output dominates that of old workers resulting in countercyclical total worker consumption as well. (The fact that total worker consumption is significantly less variable than either of its constituent components further illustrates the disparate wage patterns of the different worker groups). With capitalists' consumption (dividends) small relative to worker consumption, this same countercyclical effect carries over to the total consumption series as well. Note that this latter property is, to a significant extent, a

phenomenon of detrending, and is not evident in the unfiltered data. That being said, it must nevertheless be acknowledged that even for unfiltered data, the total consumption - output correlation is too low relative to the data, a fact again attributable to the excessive power of our income stabilization mechanisms. A weaker mechanism is apparently called for. Alternatively, this unattractive feature of the model could also be eliminated if the proportion of the worker population represented by young unskilled workers were reduced to a more realistic level.

5. Dunlop (1938) and Tarshis (1939) have criticized both classical and Keynesian labor market theories for predicting, counterfactually, that average productivity and hours worked should be strongly negatively correlated with output. As documented by Christiano and Eichenbaum (1990), this same correlation is highly positive - normally above .9 - for typical RBC models. For the U.S. economy, however, these series are essentially uncorrelated. A similar result is observed for our model as well as in the case presented in Table (2), the correlation of hours and productivity is -.13, a dramatic improvement vis-a-vis earlier models. Excessive emphasis should not perhaps be placed on replicating this particular stylized fact, however, as it appears peculiar to the U.S. alone. Indeed, Danthine and Donaldson (1990b) report, for a large selection of countries that the hours - average productivity correlation is typically strongly negative; for example, this correlation is -.55 for Germany and -.50 for the United Kingdom while even ranging to a level of -.70 for Australia. These latter observations appear to increase the attractiveness of our modelling perspective vis-a-vis standard RBC formulations.

With regard to the correlation of total wages and hours, our model predicts a value of $-.81$, which again differs substantially from Walrasian RBC models where it is highly positive. The data is again somewhat ambiguous in this regard: Danthine and Donaldson (1990b) report a range of $-.58$ to $+.41$ for this variable across their sample of countries. Generally speaking, however, they find this correlation to be mildly positive. Our result again reflects the extreme nature of our assumptions. Note that for our formulation, the correlations of wages and hours and average productivity and hours can differ substantially (as reported) because under contracting wages need bear no direct relationship to marginal (and thus average) productivity.

6. Lastly, let us emphasize the impact of agent heterogeneity by comparing the behavior of the different components of aggregate consumption. In particular, while the standard deviation of aggregate consumption generally conforms to U.S. observations, it is simultaneously compatible with one segment of the population, profit earnings, experiencing a much larger consumption variability. This property, in our view, has potentially interesting implications, especially for the magnitude of equity premium. In the same vein - and as noted earlier - the variation in total wages is much smaller than the variation in either of its components.

5. Concluding Comments

In this paper, we have proposed a dynamic macroeconomic model with non-Walrasian features that is able successfully to mimic the major stylized facts of the business cycle. The income and risk sharing considerations we have introduced furthermore appear to provide a natural resolution to one of the outstanding puzzles: the wage-

employment variability paradox. By constructing a non-Walrasian model with a suboptimal equilibrium path which is observationally equivalent to existing pure Walrasian RBC models, we have also substantially softened the assertion that business cycle fluctuations of necessity result from the optimal responses of economic agents to uncertainty in the rate of technological change. We have also demonstrated the versatility of the RBC methodology and the possibility of extending it to accommodate a wide range of phenomena.

Footnotes

¹Kydland and Prescott (1988b) argue that this is due in large part to mismeasurement and that quality adjusted hours -- which they argue persuasively is the more appropriate measure of labor input -- vary much less than aggregate hours.

²We intend that the infinitely lived dynasty be a proxy for a family for which each generation internalizes the utility of its heirs. Barro (1974) demonstrates that such an organization will behave collectively like the infinitely lived agent we postulate.

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