



**HAL**  
open science

## How do writings in the astral sciences document mathematical practices and practitioners

Matthieu Husson, Kremer Richard

► **To cite this version:**

Matthieu Husson, Kremer Richard (Dir.). How do writings in the astral sciences document mathematical practices and practitioners. 2016, How do Writings in the Early Astral Sciences Reveal Mathematical Practices and Practitioners, 10.1111/1600-0498.12117 . hal-01538659

**HAL Id: hal-01538659**

**<https://hal.science/hal-01538659>**

Submitted on 28 Jun 2017

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## How do writings in the astral sciences document mathematical practices and practitioners

**Matthieu Husson**

(CNRS, SYRTE Observatoire de Paris - ERC project SAW)

&

**Richard Kremer**

(Dartmouth College, USA - ERC project SAW)

**Editors**

*Publisher Centaurus - 2016*

### **Table of contents**

#### **Introduction. Matthieu Husson & Richard Kremer**

*On recovering mathematical practices from Sources of the early astral sciences*

#### **Chapter 1. Sebastian Falk**

*Learning medieval astronomy through tables: the case of the Equatorie of the Planetis*

#### **Chapter 2. LI Liang**

*Template tables and computational practices in Early modern chinese calendric astronomy*

#### **Chapter 3. Matthieu Husson**

*Exploring the temporality of complex computational practice: two eclipse Notes by John of Murs in the ms Escorial O II 10*

#### **Chapter 4. HIROSE Sho**

*Two versions of a description of the armillary sphere in Paramesvara's Goladipika*

#### **Chapter 5. Daniel Patrick Morgan**

*Sphere confusion: a textual reconstruction of astronomical instruments and observational practice in first-millennium CE China*

#### **Chapter 6. Richard Kremer**

*Playing with geometrical tools: Johannes Stabius's astrolabium imperatorium (1515) and its successors*

NB: This chapter is to be downloaded from the Centaurus website

\*\*\*

*This research was supported by the European Research Council under the European Union's Seventh Framework Program (FP7/2007-2013)/ERC Grant agreement n. 269804, in the context of the project Mathematical Sciences in the Ancient World, acronym SAW.*

## How do Writings in the Early Astral Sciences Reveal Mathematical Practices and Practitioners?

Matthieu Husson (CNRS, SYRTE-Observatoire de Paris, SAW project);

Richard Kremer (Dartmouth College, SAW project)

Predicting eclipses and planetary phenomena has always been a desideratum for the astral sciences, whether in China, India, Babylonia, Egypt, Persia, or Italy. Indeed, we might say that it was largely by inventing mathematical procedures for these tasks that sky watchers became astral scientists. When sources have been preserved that allow us to observe these procedures, we can often learn not only about early astronomical assumptions but also about the mathematical practices of the astronomers. Such practices might include the design of metrologies and units, types of numbers and their arithmetics (e.g., fractions, sexagesimals), computational techniques, or the use of accessories such as diagrams, instruments, or numerical tables.

Consider, for example, the case of the armillary sphere, for centuries an emblem of mathematical astronomy. A great diversity of ancient and medieval sources describe this device and its uses. For example, official Chinese inventories of the holdings of particular astronomical institutions may discuss different kinds of ancient armillary spheres and their uses. Spheres can be described in versified Sanskrit treatises with commentaries, describing step-by-step the construction of the instrument and defining the astronomical meanings of its parts. Or advanced technical treatises may explore intricate cosmological and mathematical questions. Such a variety of sources can reveal a corresponding variety of mathematical practices linked to the armillary spheres and thus may constitute a methodological challenge for the historian seeking to understand the mathematical, astronomical or cultural significance of this instrument.

Similarly, early astronomers were often confronted by the management of long, complex computations, performed individually or collectively, with serious political and social consequences in case of perceived mistakes or inaccuracies. Again, a great variety of computational practices are attested in the sources. For instance, manuscripts assembled over a few years by John Westwyk, a 14th-century monk at the St Albans monastery and Tynemouth priory shows its owner learning mathematical astronomy through the compilation of various tables and instruments that he progressively adapted for his own purposes and computational habits. A much more institutionalized type of source, addressing the same kind of issues, is the ‘template table’ of Ming China, a printed sheet offering a grid with blank spaces for writing in numbers. Officially prepared by the Astronomical Bureau, the templates were designed to record intermediate results of an eclipse calculation. Textual labels on the rows and columns guide the user at each step of the computation. Here we find a writing support that normalizes mathematical practices, removes uncertainty or variation, and presumably would allow less experienced practitioners to compute eclipses even if they did not understand the ‘meaning’ of the algorithmic steps. The Bureau's template sheets, like the manuscript of John Westwyk, enable us to explore mathematical practices, to learn more than we would simply by considering the ‘theories’ of the astral sciences.

All of these instances as well as others are analyzed in this collection of papers. The articles in this issue arose within a collaborative research project conducted in Paris, *Mathematical*

*Sciences in the Ancient World* (SAW), funded by the European Research Council for 2012–2016 and directed by Karine Chemla. Focused on the analysis of sources for the early astral sciences, the special issue seeks to build a new awareness of specific features in such sources that can allow historians to recover often overlooked information about mathematical practices and practitioners. These practices could vary widely across time and geography; some travelled with texts or individual people; others remained local and particular.

The written sources in which such mathematical practices can be found encrypted can range widely, from documents produced in imperial offices to computational notes inserted by individuals into the blank folios of manuscripts filled with other materials. The more explicit texts of the astral sciences include numerical tables, diagrams, instructional canons or astrological charts, each laid out or formatted according to local or individual variation as well as knowledge of the writing technologies found in earlier astral texts. The sources of the early astral sciences often attest to very rich mathematical and writing practices.

Guided by these concerns, the authors of these case studies seek to explore, in new ways, various sources from the early astral sciences. Rather than merely explicating the astral sciences themselves, e.g., astronomical theory or computational algorithms, they are interested in documentary questions. How do the writing formats and medial conventions shape and reveal mathematical practices? Can similarities and differences in mathematical reasoning, computational techniques, metrology and units, algorithmic recipes, or the visual grammar of diagrams and pictures be explained by reference to writing practices in the early mathematical cultures specific to the milieus producing the sources of astral sciences in China, India, and Latin Europe?

Considering the material aspects of documents in the study of early astral and mathematical sciences is not new, as can be seen in the work of earlier (Paul Tanery, J.L. Heiberg, Otto Neugebauer or David Pingree) or more recent (Alexander Jones, Christine Proust or John Steele) scholars. Building on their erudition and results, the authors of the contributions in this issue seek to explore new methods to recover mathematical practices that go beyond the establishment and dating of definitive texts. The idea to analyse together the physical and textual features of historical sources in order to expose various cultural practices of authors, scribes, readers, and owners has its own history, stretching across disciplines too complex to trace here. Nevertheless, we find our approach informed by the ‘new bibliography’ exemplified by D. F. McKenzie, who in his Panizzi Lectures of 1985<sup>1</sup> called for studying the ‘sociology of texts,’ i.e., the social processes of their production, transmission, and consumption and the range of symbolic meanings evoked by the signs on the page. We also have learned from the seminar ‘History of Science, History of Text’ of the SPHERE Laboratory (Université Paris 7) that has, for the past two decades, investigated such issues in the history of science<sup>2</sup>. In this historiographic context, we hope that this special issue will contribute to the history of mathematics and astronomy as well as to the cultural history of science by attending to physical and textual features of sources for the astral sciences.

The case studies of this special issue are organised around two themes. Three articles focus on computational practices; the other three consider uses of diagrams and instruments. In the first set, Sebastian Falk analyses the tabular portion of a manuscript in Cambridge, Peterhouse MS 75.I. Known as the *Equatorie of the Planetis*, this late 14th-century English manuscript is

---

<sup>1</sup> D. F. McKenzie, *Bibliography and the sociology of texts*, London: British Library, 1986

<sup>2</sup> Chemla, K. and Virbel, J. (eds.) (2015) *Text, Textual Acts and the History of Science* (Basel: Springer).

written in two hands and two languages plus a secret cipher. Falk argues that an actor probably from a monastic context invented a novel astronomical computational tool, brought together in the same codex different and sometimes redundant numerical tables to use with the instrument, corrected some of these tables and probably computed others as his expertise increased in the process of assembling the codex. Falk finds an author experimenting with his material, exploring practices such as rounding and the accuracy and precision of various types of astronomical computation.

Li Liang also examines astronomical computation but in the context of late early modern astronomical imperial offices in China and Korea. From the Ming dynasty onward, very detailed template tables were developed and published by the imperial astronomical offices as a means to standardise astronomical computation. Through an examination of these printed documents Liang argues that, despite the profound differences in astronomical theory and cosmological beliefs, computations were performed with the same tools by actors making the traditional Chinese calendar and Arabic and European ephemerides. An analysis of errors in extant computations allows Liang to recover important features of rounding and the ways in which tables were read. The template tables reveal fundamental features of the computational practices in the imperial bureau of astronomy

Matthieu Husson addresses a solar eclipse computation recorded in 1333 by the Parisian astronomer John of Murs in several leaves left blank by the main scribe of a manuscript now in the Escorial Library. This document shows how a complex computation is managed by an astronomer working in the context of the medieval university. Husson considers multiple writing supports, references to astronomical tables built on conflicting parameters, various procedures employed at different steps of the computation, and the major 'choices' of the computation, paying close attention to the temporal dimension of writing. The analysis of the document allows Husson to approach mathematical practice on very short time scales. Husson wants to know when his actor wrote, turned to an auxiliary codex, perused his memory, corrected mistakes, or thought about how he might, in the future, consult the computation currently being made.

The second set of case studies addresses mathematical practices in diagrams and instruments. Sho Hirose examines two textual descriptions of the armillary sphere handed down in two distinct versions of Paramesvara's *Goladīpikā* (circa 1440). These texts were both produced by the same author in south India, under the same title. Their contrasts reveal how mathematical practices surrounding the same instrument may be presented in complementary ways, according to genre and intention. The armillary sphere thus appears at the centre of complex mathematical practices. On one side, they involve the three-dimensional material object itself. On the other, they employ different kinds of textual representations of the instrument. These practices allow the instrument's use to be directed toward the definition of fundamental astronomical concepts, such as latitude or the design and justification of complex computations, or the discussion of cosmological issues.

Daniel Morgan also considers armillary spheres but in the context of imperial Chinese scholarship of the first millennium, with a focus on their differing uses. From documents of a very specific type, imperial histories, Morgan demonstrates that the Chinese texts actually describe two different kinds of armillary spheres, those provided with sights through which users could view and measure astronomical phenomena and those that represented celestial motions and may have been used for time keeping. Each type of armillary sphere is associated with a specific set of mathematical practices that can be partially excavated from the documents.

Finally Richard L. Kremer examines a paper instrument printed as a broadside in 1515 in Nuremberg, designed by Johann Stabius, historian and mathematician at the court of Emperor Maximilian I. The extant sheet offers no proofs or recipes for constructing the instrument and only abbreviated instructions for its use. It is nonetheless possible to deduce from Stabius's broadside the different geometrical tools that Stabius and his contemporaries would have accepted as legitimate and reliable even if they are nowhere discussed, justified, explained or demonstrated in the written sources. Innovation in this community of cosmographers could occur as practitioners manipulated and rearranged those geometrical tools to solve various computational tasks via paper instruments.

In addition to uncovering usually hidden features of mathematical practices in sources of the early astral sciences, these essays illustrate different approaches to the analysis of the documents. For example, several of these studies rely on a close analysis of the materiality of the extant written sources. How do writing supports shape the production of texts? How did actors interact with their writing supports in order to perform computation or other operations, reasonings, algorithm construction, or the making of three-dimensional instruments? Did writing supports in the astral sciences interact with practice differently from those in other domains such as bureaucracy, poetry, or law? Such broad questions about the writings cultures which produced the sources examined here can build a greater awareness of details that may offer important clues for the recovery of mathematical practices. They also allow scholars to critically assess the importance of such sources and what it meant for actors to produce such documents.

Other essays in this issue employ more traditional textual approaches to their sources, including palaeography, philology, consideration of technical terminology, grammar, thematic analysis, etc. Such methods enable the contributors to assess mutual dependencies among sources, relations of their different parts, intellectual tensions around which they were built, multiple layers of meanings and the various ways that the sources could have been read and used. When coordinated with the material approaches, such textual studies can help us to identify trajectories of use for given documents.

Finally, our contributors also have drawn insights from the history of mathematics. These enable the reconstruction of the flow of computations sometimes incompletely recorded in the documents. They offer clues about types of numbers, rounding, and tabular interpolation. They enable us to assess the significance of specific geometrical arrangements in paper instruments or the intricacies of graduating a circle on an armillary sphere. Using mathematical approaches, along with the textual and material, we can begin to imagine the ways in which texts, tables and geometrical tools were deployed by the actors, how a terse procedural text complements a template table, how an incomplete set of tables is linked to a well-designed diagram or how an instrument may depict fundamental astronomical ideas.

We hope that the papers in this special issue may inspire other efforts to recover mathematical practices from documents or other materials that comprise the sources of the early astral sciences. Conversely we wish to promote the idea that analyses of mathematical practices also can yield important insights about the material and textual dimensions of sources.

### **Acknowledgements**

The research leading to these results received funding from the European Research Council under the European Union's Seventh Framework Program (FP7/2007-2013)/ERC Grant

agreement n. 269804 and was conducted in the context of the project SAW: Mathematical Sciences in the Ancient World (SAW).

# Learning Medieval Astronomy Through Tables: The Case of the *Equatorie of the Planetis*

SEB FALK\*

*Abstract.*<sup>†</sup> Medieval tables can be rich sources of evidence about the practices of the mathematicians and astronomers who used them. This paper analyses an important set of tables, revealing their compiler's learning practices and elucidating a valuable document of inexpert science. Peterhouse, Cambridge MS 75.I, 'The Equatorie of the Planetis', is a late-fourteenth-century compilation. It contains a treatise describing the construction and use of an equatorium (an astronomical instrument that computes the positions of the planets), bound with a collection of related astronomical tables. It was long thought to be written by the English poet Geoffrey Chaucer, but has recently been shown to be the work of a Benedictine monk, John Westwyk. This paper reassesses the manuscript as a monastic compilation. Westwyk copied a set of astronomical tables that suited his needs; their use supported and complemented the equatorium he describes in his treatise. He experimented with different techniques, cited astronomers whose work he admired (including Chaucer) and refined his tables in order to obtain the greatest possible precision. By reconstructing Westwyk's mathematical practices in compiling, computing and using tables that required and enabled a range of astronomical techniques, this paper paints a vivid picture of inexpert science in medieval Europe.

*Keywords.* Tables, astronomy, medieval, instruments, practices

The fifthe partie shal be an introductorie, after the statutes of oure doctours, in which thou maist lerne a gret part of the generall rewles of theorik in astrologie. In which fifthe partie shalt thou fynden tables of equaciouns of houses after the latitude of Oxenforde; and tables of dignitees of planetes, and othere notefull thinges [Middle English quotations are translated in endnotes].<sup>1</sup>

Geoffrey Chaucer (1988, p. 663)

Chaucer's desire to help his son Lewis – and perhaps other readers – 'lerne sciences touching nombres and proporciouns' is familiar to readers of his *Treatise on the Astrolabe* (Chaucer, 1988, p. 662).<sup>2</sup> But while the potential for learning through the use of an instrument has been accepted and widely discussed since Chaucer's time, less has been written about the connection between tables and learning practices in astronomy. Tracing past learning processes is always

---

\* Department of History and Philosophy of Science, University of Cambridge, Free School Lane, Cambridge, CB2 3RH, UK. E-mail: sldf2@cam.ac.uk

<sup>†</sup> For comments on earlier drafts of this article, I would like to thank José Chabás, Matthieu Husson, Richard Kremer and Richard Oosterhoff, as well as the *Centaurus* editors and reviewers. I am also grateful to Karine Chemla, Matthieu Husson and Richard Kremer for organising a workshop at which the ideas in this article were refined. This was supported by the European Research Council under the European Union's Seventh Framework Program (FP7/2007-2013) / ERC Grant agreement n. 269804, in the context of the project SAW: *Mathematical Sciences in the Ancient World*. This article is based on material from my University of Cambridge PhD thesis, funded by the Arts and Humanities Research Council. I would particularly like to thank my supervisor Liba Taub and advisor Nick Jardine.



difficult, particularly if we wish to focus on the reception and absorption of knowledge rather than the mechanisms and institutions through which knowledge was communicated: learning, rather than teaching (Bernard and Proust, 2014).<sup>3</sup> Moreover, although the abundance of tables in medieval scientific manuscripts is testament to their popularity, identifying the methods by which astronomers learnt to use tables, or learnt theories and techniques through tables, is especially difficult since they are rarely accompanied by didactic text; explicatory canons which do sometimes direct their users are invariably written in spare instructional prose, and there are few clues as to how, or by whom, such canons were followed.

Nevertheless, medieval tables can be rich sources of evidence about the practices of the astronomers who used them. And where those astronomers lack expertise, we can draw conclusions about the ways that they learnt and practised mathematical techniques through such use. The *Equatorie of the Planetis* (Peterhouse, Cambridge MS 75.I) is a valuable document of such unpolished astronomy. It comprises a fourteenth-century draft treatise describing the construction and use of an equatorium, an instrument that computes the positions of the planets, bound with a collection of related astronomical tables. Its first editor, Derek Price, suggested that this treatise was ‘obviously intended for the amateur rather than the professional’ reader (Price, 1955, p. 159). His implication, supporting his contention that the *Equatorie* represented Chaucer’s completion of his *Treatise on the Astrolabe* (it incorporates much of the content Chaucer had promised for the *Astrolabe*’s third, fourth and fifth parts), was that its author was a competent astronomer writing for a less learned pupil. But Price, concerned above all to prove Chaucer’s authorship of the treatise, did not consider it in its codicological context. He dismissed the tables that comprise the bulk of the manuscript as ‘of comparatively slight interest since they are a simple modification of the well-known Alfonsine tables’, and thought it ‘only necessary to indicate their content and the manner in which they have been modified’ (Price, 1955, p. 75). Similarly, John North, despite stating that ‘the sheer aptness of *all* the tables in the codex for use with the equatorium cannot be too strongly emphasized’, gave almost no explanation of that use (North, 1988, p. 176; his emphasis).

North surmised that the *Equatorie of the Planetis* was the work ‘of a generally competent if not fully confident astronomer’ (North, 1988, p. 170); that astronomer has recently been identified, on palaeographical grounds, as John Westwyk, a monk of St Albans monastery and Tynemouth priory (Rand, 2015). This paper will analyse the fascinating, varied tables, all either written or annotated by Westwyk, alongside the instrument they accompany. When one examines the tables closely and considers their use both with and without the instrument, their heterogeneity stands out, and the conclusions of Price and North quoted above begin to seem

rather bold.<sup>4</sup> These scholars, influenced by the seminal history of mathematical astronomy of Otto Neugebauer, rightly saw the *Equatorie* within a wide-ranging, enduring network that communicated astronomical theories and instruments across the medieval world. I take a different approach. Rather than emphasizing the continuity visible through these tables, this paper will emphasize the individuality of their production, the specific historical context of their use. It takes the manuscript's 78 folios together as a personal compilation, revealing much about its producer's priorities, learning processes, and level of expertise. At the same time, this paper has a mathematical focus, exploring what we can learn through a reconstruction of Westwyk's practices in compiling, computing and using tables that required and enabled a range of astronomical techniques. It is hoped that this combination of computational and contextual methodologies will provide new insights into these astronomical tables and instrument, as well as the man and environment that produced them.<sup>5</sup>

### **The *Equatorie*: instrument and tables**

John Westwyk's equatorium (Figure 1) was, he writes, 'compounded the yer of Crist 1392 complet the laste meridie of decembre' (Peterhouse, Cambridge MS 75.I, f. 71v; all folio

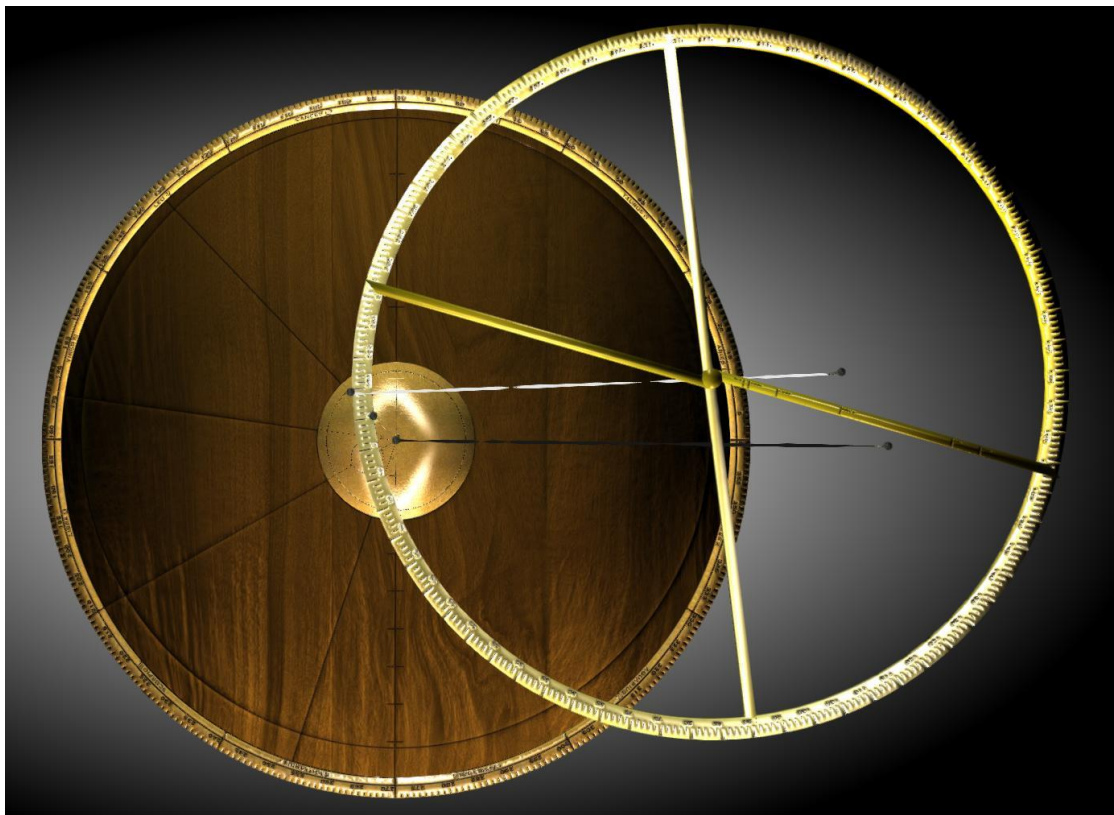


Fig. 1: Virtual model made according to the instructions in Peterhouse MS 75.I, by Ben Blundell and Seb Falk for the Cambridge Digital Library. See <http://cudl.lib.cam.ac.uk/view/MS-PETERHOUSE-00075-00001> for further explanation and interactive model. See also Price (1955), pp. 93-118. Reproduced by permission of the Master and Fellows of Peterhouse, Cambridge.

references will be to this manuscript, unless stated otherwise).<sup>6</sup> Like other equatoria, this instrument requires an input of mean motion data in order to compute the longitudes of the planets (including the Sun and Moon); the necessary tables of mean longitudes and mean anomalies, with radices for 1392, are found, written in Westwyk's hand, in the first folios of the manuscript.<sup>7</sup> Westwyk names Chaucer as a source for these radices and elsewhere cites the *Treatise on the Astrolabe*; the influence of Chaucer's remark that 'smallist fraccions ne wol not be shewid in so small an instrument as in subtile tables calculated for a cause' (Chaucer, 1988, 663)<sup>8</sup> is apparent in Westwyk's opening statement that 'the largere þat thow makest this instrument, the largere ben thi devisiouns; the largere þat ben tho devisiouns, in hem may ben mo smale fraccions; and evere the mo of smale fraccions, the ner the trowthe of thy conclusiouns' (f. 71v).<sup>9</sup> From the very beginning of the treatise, then, Westwyk shows awareness that instruments and tables represented competing (and complementary) methods of computing planetary positions; these methods had to balance speed and convenience against precision.<sup>10</sup> It may also be suggested that learning different techniques was an objective in itself, separate from the ultimate outcome of finding positions. Peterhouse MS 75.I reveals how Westwyk tried two alternative techniques: the use of an equatorium with tables, and the use of tables alone. For the former, the first set of tables in Westwyk's hand (folios 1r-13v) is perfectly sufficient; of them it would be correct to say, as North rather exaggeratedly said of the whole codex, that they are entirely apt for use with the equatorium, and it seems likely that Westwyk drew them up for that purpose. They are broadly standard tables in the Parisian Alfonsine tradition, supplying daily and annual changes in position of three sets of data: the planetary apogees, mean longitudes and mean anomalies (see Table 1).<sup>11</sup> The equatorium incorporates two further sets of data – the eccentricity of each planet's deferent circle, and the relative sizes of their deferents and epicycles – so that the user is required only to extract radices and mean motions from the tables, perform some simple additions or subtractions, and lay out the instrument's brass ring and black and white threads as Westwyk explains, in order to read the true planetary longitudes on the ecliptic scale on the equatorium's limb. The 72-inch diameter Westwyk stipulates would allow planetary longitudes to be read at a precision of around 2' of arc; the whole process for each planet can be accomplished within a few minutes. It would probably have taken him somewhat longer to carry out the calculations and interpolations involved in using the tables on their own. (Those methods, using the tables on ff. 45r-61r, are discussed later in this article.)

Table 1: Contents of Peterhouse MS 75.I

<b>ff. 1r-13v in John Westwyk's hand</b>	
1r	Note with sexagesimal equivalent of 1392, 1393 years; table to convert years to days Radices of mean longitude and mean anomaly of planets for 31 Dec 1393 Table of annual motion of deferent centre of Moon (radix for Incarnation, London)
1v-3r	Annual motion of mean longitude and mean anomaly of Moon, Caput Draconis and planets, with radices for Incarnation, London
3v	Annual values for mean motus of ascendant for latitude 51;34°; radix for 28 Feb 1393
4r-4v	Ascensions of signs for latitude 51;50° Tables to convert between hours and sexagesimal fractions of day
5r	Radices of mean longitudes and anomalies, 31 Dec 1392 and Incarnation, London Radices of mean apogees, Incarnation, London
5v	'Radix chaucer' note giving number of days in 1392 years; note with days in 1395 years Multiplication table for orders of sexagesimals; table to convert between years and days
6r	Daily motion of mean argument of 8th sphere
6v	Apogees of planets for Incarnation, 1392, 1400 Radix for mean elongation of Moon, Incarnation, Toledo
7r	Annual motion of mean longitude of apogees; annual motion of mean argument of 8 <sup>th</sup> sphere
7r-13r	Annual and daily motions of mean longitudes and mean anomalies of Sun, Moon, Caput Draconis and planets. Radices for 13 Dec 1392, London
13v	Daily motion of apogees (linear precession)
<b>ff. 14r-62r in 'Hand S', with annotations by 'Hand A' and Westwyk<sup>12</sup></b>	
14r-16v	Calendar for motions of apogees; table of equation of the 8th sphere
16v-30r	Calendars for mean longitudes and mean anomalies of Sun, Moon, Caput Draconis and planets, and for motion of deferent centre of Moon
30v-31v	Calendar for mean centre of Moon; radix for 28 Feb 1392
32r-38r	Tables for latitudes of Moon and planets
38v-44v	Tables of proportion for multiplication of sexagesimal numbers
45r-61r	Double-entry tables for planetary longitudes (headed 'Equatio [name of planet]')
61v	Table of ascensions of signs and houses for latitude 50;50°
62r	Table of precession for 1349-1468, at rate of 1° in 98-99 years
<b>ff. 62v-78v in John Westwyk's hand</b>	
62v	Hourly values (excess degrees) for motion of Moon, 1-12 hours <sup>13</sup>
63v	Difference in length of half of longest day over equinoctial day, for latitudes 0-60° Planetary longitudes and latitudes for 31 Dec 1393 (attributed to John Somer, Oxford)
64r	Solar declination and differences in ascensions of signs for latitudes 0-60° Radices (including mean centre), 28 Feb 1394, London
64v	Horoscope with accompanying Latin text (Kennedy, 1959)
65r-70v	Ascensions of signs for latitude of Oxford, 51;50° (John Walter's tables) (North, 1988, p. 191)
71r	Incomplete star table, with altitudes at Oxford (partial) and London
71v-78v	Canon to the equatorie of the planetis

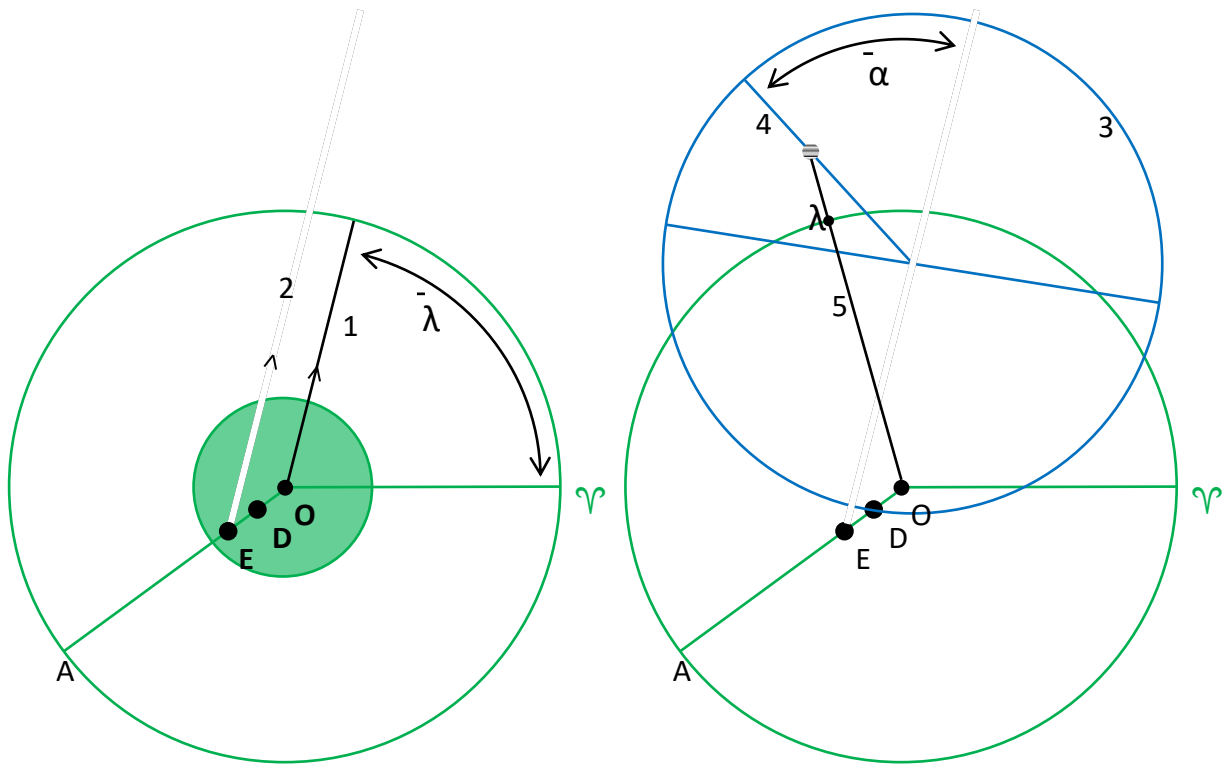


Fig. 2: Steps (numbered) in the use of the Peterhouse equatorium to find the true ecliptic longitude ( $\lambda$ ) of a superior planet.

### Calculation and copying; precision and accuracy

Before the mean longitude and mean anomaly of a planet were laid out to find its true longitude (as shown in Figure 2), the equatorium could be calibrated so that the lines marking the planetary apogees, on which lay the deferent centre and equant, were up to date. This task was not particularly important for a user soon after the equatorium's production, and it did not have to be done every time it was used, but it is clear that John Westwyk attached some importance to it. The parts of the tables and treatise pertaining to this task shed important light on Westwyk's methods and priorities in composing and compiling his manuscript.

The Alfonsine apogees were thought to move in two ways: a linear precession, increasing in longitude by one revolution every 49,000 years, also known as the mean motus of apogees and fixed stars; and accession and recession of the eighth sphere, an oscillating motion of up to  $9^\circ$  in each direction, with the period of oscillation being 7000 years (Dobrzycki, 1965). The relevant tables are on ff. 5r-7r and 13v. Radices are given for *anges medie* (the apogees incorporating only linear precession) and *anges vere* (apogees fully corrected to include accession and recession of the eighth sphere). To find the apogee for the desired date, the radices were to be corrected first by the addition of the linear component. This was provided in tables of annual and daily motion (on ff. 7r and 13v); the former was laid out with 1-3 years of 365 days, followed by 4, 8, 12... 56 years of 365.25 days, and then 1-3 years of 365.25 days; the latter as 1-59 days. The linear movement

of the apogee since the date of the radix could be added to the radix value to give the ‘mean apogee’.

Calculating the ‘true apogee’ was slightly more complicated, as none of the tables in Westwyk’s hand gives the oscillating component directly. Instead, he wrote out daily and annual tables of what is called *argumentum medium vel accessus et recessus 8<sup>e</sup> spere* (ff. 6r, 7r).<sup>14</sup> These tables, which are laid out in the same way as those just mentioned, give daily and annual fractions of a complete revolution in 7000 years. The values are therefore 7 times those in the tables of linear precession. To convert these fractions of a complete revolution into the correct fractions of a complete oscillation of  $\pm 9^\circ$ , Westwyk initially intended to use his equatorium. He instructs his reader to divide ‘the line þat goth fro centre aryn to the hed of capricone which lyne is cleped in the tretis of the astrelabie the midnyht line’ into 9: ‘thise last seid 9 divisious in the midnyht lyne shollen seruen for equacioun of the 8e spere’ (f. 72v).<sup>15</sup> However, he does not explain the technique for using these divisions to compute the equation of the eighth sphere from the mean argument of the eighth sphere.

Why might he have left the treatise unfinished in this way? Beyond lack of opportunity or lack of knowledge, there are several reasons why Westwyk may have chosen not to explain this technique in full. First, although it is important for the long-term maintenance of the equatorium’s capabilities, the effects of precession would only be noticed after some years; the explanation of this function was thus hardly likely to be a priority.<sup>16</sup> Secondly, the technique would have been analogous to that of computing the latitude of the Moon on the radius opposite the midnight line, which Westwyk explained at great length; he may have felt it unnecessary to explain a similar principle again, presuming that a reader could infer the analogy.<sup>17</sup> It should be noted that the function of accession and recession of the eighth sphere was not as simple as that for the latitude of the Moon, so a third (somewhat remote) possibility is that Westwyk realized that the same technique would not work so well for the latter function, and abandoned his attempt to use the equatorium in this way. However, a more likely explanation is that he found a simpler source of the necessary data. The large set of tables that are not in Westwyk’s hand (ff. 14r-62r) contain a table of the equation of the eighth sphere (f. 16), as well as a smaller table containing additions to be made to the apogees for each year for 1349-1468 (f. 62r). The latter, which is computed using the two-component Alfonsine precession, functions as a ready-reckoner to allow the true apogee to be easily obtained. These ‘Hand S’ tables had been annotated by another hand (‘Hand A’) before Westwyk began to use them;<sup>18</sup> but Westwyk’s annotations, and the repetition of some material, suggest that he at least began making his own set before using them (North, 1988, p. 176). The fact that he instructed his readers to mark the tool for the

equation of the eighth sphere on the face of the equatorium, but did not explain how to use it, suggests that he may have obtained the larger set of tables before he completed the treatise, and realized that they obviated the need for that tool. Nevertheless, a reader who could work out its use (with or without reference to the explanation of lunar latitude), could still use it; Westwyk may have been presenting his reader the same choice of techniques that he enjoyed.

The tables of linear precession that appear in both Westwyk’s own set of tables and those in ‘Hand S’ raise some important questions. In the first place, it may reasonably be wondered what the purpose was of tabulating daily values for an astronomical variable that changed by less than half a minute of arc in a whole year, an amount that could not be read on an equatorium even if it were constructed at the scale Westwyk recommends. More striking still is the fact that those daily values – and indeed many others in the codex – are given to a precision of sexagesimal ninths. The 37 that appears in the column of ninths for one day’s motion of the apogees (f. 13v) is equal to one 98,000,000,000,000th part of a complete circle; an equatorium capable of displaying such precision would have to be around nine trillion times the size specified in Westwyk’s description. Such precision clearly does not reflect observational accuracy, but arising from calculations carried out by standard methods in accordance with Ptolemaic theory, it was difficult to discard. And the same principle gives us the reason for the table of days: smaller divisions of the basic unit of one revolution in 7000 years simply seemed more precise.

This greater precision is a paradoxical indicator of an amateur compilation: perhaps partially motivated by the satisfaction of correct – albeit observationally meaningless – calculation, but lacking the sophistication necessary for purposeful rounding.<sup>19</sup> For historians, on the other hand, it is highly valuable, as it may indicate how the tables were adapted from earlier, more rounded versions, and the order in which they were produced. We can see this in the example of annual and daily motions of the mean motus of apogees. In Westwyk’s table on f. 7r we find the motion in one year as 0;0,26,26,56,20,0,0,1,44,15°. It can immediately be seen, in the middle row of Figure 3, that the final 15 was added after the rest of the table was written. The two columns of zeroes in the middle of the figure also attract attention, suggesting that the

42		22	44	20	29	20	2	1	30	21		
46	0	0	29	21	8	39	40	2	1	38	18	
1		0	26	26	46	20	0	0	1	42	14	
2	0	0	0	42	43	42	20	0	0	3	28	30
3	0	0	1	19	20	29	0	0	0	4	12	24

Fig. 3: Last five rows of table of annual motion of mean motus of apogees. Peterhouse, Cambridge MS 75.I, f. 7r. Reproduced by permission of the Master and Fellows of Peterhouse, Cambridge.

number was rounded at an intermediate stage. A full revolution divided by 49,000 years is approximately  $0;0,26,26,56,19,35,30^\circ$ ; it is clear that this was at first rounded to  $0;0,26,26,56,20^\circ$ . We may identify the source of the extra 1,44 by comparing the values in Westwyk's table of daily motions (f. 13v). The value for one day found there ( $0;0,0,4,20,41,17,12,26,37^\circ$ ) is exactly equal to  $0;0,26,26,56,20^\circ \div 365.25$  (or  $6,5;15$ , as it would have been rendered), to the precision of sexagesimal ninths that seems to have been preferred by the creator of these tables. If that daily figure is multiplied by 365.25, again using nine sexagesimal places, we obtain  $0;0,26,26,56,20,0,0,1,44^\circ$ , which was the figure Westwyk first wrote. It seems most likely, then, that the table of annual linear precession was produced from a table of daily motions, which itself had been based on a rounded value for the annual motion. That may not have been done by Westwyk (though I know of no other extant tables, from which he could have copied, with as many sexagesimal places as his). But the last step is clearly Westwyk's own. He apparently noticed – perhaps as he was rubricating the table – that the figure for 4 years ( $0;1,45,47,45,20,0,0,6,57^\circ$ ) does not match the figure for a single year: the figure ending in 44, multiplied by 4, could not result in a number ending in 57. It was a simple exercise for Westwyk to split the difference, squeezing an extra column into the final three rows of the table and writing 15, 30 and 45. He thus made the table appear internally consistent – and gave it a precision of sexagesimal tenths.

Examination of such precise tables can also reveal how carefully they were computed. Here a useful source are the radices of planetary mean longitudes and mean anomalies for *era Christi* (noon, 31 December preceding AD 1), whose values were consistent across manuscripts based on the Parisian Alfonsine Tables (Chabás and Goldstein, 2012, pp. 59-61). These were generally given for the meridian of Toledo, but in the tables Westwyk wrote out for use with his equatorium, they are recomputed for the meridian of London. This was achieved by subtracting  $8;26^\circ$  of longitude ( $0;33,44$ h of time), which was thought to be the difference in longitude between London and Toledo (Price, 1955, pp. 80-82). In Table 2 Westwyk's radices for *era Christi* are shown alongside Toledo values at the same epoch.<sup>20</sup> They should differ by an amount corresponding to the correction for longitude, but this is not always the case: there are small scribal errors in six of the ten radices for the era of Christ.

The quantity of these errors is not unusual for a table of this kind; any theory as to their origin must be speculative. The most likely cause is wavering concentration during the copying of so many seemingly random digits. In some cases, such as the confusion of 12 for 13 in the mean longitude of Caput Draconis, it may be suggested that the source text was misread by the copyist. In a few others, it is just possible that errors arose when the calculation was first carried



Table 2: Radices of mean longitude and mean argument ‘ad eram Christi’, adapted from Toledo values (f. 5r)

	Toledo values (°)	Westwyk’s radix (f.5r) (°)	Value recomputed for meridian of 8;26° (°)
Argument of the 8 <sup>th</sup> sphere	5,59;12;34	5,59;12,33,59,17,15,7,7,40	(as left)
Mean longitude of the Sun, Venus, Mercury	4,38;21,0,30,28	4,38;19,37,23,6,45,12,37,4,39	4,38;19,37,23,5,45,12,38,4,39
Mean longitude of the Moon	2,2;46,50,16,40	2,2;28,19,4,8,34,9,25,53,28	2,2;28,19,4,6,34,19,25,53,28
Mean anomaly of the Moon	3,19;0,14,31,17	3,18;41,52,42,26,30,20,23,4,24	(as left)
Mean longitude of Caput Draconis	1,31;55,52,41	1,31;55,48,12,3,5,8,50,13,37	1,31;55,48,13,3,5,8,50,13,37
Mean longitude of Saturn	1,14;5,20,12	1,14;5,17,22,30,23,29,37,9	(as left)
Mean longitude of Jupiter	3,0;37,20,44	3,0;37,13,43,22,36,52,36,10,3,20	(as left)
Mean longitude of Mars	0,41;25,29,43	0,41;24,45,31,12,48,59,38,20	0,41;24,45,31,12,58,59,38,20
Mean anomaly of Venus	2,9;22,2,36	2,9;21,10,56,45,49,16,40,45,40	2,9;21,10,36,25,49,16,40,45,40
Mean anomaly of Mercury	0,45;23,58,0	0,45;19,32,0,5,9,40,33,34,40	0,45;19,36,0,5,9,40,33,34,40

out, and its results transcribed from an abacus or set of counting stones. It would have been easy, for example, to miscount 7 for 8, as in the mean longitude of the Sun, Venus and Mercury. Whatever the cause of the errors, they need not have been made by John Westwyk: the fact that he is known to have been a careful copyist may make us suppose that he was copying an already faulty pre-existing table.<sup>21</sup>

On the other hand, another table adjusted for the longitude of London reveals how easily copying errors could slip in, even for a copyist as diligent as Westwyk. Figure 4 shows a small table of the radices of the mean apogees of the planets. Because the apogees moved at the same rate owing to precession, each radix was adjusted by the same amount: 8;26°/360° multiplied by the daily motion of 0;0,04,20,41,17,12,26,37°, which is given in a table on the same folio. Because they were adjusted to a greater level of precision than the original Toledo radices, the final seven columns in the table are the same for each planet. Yet in the penultimate column the final two rows show 4 instead of 8, which must have arisen from a lapse in concentration when copying. (An identical table on f. 5v repeats this error.) Such a copying error does not prove that the re-computation was not the work of John Westwyk: he could have miscopied from his own earlier calculations. But it does reveal how easily mistakes could be made. The fact that such a

	auges medie	ad temp xpi	londona	8	8	8	8	8	8		
Sat n	3	43	23	22	3	43	43	34	18	8	41
Jupit	2	33	38	0	3	43	43	34	18	8	41
Mars	1	44	12	13	3	43	43	34	18	8	41
Sol ve	1	11	24	22	3	43	43	34	18	4	41
Mercur	3	10	39	33	3	43	43	34	18	4	41

Fig. 4: Table of mean apogees ‘ad tempus Christi’, adapted from Toledo values. Peterhouse, Cambridge MS 75.I, f. 13v. Reproduced by permission of the Master and Fellows of Peterhouse, Cambridge.

noticeable error was not corrected may suggest that Westwyk made little further use of this table, or that he did not care about the later sexagesimal places when using it.

### Experimentation for learning

Westwyk did not always challenge himself to perform calculations; after all, his equatorium was designed to minimise the need for such tasks. What he did do was try out a range of instrumental and computational techniques for obtaining astronomical answers at different levels of precision. We have already seen that the set of tables he collected for his manuscript included not only precise tables of the linear and oscillating components of precession, but also a table in degrees and minutes, which functioned as a ready-reckoner to adjust the apogees. Westwyk clearly used both. Above the ready-reckoner, we find a *signe-de-renvoi* (the geomantic figure for Fortuna Major); the same sign appears eighteen folios earlier (45r), together with a note in Westwyk's hand instructing the reader to use the ready table of additions. (As Figure 5 shows, the reference to the eighteenth folio following has been emended in a different hand, suggesting that the tables may not have been in their current order when Westwyk wrote the canon.)

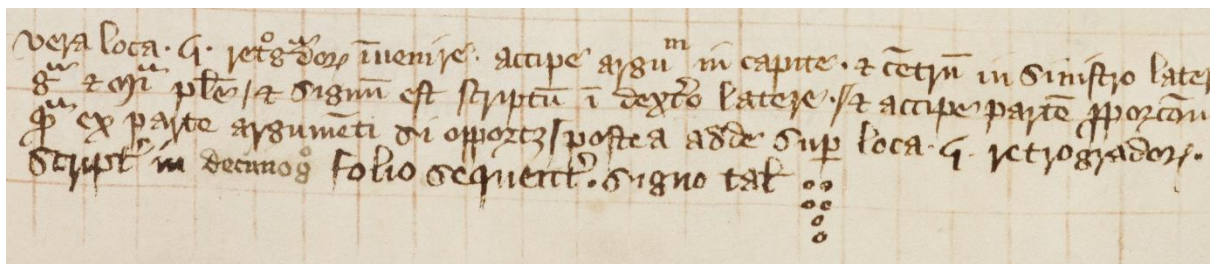


Fig. 5: Part of canon, with *signe-de-renvoi* and corrected folio reference. Peterhouse, Cambridge MS 75.I, f. 45r. Reproduced by permission of the Master and Fellows of Peterhouse, Cambridge.

Westwyk's canon describes the adjustment of the apogees as the final stage in a computation of planetary positions that was a complete alternative to the use of his equatorium. Instead, this technique used the tables on ff. 45r-61r, written in 'Hand S'. Entitled 'Equatio [name of planet]', they are double-argument tables at intervals of  $6^\circ$ , allowing the user to find the true longitude directly from the mean centre (down the left hand side of the table) and the mean anomaly (along the top). The longitude is given in degrees and minutes, with the names of the signs written down the right hand side and demarcated by lines across the table; annotations underneath indicate phases of direct and retrograde motion, stations and conjunctions. The tables are the '1348' tables associated with Oxford (North, 1977, pp. 279-284; North, 1988, p. 188); the only significant difference is that the Oxford tables, following John of Lignières, were given with signs of  $30^\circ$ , whereas Westwyk's tables use signs of  $60^\circ$  for the mean centre and mean anomaly.<sup>22</sup>

Although Westwyk's canon details the procedure for use of these tables, he does not explain how the mean centre, which is not tabulated anywhere in the codex, was to be found. Although this could be calculated simply by subtracting the apogee from the mean longitude, it represents an added step in the process and an inconsistency in the tables: the mean longitudes provided elsewhere in the codex are perfect for use with his equatorium design, as we have seen, but are not ideally suited for the use of these Oxford-style tables of equations. However, once the mean centre was obtained, the tables of equations could be used to give an approximation of the longitude of the planets in a single step. However, that would only be a very rough approximation, since the tables, like the Oxford tables from which they were presumably copied, give mean centres and mean anomalies in  $6^\circ$  increments. Westwyk does not specify how or when he thought interpolation should be used to obtain more precise results: his canon merely advises the reader to 'take the proportional part corresponding to the centre or corresponding to the argument if necessary'; a suitable table of proportions appears on the preceding few folios (38v-44v).<sup>23</sup> We cannot be certain how often Westwyk would have deemed it necessary to use the table of proportions, but its use had the potential to add significant labour to the procedure of computing the longitude. If, as is most likely, both the mean centre and mean anomaly fell between  $6^\circ$  values, the table of proportions would have had to be used for two sets of sexagesimal multiplication; for each, the table would have to be consulted four times and the resulting four figures added together, taking care to ensure that they were kept in the correct sexagesimal column. Including the addition of the final interpolated figure to the rounded value drawn from the table of equations, interpolation could involve up to eight multiplications using the tables of proportion and three additions: a time-consuming and error-prone process. There is evidence in the manuscript that Westwyk attempted, and experienced difficulties with, these interpolations: a note in ciphered Middle English on the first page of the table of proportions (f. 38v) emphasizes that for the planets, the proportions of  $6^\circ$  should be used (the table also permits working with proportions of  $3^\circ$ ). On the same page Westwyk corrected a note made (in Latin) by an earlier user of the 'Hand S' tables, reminding the user that degrees multiplied by degrees yield degrees (rather than minutes as originally stated), minutes multiplied by minutes yield seconds, and so on. A similar reminder is conveyed by the small multiplication tables that Westwyk added to ff. 1v, 5v and 7r. A note on the last of these folios, concerning the adaptation of planetary longitudes to account for the equation of days, refers to the tables of equations, suggesting that Westwyk used the tables he had compiled and those in 'Hand S' together.

It is clear, then, that even if Westwyk originally obtained the 'Hand S' tables to facilitate calculation with his equatorium – and his note 'pro instrumento equatorii' on a calendar of the

daily motion of the Moon's deferent centre (f. 20r) is evidence that he did use them in that way – he also took advantage of the opportunity they provided to compute positions without the equatorium, using interpolation techniques on some occasions to obtain more precise results. The fact that Westwyk used both methods indicates that he was learning, or trying out, different techniques, perhaps at different times or for different purposes. For a very rough approximation the tables could be used more quickly than the instrument, and were more portable; on the other hand, they could perhaps give greater precision, but only via complex and time-consuming calculations. As we have seen, the equatorium provided for a good balance of speed and precision, and although it needed instructions for use, so did the tables of equations, as demonstrated by the canons that Westwyk added to them. And of course they could be used to learn different techniques, or to emphasise different theoretical points.

The diversity of methods and content is most obvious in the manuscript's final set of tables, written in Westwyk's own hand. Few of these tables are closely related to the equatorium, because they are not planetary; some, indeed, are more suited to use with an astrolabe, an instrument with which Westwyk was clearly familiar. They are, however, squarely astrological and are thus related to the planetary tables. Most obvious in this regard is the horoscope of Māshā'allāh that appears on f. 64v (Kennedy, 1959), but the tables of right and oblique ascensions on ff. 65r-70v should also be noted. The latter are based on John Walter's tables of astrological houses (North, 1986, pp. 128-130; North, 1988, p. 191), and this set of tables gives the strong impression of having been compiled from a wide variety of sources that caught Westwyk's eye. Their variety, and discrepancy with tables earlier in the manuscript, is striking. Most obvious is the fact that the majority of this set were explicitly produced for Oxford, in contrast with Westwyk's first set of tables where it is stated that the radices are for London. Yet this discrepancy is not new: it exists even within the first set, where on f. 3v we see that the table of revolutions of years is for latitude 51;34° (suitable for London), while the facing page has a table of ascensions of signs for latitude 51;50°, which was probably Oxford. (St Albans, Westwyk's sometime home, was ascribed a latitude of 51;38°. <sup>24</sup>) But other inconsistencies are new. The table on f. 63v, which gives the differences in half the length of the day between the equinox and solstice for latitudes from 0 to 60°, incorporates an ecliptic obliquity of 23;35°, which contrasts with the figure that appears directly on f. 64r, which is 23;33,30°. <sup>25</sup> Finally, a list of radices on f. 64r, computed for 28 February 1394, at London, incorporates a longitude of 8° east of Toledo. This contrasts with Westwyk's first set of tables which, as we saw, were adapted from Toledo tables by the subtraction of an arc equivalent to 8;26° of time. It is likely, therefore, that rather than updating his own radices by the addition of a year's (or in this case a year and

two months’) motion to previous radices, Westwyk took these radices ready-prepared from another source.

The source of Westwyk’s radices is significant because the (now settled) arguments about Chaucer’s possible authorship of the manuscript pivoted around the ‘Radix Chaucer’ note on f. 5v. That note expresses 1392 years sexagesimally and remarks that it is ‘deffe<sup>a</sup> xpi & R<sup>x</sup>a chaucer’ – the difference between [the era of] Christ and the radix of Chaucer. John North argued that this was Chaucer writing his own name, because no astronomer would cite another for such a simple radix; it was, North stated, ‘a trifling matter for anyone who was capable of calculating with such a set of tables as we have here, to produce fresh radices for each year’s end’ (North, 1988, p. 173). But the evidence we have already seen suggests that, for an amateur astronomer such as John Westwyk, that was not the case. Westwyk was not as capable as North supposed; on the other hand, he was keen to draw on a wide range of material, and to cite his sources. A small table of planetary positions for the end of 1393 (f. 63v) is headed ‘J. Somer, oxonia’, undoubtedly the same John Somer whose calendar inspired Chaucer (Chaucer, 1988, p. 663; Mooney, 1998; O’Boyle, 2005). On the facing page (f. 64r) is the comment, above a table of declinations, that ‘istae sunt declinationes arsachelis ut estimo // verum est quod R.B.’<sup>26</sup> Arzachel (or al-Zarqālī) was and is well known as a leading contributor to the Toledan Tables; R.B. may refer to Roger Bacon, who was known to have drawn up tables, and is cited in identical terms in other scientific manuscripts of this period (Bacon, 1897, pp. 208-210; Millás Vallicrosa, 1943; Voigts, 1990).<sup>27</sup> Finally, on the penultimate page of the table of ascensions, itself the penultimate table of the codex, a note appears referring to the Jewish astronomer Jacob ben Makhir Ibn Tibbon (d. 1304), known to Westwyk as Profatius (f. 70r). The note (shown in Figure 6) gives the maximum and minimum values for the equation of days, which is related to the modern equation of time (North, 1986, p. 128-29). The maximum equation is stated to be when the Sun is at Scorpio 8-9°, and it cannot be coincidental that the note appears beneath the section of the table for an ascendant in Scorpio, where the maximum value is indeed at 8-9°. However, the two maximal values for the equation are different: the note says 7;57°, while the table gives 7;54°.<sup>28</sup> Judging by its appearance before a wedge paragraphus and to the left of an

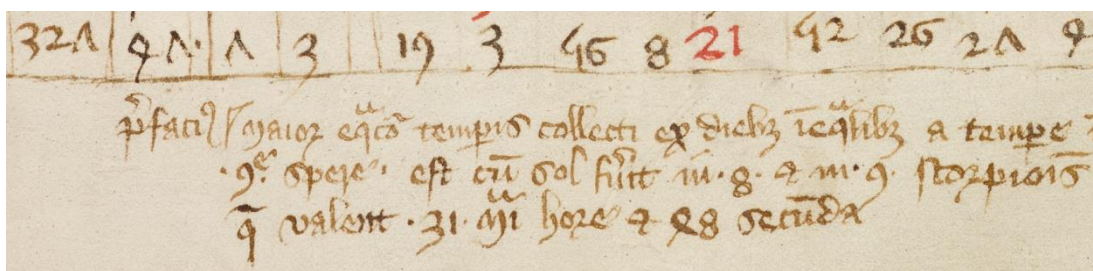


Fig. 6: Note on referring to Profatius. Peterhouse, Cambridge MS 75.I, f. 70r. Reproduced by permission of the Master and Fellows of Peterhouse, Cambridge.

otherwise aligned body of text, the reference to Profatius as an authority appears to have been added later. It is possible that Westwyk computed his own value and called upon Profatius as an authority, but this seems unlikely to have been within his astronomical capabilities.<sup>29</sup> More likely is that, as a diligent student and copyist, he had spotted a discrepancy in two sources he was using. He maintained the value he found in John Walter's tables, but noted that 'Profatius' had used a different value. The tables commonly misattributed to Profatius are now known to be by Peter of St. Omer, but Westwyk was not alone in this confusion, which arises in several British copies of Peter's *Tractatus de semissis* (Pedersen, 1983-84, p. 43.)

Westwyk's diligence as a copyist and table-maker is demonstrated by a curious duplication that occurs on f. 62v. This is the first table of Westwyk's latter set, and the only one of this group that could be used – albeit indirectly – with the equatorium: it is a division table allowing the user to interpolate hourly values for the longitude of the Moon. This table, which was useful for the prediction of eclipses, has no equivalent in Westwyk's first set of tables, and it seems that he may have chosen to add it later. A note in Latin instructs the user to first calculate the daily motion of the Moon from two successive noon positions 'in almenac'. The user then looks for this 24-hour difference (in degrees and minutes) on the far right of the table, and can then interpolate the motion in 1-12 hours within the table. Although it is unusual to find a table whose entry is on the right, its content is straightforward; however, what is strange is that the table appears to be missing every fourth row.<sup>30</sup> Such regular omissions are unlikely to be inadvertent; nonetheless it was probably dissatisfaction with those gaps that motivated Westwyk to redraw the table immediately beneath, identical but for the insertion of the missing rows, and a slightly different range.<sup>31</sup>

It is clear from this table, as well as from the radix Westwyk added to the 'Hand S' calendar of the double elongation of the Moon (f. 30v), that he was particularly interested in lunar positions and eclipses. We should not be surprised, therefore, that his equatorium included a tool to compute the latitude of the Moon. He explained this tool in staggering detail, covering three-and-a-half pages of the manuscript, with emphatic repetitions and three worked examples (ff. 77r-78v, for 17 December and 19 and 23 February 1391). The level of worked detail indicates that Westwyk lacked confidence with these techniques, and this is supported by some errors in his explanation. He states that Caput and Cauda Draconis are each confined to one half of the zodiac, when in fact they both rotate through the zodiac, always opposite each other. There is also a mistake in the last of his three worked examples: he gives the latitude as 1;22° N, when it was in fact southerly. This is an understandable error caused, perhaps, by the fact that northerly and southerly latitudes were read on the same  $\pm 0-5^\circ$  scale on the equatorium.<sup>32</sup>

Overall, it is hard to escape the conclusion that Westwyk was himself learning these methods as he was carefully teaching them to his reader, perhaps inspired by Seneca’s dictum ‘homines dum docent discunt’.<sup>33</sup>

The sense of learning through experimentation, at the same time as providing instruction for future readers, is perhaps most apparent in the comments in cipher that appear in five places within the later sets of tables (ff. 14r, 30v, 38v, 62v, 63v; Price, 1955, pp. 182-187). Not only is the fairly basic substitution cipher itself evidence of experimentation with different techniques and ideas; the contents of the ciphered passages suggest incipient understanding of the tables being copied and commented on. For example, the ciphered comment on the table of half-day lengths on f. 63v (Figure 7) reads ‘this is how mochel the half ark of the lengest dai is more than six houris’, which is a straightforward description of a fairly simple table.<sup>34</sup> We thus have a glimpse of Westwyk’s enjoyment of the parallel process of learning the use of the tables and of cipher. In cipher, in Latin and in plain English he makes notes on what he sees and copies, cites authorities whose achievements he respects, and comments on the results of his computations.

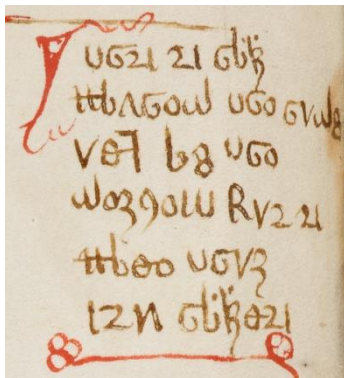


Fig. 7: Ciphered Middle English text. Peterhouse, Cambridge MS 75.I, f. 63v. Reproduced by permission of the Master and Fellows of Peterhouse, Cambridge.

## Conclusion

Peterhouse MS 75.I is not an astronomer’s rough workbook. Although his equatorium treatise is a draft, and some of his calculations contain errors, John Westwyk clearly took pride in his compilation. His diagrams are carefully drawn, and the radices he compiled (apparently at the same time) for 1392 and 1400 (f. 6v) demonstrate his intention to continue using the tables for years into the future. He surely realized that he still had techniques to learn; the absence of his annotations on some of the more complex ‘Hand S’ tables, such as the double-argument tables of latitudes, suggest the limitations of his abilities or interests. And although he perhaps lacked the sophistication to realize that great precision did not equate to ‘the trowthe of conclusiouns’, and his treatise contains some errors, he was far from incompetent, capable of explaining the construction and use of an equatorium in clear prose.

More significant than his mistakes is his malleability. He was willing to try different forms of presentation such as signs of 60° and 30°, and years ending on 31 December and 28 February; different layouts such as tables of numbered days and calendars grouped by months; even entirely different calculation techniques, using tables as well as the equatorium he had designed or adapted. Such variety may have been forced on him by the sources available for his compilation, but he was quite willing to use, and perhaps learn from, them. His suggestibility extends to language, where he adopts Latin and Arabic terms from his source texts and incorporates them into his own Middle English (whether plain or ciphered). Sometimes this was for lack of an existing term in the vernacular; but in other cases, such as his use of ‘retrogradorum’ when he could easily have written ‘planetis’ (f. 38v), we again have the sense of a keen learner trying out new ideas and techniques as he computes, compiles and composes a new treatise.<sup>35</sup> This willingness to experiment was not, perhaps, common to university scholars in this period. Rather, it is the hallmark of the amateur: a monk producing an idiosyncratic compilation, perhaps for use in his community.<sup>36</sup>

If the mistakes and inconsistencies that do occur in Westwyk’s work diminish the astronomical value of the treatise and tables very slightly, they enhance its historical value hugely. Such imperfections, as is often the case, tell us far more than a faultless document or object would do. In the first place, they remind us of the their author’s humanity and individuality. We still know little about John Westwyk: his life after he returned from crusade against Flanders in 1383, why he produced tables computed for London, and for whom he was writing. Such questions cannot be answered solely with reference to the astronomical content of Peterhouse MS 75.I. But an analysis of this manuscript has told us much about his abilities, interests and the methods through which he learned the science of astronomy. More broadly, mathematical analysis of Westwyk’s tables has revealed important details of the processes of transmission, compilation and computation that went into this manuscript and others like it. Westwyk was an individual monk, but one learning the tools and techniques of a mathematical astronomy that extended across medieval Christendom, and beyond. And we too can learn through these tables, as such computational case studies offer new insights into the practices of the non-elites who learned, developed and communicated the ideas and instruments of medieval astronomy.

---

<sup>1</sup> ‘The fifth part will be an introduction according to the rules of our experts, in which you may learn a great part of the general rules of astrological models. In this fifth part you will find tables of equations of houses for the latitude of Oxford, and tables of planetary dignities, and other useful things.’

<sup>2</sup> The explicit and potential audiences of the *Astrolabe* have been discussed by many scholars: see, for example, Laird (2007); Mead (2006).



---

<sup>3</sup> See particularly the ‘General Introduction’ and contribution by Alain Bernard (2014).

<sup>4</sup> The only other examination of the manuscript from a technical perspective is that of Emmanuel Poulle (1980, pp. 161-165), but this is limited to an analysis of the equatorium design.

<sup>5</sup> This dual approach is influenced by Soler et al. (2014). See especially the contribution by Karine Chemla.

<sup>6</sup> The manuscript has been fully digitised and is freely accessible at <http://cudl.lib.cam.ac.uk/view/MS-PETERHOUSE-00075-00001>.

<sup>7</sup> Medieval planetary theory (based on models set out in Ptolemy’s *Almagest*) mapped the motion of the planets in the plane of the ecliptic. Mean motions in this theory are the mean anomaly (also known as mean argument), which gives the position of the planet on its epicycle; and the mean longitude (or mean motus), which gives the position of the epicycle centre on its path around the deferent circle, measured from the vernal point (head of Aries). Tables supplied daily and annual increments of the mean motions, which were added to the radix (the value at some epoch, such as the Incarnation of Christ) to give the mean motions at the desired date. See Chabás and Goldstein (2012).

<sup>8</sup> ‘the smallest fractions will not be shown [as well] on such a small instrument as in ingenious tables calculated for a cause’.

<sup>9</sup> ‘The larger you make this instrument, the larger your main divisions will be. The larger those divisions, the smaller the fractions into which they can be divided; and the smaller these fractions, the nearer the truth of your calculations.’

<sup>10</sup> On the importance of convenience to table-makers, see Chabás and Goldstein (2013).

<sup>11</sup> On the forms and contents of the Parisian Alfonsine Tables, see Chabás and Goldstein (2012), pp. 53-61.

<sup>12</sup> Price (1955), p. 75, designated the two main hands of the manuscript as Hand C and Hand S. Subsequent scholars have followed this usage, but Hand C has now been identified as that of John Westwyk. ‘Hand A’, a hand roughly contemporary with Hand S, added canons on two folios; Westwyk subsequently annotated one of those. Rand Schmidt (1993), pp. 111-112.

<sup>13</sup> The fact that f. 63r is blank may suggest that the table on f. 62v is incomplete: it does not contain values for 13-24 hours. However, the canon on f. 62v explains how to use the table as it stands to find the motion in 13-24 hours.

<sup>14</sup> The *argumentum medium* tabulated in the Parisian Alfonsine Tables corresponds to arcs of small circles which, in the theory of accession and recession attributed to Thābit ibn Qurra and incorporated into the Toledan Tables, carried the Aries and Libra points of the eighth sphere back and forth, causing the oscillating precession. See Dobrzycki, 1965; Chabás and Goldstein (2012), pp. 43-52.

<sup>15</sup> ‘the line that goes from centre *aryn* [the centre of the equatorium] to the head of Capricorn, which line is called the Midnight Line in the *Treatise of the Astrolabe* ... the nine divisions of the midnight line just mentioned will serve for the equation of the eighth sphere.’

<sup>16</sup> The combination of linear and oscillating precession led to a total correction of 1° in about 65 years; the oscillating term accounted for around half of this. See Price (1955), pp. 104-107.

<sup>17</sup> The Moon’s latitude ( $\beta$ ) can be computed from the Moon’s distance in longitude (L) from its node, where its orbit crosses the ecliptic, by the relationship  $\beta = 5\sin L$  (North (1988), pp. 165-168), which was easily modelled on the face of the equatorium. Westwyk described a tool to perform this function, on the upper half of the instrument, in great detail (ff. 77r-78v). The relationship between the equation of accession and recession of the eighth sphere ( $\psi$ ) and the mean argument of the eighth sphere ( $\theta$ ) was of the form  $\sin\psi = \sin 9.\sin\theta$  (Chabás and Goldstein (2012), p. 51). Nevertheless, an astronomer working to a precision of minutes could use the approximation  $\psi = 9\sin\theta$  to satisfactory effect.

<sup>18</sup> Westwyk made additions to the Hand A notes on ff. 31v and 38v.

<sup>19</sup> The word ‘amateur’, although sometimes used by historians of medieval science (quoted, for example, in the introduction to this article), is problematic. Here I am using it to denote lesser expertise, along with the freedom to pursue personal interests and satisfaction. For a discussion of the difficulties of defining professional and amateur status, see Berman (1975). Berman’s definition focuses on the early nineteenth century, when these categories became contentious.

<sup>20</sup> These are taken from the first printed edition of the Alfonsine Tables (Alfonso, 1483). It is highly likely that these very common Toledo values were used, but even if not, it would not make any difference to most of the results, as most columns in the table were evidently subtracted from zero (because the correction was carried out to a greater number of sexagesimal places than the initial Toledo radix).

<sup>21</sup> Westwyk’s copy of the tables in Richard of Wallingford’s *Tractatus albionis* (Bodleian Library MS Laud Misc. 657, ff. 32r-45r, is remarkably free from errors.

<sup>22</sup> An example of the more usual 30° presentation is in Cambridge University Library MS Ii.1.27, ff. 23r-33v. This manuscript (dated 1424) also contains canons ascribed to Lignières.

<sup>23</sup> ‘accipe partem proporcionalem tam ex parte centri quam ex parte argumenti si oportet.’ The table allows the user to multiply two numbers from 1 to 60, with the results given as a proportion of 6°. For example, 5 x 5 gives the result 4,10.

<sup>24</sup> Bodleian Library MS Laud Misc. 674, f. 74r

<sup>25</sup> North (1986), pp. 128-30, discusses the table of differences in half-day length.

<sup>26</sup> ‘These are the declinations of Arzachel, I believe. Correct, according to R.B.’

---

<sup>27</sup> The tables that survive with the *Opus Majus* (Bacon, 1897, pp. 208-210) are for calculating the date of Easter; however, Bacon refers to other tables which are not extant.

<sup>28</sup> A maximal value of 7;57 does indeed appear in work commonly attributed by medieval astronomers to Profatius (in fact it is by Peter of St Omer (fl. 1289-1308)); see Pedersen (1983-84), pp. 42-43, 702; Pedersen (2002), pp. 984-985. John of Lignières also uses 7;57. 7;54 is the figure used by al-Battani, the Toledan Tables and Parisian Alfonsine Tables. See Chabás and Goldstein (2012), p. 40.

<sup>29</sup> In order to check his result, we would need to know what value he was using for the solar eccentricity, which is not certain; the value incorporated into the equatorium design was 1/30<sup>th</sup> of the solar radius, but this was quite different from the value common to authorities in this period. See Chabás and Goldstein (2012), p. 66.

<sup>30</sup> The leftmost column gives 24 hours' motion in minutes; since the maximum value given is 1080', there may be some relation with the common division of one hour into 1080 points (*helaqim*). *Helaqim* are used in some tables of Jewish origin, and this, as well as the fact that the table is entered on the right, may suggest a Hebrew source. See Chabás and Goldstein (2012), p. 141.

<sup>31</sup> The rows are at intervals of 24'. The range of the first table is 10;0-17;12°/day; the second is 11;12-18;0. Both ranges exceed anything possible according to Ptolemaic lunar theory. Values for maximum and minimum daily lunar motion varied, but on the equatorium the range of achievable values was certainly no greater than 11;36-14;48°/day, so in that sense either table would have been quite sufficient. See Goldstein (1992).

<sup>32</sup> In Westwyk's defence, we may note that errors which Price claimed to have identified in his explanation were, in fact, correct; Price mixed up figures for the retrograde motion of Caput Draconis and the resulting position, which was obtained by subtracting the motion from 360°. We may therefore reasonably conclude that no scholar is immune to such errors, and we should not judge Westwyk's performance too harshly.

It is also worth noting that Westwyk had almost certainly not computed these exceptionally accurate (barring his one cardinal error) positions on the equatorium, but rather taken them from tables for use in his worked examples. See arguments in Price (1955), pp. 72-3, and North (1988), pp. 168-9.

<sup>33</sup> This is an impression sustained throughout the treatise, which is a model of pedagogical writing while still containing significant theoretical errors. Seneca (1917), VII:8. Seneca's writings were very popular in this period, and his influence on Chaucer has been noted (Wilson (1993)).

<sup>34</sup> "This is the amount by which the half-arc [half the daylight hours] of the longest day is more than six hours."

<sup>35</sup> On this use of 'retrogradorum', which may be influenced by John Somer, see North (1988), p. 188.

<sup>36</sup> Westwyk's use of the vernacular for the *Equatorie* treatise may well have been an act of devotionally inspired charity. See Getz (1990).

## REFERENCES

- Alfonso X. (1483) *Alfontii regis Castellae illustrissimi caelestium motuum tabulae* (Venice: Erhard Ratdolt).
- Bacon, R. (1897). *The Opus Majus of Roger Bacon*. [Ed. Bridges, J. H.]. (Oxford: Clarendon Press).
- Berman, M. (1975) "Hegemony" and the Amateur Tradition in British Science, *Journal of Social History*, 8 (2), 30–50.
- Bernard, A. (2014) In What Sense Did Theon's Commentary on the Almagest Have a Didactic Purpose?, in: A. Bernard and C. Proust (eds.) *Scientific Sources and Teaching Contexts throughout History: Problems and Perspectives* (Dordrecht: Springer), pp. 95-121.
- Bernard, A. and Proust, C. (eds.) (2014) *Scientific Sources and Teaching Contexts throughout History: Problems and Perspectives* (Dordrecht: Springer).
- Chabás, J. and Goldstein, B. R. (2012) *A Survey of European Astronomical Tables in the Late Middle Ages* (Leiden: Brill).
- Chabás, J. and Goldstein, B. R. (2013) Computing Planetary Positions: User-Friendliness and the Alfonsine Corpus, *Journal for the History of Astronomy*, 44, 257–276.
- Chaucer, G. (1988) *The Riverside Chaucer*. [Ed. Benson, L. D.] (Oxford: Oxford University Press)

- 
- Dobrzycki, J. (1965) The Theory of Precession in Medieval Astronomy, translated in: *Selected Papers on Medieval and Renaissance Astronomy, Studia Copernicana* 43 (Warsaw: Instytut Historii Nauki, 2010), pp. 15–60.
- Getz, F. M. (1990) Charity, Translation, and the Language of Medical Learning in Medieval England, *Bulletin of the History of Medicine*, 64, 1-17.
- Goldstein, B. R. (1992) Lunar Velocity in the Ptolemaic Tradition, in: P. M. Harman and A. E. Shapiro (eds.) *The Investigation of Difficult Things: Essays on Newton and the History of the Exact Sciences in Honour of D.T. Whiteside* (Cambridge: Cambridge University Press), pp. 3–17.
- Kennedy, E. S. (1959) A Horoscope of Messehalla in the Chaucer Equatorium Manuscript, *Speculum*, 34, 629–630.
- Laird, E. (2007) Chaucer and Friends: The Audience for the “Treatise on the Astrolabe”, *The Chaucer Review*, 41, 439-444.
- Mead, J. (2006) Geoffrey Chaucer’s Treatise on the Astrolabe, *Literature Compass*, 3, 973-991.
- Millás Vallicrosa, J. M. (1943) *Estudios Sobre Azarquiel* (Madrid: Consejo Superior de Investigaciones Científicas).
- Mooney, L. R. (1998) *The Kalendarium of John Somer* (Athens: University of Georgia Press).
- North, J. D. (1977) The Alfonsine Tables in England, in Y. Maeyama and W. G. Saltzer (eds.) *Prismata: Naturwissenschaftsgeschichtliche Studien: Festschrift für Willy Hartner* (Wiesbaden: Steiner), pp. 269–301.
- North, J. D. (1986) *Horoscopes and History* (London: Warburg Institute).
- North, J. D. (1988) *Chaucer’s Universe* (Oxford: Clarendon).
- O’Boyle, C. (2005) Astrology and Medicine in Later Medieval England: The Calendars of John Somer and Nicholas of Lynn, *Sudhoffs Archiv*, 89, 1–22.
- Pedersen, F. S. (ed.) (1983-84) *Petri Philomenae de Dacia et Petri de S. Audomaro opera quadrivialia* (Copenhagen: Danish Society of Language and Literature).
- Pedersen, F. S. (2002) *The Toledan Tables: A review of the manuscripts and the textual versions with an edition* (Copenhagen: Royal Danish Academy of Sciences and Letters).
- Poulle, E. (1980) *Les instruments de la théorie des planètes selon Ptolémée : équatoires et horlogerie planétaire du XIIIe au XVIe siècle* (Genève: Droz).
- Price, D. J. (1955) *The Equatorie of the Planetis* (Cambridge: Cambridge University Press).
- Rand Schmidt, K. A. (1993) *The Authorship of the Equatorie of the Planetis* (Cambridge: D. S. Brewer)
- Rand, K. A. (2015) The Authorship of The Equatorie of the Planetis Revisited, *Studia Neophilologica*, 87, 15-35.
- Seneca. (1917). *Epistles, Volume I: Epistles 1-65* [Trans. Gummere, R. M.] (Cambridge: Harvard University Press).
- Soler, L. et al. (eds.) (2014) *Science After the Practice Turn in the Philosophy, History, and Social Studies of Science* (New York: Routledge).
- Voigts, L. E. (1990) The “Sloane Group”: Related Scientific and Medical Manuscripts from the Fifteenth Century in the Sloane Collection, *British Library Journal*, 16, 26–57.
- Wilson, G. G. (1993) “Amonges Othere Wordes Wyse”: The Medieval Seneca and the “Canterbury Tales”, *The Chaucer Review*, 28, 135–45.

# Template Tables and Computational Practices in Early Modern Chinese Calendric Astronomy

Liang Li

Abstract: This article introduces a writing format, the “template table” (*suanshi*, 算式) that was designed to guide the process of calendrical astronomical calculations in early modern China. In conjunction with another kind of text, known as “detailed procedures” (*xicao*, 細草), users could perform calculations easily by operating the “template table” and extracting data from given numerical tables. This method, that not only normalized the use of numerical tables but also linked instructions with the corresponding tables in computational practices, became widespread from the Ming period (1368-1644) onwards. Wanting to acquire this computational regimen, the Joseon court of Korea (1392-1897) even sent skilled officers to China to learn it secretly. The circulation of the template method beyond China suggests its significance. The article also discusses the advantages and disadvantages of using this method.

Keywords: template table, detailed procedures, calendrical systems, astronomical tables, transmission of astronomy from China to Korea

## 1. Introduction

Many resources related to ancient astral sciences survive in China. For mathematical astronomy, there were theoretical texts and tables to make computations. But theoretical texts and tables alone do not suffice; skills to manipulate tables and procedures to carry out mathematical practices are also needed.<sup>1</sup> The official astronomers were very creative in inventing template table to standardize computational practices. By using such writing formats, users needed only to add and subtract, multiply and divide. Handbooks provided detailed instructions for the templates, outlining the procedures step-by-step. When more than elementary operations were necessary, the handbooks provided the calculators with tables in which they merely looked up a quantity as the instructions demanded. As long as they followed instructions, memorized the names of constants and variables and recorded these data in the template table, users had no need to suffer from the complicated underlying astronomical theories. These algorithmic practices were used in many astronomical systems (traditional Chinese, Islamic and European systems used in China); even if these traditions employed different “theories,” they shared some similar mathematical practices. In this paper I want to show how template tables were used as a tool in early modern Chinese astral sciences and to reveal the mathematical

---

<sup>1</sup> For example, the management of large sets of tables, management of long and complex algorithms, and decisions about the precision of the computation and rounding, etc..

practices behind them.

As the complexity of astronomical computation practices increased over time, users became more inclined to rely on tables.<sup>2</sup> Ming scholar, Lang Ying (郎瑛, 1487-1566), once wrote that calculations carried out using tables did not require as high professional skills as earlier: “If one is provided with (tables) to carry out (astronomical) computations, even ordinary people can do them” 若夫授以成算，則中人可為。<sup>3</sup>

In order to avoid computational errors that might cause serious political disasters, the official astronomers in the Astronomical Bureau (*Qintian jian* 欽天監)<sup>4</sup> were inclined to rely on established procedures and template tables. Zhou Xiang 周相 (fl. 1560s), the director of Astronomical Bureau, once pointed out that the canon of the *Season granting calendrical system* (*Shoushi li* 授時曆, used between 1280 and 1385),<sup>5</sup> the most sophisticated Chinese calendrical system, was thought to be abstruse and difficult to understand 玄奧而難明 and even the official astronomer suffered from the calculations 曆官難於考步。<sup>6</sup> To address this problem, Yuan Tong (元統, fl. 1380s), director of Astronomical Bureau in the early Ming, was assigned to design a handbook to make the calculations easier. He wrote the *General Rules for the Great Concordance Calendrical System* 大統曆法通軌 (henceforth *General Rules*) and ordered the Bureau staff to follow this method. The Great concordance calendrical system was mainly based on the Season granting calendrical system, even though there were a few differences between them, especially in the part devoted to tables. The former only relied on handy tables and explained how to use them. The use of tables could simplify calculations, especially when eliminating complicated calculations such as a lot of exponentiation calculations. Users only needed to know how to carry out addition, subtraction, multiplication, and division to follow the

---

<sup>2</sup> The calculations required in ancient Chinese calendrical systems could use two methods: procedures described in texts or specific types of tables named *licheng* (立成, pick-up table or handy table). It seems that there is a correlation between the use of quadratic interpolation and the making of tables of a *licheng* type. With the term *licheng* one is probably closer to a pick-up table expressing a relationship, avoiding computations except for linear interpolation.

<sup>3</sup> *Qixiu leigao* 七修類稿, Vol.5.

<sup>4</sup> The Astronomical Bureau was a middle-ranking institution serving the emperor. Its main tasks included calculation of the calendar, forecasting solar and lunar eclipses and observation of abnormal astronomical phenomena. Normally, the Astronomical Bureau had several different departments including the department of Calendar (*li ke* 曆科), the department of Heavenly Signs (*tianwen ke* 天文科) and the department of the Water Clock (*louke ke* 漏刻科). The Calendar department got its name from its duty of making the state calendar and was mainly focused on calendrical calculations. The department of Heavenly Signs was in charge of observing celestial phenomena and presented their astrological interpretations to the emperor. Time-keeping was the simplest service that the Water Clock department provided to the court. For more information on the Astronomical Bureau in China see Chang, Ping-Ying (2015) and Deane, Thatcher Elliott (1989).

<sup>5</sup> Most astronomical treatises on Chinese calendrical systems, in their surviving forms, contain a canon. Canons, sets of instructions for minimally skilled users, neither explain nor justify their procedures. Once a user had worked his way through the canon, he could carry on various calculations such as the prediction of the winter solstice, solar and lunar eclipses and planetary phenomena (Sivin, 2009, p.40).

<sup>6</sup> *Daming datong lifa* 大明大統曆法 (Great concordance calendrical system of Great Ming), 4b.

General Rules.<sup>7</sup> By contrast, the canon of Season granting calendrical system presented both procedures and methods based on tables, which were introduced by the expression “*youshu* (another procedure, 又術)”.

Handbooks in the type of *General Rules* (*Tonggui*, 通軌), generally named *Detailed Procedures* (*Xicao*, 細草) in the Qing dynasty (1644-1911), were compiled to guide users to operate templates and numerical tables normatively. In the early 17<sup>th</sup> century, the Jesuits sought to improve the Chinese calendrical system by referring to western astronomy. When western astronomical system was adopted in China, the method of using detailed procedures and template tables were also borrowed by Jesuits serving in the Astronomical Bureau. Hence, Chinese computational practices, in the 17<sup>th</sup> century, were combined with European astronomical theory.

During the late Koryo dynasty (1392-1394) of Korea, the Season granting calendrical system was introduced to Korea. However, it appears as if the Korean astronomers at first could not wholly take advantage of the new knowledge. In order to develop calendrical astronomy in his kingdom, King Sejong (r.1418-1450) imported the new Chinese computational techniques and ordered his astronomers to carry out systematic research on the Great concordance calendrical system.<sup>8</sup> Meanwhile, some competent astronomers were sent to China; they brought back many astronomical works, including the *General Rules*, and reprinted these works in Korea.

Soon after the Qing court promulgated the *Xiyang xinfǎ lishu* (西洋新法曆書, *Books on Calendrical Astronomy According to the New Western Method*)<sup>9</sup> in 1644 as its official calendrical system, the Joseon (1392-1910) court launched a project to master the methods of the new western astronomy. For over a century, a series of official missions were dispatched to Beijing to acquire astronomical texts and instruments as well as to learn calculation methods.<sup>10</sup> During the Ming and Qing period, the *General Rules* and *Detailed Procedures* thus played an important role in the transmission of astronomy from China to Korea.

## 2. The “template table” in the Qing period (1644-1911) and its origin in the Ming period (1368-1644)

In the Bibliothèque nationale de France (National Library of France), we can find a printed Chinese sheet entitled *suanshi* (算式, calculation form, shelf mark Chinois 5015)<sup>11</sup>, containing two sets of tables for the calculation of solar and lunar eclipses

---

<sup>7</sup> Li Liang (2011), p.12.

<sup>8</sup> Shi Yunli (2003).

<sup>9</sup> *Xiyang xinfǎ lishu* is the revised version of *Chongzhen Lishu* 崇禎曆書 (*Books on calendrical astronomy of the Chongzhen reign*) which was finished between 1629 and 1636. *Chongzhen Lishu* is the main achievement of the astronomical reform under the leadership of Xu Guangqi 徐光啓 (1562-1633). This set of books introduce a system of computational astronomy based on Tycho's system of planetary motions and many new achievements in astronomy since Copernicus.

<sup>10</sup> Lim Jongtae (2012).

<sup>11</sup> The *suanshi*, sometimes named *chengshi* 程式 (procedure form), is a technical term which

according to the newly introduced western astronomical methods. These template tables are printed in red ink, which means that their users were official astronomers in the Chinese Astronomical Bureau. The content and layout of the tables show that they were specifically designed to guide the process of calculation and to record the results of each step as the user filled the table with data extracted from appropriate astronomical tables. The template table of solar eclipse, for example, reads: “Conjunction day in [blank] year [blank] agricultural year [blank] month under the Kangxi reign, the magnitude with the directions of first and last contact of the solar eclipse”康熙\_\_年\_\_歲\_\_月初一日朔,日食分秒及起復方位 (Fig. 1).

		日宮時度	分	秒			
首朔根					康熙 年 歲 月初一日 朔日食分秒時刻并起復方位	日食算式	
紀日							
朔策							
平朔							
太陽引根							一求諸平行
朔策							
太陽平引							
太陰引根							
朔策							
太陰平引							
交周度根							
朔策							
交周平行							
太陽經度根							
朔策							
太陽經平行							

refers to a type of template tables for calendrical calculations.

	Day/ Sign	Hour/ Degree	Minute	Second			
Radix time for the first mean conjunction after the winter solstice ( <i>shoushuo gen</i> )					[Step] one: Calculate different mean motions	Conjunction day in [blank] year [blank] agricultural year [blank] month under the Kangxi reign, the magnitude with the directions of first and last contact of the solar eclipse	Template table for solar eclipse
Date counting ( <i>jiri</i> )							
Lunation factor I ( <i>shuoce</i> I)							
Mean conjunction time [at selected conjunction] ( <i>pingshuo</i> )							
Radix position of the solar epicycle at the first mean conjunction ( <i>taiyang yingen</i> )							
Lunation factor II ( <i>shuoce</i> II)							
Mean position of solar epicycle Mean position of the solar epicycle from perigee [at selected conjunction] ( <i>taiyang pingyin</i> )							
Radix position of the lunar epicycle at the first mean conjunction ( <i>taiyin yingen</i> )							
Lunation factor III ( <i>shuoce</i> III)							
Mean position of the lunar epicycle from apogee [at selected conjunction] ( <i>taiyin pingyin</i> )							
Radix position of the ascending node at the first mean conjunction ( <i>jiaozhou dugen</i> )							
Lunation factor IV ( <i>shuoce</i> IV)							
Mean position of the ascending node [at selected conjunction] ( <i>jiaozhou pingxing</i> )							
Radix position of the sun at the first mean conjunction ( <i>taiyang jingdugen</i> )							
Lunation factor V ( <i>shuoce</i> V)							
Mean position of the sun [at selected conjunction] ( <i>taiyang jingpingxing</i> )							

Fig. 1 The first page of the template table, BnF Chinois 5015 (above), and my translation (below).<sup>12</sup>

This template table contains sixteen steps, each with several sub-steps. This hierarchy of step and sub-steps in the template table gives a structure of a long computation in line with the instructions of procedure. This structure might help users to understand the process and to search for mistakes. The layout shows that the left part of the table presents the title of each sub-step, the top lists the units (day/sign 日/宮, hour/degree 时/度, minute 分 and second 秒), and the right part gives many blank cells named “ge” 格 to fill with data.

Let us take the first step, “calculate different mean motions” 求諸平行, as an example. This step requires the user to calculate five different mean motions using astronomical tables, including the mean conjunction time (*pingshuo*, 平朔), mean position of the solar epicycle from perigee (*taiyang pingyin*, 太陽平引), mean

<sup>12</sup> In order to distinguish various *shuoce* 朔策 (lunation factor), we use lunation factor I to lunation factor V in the translation. This figure illustrates step one of the template table.



position of the lunar epicycle from apogee (*taiyin pingyin*, 太陰平引), mean position of the ascending node (*jiaozhou pingxing*, 交周平行), mean position of the sun (*taiyang jingpingxing*, 太陽經平行). To finish these calculations, the user should refer to a numerical table named “Table of five motions lasting two hundred years after the calendrical epoch” (曆元後二百恆年五行表, *liyuanhou erbai hengnian wuxing biao*)<sup>13</sup>. This table (Fig. 2) has seven parts divided with bold lines; the top row is the argument “year counting” (*jinian*, 紀年)<sup>14</sup> and the following six parts are different “radix” (*gen*, 根) for different mean motions—radix time for the first mean conjunction after the winter solstice (*shoushuo gen*, 首朔根), radix position of the solar epicycle at the first mean conjunction (*taiyang yingen*, 太陽引根)<sup>15</sup>, radix position of the lunar epicycle at the first mean conjunction (*taiyin yingen*, 太陰引根), radix position of the ascending node at the first mean conjunction (*jiaozhou dugen*, 交周度根), radix position of the sun at the first mean conjunction (*taiyang jingdugen*, 太陽經度根), “lodges” (*xiu*, 宿)<sup>16</sup> and “date counting” (*jiri*, 紀日).

寅	戌	丑	丁	子	丙	亥	乙	戌	甲	酉	癸	申	壬	年	紀
一	二	三	〇	三	一	四	二	五	〇	七	一	七	二	日	首朔
四	〇	六	〇	一	二	三	一	五	一	六	〇	一	二	時	朔
六	一	三	四	五	五	六	〇	三	三	五	四	六	五	分	
四	二	三	四	五	〇	七	二	六	四	八	〇	〇	三	秒	
〇	〇	一	一	〇	〇	〇	〇	一	一	〇	〇	〇	〇	宮	太
四	一	六	二	七	〇	八	一	九	二	〇	一	一	二	度	陽
五	五	三	三	七	一	一	〇	八	三	二	二	六	〇	分	引
八	四	九	一	一	一	三	〇	四	三	六	二	八	一	秒	數
二	〇	三	〇	四	〇	六	〇	七	〇	九	〇	〇	一	宮	太
四	一	九	〇	九	二	九	一	三	一	四	〇	四	二	度	陰
九	四	二	一	四	二	六	三	九	五	一	一	三	二	分	引
四	四	三	四	二	四	一	四	〇	四	九	三	八	三	秒	數
二	〇	一	〇	一	〇	一	〇	一	一	一	一	一	一	宮	交
六	二	七	一	九	〇	一	〇	三	二	四	一	六	〇	度	周
一	三	八	四	五	四	三	四	〇	〇	七	五	四	五	分	度
三	四	二	四	五	五	八	〇	七	〇	〇	二	三	三	秒	
〇	〇	〇	〇	〇	〇	〇	〇	〇	〇	〇	〇	〇	〇	宮	太
一	二	三	〇	四	一	四	二	六	〇	七	一	七	二	度	陽
八	四	五	二	八	〇	一	五	八	二	一	一	四	五	分	經
五	二	九	〇	七	一	五	二	九	〇	七	一	五	二	秒	度
五	二	三	二	二	二	一	二	〇	二	八	一	七	一		宿
三	二	七	一	三	一	七	〇	二	〇	六	五	一	五	日	紀

<sup>13</sup> The epoch was the mean time of winter solstice for the first year under the Chongzhen reign (1628-1644), i.e., 23 December 1627 in the Gregorian calendar.

<sup>14</sup> A Chinese coordinate system for year counted in heavenly stems and earthly branches 干支, which has a circle of sixty. This system also can be used for counting dates and was called “*jiri*” (紀日, date counting).

<sup>15</sup> Here, the first conjunction means the first conjunction of the sun and moon after the winter solstice (*tianzheng dongzhi*, 天正冬至).

<sup>16</sup> The Chinese had several different coordinates for locating points in space and time. One coordinate system for dates was the twenty-eight lodges, a scheme essential for Chinese astrology.

<i>wuyin</i> 15	<i>dinchou</i> 14	<i>bingzi</i> 13	<i>yihai</i> 12	<i>jiayu</i> 11	<i>guiyou</i> 10	<i>renshen</i> 09	Year counting	
21	03	13	24	05	17	27	Day	Radix time for first mean conjunction after the winter solstice
04	06	21	13	15	06	21	Hour	
16	43	55	06	33	45	56	Min.	
24	43	05	27	46	08	30	Sec.	
00	11	00	00	11	00	00	Sign	Radix position of the solar epicycle at the first mean conjunction
14	26	07	18	29	10	21	Deg.	
55	33	17	01	38	22	06	Min.	
48	19	11	03	34	26	18	Sec.	
02	03	04	06	07	09	10	Sign	Radix position of the lunar epicycle at the first mean conjunction
14	09	29	19	13	04	24	Deg.	
49	12	24	36	59	11	23	Min.	
44	43	42	41	40	39	38	Sec.	
01	01	01	01	11	11	11	Sign	Radix position of the ascending node at the first mean conjunction
26	17	09	01	23	14	06	Deg.	
31	48	45	43	00	57	54	Min.	
43	42	55	08	07	20	33	Sec.	
00	00	00	00	00	00	00	Sign	Radix position of the sun at the first mean conjunction
21	03	14	24	06	17	27	Deg.	
48	25	08	51	28	11	54	Min.	
25	09	17	25	09	17	25	Sec.	
25	23	22	21	20	18	17		lodges
23	17	13	07	02	56	51		Date counting

Fig. 2 “Table of five motions lasting two hundred years after the calendrical epoch” (above), excerpt in *Xiyang xinfa lishu* (Vatican Library, R.G.Oriente.III.240 int01, 1.7b) and and my translation (below).

Each type of mean motion has its own “lunation factor” (*shuoce*, 朔策), which means the mean displacement of each motion during a mean synodic month.<sup>17</sup> The basic operation of the first step in the template table is to pick-up different “radices” of mean motions from the table and adds them to various lunation factors to get each mean position. That is, by adding the radix position of each mean motion at the first

<sup>17</sup> The values of *shuoce* in this system refer to Tycho Brahe. The lunation factor for time of mean conjunction, displacement of the solar epicycle, lunar epicycle, ascending node and mean motion of sun (in a mean synodic month) are 29 days 12 hours 44 minutes 3 seconds, 0s;29°6'21", 0s;25°49'0", 1s;0°40'14", 0s;29°6'24", respectively.

conjunction and its displacement between the first and chosen conjunctions, the user can find the mean positions at any time of mean conjunction.

For example, suppose we need to calculate the “mean position of the solar epicycle from perigee” at the Month I conjunction in the thirty-first year under Kangxi reign 康熙三十一年正月 (17th Feb., 1692 in Gregorian calendar). We extract “radix position of the solar epicycle at the first mean conjunction”  $0s;21^{\circ}06'18''$  in the table (see Fig. 2), and add it to its lunation factor  $0s;29^{\circ}06'21''$ , giving  $1s;20^{\circ}12'39''$  for the “mean position of the solar epicycle from perigee”. If we need to calculate the Month II, we just need to add the lunation factor twice and get  $2s;19^{\circ}19'0''$ .

The form BnF Chinois 5015 is unique. Official astronomical publications in this period often include two parts, “the *Lizhi* (The gist of the calendrical system, or theoretical part) was used to clarify its principles, whereas the table part was used for placing the data” 曆指以明其理，表以着其數.<sup>18</sup> But the BnF sheet was used neither for introducing the theories nor for offering data. Rather, it is a specific tool to guide the user to follow procedures and select data from given tables.

Another template table similar to Chinois 5015 can be found in the book *Jiaoshi Mengqiu dingbu* 交食蒙求訂補 (*Amendments of elementary course for eclipse calculations*) by Chinese mathematician Mei Wending (梅文鼎, 1633-1721). It was included in the appendix to this book and has not been studied previously by scholars. Noteworthy is the fact that, according to Mei Wending, this book is a supplement with annotations to a handbook named *Jiaoshi Mengqiu* 交食蒙求 (*Elementary course for eclipse calculations*). The *Elementary course for eclipse calculations* is one volume in the book *Chongzhen lishu* 崇禎曆書 (*Books on calendrical astronomy of the Chongzhen reign*) which introduces systematically the theories and methods of the classic European astronomy. The *Elementary course for eclipse calculations* was finished by Jesuit astronomer Johann Adam Schall von Bell (1592-1666), first presented to the emperor Chongzhen (崇禎, r. 1627-1644) in 1634, but remained unpublished and later became lost. That is, the extant information suggests that Chinois 5015 has some links to the *Elementary course for eclipse calculations* which belongs to the *Books on calendrical astronomy of the Chongzhen reign*. Mei Wending also pointed out that the *mengqiu* 蒙求 has another name *xicao* 細草: “The [*Chongzhen*] *lishu* (*Books on calendrical astronomy of the Chongzhen reign*) has a *xicao* for the convenience of calculations just as the Season granting calendrical system has the *General Rules*” 《曆書》之有《細草》，以便入算，亦猶《授時曆》之有《通軌》也.<sup>19</sup>

That is, template tables are bridges that connect a set of instructions and the corresponding tables in computational practices.

The normalization of tabular use can be perceived in the title of the *General Rules for the Great Concordance Calendrical System* by Yuan Tong. The term *tonggui* (通軌, general rules) suggests that the book prescribes how to perform calendrical

<sup>18</sup> *Xiyang xinfa lishu* 西洋新法曆書, “Tables for planets” 五緯表, vol. 1, 1.1a.

<sup>19</sup> *Wuan lishu* 勿庵曆算書目 (*Wuan's bibliography on calendrical system and mathematics*), 12a.

calculations with tables quickly and easily. In fact, this book even presents the steps of calculations in tabular form. The work has six volumes, covering the calculation of solar terms and syzygy, position of the sun, moon, five planets, and *siyu* 四餘 (that is, four hypothetical celestial bodies that were important in astrology) as well as eclipses. Each volume contains four parts. Before introducing the procedures in detail and offering the appropriate astronomical tables, it gives the *yongshu mulu* 用數目錄 (list of the necessary parameters or constants, the values we already have achieved) and the *chengshi* 程式 (procedure form, a type of template table). The template table *chengshi* lists the values we need to calculate and provides blank entries for users to fill in the data extracted from appropriate numerical tables. The result of each step is recorded in the appropriate cell of the template table, thereby normalizing the process.

Let us take the planetary part of this book as an example. It emphasizes that “for the calculation of planets, [the user] have to follow the form *shi*<sup>20</sup> marked with boundary and fill in [the data]” 凡推算五星者，依此式界劃填寫. The book also emphasizes that other calculations should refer to these forms 皆當仿此式. This method of calculating with template table was thus an established rule in the Astronomical Bureau at least since the Ming period. Beside the *General Rules*, another book *Weidu taiyang tongjing* 緯度太陽通徑 (*Gateway for the position of sun with Islamic method*) also has a template table *chengshi* used as a sample to calculate the true position of the sun by *Huihui lifa* 回回曆法 (Islamic calendrical system in Ming China), which was an official calendrical system used in parallel with the Great concordance calendrical system.<sup>21</sup>

In sum, these two books describe how to use prearranged printed forms for computations required by the Great concordance calendrical system and the Islamic calendrical system, respectively. The kind of template table called *chengshi*, later sometimes named *suanshi* (算式, calculation form), specifically designed to guide a process of calculation and to record the results of each step, was widespread from the 14th century onwards. This technique helps to normalize the use of numerical tables and the computational procedures; it also enables users to check the results of each step and reduce chances of error.

### 3. The “detailed procedures” and the use of “template tables”

The term *xicao* (detailed procedures) initially came from ancient mathematical texts and referred to a kind of notebook used to explain algorithms or give detailed comments on classic mathematical texts. For example, extant editions of *Zhang Qiujian Suanjing* 張邱建算經 (*Mathematical Classic by Zhang Qiujian*), initially composed between 466 and 485, contain “detailed procedures” (草, *cao*). This form of text occurs here for the first time in the extant mathematical texts; in the 11th century we have a second occurrence in Jia Xian’s 賈憲 *Huangdi Jiuzhang Suanshu Xicao* 黃帝九章算術細草 (*Detailed procedures of Huangdi’s Canon of the Nine*

<sup>20</sup> Sometimes *chengshi* is also called *shi* 式 (form) in the astronomical texts.

<sup>21</sup> That is, different types of calendrical systems have their own template tables but handle their computational practices in similar ways.

*chapters on mathematics*). Most of the extant mathematical books in the 18th century and 19th century offer this type of detailed procedures as do books on the astral sciences in the same period.<sup>22</sup> The detailed procedures thus occurs both in mathematics and in the astral sciences.

However the detailed procedures are rare in the extant ancient Chinese astronomical texts, this may be because the astral sciences were strictly controlled by the imperial court and were only accessible to members of the Astronomical Bureau. In this section, I will present the *Suan qizheng jiaoshi lingfanfa* 算七政交食凌犯法 (*Calculation methods for seven luminaries, eclipses and encroachments*)<sup>23</sup> and *Xuanxiang xinfa xicao leihui* 玄象新法細草類匯 (*Collection of detailed procedures for new celestial methods*) as examples to introduce the contents and circulation of *xicao* in astronomy. The former text, preserved in the Library of Forbidden City, is a “detailed procedures” transcribed in the middle period of the Kangxi reign (1661-1722). The latter one, now in Kyujanggak Library in the Seoul National University, is a manuscript “secretly purchased” 密買 by Korean astronomers from China in the early 18th century.

The *Collection of detailed procedures for new celestial methods* gives a set of detailed procedures to calculate the position of the sun, the moon and five planets, solar and lunar eclipses as well as the lunar and planetary encroachments (the passing of the moon or planets through an asterism). In the part on eclipses, we find that the procedures the book introduces are fully consistent with the template table recorded in BnF Chinois 5015. For example, the “sixteen steps” and “twelve steps” for solar and lunar eclipses respectively in these two documents are identical. In both cases the procedures for determining the mean motions is identical.

In order to illustrate how the detailed procedures works, let us take the “solar part” in the *Collection of detailed procedures for new celestial methods* as an example to compare the process of calculations with the “detailed procedures” and “template table”. In this book, the “solar part” gives eight steps to calculate the true position of the sun according to the western method introduced by Jesuits. The text says:<sup>24</sup>

Method for calculating the thread of the sun

Radix of the year (*niangen*, 年根): First, to pick up the radix of the current year, note: The radix of the year corresponds to a computation that starts from winter solstice and does not mean it begin calculating

---

<sup>22</sup> In mathematical texts published in the 18th century and 19th century, we can find *Detailed procedures and illustrated description for nine chapters on the art of mathematics* 九章算術細草圖說 and *Detailed procedures of sea island mathematical manual* 海島算經細草圖說 by Li Huang 李潢 (1746-1812), *Detailed procedures of sea mirror of circular measurement* 測圓海鏡細草, *Detailed procedures of the procedure of calculation of arcs and sagittas* 弧矢算術細草 by Li Rui 李銳(1768-1817), and *Detailed procedures of Reflections on mathematics up to four variables* 四元玉鑿細草 by Luo Shilin 羅士琳(1789-1853). These books all belong to the type of “detailed procedures” for the ancient classic mathematical texts.

<sup>23</sup> Actually, the initial handbook *Calculation methods for seven luminaries, eclipses and occultation* has no title and preface; a librarian added this title later, based on its contents.

<sup>24</sup> The reader who doesn’t care for the technical sides and the translation can go forth to the next section of this paper. You will find an example in the next section which can help you understand this texts better.

from the first month [of the lunar year]; input this data into the cell “radix of the year”. Alongside, record the position of perigee [at winter solstice] (*gaochong*, 高衝) this year and write on the [blank] paper which put aside.

Motion in days (*rishu*, 日數): Second, to check how many days there are from the day after the winter solstice of the current [year] to a chosen day, then use the number of the days [as the argument]. See the table of annual mean motion of the sun, which in volume one, page thirteen, When the degrees, minutes and seconds of a sign go beyond thirty degrees, move ahead by one sign, and record it in the cell “motion in days”. By the way, to record the motion of perigee [from winter solstice] (*gaoxing*, 高行) up to this day in the paper put aside, add it [with *gaochong*, the position of perigee at winter solstice], record the result.

Motion in hours (*shishu*, 時數): Third, from the mid-night to the time (hours) of the chosen day, use the number of the hours [as the argument]. See the table of mean position of the sun hours in one day, which is in volume one, page fifty-five, and record the motion in hours in the cell according to the units in degree and second.

Mean position (*pingxing*, 平行): Fourth, add the data radix of the year, motion in days and motion in hours together in degree, minute and second, record [it] in the cell “mean position”.

Position of perigee (*gaochong*, 高衝) : Fifth, pick up the sum of the two data that have been added in the paper put aside, record [it] in the cell “position of perigee”.

Epicyclic mean anomaly (*yinshu*, 引數, literally argument): Sixth, subtract from the mean position the value of the position of perigee; if too small to be subtracted, add it with twelve signs and then subtract, [the result] as the epicyclic mean anomaly, record [it] in the cell “epicyclic mean anomaly”.

Equation of the sun (*junshu*, 均數): Seventh, use the [data of] epicyclic mean anomaly [as the argument] in sign, degree and minute, see the table for additive and reductive corrections [of the tread of the sun] (equation table) in volume two, to pick up its corresponding equation of the sun, remember to add the plus sign or minus, record [it] in the cell “equation of the sun”. To check the equation [in which sign], [if] in the first half six signs, which are sign 0,1,2,3,4,5, use the upper side and [read the table] forward; if in the second half of the six signs, which are sign 6,7,8,9,10,11, use the lower side and [read the table] backward. The upper side [results are positive] sign +, the lower side [results are negative] sign -. To use the proportional method for interpolation (*bilifa*, 比例法), subtract the pick up value with the value of next entry, get the remainder in the unit of seconds. Multiple the data with the remainder of the epicyclic mean anomaly, remove

the mantissa, then add or subtract it with the pick up value. If the pick up value is larger than the value of next entry, use the sign -, if smaller, use the sign +. Then get the equation of the sun for the corresponding epicyclic mean anomaly, record [it] in the cell “equation of the sun”.

True position (*shixing*, 實行): Eighth, add or subtract the mean position with the value of equation according to the plus or minus sign [before the equation value], [the result] is the true position, record [it] in the cell “true position”.<sup>25</sup>

推日躔法

年根:一、察本年根,按各年根冬至起算,非各正月起算也,寫于年根格內。隨錄本年高衝書於傍紙。

日數:二、察於本冬至後一日至某日,有幾日即用幾日之日數。至太陽周歲平行表內,一卷三十三張,其宮度分秒之數凡滿三十度,則進一宮,寫於日數格內,隨錄本日高行於傍紙下,相加得數記之。

時數:三、從子正起至某時,止有幾時,用此數。至周日時對準日行表,在一卷五十五張內,相對分秒寫于時數格內。

平行:四、年根日數、時數、宮度分秒,一併相加于平行格內。

高衝:五、將傍紙所記加得高衝之數,寫于高衝格內。

引數:六、平行內減去高衝之數,若不足減者,加十二宮減之,為引數,寫於引數格內。

均數:七、以引數宮度分至二卷加減差表內,察相對之均數,記書加減號,得數寫於均數格內,但察均數前六宮,〇、一、二、三、四、五宮者,其度分在上順察,後六宮,六、七、八、九、十、十一宮者,其度分在下逆察。在上號加,在下號減,若用比例將先均數與後均數,相減所得之秒數,用此秒與引數之餘分相乘,所得數去尾數與先均數或加或減。若先均數大於後均數,用減。若小於後均數,用加,即得。本引數之均數寫於均數格內。

實行:八、均數依加減號,或加、或減于平行為實行,寫於實行格內。

From the texts above, we can find some features of this text detailed procedures. First, each step has its own title in the beginning, such as the titles “radix of the year”, “motion in days” and “motion in hours” for the first several steps and each step of calculation is designed in the form of given “cell”. The user only needs to know how to extract data from the appropriate tables and how to fill each cell with intermediate data. The final result is then secured by simple arithmetic calculation.

Second, some steps identify the title of table that should be referred to and even give the relevant page number. For example, the step “Motion in hours” says “refer to the table of annual mean motion of the sun, in volume one, page thirty-three” 至太陽周歲平行表內,一卷三十三張。 Because this detailed procedures only gives the procedures without the whole tables, we do not know to which tables they refer. But

<sup>25</sup> *Suan qizheng jiaoshi lingfanfa* 算七政交食凌犯法 (Calculation methods for seven luminaries, eclipses and occultation), 1a.

with the help of the titles and page numbers, we discover that this set of detailed procedures fit the tables in the *Books on Calendrical Astronomy According to the New Western Method* (finished in 1645) instead of the *Books on calendrical astronomy of the Chongzhen reign* (finished between 1629 and 1636) which is the first edition of the former.

Third, we find that the procedure requires that some values be recorded in the template table, while a blank sheet called *pangzhi* (a paper put aside, 傍紙) is required in the process of calculations in addition to the “template table”. Obviously, other values such as temporary data only need to be written on the paper put aside.<sup>26</sup>

The contents of “detailed procedures” in the *Calculation methods for seven luminaries, eclipses and encroachments*, numerical tables in the *Books on Calendrical Astronomy According to the New Western Method* and the corresponding “template table” in BnF Chinois 5015 suggest that these three documents were used together for the calendrical calculations. Users who were not professionals or did not understand the fundamentals of the western astronomy could still finish the computations with the help of a “template table” if they followed the “detailed procedures” and knew how to select data from the tables.

#### 4. An example for the operation of “template tables”

In order to show how the template table is used with appropriate detailed procedures and numerical tables, we will give an example: follow the instructions of detailed procedures and pick up data in the given astronomical tables (from fig. 3 to fig. 5) to finish the computational practice and complete a template table *suanshi* (in Table 1). Suppose we want to find the true position of the sun at midnight in Beijing on the first day in Month One in the fourth year under the Chongzhen reign (1st Feb. 1631 in Gregorian calendar).<sup>27</sup>

In this calculation, three numerical tables are needed: *liyuanhou erbai hengnian biao* 曆元後二百恒年表 (Table for lasting two hundred years after the calendrical epoch), *taiyang zhousui pingxing biao* 太陽周歲平行表 (Table for annual mean motion of the sun) and *richan jiajiancha biao* 日躔加減差表 (Table for additive and reductive corrections of the tread of the sun, which is the equation table of the sun). The first table gives both the “radix of the year” (mean position of the sun at the winter solstice) and “mean position of perigee” (*gaochong*) at the winter solstice.<sup>28</sup> The second gives the displacement of mean motion of the sun and the mean motion of solar perigee in a whole year by days. The third gives the correction from the mean to the true position of the sun.

---

<sup>26</sup> The step and sub-steps in the template *suanshi* were designed according to the instructions of procedure. When we pick up data from astronomical tables, we get the data needed in the current step and put it down in the *suanshi* directly. If the table has offered the data we required in the subsequent steps, they can be written on the paper *pangzhi* temporarily and taken back later when needed.

<sup>27</sup> Here, the Month One means the first month in Chinese lunar month system.

<sup>28</sup> This table also gives the date of *xiu* (宿, lodges) and *jiri* (紀日, date counting), but they are not necessary in this calculation and we can ignore them here.



According to the detailed procedures, the operation of these tables is as follows. First, we choose the fourth year under the Chongzhen reign, the year *xinwei* 辛未<sup>29</sup>, in the first table. We find that the values for “radix of the year” and position of perigee are “ten *fen* thirty-seven *miao* thirty-three *wei*” (10'37"33) and “six *du* two *fen* fourteen *miao*” (6°2'14") (see Fig. 3).<sup>30</sup> So we record the former value in the blank cells of “radix of the year” in template table and the latter value on the paper put aside (see Table 1).

Because the first day of Month One is the 41th day after the winter solstice this year, we search “41” at the entry of “days” to obtain the two values under this column. It says “forty *du* twenty-four *fen* forty-one *miao* thirty-three *wei*” (40°24' 41"33) and “eight *miao*” (8") (see Fig. 4). Then we convert each thirty degrees to one sign, and get “1s;10°24'41"33” for the former value. Let us record them in blank cells of “motion in days” in template table and “motion of perigee” on the paper put aside, respectively. Because the time we chose in this example is midnight, or the beginning of a day in Chinese custom, we do not need to take the “motion in hours” into consideration here. After adding the values “radix of the year” and “motion in days”, we get “sign one ten *du* thirty-five *fen* nineteen *miao* six *wei*” (1s;10°35'19"6) or the “mean position of the sun” at the chosen time that we record in the detailed procedures. Now we turn to the paper put aside and add the “position of perigee [at winter solstice]” (*gaochong*) and “motion of perigee [from winter solstice]” (*gaoxing*) to get the “position of perigee” at the chosen time. The result is “six *fen* two *miao* twenty-two *wei*” (6'2"22) and we record it in the template table as well (see Table 1).

Subtracting the “position of perigee” (*gaochong*) from the “mean position” (*pingxing*), we get the “epicyclic mean anomaly” for the third table. The result of it is “sign one four *du* thirty-two *fen* fifty-seven *miao* six *wei*” (1s;4°32'57"6) that we enter in the template table (see Table 1). The third table we rely on is the equation table of the sun, whose argument is given in ten-minute intervals from the first to thirtieth degree of each sign. The table divides the twelve zodiacal signs in two, with the first six running right-to-left across the top of the table and the last six running left-to-right at its bottom. With the “epicyclic mean anomaly” (1s;4°32'57"6), we refer to the entries “4°30” and “4°40” in sign one and find “one *du* ten *fen* forty-seven *miao*” (1°10'47") and “one *du* eleven *fen* four *miao*” (1°11'4") (see Fig. 5). After using the ‘proportion method’ (*bilifa* 比例法, an operation similar to the linear interpolation)<sup>31</sup> to perform interpolation, we can get the equation “one *du* ten *fen* fifty-two *miao*” (1°10'52"). Finally, according to the detailed procedures, we can obtain the true position of the sun “sign one and eleven *du* forty-six *fen* eleven *miao* six *wei*” (1s;11°46'11"6) after adding the mean motion and its correction (see Table 1).

<sup>29</sup> The year *xinwei* 辛未 is the eighth year in the sexagesimal circle in heavenly stems and earthly branches.

<sup>30</sup> Here, the Chinese writings adopted the sexagesimal system in degree, minute and second. The units *du* 度, *fen* 分, *miao* 秒, *wei* 微 are equivalent to degree, minute, second and one-sixtieth second.

<sup>31</sup> Linear interpolation was well known and appears for instance as the *Yingbuzu* 盈不足 (Excess and deficit) in the Chinese classical *The Nine Chapters on the Mathematical Art* 九章算術 which dates probably from the 1st century CE (Martzloff, 2006, pp. 336–338).

Because the Chinese custom takes the winter solstice (270 °) as the starting point, the true sun's position is equivalent to 10s;11°46'11"6 from Aries after converting.

Table for continuous two hundred years after the calendrical epoch											
Year counting	Deg.	Ten [Min.]	Min.	Ten [Sec.]	1/6 Sec.	1/60 Sec.	Position of Perige, Deg.	Ten [Min.]	Ten [Sec.]	Lodges	Date counting
wuchen 05		5	3	3	5	3	5	5	9	Well 22	jimao 16
jisi 06		3	9	1	6	1	6	0	4	Devil 23	jichan 21
gengwu 07		2	4	5	6	5	6	0	2	Willow 24	jichou 26
xinwei 08		1	0	3	7	3	6	0	1	Stars 25	jiawu 31
renshen 09		5	5	2	6	3	6	0	5	Wings 27	gengzi 37
guyou 10		4	1	0	7	0	6	0	4	Baseboat 27	yisi 42
jiawu 11		2	6	4	7	7	6	0	9	28 Hom	gengxu 47

曆元後二百恒年表		
紀年度	十分	十秒
戊辰	五三三	五三九
巳巳	三九一	六一七
庚午	二四五	六五五
辛未	一〇三	七三三
壬申	五五二	六三一
癸酉	四一〇	七〇九
甲戌	二六四	七四七
宿紀日	五五九	五九九
井巳卯	六〇〇	四四四
鬼甲申	六〇一	二九九
星甲午	六〇二	二一四
翼庚子	六〇三	四四四
軫乙巳	六〇四	二九九
角庚戌	六〇五	二九九

Fig. 3. Table for lasting two hundred years after the calendrical epoch (*liyuanhou erbai hengnian biao*, 曆元後二百恒年表) with translation (Vatican Library, R.G.Oriente.III.236 int02, 1. 21a)

Days	Mean motion One hundred [Deg.]	Ten [Deg.]	Deg.	Ten [Min.]	Min.	Ten [Sec.]	Sec.	1/6 Sec.	1/60 Sec.	Motion of perige sec.
41		4	0	2	4	4	1	3	3	8
40		3	9	2	5	3	3	1	3	7
39		3	8	2	6	2	4	5	3	7
38		3	7	2	7	1	6	3	4	7
37		3	6	2	8	0	8	1	4	7
36		3	5	2	8	5	9	5	4	7
35		3	4	2	9	5	1	3	4	6
34		3	3	3	0	4	3	1	4	6
33		3	3	3	1	3	4	5	4	6

日数	平行百十度十分十秒十微	最高行十秒
三三	三二二一三四五四	六
三四	三三二〇四三一四	六
三五	三四二九五一三四	六
三六	三五二八九五四	七
三七	三六二八〇八一四	七
三八	三七二七一六三四	七
三九	三八二六二四五三	七
四十	三九二五三三三三	七
四一	四〇二四四一三三	八

Fig. 4. Table for annual mean motion of the sun (*taiyang zhousui pingxing biao*, 太陽周歲平行表) with translation (Vatican Library, R.G.Oriente.III.236 int02, 1. 35a)

	40	30	10	10	<sup>10deg</sup> 4	50	Add 40	Argument	
	0	0	0	0	0	0	0	Deg.	Sign 11
	10	9	9	9	8	8	8	Min.	Sign 0
	13	51	30	8	46	24	2	Sec.	
	1	1	1	1	1	1	1	Deg.	Sign 10
	11	10	10	10	9	9	9	Min.	Sign 1
	4	47	29	11	53	35	17	Sec.	
	1	1	1	1	1	1	1	Deg.	Sign 9
	52	52	51	51	51	51	51	Min.	Sign 2
	12	3	54	45	36	27	17	Sec.	
	2	2	2	2	2	2	2	Deg.	Sign 8
	2	2	2	2	2	2	2	Min.	Sign 3
	34	36	38	40	42	44	45	Sec.	
	1	1	1	1	1	1	1	Deg.	Sign 7
	40	40	40	40	41	41	41	Min.	Sign 4
	20	33	45	57	10	22	34	Sec.	
	0	0	0	0	0	0	0	Deg.	Sign 6
	51	52	52	52	53	53	53	Min.	Sign 5
	51	10	29	48	7	26	45	Sec.	
	20	30	40	50	<sup>10deg</sup> 26	10	20	Argument	

Fig 5. Table for additive and reductive corrections of the tread of the sun (*richan jiajiancha biao*, 日躔加減差表) with translation (Vatican Library, R.G.Oriente.III.236 int03, 2.2b)

Table 1. The example template table for the calculation of 1st February 1631 (above) and appropriate paper put aside (below)

	日/宮 Day/Sign	時/度 Hour/Deg.	分 Min.	秒 Sec.	微 1/60 Sec.	日躔算式 Template table for the calculation of the thread of the sun
年根 Radix of the year			10	37	33	
日數 Motion in days	1	10	24	41	33	
時數 Motion in hours		0	0	0	0	
平行 Mean position	1	10	35	19	6	
高衝 Position of perigee		6	2	22	0	
引數 Epicyclic mean	1	4	32	57	6	

anomaly (Argument)						
均數 Equation		1	10	52	0	
實行 True position	1	11	46	11	6	

position of perigee (*gaochong*) this year 本年高衝: 6 deg. 2 min. 14 sec.  
 motion of perigee (*gaoxing*) up to this days 本日高行: 8 sec.

### 5. Transmission of the “template table” and “detailed procedures” from China to Korea

Because the Qing court imposed some restrictions on the circulation of the astronomical knowledge, it was difficult for Korean scholars to acquire astronomical manuals and instruments in the early Qing period. Even though some official astronomical books on theories and tables could be purchased through legal trade, a flexible solution was sought by the Joseon court to secure unpublished “top secrets” texts such as the detailed procedures. The Joseon king ordered some officers, who were actually experts in astronomy, to disguise themselves as normal diplomatic officers to the Qing court and to learn the calendrical system secretly. Korean astronomers Song Innyong 宋仁龍 and Kim Sangbeom 金尚范 were dispatched to Beijing where they eventually learned some basic methods from Adam Schall von Bell and Chinese astronomers in the Bureau.<sup>32</sup> Consequently, in the year 1654 the Korean court was able to issue its own civil calendar consistent with the Qing court. But they only learned the method of calculating the solar positions; methods for calculating eclipses and positions of the planets were not obtained.

Fifty years later, obvious discrepancies arose between the calendars of the Qing and Joseon courts for the year 1705 because the Chinese had improved their method and revised some parameters. The Joseon court thus sent an astronomer named Heo Won 許遠(fl. 1708) to Beijing, hoping that he could learn the new method. Heo Won was able to establish a favorable relationship with a Qing astronomer named He Junxi 何君錫 and to learn the calendrical system privately from him.

According to Korean records in *Seungjeongwon ilgi* (承政院日記, *The daily records of the royal secretariat*), Heo Won brought back many books besides the detailed procedures after paying considerable rewards to He Junxi for access to the forbidden knowledge.<sup>33</sup> The preface of Heo Won’s book, *Collection of detailed procedure for new celestial methods*, gives more details about the progress of securing these detailed procedures editions:

<sup>32</sup> Lim Jongtae (2012).

<sup>33</sup> The electronic texts of *Seungjeongwon ilgi* are available on the official website of the National Institute of Korean History, <http://sjw.history.go.kr>. See item King Sukjong 肅宗 (1708/12/29).

Humble servant Heo Won conducted the mission to Yanjing (Beijing) and transcribed several kinds of calculating methods for two calendrical systems from the [Chinese] Astronomical Bureau officer He Junxi. The Chinese literatures and books were sought and purchased with no missing. But these things are forbidden secrets. The study of the annual radix for Venus and Mercury, solar epicyclic motion and eclipse calculations have not been studied yet, so [I was] dispatched [to Beijing] again in 1708....For these methods are complex and difficult [to learn], [we] questioned and answered by [brush talk, writing Chinese characters] on literary notebook or small piece of papers during the teaching. [Finally, the notes] were collected into volumes and named the *xicao leihui* (Collection of detailed procedures)...This book is a standard for the calendrical scholar just as the yardstick and rule are to the craftsman.<sup>34</sup>

臣遠受命而往燕京，從欽天監官何君錫，書得兩曆法推步之術多種，文法書冊質覓無遺，而事系禁秘，金、水年根、日躔高衝及交食推解之法，猶有所未盡學得，又於戊子冬再往，....蓋此法艱巨，授受之際，隨端問答，或以片劄，或以小紙者有之，故合成卷軸，名之曰《細草類匯》。... 此書之於曆家猶之工師之準繩規矩。

Because the *xicao* were essential in its calendrical calculation, the Joseon court spared no efforts to appropriate the new version of it. In 1727, officers again were dispatched to Beijing to calibrate the calendar and they brought back some astronomical books without the latest version of detailed procedures. Even though this book was believed to be the utmost urgency and importance one 甚緊要, it must be “purchased” in time 不可不及時買來.<sup>35</sup> Several decades after Heo Won, the Qing courts again rectified its calendar. As an essential book for calculations 推步之緊要方書, the latest detailed procedures was secured in 1735 and printed soon thereafter in Korea.<sup>36</sup> This episode about the transmission of “template tables” and “detailed procedures” from China to Korea helps us to realize the important status of these computational tools in the maintenance of the calendrical system.

## 6. Final Remarks

As we have seen, a good calendrical system should be precise enough and easy to use. The *Books on calendrical astronomy of the Chongzhen reign* emphasizes that the theoretical part of the calendrical system is used to clarify its principles and the tabular part is used for placing the data. If the usage was not clarified 不明其用, users

<sup>34</sup> *Xuanxiang xinfa xicao leihui* 玄象新法細草類匯 (*Collection of detailed procedures for new celestial methods*), 1b.

<sup>35</sup> *Seungjeongwon ilgi*, see item King Yeongjo 英祖 (1728/10/24).

<sup>36</sup> *Seungjeongwon ilgi*, see item King Yeongjo 英祖 (1735/5/19).

could not handle the calculations 算者無從下手.<sup>37</sup> Actually, the *Books on calendrical astronomy of the Chongzhen reign* is a work compiled in haste; its principles do not fit its tables well and many contents contradict each other. Even though its revised edition, the *Books on Calendrical Astronomy According to the New Western Method*, made a few rectifications, the book remains far from perfect and scholars always puzzle over how its procedures work. The detailed procedures shed light on this problem to some extent; it shows that the Astronomical Bureau mainly relied on the unpublished detailed procedures to normalize the calculations in practice, rather than referring to the implicit method introduced in the published *Books on calendrical astronomy of the Chongzhen reign*. That is, in ancient Chinese astral sciences, especially the calendrical system, it was not enough just to have theoretical texts and tables; skill in manipulating tables and mathematical practices were also necessary.

Later scholars felt that the decline of astronomical knowledge in China during the Ming dynasty could be blamed on the excessive reliance on tables. As Zhou Xiang 周相 (fl. 1560s), who attempted to reform the *Great concordance calendrical system* (used between 1386-1644), wrote, the staff in the Astronomical Bureau only “respectfully keep the methods handed down from generation to generation and rely on its established practical rules” 謹守世業，據其成規.<sup>38</sup> A late Ming scholar, Tang Shunzhi (唐順之 1507-1560)<sup>39</sup>, claimed that this led to a degeneration of knowledge regarding use of tables. “The [methods] such as using *licheng* tables and [what was introduced] in the *General Rules [for the Great concordance calendrical system]*, he claimed, are inferior knowledge based on the writings [of the astronomer] Guo [Shoujing (郭守敬, 1231-1316)]<sup>40</sup>, but they are *si shu* (dead numbers)” 《立成》、《通軌》云云者，郭氏之下乘也，死數也.<sup>41</sup> Scholars of the subsequent Qing dynasty concurred with this assessment, perceiving that the tables that had once been convenient for calculations had hindered the developments of astronomy. Mei Wending (梅文鼎, 1633-1721) pointed out that “The descendants of mathematicians and astronomers all followed the *General Rules* by Yuan Tong (fl. 1380s, director of Astronomical Bureau in the early Ming), they gradually forgot the ultimate source [theories]” 疇子子弟皆以元統之《通軌》入算，逐末忘源.<sup>42</sup> He also worried that the calculators yearned unilaterally for the simple and convenient, they set aside the canon and neglected it 算者貪其簡便，而全部《曆書》或度高閣.<sup>43</sup> This is one of the

<sup>37</sup> *Chongzhen lishu*, “Tables for planets” 五緯表, vol. 1, 1.1a.

<sup>38</sup> *Daming datong lifa* 大明大統曆法 (Great concordance calendrical system of Great Ming), 5a.

<sup>39</sup> Tang Shunzhi was a famous Ming mathematician and advocate of the ancient prose style, many of his works underlined importance of calendrical studies and mathematics.

<sup>40</sup> Guo Shoujing was a famous Chinese astronomer, engineer, and mathematician who lived during the Yuan Dynasty (1271-1368). His main achievement is the compilation of the most sophisticated Chinese calendrical system *Season granting calendrical system* (Shoushi li 授時曆, used between 1280 and 1385).

<sup>41</sup> *Jingchuan xiansheng wenji* 荆川先生文集, Vol. 7.

<sup>42</sup> *Wuan lisuan shumu* 勿庵曆算書目 (*Wuan's bibliography on calendrical system and mathematics*), 8a.

<sup>43</sup> *Wuan lisuan shumu* 勿庵曆算書目 (*Wuan's bibliography on calendrical system and mathematics*), 12b.

reasons why Mei Wending revised detailed procedures and supplemented it with annotations about theory. When he participated in the compilation of the “Monograph for the pitch-pipes and calendar” (*Lilizhi*) for the *History of Ming*, he also insisted on supplementing the theoretical portion and divided the book into three parts: *fayuan* 法原 (theories), *licheng* 立成 (tables) and *tuibu* 推步 (procedures) instead of consulting the existing book *General Rules* directly. That is, creative moves in mathematical practices, such as using the template table to carry on computations, had their own advantages. But when this method became a bureaucratic routine, users merely knew the practical skill but could not grasp the astronomical theories behind these practices.

#### Acknowledgements

The research leading to these results received funding from the European Research Council under the European Union’s Seventh Framework Program (FP7/2007-2013)/ERC Grant agreement n. 269804 and was conducted in the context of the project SAW: Mathematical Sciences in the Ancient World (SAW).

#### REFERENCES

- Chang, Ping-Ying (2015). *Chinese hereditary mathematician families of the Astronomical Bureau, 1620-1850*. Ph.D. diss. City University of New York.
- Deane, Thatcher Elliott (1989). *The Chinese Imperial Astronomical Bureau: Form and Function of the Ming Dynasty “Qintianjian” from 1365 to 1627*. Ph.D. diss., University of Washington.
- Jean-Claude Martzloff (2006). *A history of Chinese mathematics*. Berlin: Springer-Verlag.
- Sivin, Nathan (2008). *Granting the seasons: the Chinese astronomical reform of 1280, with a study of its many dimensions and a translation of its records*. Springer Science & Business Media.
- Li Liang 李亮 (2011), *Computer Simulation Analysis and Comprehensive Researches on Ming Dynasty Astronomy* 明代曆法的計算機模擬分析與綜合研究, Ph.D. diss., University of Science and Technology of China.
- Lim Jongtae (2012), Learning Western Astronomy from China: Another Look at the Introduction of the *Shixian li* 時憲曆 Calendrical System into Late Joseon Korea, *The Korean Journal for the History of Science*, vol. 34: pp.205-225.
- Shi Yunli (2003), The Korean Adaptation of the Chinese-Islamic Astronomical Tables, *Arch. Hist. Exact Sci.* 57, pp: 25–60.

#### TRADITIONAL WORKS IN CHINESE

- Chongzhen lishu* 崇禎曆書 (*Books on calendrical astronomy of the Chongzhen reign*), “Tables for planets” 五緯表, vol. 1, 1.1a, finished between 1629 and 1636, Shanghai: Shanghai guji chubanshe, reprint 2009.
- Daming datong lifa* 大明大統曆法 (Great concordance calendrical system of Great Ming), compiled by Zhou Xiang 周相 (fl. 1560s) in 1569, copy preserved in the Interior Department of the National Archives of Japan.



*Jingchuan xiansheng wenji* 荆川先生文集 (*Collected works of the scholar Jingchuan*), compiled by Tang Shunzhi (唐順之 1507-1560), Wanli edition (1573-1620), Ming period 明萬曆刻本, vol. 7.

*Qixiu leigao* 七修類稿 (*Classified works of seven categories*), compiled by Lang Ying 郎瑛 (1487-1566), Jiajing edition (1507-1567), Ming period 明嘉靖刻本, Vol.5. *Suan qizheng jiaoshi lingfanfa* 算七政交食凌犯法 (*Calculation methods for seven luminaries, eclipses and occultation*), anonym, ca. 1690s, copy of Library in Forbidden City.

*Wuan lisuan shumu* 勿庵曆算書目 (*Wuan's bibliography on calendrical systems and mathematics*), Mei Wending 梅文鼎 (1633-1721), ca. 1692; Kangxi period edition 康熙刻本 preserved in the library of Tsinghua University.

*Xiyang xinfa lishu* 西洋新法曆書 (*Books on Calendrical Astronomy According to the New Western Method*), “Tables for eclipses” 交食表, vol. 1, Vatican Library, R.G.Oriente. III.240 (int01).

*Xiyang xinfa lishu* 西洋新法曆書 (*Books on Calendrical Astronomy According to the New Western Method*), “Tables for planets” 五緯表, vol. 1, Vatican Library, R.G.Oriente. III.242 (int01).

*Xiyang xinfa lishu* 西洋新法曆書 (*Books on Calendrical Astronomy According to the New Western Method*), “Tables for the sun” 日躔表, vol. 1, Vatican Library, R.G.Oriente. III.236 (int02).

*Xuanxiang xinfa xicao leihui* 玄象新法細草類匯 (*Collection of detailed procedures for new celestial methods*), Heo Won, 1710, copy of Kyujanggak Library in Seoul national university.

# **Exploring the Temporality of Complex Computational Practice: Two Eclipse Notes by John of Murs in the ms Escorial O II 10**

Matthieu Husson (CNRS, SYRTE-Observatoire de Paris and SAW project)

## **Abstract**

Manuscript Escorial O II 10 is a late 13th-century document containing a well-known collection of astronomical texts from the arts faculty context. During the first half of the 14th century, this manuscript belonged to John of Murs, an important master of art of the Paris University, responsible, with others for the establishment of the Parisian Alfonsine Tables. John of Murs used the Escorial manuscript to record a wide range of notes over a 20-year period. Among those notes I examine here one concerned with two solar eclipses. Although I will review the relevant information concerning eclipse theory and mathematical practices of European astronomers in the 14th century, this essay will not focus directly on such matters. Rather I am interested in a documentary question: looking at a specific astronomical source I seek clues about the temporal dimensions of a computation as it was recorded in the codex. This focus will help assess the computation practices of John of Murs and will allow an understanding of the meanings such a computational record could have both for its author and in the more general context of early Alfonsine astronomy.

## 1. Introduction

Among mathematical practices, computation is fundamental and probably one of the most ubiquitous. All sorts of computations are and were performed in various scholarly and professional milieus. Computation is a performance embedded in many activities: material, temporal, semantic, individual and collective. The semantic dimension of computations—analysing the kind of numbers used, their relation to measure and metrology, the type of manipulations and operations made with them—is an emerging topic in historiography but will not be at the centre of attention here. The material dimension of computation is also an important aspect: are actors computing on an abacus, of which kind, computing on paper, on dust or wax tablet, with rods, with a combination of tools? The source examined here will give little direct indication about this dimension. Another interesting aspect of computation, which in some cases can be addressed, is the relation between the individual and the collective. Are the actors computing alone, or in a group? If the latter, how is the computation shaped to allow for a collective to perform a computation? On this aspect also our source is mostly silent.

Computation always takes time. Some of them are thus worthy of being communicated and remembered, especially complex computations required to predict some astronomical phenomenon like eclipses. This not often studied aspect of ancient computation will be examined here and designated by the term ‘temporality’. The temporal dimension of eclipse computation will be addressed in two venues: the relation of temporality to computation through memory and storage and the relation of temporality to computation during the execution or manipulation of numerical quantities.

Although relevant information concerning eclipse theory and mathematical practices of European astronomers in the 14th century will be mentioned, this essay is not aiming at these as such. Rather I am interested in a documentary question: can a specific astronomical source offer traces about the temporal dimensions of a computation? The way in which the manuscript stores intermediate results of computations will be used to consider the relation of computation to memory: the execution of computation will be explored by following in the manuscript the sequential steps of the computation.

The astronomical source for these investigations will be Escorial O II 10, which contains John of Murs's computations for the solar eclipses of 1333 and 1337. The first part of our essay will examine the manuscript as a storage device; the second part will trace the temporal performance of John's computation.<sup>1</sup>

## 2 Temporality and Memory in Astronomical Computation: The Manuscript as Storage Device

Escorial O II 10 is a parchment codex of 226 folios, 319 × 206 mm, written in a late 13th-early 14th century hand (Guillermo, 1911, pp. 4: 209–211).<sup>2</sup> It is well copied with large marginal spaces and contains a usual collection of texts covering mathematical astronomy as it was conveyed in medieval arts faculties (Pedersen, 2002, 114):

- Treatises and tables on computistics
- Sacrobosco: *Algorismus, De sphaera*
- Robert Anglicus, *De quadrantis*
- Messahala: *De astrolabii*
- *Theorica planetarum gerardi*
- Thebit, *De motu*

---

<sup>1</sup> John of Murs's astronomical notes of Escorial O II 10 are not yet edited; this paper offers preliminary work toward but not a full edition.

<sup>2</sup> All analyses presented here are made from a microfilm copy of the manuscript.

- Toledan Canons and tables
- Eclipse tract from the Toledan canons

Many comments could be made about this collection of texts. Let me note only a few points, setting a background for the analyses to come.

To begin, the interest for mathematical astronomy is related here to questions concerning the calendar and the determination of dates for moveable feasts like Easter. This connection was already old in the late 13th and early 14th century and would remain strong for at least two more centuries. Of course computus is not the only reason why Latin medieval societies were interested in mathematical astronomy; astrology is another obvious motivation, but the former should never be overlooked.

Another point of interest is the presence of Sacrobosco's *Algorismus*. This famous adaptation for the medieval university context of al-Khwārizmī's eponymous text instructs readers about computation on paper with indo-arabic digits. Latin medieval astronomers were computing with sexagesimal numbers using integers written with indo-arabic digits. The codex begins with the fundamental text regarding the manipulation of these integers.

The codex includes several texts on the geometrical bases of mathematical astronomy. In the Escorial O II 10 the sequence is particularly rich as it has both instrument texts (on the quadrant and the astrolabe) and theoretical texts (Sacrobosco, *De sphaera*, the *Theorica planetarum*, and *De motu*). Together these texts cover the theory of daily, planetary and stellar motions. It is not uncommon to see only parts of these texts in a manuscript and is not rare to find only texts on instruments. Both types of compilations offer their readers a qualitative grasp of the different circles and spheres composing the medieval cosmos and their possible relative motions. These texts also define the technical vocabulary of mathematical astronomy and its geometrical frameworks (Pouille, 1981, 52). If the reading of these texts were accompanied by manipulation of actual instrument like the astrolabe or quadrant, readers' qualitative understandings would have been more concrete.

The Toledan Tables and their canons constitute the majority of the codex. This set of mathematical tools enables users to compute positions of the celestial objects at any time. It also facilitates computation of their various 'accidents' as being visible or not, direct or retrograde, in conjunction or opposition, eclipsed, etc. The Toledan Tables, translated several times from Arabic in the twelfth century and adapted in various ways in Latin and vernacular up into the 13th century, were the main computational tools of mathematical astronomy in Europe during the 13th and early 14th century (Pedersen, 2002), until they were gradually replaced, and partly incorporated, into the Alfonsine Tables (Chabás and Goldstein, 2003; Pouille, 2005).

Among the various computations an astronomer could make with such a set of instructions and tables, eclipse computations were unmistakably among the most difficult and complex. This explains the existence of several eclipse tracts both in the Toledan and Alfonsine traditions. In this section of the manuscript an unidentified hand copied a fragment of John of Lignères canon on syzygies.<sup>3</sup>

If Escorial O II 10 is a noteworthy manuscript, it is not for its highly coherent collection of texts, which can be found in other 14th-century codices, but for the exceptional set of marginal notes which it contains. Antolin briefly acknowledged the presence of these marginal notes and their relation to eclipses (Guillermo, 1911, pp. 3:211), but it was G. Beaujouan who made the real discovery about half a century later in 1962. He identified most of these notes as in John of Murs's hand (Beaujouan, 1964). John of Murs was a very important Parisian master of arts during the first half of the 14th century (Pouille, 1973). He contributed important works in each field of the *quadrivium*, arithmetic, geometry, music and astronomy. The notes reveal that Escorial O II 10 belonged to John of Murs for about 25 years.

---

<sup>3</sup> That is the computation of new and full Moon and the first stage of eclipse computation, fol.186r–189v.

According to L. Gushee, who made a comprehensive study of those notes, focusing on their biographical content, the topics covered are broad and include (Gushee, 1970):

- Commentaries on the principal texts of the manuscript
- Extensive notes on astronomical subjects
- Calculation or observational reports of astronomical phenomena
- Records of book loans
- Records of financial transactions
- Miscellaneous

John of Murs owned the Escorial manuscript for many years and probably carried it on his many travels. He used it partly as a notebook to record notes of various elements from his daily life, many of which reflect John of Murs's relation to other individuals (books loans) or institutions (financial transactions with the *College of Sorbonne*). Yet most of the notes deal with the content of the manuscript itself, i.e., astronomy. They are either directly related to the main texts of the codex or are various *addenda* proposed by John to complement the information in the treatises.

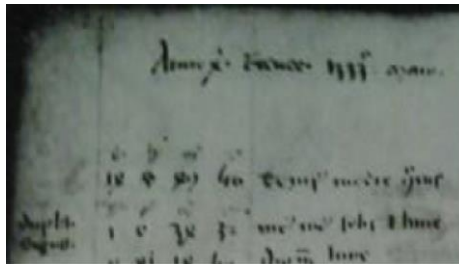
We might think of John's commentaries as arising from a cursive reading of the texts; he would be annotating the pages while he read them, as a way to build his own understanding or to record for later occasions his evaluation of the texts. This kind of reading with 'pen in hand' certainly does not apply, however, to the *marginalia* where John of Murs inserted into the codex elaborate astronomical content, including his own computations and observations.

For example, John of Murs's notes on eclipses are placed in the manuscript exactly where they are pertinent, near the canons to the Toledan Tables. Two folios left blank by the original scribe (92v–93v) were available just before the beginning of the Toledan canons where he inserted his observations of solar eclipses for 1333 and 1337. Another blank folio (123r-v) was available at the end of the canons on which he inserted notes on a lunar eclipse in 1334 (Pedersen, 2002, 114). Moreover in the first three folios of the manuscript John provided a summary of the different computational and observational notes he added in codex. These notes undoubtedly reflect John's intention not only to keep a memory of these computations for himself but also to position them where they would be understandable by any later competent reader of the codex. This suggests that John of Murs was probably not only writing solely for himself. He wanted his notes, computations and observations, to be remembered in the long term. He used the margin and available folios of his manuscript for this end and revealed this intention by the places in the manuscript where some of these notes are recorded. This is not only true of strictly astronomical notes; for instance, notes in the margin of the Sacrobosco *algorism* may reflect John of Murs's teaching of the topic and thus his desire for their long-term remembrance. In this respect, of course, John of Murs reflects the late medieval scholarly culture where an author is someone who adds to the existing texts and knowledge. These additions can take all types of textual forms, from humble marginal notes to vast treatises, but none should be considered insignificant.

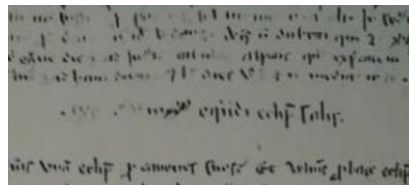
Marginal notes are arguably less elaborate than full treatises, but some of them are more structured than others. In the case at hand, John of Murs showed that he intended his computation and observational notes on the 1333 and 1337 solar eclipses to be remembered by the way he organized the notes themselves. Thus we have not only external clues about the relation of these computations to the astronomical tradition, derived from their position in the manuscript and the habits of late medieval scholarly cultures, but also internal clues regarding the way the notes are shaped.

Describing these computations and observations records as marginal notes, however, is misleading. Because of the dimensions of the text that extend over three full pages on two folios, we have here more than a small annotation in the margin of the Toledan canons. The text of John of Murs fills the full space of the pages and is not related directly as a marginal note to any pre-existing text of the manuscript. The text also features another formal aspect untypical of marginal notes and commonly used to mark the

limits in a codex of an independent treatise. It has, in John's hand, a title, *Anno Christi currente 1333 maio* (Figure 1), and an explicit, *Explicit verum modum equandi eclipsis solis* (Figure 2). The title, or *incipit*, is not very informative about the content of the text but the explicit is very interesting. In writing down this text John of Murs sought to explore the various possible methods of solar eclipse computation and then selected his own. This explains why the notes are positioned at the opening of the Toledan canons, i.e. a treatise instructing readers how to use astronomical tables. Finally John of Murs's text has its own internal set of marginal notes, managed by specific tie-marks giving corrections, comments or alternative computations. All these features taken together suggest that John Murs's text here is closer to a short treatise than to a marginal note.



**Figure 1.** Escorial O II 10 f. 92v (extract) (Solar Eclipse 1333: title of the text).



**Figure 2.** Escorial O II 10 f. 93v (extract) (Solar Eclipse 1333: *explicit* of the text).

These efforts to shape the text and position it within a relevant manuscript reveal John of Murs's intention not only to use the manuscript as a writing support where he could perform his computation. John of Murs here acts truly as a medieval author, adding new information to the tradition and to this end carefully deploying the codes of a scholarly culture.

In fact, these records of John of Murs are rare in Latin medieval astronomy. The Parisian astronomer could not draw on any well-established genre to convey his astronomical computations and observations. John of Murs's text is thus an important element in the history of the relation of astronomy and natural philosophy to empirical evidence (Goldstein, 1972). It is interesting to see that for John of Murs the purpose of the observation is not primarily to adjust the geometrical model or its parameters but to determine a correct procedure for using the tables. In this endeavour, notably William of Saint Cloud, a generation earlier in Paris, preceded him (Poulle, 1976; Pedersen, 2014). John of Murs added to William by giving much more information about his computations.

John of Murs had probably many motives that drove him to record in such a careful way these solar eclipse computations and observations. Clearly he hoped to shape and position his record so as to ensure its remembrance in the tradition. It was possible for him to do so because the manuscript culture in which he worked had certain habits and codes that he had mastered and used. First, even if codices often convey miscellaneous collection of texts, these sets of texts are assembled, in most cases, through fixed patterns with a clear intellectual meaning. This allowed John of Murs to position his text in a related context inside the Escorial O II 10 and so to enhance its chances of being understood and kept in the long run. Second, John of Murs used different techniques to shape his text and create for it a form between a note and a treatise that gives it an identity. In turn, it is possible for historians to grasp and interpret these features and establish that, at least in the context of astral sciences, some computations,

even linked to a particular astronomical event, were considered worthy of being remembered by a community.<sup>4</sup>

On the other hand, John of Murs did not work only for posterity. He had strong personal interests in these eclipse computations and in remembering them for his own sake. The first and obvious clue indicating such personal interest is that in 1337, 4 years after his first records of 1333, he observed a second solar eclipse and went back to the same manuscript folios to write down his conclusions after his second observation.

Hence, although John of Murs shaped his text in a very specific way so that it would be identified, understood and kept in the long term, other features of the text seem to be driven by his own personal concerns about eclipse computation and observation and his need to record them for his own efficient recall. These features also provide information about the relation of computational practice to the individual memory of the computer, in this case, an advanced mathematical performer.

John of Murs's text is recorded in two different layouts. Some parts are presented sequentially: a series of numbers is displayed in two columns, each labelled by a small caption to the right. I call this layout 'numerical' (see Figure 3). Other parts are more usual sections of text, occasionally featuring numbers. I call this layout 'narrative' (see Figure 4)<sup>5</sup>.

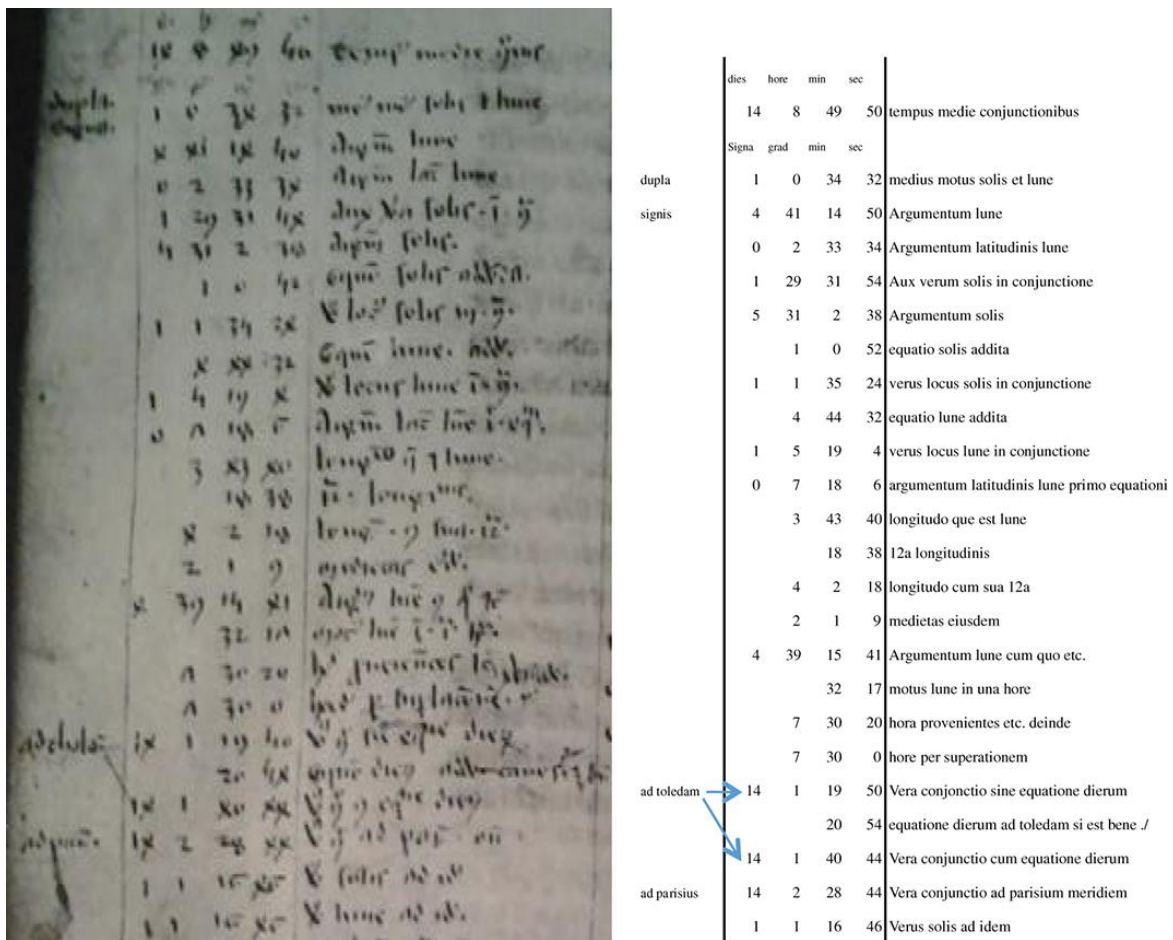
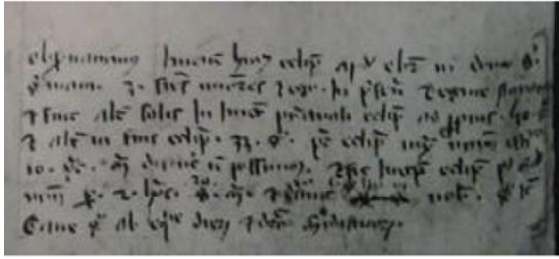


Figure 3. Escorial O II 10 f. 92v (detail). Solar eclipse of 1333, computation of the true zysgy in numerical layout.

<sup>4</sup> Astronomical tables provide another type of testimony of this point, but they store the results of computation rather than computation themselves and they are usually not linked to any particular astronomical events.

<sup>5</sup> These terms are convenient descriptive designations, for the purposes of this article, of these two types of texts. I have no intention here of elaborating a more general theoretical claim around these writing formats.



Observavimus initium eclipses apud Ebroicum in domo sancti Germani 3 fratres minores et ego in presentia Regine Navarre et fuit altitudo solis in initio punctuali eclipsis ad proprius 50 gradus et altitudo in fine eclipsis 33 gradu puncta eclipsis juxta nostrum estimationem 10 puncta, minuta discernere non possumus. Et sic incipit eclipsis post meridiem nostrum per 2 horas 20 minuti et casus (?) 144 \*\*\* tunc quere ab equatione dierum et differentia meridianorum

**Figure 4.** Escorial O II 10 f. 92v (detail). Solar eclipse of 1333, description of the eclipse observation in narrative layout.

The text begins with a long section in numerical layout filling almost all of f. 92v. A mixture of both layouts appears on f. 93r. The end of the text is entirely presented in the narrative layout which is the only content on f. 93v. This arrangement is not fortuitous. It allows a quick understanding of the main lines of John's thought without necessitating a detailed reading of each number and comment. This possibility was exploited by G. Beaujouan who was able to summarise John of Murs's records by picking up relevant numbers and sentences, guided by the arrangement of the text. The result of this reading can be paraphrased as follows (Beaujouan, 1974).

According to Beaujouan, John of Murs's first computation led him to believe that the solar eclipse was to begin 2;37 h after noon on 14 May 1333. He observed, however, that the eclipse in Evreux began 17 min earlier. The discrepancy resulted in a set of corrections to the first computation as well as in a series of comments about its possible origin. John of Murs changed his procedure to compute the time of syzygy, equation of day, lunar parallax and the location in the computation where he shifted meridians from Toledo to Paris. Some of these corrections are presented as marginal notes to the first numerical section of f. 92v, others produce a new numerical section on f. 93r. These alternatives are pondered in a final paragraph on f. 93v. Four year later, in 1337, a new paragraph is added, summing up the lessons of another solar eclipse observation and confirming John of Murs's conclusions of 1333 that the Alfonsine tables were wrong by about 20 min in time in solar eclipse prediction.

This account is generally satisfying. However the following analysis will adjust it regarding the relation of the observation to the various computational strategies explored by John of Murs: not all of which were a response to the observed data<sup>6</sup>. Yet the point of interest here is not the specifics of this account but the very fact that such a reading of John of Murs's text is made possible by the way it is written down. The structure of the text allows readers to rapidly identify the key computational and astronomical choices to be made in the course of the calculation.

Analysing the layouts of the text and the way certain types of information are thus pushed forward to the reader, suggests that John of Murs is interested to keep a memory of the structure of the computation (i.e. where are important choices to be made) rather than in the intermediate numerical values themselves. Now that this global effect of the arrangement of the numerical and narrative layouts has been identified, it is possible to deepen the examination of the content presented in each type of layout.

The narrative layout is used to convey different kinds of information. The narrative layout first occurs on f. 92v. Situated in the middle of numerical material on the page, it describes the circumstances of the actual observation (see Figure 4). Folio 93r offers more occurrences of the narrative layout. Most of the text is devoted to comments on discrepancies between the various results found using the Alfonsine tables with different procedures. Finally, f. 93v features only the narrative layout. It is used for three different kinds of content: a summary of the method of eclipse computation John of Murs selected,<sup>7</sup>

<sup>6</sup> The notes of fol. 93r are very likely to record post-observation computations; the more difficult cases of f. 92v are discussed below.

<sup>7</sup> This method is qualified in the explicit of the text as '*Verum modum equandi eclipsis solis*', f.93v.



some remaining questions about this method and the possible uses of evidence from observation in the computation,<sup>8</sup> a record of 1337 eclipse observation.

One might assume that numerical values coming from an observation should be epistemologically more weighted than predicted values and that the former should somehow act as a criterion in evaluation of methods to compute eclipses. Thus one might argue that, even if quantitatively the main content of these narrative sections were concerned with computational methods, the key element of the narrative sections should be observation. This would, moreover, nicely contrast with the content of the numerical parts devoted to computation. However, a closer look at the different numerical values discussed by John Murs quickly undermines this interpretation.

The first computed value for the initial time of the 1333 eclipse is 2;37 h<sup>9</sup> after noon, then two others are found using different computation methods. One gives 2;23 h after noon and the next 2;26 h after noon. John of Murs observed the beginning of the eclipse at 2;20 h after noon. However John rejected the method producing 2;23 h and kept the one producing 2;26 h. He made this choice by comparing the procedures especially regarding the location in the computation of the shift from Toledo to Paris. Hence, in line with what was concluded from a wide-angle analysis of the text layout, this close-up on the narrative shows that John of Murs is concerned primarily with keeping a memory of the structure of the computation and the main choices to make in its performance.

In this respect, it is interesting to compare the accounts of the 1333 and 1337 eclipses. In 1337 John of Murs did not record any details of the solar eclipse computation. He thought probably the question of the correct procedure for solar eclipse computation had been settled 4 years earlier.<sup>10</sup> Because of this John of Murs can act as if the Parisian Alfonsine Tables were producing only one value for the beginning of the eclipse.<sup>11</sup> Thus in the 1337 account John of Murs directly commented on the discrepancy between the Alfonsine and observed values for the eclipse. He concluded that the Alfonsine value is later than the actual observed value. In drawing this conclusion John of Murs was not original in the medieval context. Half a century earlier, astronomers like William of Saint Cloud (Pouille, 1976) or Peter of Saint Omer (Pedersen, 1979, pp. 59–60) had criticized the Toulouse tables (a variant of the Toledan tables for the Julian calendar) on the basis of observation. In 1333 the focus of John of Murs is on computation methods and the structure of the procedure is what matters most to him: coherence of the procedure is then more important than the proximity to observed values.

To examine the content of the sections of the text in numerical layout it is necessary to have a general idea about how a solar eclipse was computed in the context of 16<sup>th</sup> century Paris. For this it is possible to consider the various texts and tables which John of Murs was more than likely to know: the Toledan tables, John of Lignères's tables and canons for 1321 (Saby, 1987) and John of Saxony's canons of 1327 (Pouille, 1984) which address only a part of the eclipse computation. Our goal here is not to survey in detail these different sources but to give a general idea of the structure of eclipse computation and of the different features making this computation complex.

---

<sup>8</sup> These include: '*Questio utrum differentia meridianorum debeat addi post omnem operationem vel ante et si idem redeat an non. Et quomodo differentia meridianorum potest simpliciter et infallibiliter experiri. Et utrum debeat credi uni experientie ?*', f. 93v.

<sup>9</sup> In this article, numbers coming from the textual section are transcribed as in the manuscript while the positional way of rendering the order of magnitude for the numbers coming from the numerical section is noted using a semicolon. When a number represents time I add 'h' for hour; I add '°' when the number represents an arc; '°/h' indicates a velocity.

<sup>10</sup> For him only, of course, as it was never fixed in general during the period of medieval Latin mathematical astronomy.

<sup>11</sup> In fact the situation is slightly more complex. John of Murs gives three different values for the altitude of the Sun at the time of first contact but only one value each for the observed middle and the end of the eclipse.

A solar eclipse is a true conjunction of the Sun, the Moon and the dragon.<sup>12</sup> This true conjunction must happen in favourable local conditions, i.e., during the observer's daytime with lunar parallax taken into account. The first step of eclipse computation is to determine the position of the Sun and Moon at syzygy. First, one finds the mean conjunction, a conjunction of the mean luminaries (assumed to be moving at their average speeds, respectively). This is a linear problem that can be solved in a number of ways. In the astronomical context one can either use mean motion tables of the Sun or Moon and proceed by successive approximation or use specifically dedicated mean syzygy tables.

Then the true conjunction must be determined. In this second step the varying speeds of the Sun and Moon are taken into account. This difficult problem prompted much research by late medieval Arabic, Hebrew and Latin astronomers (Chabás and Goldstein, 1997). They produced many possible methods to solve the question. On one side of the spectrum are simple, dedicated tables (requiring, however, double interpolation) such as John of Murs's *Tabule permanentes* (Porres and Chabás, 2001). On the other side are successive approximation methods such as John of Saxony's 1327 canons for the Alfonsine Tables. Once the true conjunction is determined and the latitude condition for the Moon allows for a solar eclipse (i.e. the Moon is near enough to its nodes), one needs to adjust to the local conditions of observation. One must determine the position of the Sun in the local horizon at time of the true conjunction. What is computed is its distance to the meridian, to know whether the eclipse will occur during the day. This is done through spherical trigonometry, either with dedicated tables or by direct computation. The last question to be answered, before the actual computation of the eclipse duration and figure, concerns the lunar parallax. This is the most intricate of all the problems to be solved in an eclipse computation. It is solved by successive approximation methods involving specific tables for which there is at least two distinct traditions transmitted through Arabic texts: a 'Ptolemaic' method transmitted mainly by al-Battani and an 'Indian' method transmitted mainly through al-Khwārizmī (Chabás and Goldstein, 2012, pp. 127–138).

A final element of complexity in eclipse computation comes from the history of the computational tools. Local conditions are a key aspect in eclipse computation and many astronomical computational tools, tables or instruments, are fixed for a specific latitude. The performer of an eclipse computation must consider whether he requires a latitude for which no specific tables and instruments are available. He might have to combine in the same computation tools made for different latitudes, depending on his local resources.

After this description of the many steps in an eclipse computation we are surprised not by fact that John of Murs found several different values for the beginning of the eclipse in 1333 but rather by the fact that 4 years later in 1337 he was able to remember what procedures and tables to apply in order to determine a single value that he calls 'the' Alfonsine value of the eclipse.<sup>13</sup> Hence in writing the Escorial O II 10 text of 1333 he structured his memory of the computation so that this shift from several procedures to a single favoured one was made possible. A part of this work is made visible in the narrative layout section of the text but the numerical layout parts are no less important.

John of Murs did not record every single numerical value generated in the process of his computation. For example, in the record of the observation (Figure 4) John of Murs gave the measured beginning time of the 1333 solar eclipse as 2;20 h after noon. Unless you had an independent time measuring device like a water clock or a sundial this time cannot be directly observed; it is computed from the observed altitude of the Sun through specific tables. This particular step of the computation, although it is of obvious importance to the general discussion of the text, presented no difficulties for John of Murs. He

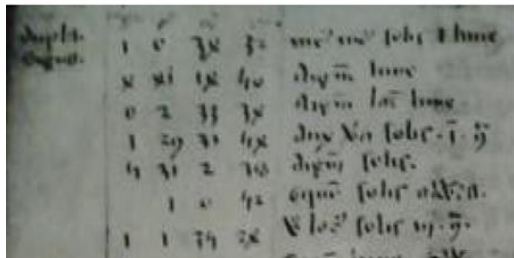
---

<sup>12</sup> The nodes of the moon which can actually be up to 12° from the conjunction for partial eclipses.

<sup>13</sup> This identification of the value as Alfonsine is historically interesting because, for the most part, the eclipse tables and procedures found in the Alfonsine corpus came from the earlier Toledan Tables. However the mean motion and equation tables differ in the Alfonsine and Toledan tables and these tables are the main tools of the first steps of a solar eclipse computation.

was probably confident in the procedure and tools he used there and the kind of accuracy they yield and thus did not record anything about it in the Escorial O II 10 notes.

The numerically laid out parts of the text show only carefully selected intermediate results that for John of Murs conveyed essential information about the computation. He cared about the value of certain quantities which allow readers to follow the computation from a geometrical perspective. A clear instance appears at the very beginning of the computation when the position of the true Sun is presented (see Figure 5).



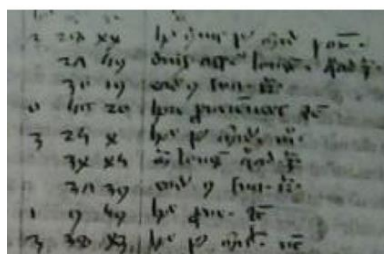
dupla	1	0	34	32	medius motus solis et lune
signis	4	41	14	50	Argumentum lune
	0	2	33	34	Argumentum latitudinis lune
	1	29	31	54	Aux verum solis in conjunctione
	5	31	2	38	Argumentum solis
		1	0	52	equatio solis addita
	1	1	35	24	verus locus solis in conjunctione

Figure 5. Escorial O II 10 f. 92v (detail, upper part of Figure 3). Solar eclipse 1333: true solar computation.

John of Murs gives here only the value of the quantities that allow one to represent the geometrical configuration, on an instrument, a diagram or even only mentally, in which the Sun is found at this particular moment. A similar geometrical reading of the tabular computation is prepared for the Moon whose case is a little more complex since the lunar latitude is also important for an eclipse.

John of Murs also recorded information about the successive approximation processes. For instance the parallax procedure used in Figure 6 is essentially that proposed by al-Baltani (Nallino, 1899-1907, pp. 297–249) that John of Murs could have accessed also in the Toledan tables (Pedersen, 2002, pp. 297–303) or in John of Lignères *Priores astrologi* (Saby, 1987, pp. 227–249).

- 2;28,44 h is the result of the true zysygy computation done before.
- 0;27,59 h is a value derived from an unknown parallax table probably for a latitude close to that of Paris in the seventh climate.
- 0;30,19 h is equal to 13/12 of 0;27,59 h. This is an approximative adjustment made in syzygy and parallax computation, based on the mean lunar and solar velocities (see *Almagest*, VI, 4).
- 0;56,20 h is the result of the division of 0;30,19 h by 0;32,17°/h which is a value for the lunar velocity (presumably taken from an unknown table of lunar velocities) at time of true syzygy computed previously. This gives a first approximation of the parallax correction.
- 3;25,4 h is the sum of 0;56,20 h and 2;28,44 h, a first corrected time of eclipse with lunar parallax.



2	28	44	Hore vere conjunctionis post meridiem parisem	
	27	59	diversus aspectus longitudo quaedam tercia	
	30	19	eadem cum sua 12a	
	0	56	20 hore provenientes etc.	
	3	25	4 hore post meridiem ****	
		34	45 minuta longitudinis quoad 3a	
		37	39 eadem cum sua 12a	
	1	9	59 hore provenientes etc.	
	3	38	43 hore post meridiem ****	

Figure 6. Escorial O II 10 f. 92v (detail). Solar eclipse 1333, part of the parallax computation.

The next four values repeat exactly the same computation process with a different value for the parallax. It can be seen in Figure 6 that values with the same or similar label are presented successively (i.e. *hore*

*vere conjunctionis post meridiem...; diversum aspectus...; eadem cum sua 12a....; hore proveniens...*). John of Murs kept track of the repetition of the computation process using consistently the labels he gave to each number so that the repetition appears clearly. This association of the numerical values (which might help identify tables) with carefully chosen labels (which display the computation procedure) is central to the way John of Murs recorded his computation.

These two first types of information -about the geometrical configuration and about the successive approximation processes- are connected with the flow of one computation at a time. John of Murs structured his memory of the computation around two things, the geometrical meaning of the arithmetical and tabular manipulations when they have one and the structure and number of steps of the successive approximation process which are often disconnected from any direct geometrical meaning. In this respect the numerical layout parts complement nicely the narrative layout parts that are more concerned with alternative ways to organize the computation. However the numerical layout parts also record these alternative computation possibilities. We will look closely in the next section at the computation of true conjunction and will examine some instances. It will be enough here to show how two different computations of eclipse duration, which were not kept in the end, are presented (see Figure 7).

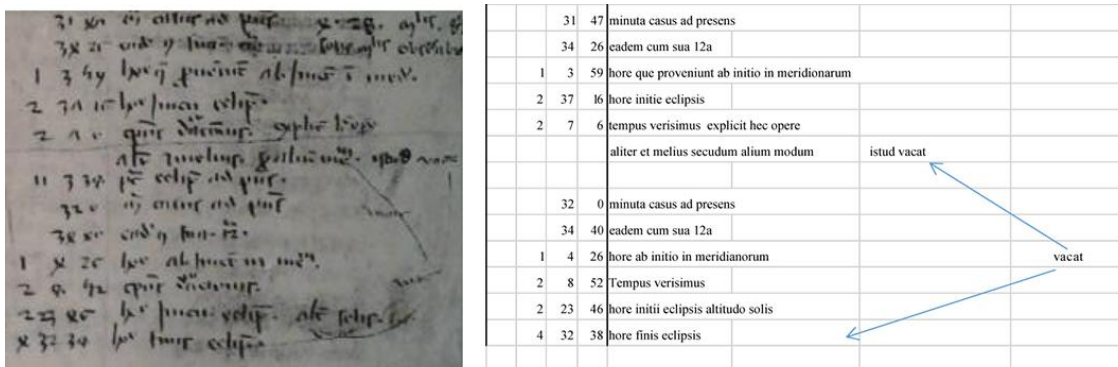


Figure 7. Escorial O II 10 f. 92v (detail). Solar eclipse 1333, computation of duration of the eclipse.

First, the same labels are repeated two times for different numbers (*minuta casus ad presens; hore ab initie eclipsis; hore finis eclipsis; tempus verisimus*, etc.) as was the case in Figure 7. However the two series are not exactly in the same order: *Tempus verisimus* (total duration of the eclipse) is computed first in the second version of the calculation while it is the final result in the initial version of the computation. Moreover values for the *minuta casus* are slightly different in the two sections (0;31,47 h vs. 0;32,0 h). This probably reflects the use of two different eclipses tables in these two versions of the computation.<sup>14</sup> Finally, the second computation is labelled *aliter et melius secundum alium modum*. This confirms explicitly what it is implied by the use of numerical values and their labels. Yet the second set of values is labeled *vacat* by John of Murs. Apparently at a later point in time he decided that this second computation was also not correct.<sup>15</sup> These two sets of computation (see Figure 8) use three ‘external’ values (derived elsewhere) to obtain the eclipse duration, following broadly the method of al-Battani, as is the case for parallax computation:

- lunar velocity at time of syzygy: 0;32,17°/h (perhaps taken from an auxiliary table and not computed by John of Murs?)
- parallax-corrected time of mid-eclipse 3;41,15 h (computed using above lunar velocity value)
- argument of lunar latitude at the parallax-corrected time of mid-eclipse 0;51,44°

<sup>14</sup> In principle the same table might have been used in both cases with rounding done differently in the interpolation. But this option seems less likely to produce a huge discrepancy (13) in the last recorded digit.

<sup>15</sup> As shown above, John of Murs kept a third value as his preference.

From the last value and an unknown solar eclipse table, John computes first the magnitude and then the duration of the eclipse, i.e., the *minuta casus ad presens* (first 0; 31,47 h, second 0;32,0 h) and adds its twelfth to the values thus obtained (first 0; 34,26 h, second 0;34,40 h). This value with the twelfth is then divided by the lunar velocity to obtain the *hore ab initio* ..., a first value for half eclipse duration (first method gives 1;3,59 h, the second method 1;4,26 h). Here John is using not only different tables but also different methods of computation. In the first method, the *tempus verisimus* is not equal to twice the *hore ab initio*. However it is used directly with the time of mid-eclipse obtained previously (3;41,15 h) to compute the time of the beginning of the eclipse (2;37,16 h). In the second method, the *tempus verisimus* is exactly twice the *hore ab initio* (first value for the half eclipse duration) and the difference between the time of the beginning and end of the eclipse is 2;8,48 h, i.e., only 4 s of time shorter than the *tempus verisimus*.

	Method 1	Method 2
Puncta eclipsis	9;42,16	11;03,38
Minuta casus	0;31,47	0;32,00
Minuta casus x 13/12	0;34,26	0;34,40
Hore ab initio	1;03,59	1;04,26
Tempus verisimus	2;07,06	2;08,52
Hore initie eclipsis	2;37,16	2;23,46

**Figure 8.** Comparison of two methods for the computation of duration of the eclipse.

These computations require that parallax corrections be made not only at the time of true syzygy, in order to obtain the time of mid-eclipse, but also at the times of the beginning and end of the eclipse. It is only when these parallax corrections are included that final values for the eclipse duration, beginning and end times are obtained. This probably explains why in the first method the *Tempus verisimus* is not twice the half eclipse duration. The difference in the *tempus verisimus* is 0;01,46, a small amount but this suggests that John of Murs cared about differences of this magnitude in his exploration of various possible computation methods. It is interesting to note that these parallax corrections to the beginning and end of the eclipse, similar in structure to those already recorded for the time of mid eclipse, were not entered by John of Murs into the Escorial O II 10 manuscript.

### *3 Temporality and Execution of the Computation: The Manuscript Page as Working Space*

A look at the various numbers so far presented suggests that there is another important aspect to this document: the manuscript page was used in the actual performance of at least parts of the computation. John of Murs corrects himself, returns to choices made in the initial computation, and adds marginal notes to his computation. This text is not cold and distant; it does not look like a clean copy of a finished computation. It is close to the original computational performance. The text carefully selects the recorded information according to John of Murs's interests; the selections are made in real time as the computation is being performed. I will argue that some parts of the computation were made if not only, at least mostly, on the manuscript. In this respect it is interesting to see that, when a number is written outside of the numerical layout parts of the text, units are noted; inside the numerical layout parts they are indicated only by position. This fact follows from the way sexagesimal computations of the period are described, either in sexagesimal algorisms like John of Lignères *Algorismus minutiarum* or in the

first chapters of canons like those of the *Tabule magne* from the same author.<sup>16</sup> This suggests that the numerical layout parts of the text can be used as space where the computation is actually performed.

Certainly Escorial O II 10 was not the only computational tool and writing support John of Murs used. He must have had in front of him codices with astronomical tables, at least a version of the Parisian Alfonsine Tables, those of John of Lignères for 1321 and material from the Toledan Tables, which he quoted explicitly, and possibly some others which he was evaluating. It is likely that he had other writing supports, perhaps wax tablets, a dust board or just rough paper in order to perform some of his computations. Such artefacts are commonly referred to in the canons of astronomical tables. My goal here is to understand how Escorial O II 10 was used as John performed his computation along with those other tools.

In places where the procedure, although perhaps long, presented no particular problems for John of Murs, Escorial O II 10 presents only a selected sample of the more complete set of values John must have written on another writing support. Such is the case, for instance, with the first computation of the true position of the Sun at the time of mean conjunction (see Figure 5). The manuscript here mentions *dupla signis*, indicating that John used signs of 60° rather than 30°. This comment may also point toward John of Saxony's canons for the Alfonsine Tables or to John of Lignères's *Quia ad inveniendum...* as tools being employed in the computation. To obtain the first values recorded in the manuscript (solar and lunar mean motions) John of Murs probably used Alfonsine mean syzygy tables which would have given him the time and the mean motions directly. John then computed the equation of the Sun as 1;0,52° from its mean argument given just above as 5 s 31;2,38° undoubtedly interpolating from a solar equation table with argument 5 s 31° and 5 s 32° (see Figure 9). These computations, not recorded in Escorial O II 10, include reading tables and adding, subtracting, multiplying and dividing (he also could have used tables of proportions). The values I find with the Parisian Alfonsine Tables match exactly those of John of Murs.

		°				
Table reading	Equatio solis					
	5s 31°	1	0	57		
	5s 32°	0	58	59		
Interpolation	Operand transformed		59	117		
	Differentia	0	1	58		
	Residui		2	38		
	2 x 1		0	2		
	38 x 1			0	38	
	2 x 58			1	56	
	38 x 58				36	44
	sum	0	3	130	44	
	proportio ad 60	0	5	10	44	
Final equation						
		1	0	57		
	Operand transformed	1	0	57		
	proportio ad 60		0	5		
	equatio solis	1	0	52		

**Figure 9.** My tentative reconstitution of John of Murs's solution of the solar eclipse

The computation necessary to obtain the other solar-related values in Figure 5 require the same or even more background computation.<sup>17</sup> Thus in this portion of the text the rhythm at which John of Murs turned to the Escorial manuscript to record intermediate values is quite slow. He is most of the time dealing with the other computing tools of his working environment; only occasionally did he return to the Escorial manuscript to enter a significant step in the computation. However when he gives the true

<sup>16</sup> The *Algorismus minuciarum* is transmitted in 19 manuscripts and edited (Busard, 1968). The canons to the *Tabule magne*, not yet edited, can be found in the BnF, lat. 7281, f. 201v–205v; BnF lat. 10263, f. 70r–78r; and Erfurt 4° 366, f. 28r–32v.

<sup>17</sup> The computation of the solar apogee is a lengthy process that includes determination of precession.

longitude of the Sun as  $1\text{ s }1;35,24^\circ$  he is simply adding the equation just obtained to the value of the mean Sun obtained earlier. It is possible that in this case the Escorial manuscript was used as the actual support of the computation. The two values to be added are written closely to each other on the Escorial manuscript. Had John of Murs used a wax tablet or dust board for the Escorial hidden computations, the solar mean motion could have been erased when the solar equation appeared; or had John used rough paper, the two values could have been written far apart each other, perhaps on different pages, and thus would have been less easy to aggregate than on the Escorial manuscript.

Places where the Escorial manuscript is closer to the actual performance of the computation for a longer period of time can also be found, for instance at the beginning of the computation of the true syzygy (see Figure 3, l. 12-16):

- $3;43,40^\circ$  is simply the longitudinal difference between the true Sun (Figure 3 l. 8) and the true Moon (Figure 3, l. 10).
- $0;18,38^\circ$  is the 12 part of  $3\text{ s }43;40^\circ$  which in sexagesimal arithmetic is the result of a multiplication by 5 and a shift of position.
- $4;2,18^\circ$  is the sum of the two preceding lines<sup>18</sup>
- $2;1,9^\circ$  is the half of  $4;2,18^\circ$ <sup>19</sup>
- Finally,  $4\text{ s }39;16,41^\circ$  is the difference of the argument of the Moon (Figure 3 l. 3) and  $2;1,9^\circ$ .

Interestingly John of Murs made a little mistake here; the final difference should be  $4\text{ s }39;13,41^\circ$ . This mistake might imply that the Escorial manuscript was here the writing support of the actual computation, showing that the computation was made sexagesimal place by sexagesimal place.<sup>20</sup> This part of the true syzygy computation adjusts the mean argument of the Moon to a specific value before the table of lunar velocities is used according to the al-Battani method for finding true syzygy. It is a convoluted point of the computation, which is why John of Murs probably was more precise in his text here than when he computed (more routinely) the true solar and lunar longitudes. John wanted to record traces of the choices he made there. However the general situation is not much different: instead of recording a key value on the Escorial manuscript by copying and rounding, John directly performed the delicate portion of the computation on the manuscript page.

John of Murs's small computation mistakes get lost in the next three steps of the computations. There are many different lunar velocities tables that John of Murs could have used (Goldstein, 1992). However the value  $0;32,17^\circ/\text{h}$  (Figure 3 l. 17), computed using  $4\text{ s }39;15,41^\circ$  as argument, undoubtedly derives from al-Battani's lunar velocity tables which produce  $0;32,16^\circ/\text{h}$ , very close to John of Murs value (Pedersen, 2002, p. 1412).

John's next two values give alternative results for the time interval between mean and true syzygy (Figure 3. 1.18-1.19). Both require several steps of computation that John did not record in the manuscript, probably because they were not problematic to him in this specific context. The value  $7;30,20\text{ h}$  on line 18 is the result of the division of line 14 by line 17, i.e., the time to true syzygy according to al-Battani's procedure. It is not clear to me what is the method John described by the expression *per superatio*.

---

<sup>18</sup> The 13/12 factor comes from Ptolemy's method (*Almagest* VI,4) for finding true syzygy and approximates the relative speed of the Sun and the Moon.

<sup>19</sup> The  $\frac{1}{2}$  factor comes from al-Battani's method for finding true syzygy and approximates the non-constant speed of the Moon between the mean and true syzygy by taking the speed of the moon at midpoint between mean and true syzygy.

<sup>20</sup> Of course another interpretation might be that the computation was made on another paper and that the mistake occurred during copying into the Escorial manuscript.

The two results differ only by 20 s of time, i.e., at least one order of magnitude beyond what John of Murs might wish to observe and, more importantly, below 1 min of arc. He keeps the lowest value of the time correction for the remainder of the computation. This is clear because the next value 14 s 1;19,50 h (Figure 3 l. 20) is the difference between 14 s 8;49,50 h (Figure 3. l. 1, the time of mean conjunction) and 7;30,0 h (Figure 3 l.19, the time correction from mean to true syzygy). These figures can be interpreted in the following way. John of Murs had planned in his initial computation to compare the two methods for finding the time correction. Seeing that they were in good agreement he decided to keep only one value. This point of comparison was important enough, however, for him to record both values in Escorial manuscript. This reveals a very important fact about John's computational performance. Not only does he know several methods to compute the same value but he compares the methods and cross-checks his result. The branching in the computation does not break the flow of the computation; the comparison of the *superatio* method and the al-Battani method seems to be planned in the initial computation.

John's determination of the equation of time 0;20,54 h (Figure 3 l.21) shows a different kind of branching in the computation, pointing to a problem in Alfonsine astronomy that will remain debated until the early 16th century. At the end of the line, the expression *si est bene* is followed by a little tie-mark './' which points to the bottom of f. 92v where we read *Secundum J. de linieris 21' 16'*.<sup>21</sup> The position of this additional marginal information suggests that when it was recorded most of the page had already been filled up writing. Here John of Murs again compared two possible values for the same quantity. However the situation is different; it seems more than likely that he considered John of Lignères's value only at a later stage, maybe even after he had observed the solar eclipse. The branching in the computation corresponds here to a fold in the temporal flow of the computation, i.e., the computation continues with the value 0;20,54 h (John of Murs uses this value at least once below l. 21). Only later did John came back and consider another possible value (differing by only 22 s!) for the equation of time.

This analysis of the temporal dimension of John's computation shows that he did not try to perform the computation as quickly as possible. Rather, he took his time, exploring possibilities and comparing methods by performing the computation in a structured way. Computation, for John of Murs, was not a rigid, mechanical activity; it was a form of mathematical thought and exploration. As such and independently of the specific numerical values, computational procedures were important to remember in the long term, both for John of Murs and for his followers, a conclusion suggested by the numbers jotted into the Escorial manuscript.

#### 4 Conclusion

The Escorial O II 10 notes of f. 92v–93v is a text designed by John of Murs to explore the procedures of solar eclipse computation and to identify the key choices in an Alfonsine algorithmic context; the notes are not merely a record of eclipse observations. First, it is not the case that the Escorial manuscript luckily happened to be there with empty writing space just when John of Murs needed a few pages to quickly compute a solar eclipse. Rather, John carefully used the conventions of his scholarly manuscript culture to construct his text. Second, John's text is not primarily about observation but rather about the correct way (*verum* according to the explicit) to perform an eclipse computation. This aspect can be seen from the way the text is shaped, with two different types of layout that emphasize specific aspects of the structure of the computation and the places where important choices must be made. Third, by analysing some of the numerical values in the text I have argued that the selectively recorded values reflect John's concern for both his personal records as well as what he believed should be remembered in the tradition,

---

<sup>21</sup> Like the preceding alternative, the discrepancy between the two values envisaged by John of Murs differ by less than a minute, indicating John Murs's standard of accuracy in this context.



i.e., for other readers, both present and future. He undoubtedly worked with several computational tools (sets of tables) and more than one writing support and performed the eclipse computations at different temporal rhythms according to his different interests as he wielded the tools. Escorial O II 10 thus shows us an early Alfonsine mathematical astronomer at work.

### **Acknowledgements**

For comments on earlier drafts of this article, I would like to thank José Chabás and Richard Kremer as well as the *Centaurus* reviewers. I am also grateful to each participant of the workshop ‘How do writings in the astral sciences document mathematical practices and practitioners’ (January 2015, Paris, SAW project) in which this work was initiated. This research was supported by the European Research Council under the European Union's Seventh Framework Program (FP7/2007-2013)/ERC Grant agreement n. 269804, in the context of the project SAW: *Mathematical Sciences in the Ancient World*.

### **Bibliography**

Beaujouan, G. (1964) *Histoire des sciences au Moyen Âge*, in: *Ecole Pratique des Hautes Etudes* (ed.) *Ecole pratique des hautes etudes IV<sup>e</sup> section* (Paris: *Annuaire 1964–1965*), pp. 259–262.

Beaujouan, G. (1974) *Observations et calculs astronomiques de Jean de Murs*. *Actes du 14e congrès international d'histoire des sciences*, pp. 27–30.

Busard, H. L. L. (1968) *Het rekenen met breuken in de middeleeuwen, in het bijzonder bij Johannes de Lineriis* (Brussels: *Paleis der Academiën*).

Chabás, J. and Goldstein, B. R. (1997) *Computational astronomy: five centuries of finding true syzygy*, *Journal for the History of Astronomy*, 28, 93–105.

Chabás, J. and Goldstein, B. R. (2003) *The Alfonsine Tables of Toledo* (Dordrecht: *Kluwer Academic Publishers*).

Chabás, J. and Goldstein, B. R. (2012) *A survey of european astronomical tables in the late middle ages* (Leiden: *Brill*).

Goldstein, B. R. (1972) *Theory and observation in Medieval Astronomy*, *Isis*, 63, 39–47.

Goldstein, B. R. (1992) *Lunar vlocity in the ptolemaic tradition*, in: P. M. Harman and A. E. Shapiro (eds.) *The investigation of difficult things: essays on Newton and the history of exact sciences* (Cambridge: *Cambridge University Press*), pp. 3–17.

Guillermo, A. (1911) *Catálogo de los códices latinos de la Real Biblioteca del Escorial* (Helenica: *Madrid Impr.*).

Gushee, L. (1970) *New sources for the biography of Johannes de Muris*, *Journal of the American Musicological Society*, 22, 3–26.

Nallino, C. A. (1899–1907) *Al-Battani sive Albatanii opus astronomicum*, 3 vols (Milan: *U. Hoepli*).

*Pedersen, F. S. (1979) Petrus de S. Audomaro tractatus de semissis, Cahier de l'Institut du Moyen Âge grec et latin, 33, 1–116.*

*Pedersen, F. S. (2002) The Toledan tables: a review of the manuscripts and the textual versions with an edition (Det Kongelige Danske Videnskabernes Selskab: Copenhagen).*

*Pedersen, F. S. (2014) William of Saint-Cloud, Almanach Planetarum: an edition of the canons, a few samples from the tables and a foray into the numbers, Cahiers de l'Institut du Moyen Âge grec et latin, 83, 1–133.*

*Porres, B. and Chabás, J. (2001) John of Murs's Tabulae permanentes for finding true syzygies, Journal for the History of Astronomy, 32, 63–72.*

*Poulle, E. (1973) John of Murs, in: C. Gillispie (ed.) Dictionary of scientific biography, Vol. 7 (Detroit: Charles Scribner's Sons), pp. 128–133.*

*Poulle, E. (1976) William of Saint Cloud, in: C. Gillispie (ed.) Dictionary of scientific biography, Vol. 14 (Detroit: Charles Scribner's Sons), pp. 389–391.*

*Poulle, E. (1981) Les sources astronomiques (textes, tables, instruments) (Turnhout: Brepols).*

*Poulle, E. (1984) Les tables alphonsines avec les canons de Jean de Saxe (Paris: edition du CNRS).*

*Poulle, E. (2005) Les astronomes parisiens au XIVe siècle et l'astronomie alphonsine, Histoire Littéraire de la France, 43, 1–54.*

*Saby, M.M. (1987) Les caons de Jean de Lignièrès sur les tables astronomiques de 1321. Unpublished thesis, Ecole nationale des Chartes, Paris.*

# Two versions of a description of the armillary sphere in Parameśvara's *Goladīpikā*

Sho Hirose \*

*Abstract.* Armillary spheres had been part of the Sanskrit astronomical tradition, and were used for understanding the structure of the heavens. *Goladīpikā* ('Illumination of the sphere') is a text in two versions by the same author which deals with structures of the armillary sphere and various topics in astronomy related to them. A close examination of the ways the armillary sphere is described in the two versions of the text will help us understand the main characteristics of the two versions of Parameśvara's *Goladīpikā* and the reasons why the author duplicated his treatise. This case study thus demonstrates how astral sciences sources from the same author may present mathematical practices surrounding the same instrument in contrasting and complementary ways according to intention.

*Keywords.* Armillary sphere, astronomical instruments, astronomy in India, manuscript, Parameśvara, Sanskrit

## 1. Introduction

Parameśvara (c.1360-1460) is an important figure in the Kerala school, a scholarly lineage of mathematicians and astronomers that flourished in the South Indian region of Kerala. He is an author and commentator on almost every discipline in *Jyotiṣa* (mathematics, astronomy and astrology). Most of his works survive in more than one manuscripts, indicating his great influence on later generations. One of his important contributions is the introduction of a new system of astronomy represented by his *Dṛggaṇita* (Observation and computation). Parameśvara mentions in this treatise that his aim is to make

---

\*Laboratoire SPHERE UMR 7219, Université Paris Diderot, Case 7093, 5 rue Thomas Mann, Paris 75251, France. E-mail: sg.kippis@gmail.com

The research leading to these results received funding from the European Research Council under the European Union's Seventh Framework Program (FP7/2007-2013)/ERC Grant agreement n. 269804 and was conducted in the context of the project SAW: Mathematical Sciences in the Ancient World (SAW).

computation agree with observation (Sarma [1963]). Such emphasis on observation is very rare in Sanskrit sources.

The only surviving records of his observations are those of eclipses given in his *Siddhāntadīpikā*, a super-commentary on Govindasvamin's commentary on Bhāskara I's *Mahābhāskarīya* (Kuppanna Sastri [1957]), where he measures the position of the sun with a gnomon. The shadow of the gnomon was an observational data which could be converted into parameters such as the longitude of the sun. Knowledge on celestial spheres is required for such computation. Concerning this topic, Parameśvara composed the *Goladīpikā*, literally 'Illumination of the sphere'. Parameśvara wrote two texts under this title, which have similar contents in different structures. The most notable common point is that both texts begin with a description of an armillary sphere.

In the *Goladīpikās*, the armillary sphere serves as a tool for explaining various circles in the sky used for locating heavenly objects. A name of a specific ring in the instrument is also used to address the corresponding celestial circle. Without knowing the rings' names, positions and motions as given in the beginning of the text, the rest of the treatise is incomprehensible. Taking into account its significance, we shall focus on this introductory part and compare the contents and styles of the two *Goladīpikās*.

Is there any difference in how the texts introduce the rings and shape the armillary sphere? What are the roles of the sections in both texts? We will try to answer these questions which will provide a clue to the final question: Are the two *Goladīpikās* different texts for different purposes, or was one of them a revision of the other?

## 2. The two *Goladīpikās*

Both of the two *Goladīpikās* are found in many manuscripts, most or all of them written in Malayalam script<sup>1</sup> on palm leaves<sup>2</sup>. I shall follow Sarma [1972] who numbered them *Goladīpikā 1* (hereafter *GD1*) and *Goladīpikā 2* (hereafter *GD2*). *GD1* was edited and translated by Sarma [1956-57] while *GD2* was edited by Sastri [1916].

As we will see in section 2.3, we do not know whether they were composed in this order.

### 2.1. *Goladīpikā 1 (GD1)*

*GD1* has 267 verses divided into four chapters. The segmentation was obviously intended by the author himself, as can be seen from the fact that every manuscript has a colophon giving the titles of the chapters at each end and that Parameśvara composed an auto-commentary indicating the same division. The auto-commentary is included in the critical edition of Sarma [1956-57].

---

<sup>1</sup>Every manuscript that has been available to me is written in Malayalam script. Among the manuscripts that I have not investigated, R.5192 of the Government Oriental Manuscripts Library (GOML) is written in Grantha script according to Pingree [1981], while Sarma [1956-57] who has incorporated its content in the edition mentions that it is in Malayalam.

<sup>2</sup>Most of the palm leaf manuscripts of the *Goladīpikās* are bundled with other texts which are often also *Jyotiṣa* treatises but sometimes of a different genre like poetry.

Chapter 1 (15 verses), called ‘Rule for constructing the sphere (*golabandhavidhi*)’ is an introduction devoted to the armillary sphere. In chapter 2 (50 verses) ‘Rule of planetary motion (*grahacāraavidhi*)’ the motion of planets along the circles given in the previous chapter, as well as the nature of the Earth, sun and moon, are explained. Chapter 3 (110 verses) ‘Thoughts on the Earth and the like (*bhūmyādicintana*)’ deals with the shape and size of the Earth with a detailed explanation of traditional cosmography in Hinduism integrated into the theory of a spherical Earth. Finally, the untitled chapter 4 (92 verses) mentions a variety of topics in astronomy that require calculation, including the gnomon, parallax, eclipses and precession. One of its verse is an example where the reader is required to perform some computations. In the last two verses (Chapter 4 91 and 92) the author mentions his name Parameśvara, that the treatise was composed in the Śaka year of 1365 which corresponds to 1443 CE and that he lives in a village whose Sine of latitude is ‘647’<sup>3</sup>, which is 10 deg 51’ in degrees. This value 647 is also used in the example of computation.

There are nine known manuscripts of *GD1*<sup>4</sup> and two of its auto-commentary<sup>5</sup>. Among the manuscripts which were available to me, the oldest one (762E) probably dates back to around the 17th century from its style of folio numbering<sup>6</sup>. The others are relatively new, possibly copied in the 19th century. There are no marginal notes, but 5864A, 8358B, 762E of the ORI& MSS and 13719 Baroda contain corrections by a later hand<sup>7</sup>.

## 2.2. *Goladīpikā 2 (GD2)*

*GD2* is composed of 302 verses<sup>8</sup> and has no chapters. The topics included in the first 67 verses are: the armillary sphere, various circles in the heavens, nature of heavenly objects, cosmology and large time periods. In verses 68 and 69 the author mentions his name, concludes the previous verses (‘thus the nature of the spheres was stated

<sup>3</sup>This is the Sine of latitude measured in a circle with a radius of 3438.

<sup>4</sup>I have examined 5864A, 8358B and 762E of the Kerala University Oriental Research Institute and Manuscripts Library (ORI& MSS), 13719 of the Maharaja Sayajirao University of Baroda Oriental Institute, and Burnell 17d of the British Library. Sarma [1956-57] mentions four more: L.1313B and T.341 in the ORI& MSS, which are lost, R.5192 of the GOML in Madras and ‘a transcript with Sri G. Harihara Sastri, Madras’ which I have not confirmed. The last transcription is probably a copy of 13719 Baroda which was sold to the institute by the same ‘G. Harihara Sastri’ and contains the same variant readings. Pingree [1981] further adds another manuscript, 3337 in volume 2 of the Catalogue of Sanskrit Manuscripts in the Punjab University Library, Lahore, which I have not confirmed.

<sup>5</sup>762F of the ORI& MSS and R.5145 of the GOML (unconfirmed).

<sup>6</sup>Personal communication with P.L. Shaji in 2013.

<sup>7</sup>Scripts are scratched on palm leaves and black powder with oil is applied afterwards for reading (Kumar et al. [2009]). Newly made corrections have none or less powder rubbed in the scratches and are easily recognizable. Corrections found in *GD1* and *GD2* manuscripts are emendations of scribal errors and do not provide new interpretations.

<sup>8</sup>This is the number given by the critical edition to the final verse, but the editor did not number every verse. There are two half-verses in separate places, and we have to assume that the editor skipped one and counted the other, or that he collectively counted them as one.

concisely by Parameśvara<sup>9</sup>) and introduces remaining topics ‘on the spheres’<sup>10</sup> such as ‘application of gnomons and the like’<sup>11</sup>. Here he also refers to his previous work, the *Siddhāntadīpikā*. Verses 68 and 69 separate the previous verses from those remaining which contain every topic appearing in chapter 4 of *GD1*, as well as new topics such as epicycles of planetary orbits and the computation of planets’ latitudes. There are six verses later in the text that contain examples, all of which use 647 as the Sine of latitude. This is equal to the value given in the example of *GD1* as well as the location of Parameśvara’s village mentioned therein.

There are eleven known manuscripts of *GD2*<sup>12</sup>, two of which include a partial commentary<sup>13</sup> by an unknown author<sup>14</sup>. One manuscript (475J) was copied in 1553 CE<sup>15</sup> while the others appear to be newer, probably copied in the 18th or 19th century. There is no commentary for the verses which we will deal with in this paper. Every manuscript that I have examined contains no marginal notes but shows corrections by a later hand, much like *GD1*. A typical example is in the latter half of verse 2, where two manuscripts (17945B and C.224F) wrongly copied *yāmyasaumyayor* (at south and north) instead of *saumyayāmyayor* (at north and south), but manuscript 17945B was later corrected<sup>16</sup>. This implies that the reader/corrector was reflecting upon the situation described in this text or had expert knowledge on the topic and payed attention to the meaning.

### 2.3. Revision or different work?

As we have seen in the previous sections, the authorship of the two *Goladīpikās* is attributed to the same Parameśvara. They have seven full verses (including two quotations from other works) and seven half verses in common. There are about a dozen half verses which are merely simple paraphrasing. This leads to the possibility that the author intended to revise his work, using parts of the former version. Parameśvara is known to have repeatedly worked on some topics, sometimes making revisions of the same content.

For instance, there are two versions of his *Grahaṇamaṇḍana* (Ornament of eclipses) left in manuscripts, one in 89 verses and the other in 100. Sarma [1965] mentions that the eleven additional verses do not introduce much by way of new material, and that

<sup>9</sup>*paramādinoktam evaṃ saṃkṣepād īśvareṇa golasya saṃsthānaṃ (GD2 68)*. Here the name is given in the form *paramādi īśvara*, which was common with this author; c.f. his commentary on the *Āryabhaṭṭīya* (Kern [1874, p.1, 100]) and on the *Sūryasiddhānta* (Shukla [1957, p.144]).

<sup>10</sup>*golagatam (GD2 68)*

<sup>11</sup>*yuktiḥ ... śaṅkvādeḥ (GD2 69)*

<sup>12</sup>I have examined 475J, 5867A, 8327A, 10583A, 13259A, 17945B, C.224F, C.1024D of the ORI& MSS and Burnell 107b, Burnell 17c and IO Sanskrit 3530 of the British Library. Pingree [1981] further lists L.1313A of the ORI& MSS which is lost. Sastri [1916] uses three manuscripts but does not provide any kind of catalog number. Based on textual analysis, I assume that two of them are 10583A and C.224F.

<sup>13</sup>Manuscripts 13259A and IO Sanskrit 3530

<sup>14</sup>Considering the style of the sentence, it is unlikely to be Parameśvara himself.

<sup>15</sup>The colophon of 475A, contained in the same bundle with 475J and written by the same hand, mentions that the date of copying was 1,699,817 days after the beginning of the *Kali Yuga*, which amounts to December 23rd, 1552. 475J must have been copied around this date too.

<sup>16</sup>C.224F was left uncorrected, and in the critical edition Sastri [1916] adopted its wrong reading.

the differences in the remaining verses are minor. Sarma concludes that the work was first composed with 89 verses and later revised with the addition of eleven verses.

Meanwhile, the *Dr̥ggaṇita* is divided into two parts and the first half of the second part begins with a restatement of part I using a different numeral notation (Sarma [1963]). In the introductory verse of this second part Parameśvara announces that his purpose is ‘to clarify (*spaṣṭīkartum*) the *Dr̥ggaṇita*’ and that it is ‘prepared for young people’s studies (*balābhyāsahitaṃ*)’. In this case the second part is not a mere revision but a text intended for different students in different levels.

Some aspects of the two *Goladīpikās* suggest that they might also be individual texts for different curricula: *GD1* is segmented and arranged while *GD2* is continuous; *GD1* has a commentary for elucidation by the author himself while *GD2* is mostly un-commented; *GD2* contains more topics on computation than *GD1*.

Both texts were copied and read in many manuscripts, and there is no sign that one was more popular than the other. The manuscripts of *GD1* and *GD2* are preserved in different bundles with only two exceptions<sup>17</sup>, suggesting that they spread independently as two different treatises.

Sarma was the first to reflect on the two versions of the *Goladīpikā*. He does not mention whether one is a revision of the other, but he seems to think that *GD2* was composed later, as he writes ‘In the *Goladīpikā* published in the Trivandrum Sanskrit Series (= *GD2*), ... some topics like cosmogony are left out; others, like the conception of the *yuga-s* and calculation of the latitudes of planets, are newly introduced’ (Sarma [1956-57, page 3]). Probably this is the reason why he numbered them *GD1* and *GD2* in his survey (Sarma [1972])<sup>18</sup>. Sarma gives no arguments for their order otherwise. *GD2* is undated and we have no definitive clue for the chronological order of the two texts. Pingree [1981] comments that *GD2* refers to *GD1*<sup>19</sup>, but this is not correct.

In the following I will argue that these two texts are two independent treatises, treating the same topic in two distinct fashion. This, we will see, can be revealed by the different ways in which they treat the armillary sphere.

### 3. The structure of an armillary sphere

The Sanskrit word used for ‘armillary sphere’ in *GD1* and *GD2* is *gola*, which can refer to all kinds of spheres such as celestial spheres, heavenly bodies with the form of a sphere, or even the name of a topic in astronomy or cosmography concerning them. Treatises often refer to an extremely complex system of ‘*gola*’, such as the system with fifty-one moving circles in Brahmagupta’s *Brāhmasphuṭasiddhānta* (Ikeyama [2002]), which

<sup>17</sup>L.1313B and L.1313A of the ORI& MSS which is lost and cannot be confirmed, and Burnell 17d and Burnell 17c of the British Library which were copied for A.C. Burnell (Keith [1935])

<sup>18</sup>Meanwhile he first numbered the texts in reverse order (Sarma [1960], Sarma [1963], Sarma [1965]), possibly due to the order the editions were published

<sup>19</sup>‘A *Goladīpikā* in 302 verses in which Parameśvara refers to his first *Goladīpikā* and his *Karmadīpikā* on the *Mahābhāskarīya*.’ (page 191) However, *GD2* 69 refers to the *Siddhāntadīpikā*, Parameśvara’s super-commentary to the *Mahābhāskarīya*, but not to the *Karmadīpikā* which is his direct commentary on the treatise.

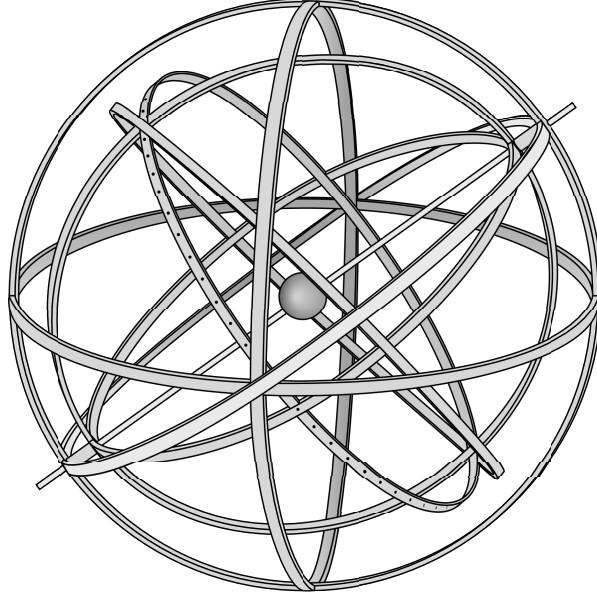


Figure 1: A representation of the entire armillary sphere (All representations are drawn by the author and do not appear in manuscripts and critical editions)

is unlikely to have been actually built. Meanwhile, whenever the word *gola* is used in combination with *yantra* (instrument), or has some reference to its material (typically wood or bamboo), the object described is much simpler<sup>20</sup>.

The armillary sphere appearing in *GD1* and *GD2* consists of two layers of rings connected by an axis (Fig. 1). The inner set of rings showing the coordinates of stars and planets revolves on the axis while the outer set of rings are fixed and represent the observer's horizontal coordinate. This double-layered armillary sphere appears to have been common, and can be seen in older texts such as the commentary on the *Āryabhaṭīya* by Bhāskara I (629 CE), the *Śiṣyadhīvrddhidatantra* (8th century) by Lalla, the later *Sūryasiddhānta* (c. 800 CE), the *Siddhāntaśekhara* (1039) by Śrīpati and the *Siddhāntaśiromaṇi* (1150) by Bhāskara II.

In most of the texts, including the two *Goladīpikās*, the armillary sphere is only used for demonstration (in all senses of the word, that is to exhibit properties, manifest and prove them)<sup>21</sup>.

The following description of an armillary sphere applies not only to *GD1* and *GD2* but also to the aforementioned treatises in general.

The inner set of rings called the 'stellar sphere (*bhagola*<sup>22</sup>)' (Fig. 2) contains three

<sup>20</sup>For example, the *Śiṣyadhīvrddhidatantra* has a chapter labeled *yantra* where among other instruments a *gola* made of wood with only two sets of circles is described, while in another chapter called *golanibandha* (constructing the *gola*) the *gola* is complex and has fifty-one moving circles like the *Brāhmasphuṭasiddhānta*.

<sup>21</sup>However, Ohashi [1994] quotes from the *Siddhāntaśekhara* what appears to be a measurement of the sun's position in the sky using an armillary sphere.

<sup>22</sup>Each part of the armillary sphere is often called by different Sanskrit terms in different texts and



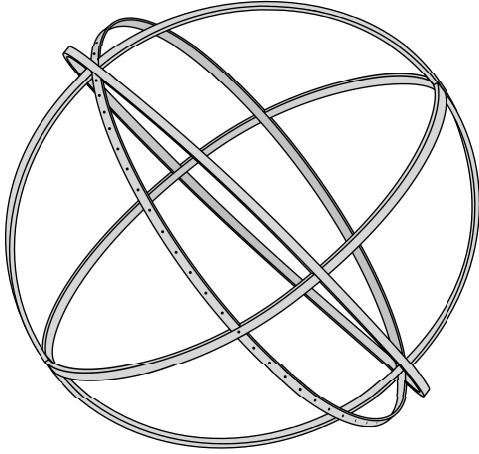


Figure 2: Stellar sphere

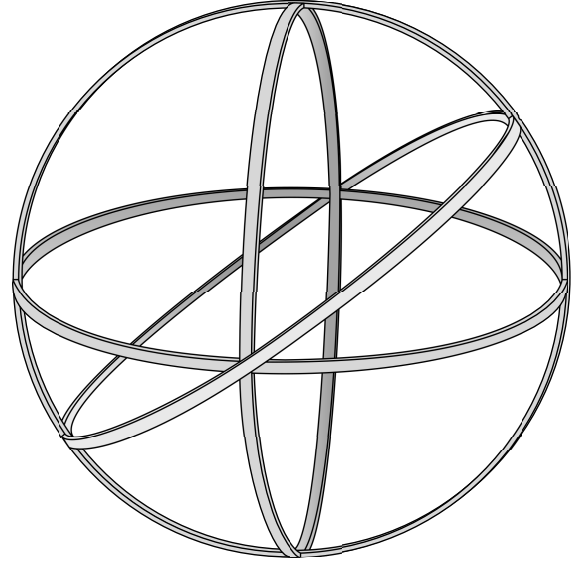


Figure 3: Celestial sphere

rings representing the equatorial coordinates: celestial equator (*ghāṭika*), solstitial colure (*dakṣiṇottara*) and equinoctial colure (*viṣuvat*). A fourth ring tilted 24 degrees against the celestial equator represents the ecliptic (*apama*), the path of the sun in a solar year. Each of the rings are graduated: The celestial equator has 60 marks, each representing one *ghaṭikā*, a time unit equivalent to one sixtieth of a day. The two colures and the ecliptic have gradations for 360 degrees. Optionally, diurnal circles (*svāhorātra*) parallel with the celestial equator that are approximations of the path of the sun on a given day<sup>23</sup> and orbits of the moon and other planets (*vikṣepamaṇḍala*) can be added. An axis (*daṇḍa*) pierces the stellar sphere in the two celestial poles, i.e. the intersections of the two colures, so that the whole sphere can rotate to represent the geocentric motion of heavenly objects. A miniature Earth made of wood or clay is placed in the middle of the axis (see Fig.1).

The outer set of rings, or the ‘celestial sphere (*khagola*)’ (Fig. 3) represents the horizontal coordinates with the prime vertical (*samamaṇḍala*), the prime meridian (*dakṣiṇottara*) and the horizon (*kṣitija*). The polar axis carrying the stellar sphere is attached to the prime meridian, tilted so that the celestial north pole is elevated against the horizon by an angle corresponding to the local latitude. Two pieces of reed on the axis separate the two sets of rings. Finally a fourth ring is attached to the celestial sphere so that it goes through the horizon at the east and west and through the two tips of the axis. This is the six o’clock circle (*unmaṇḍala*)<sup>24</sup>.

---

more than one term is used even within the works of Parameśvara. In such cases I only give the term most frequently used by Parameśvara.

<sup>23</sup>An approximation in the sense that the sun is assumed not to move along the ecliptic in the course of that day. Otherwise it could not form a single closed loop.

<sup>24</sup>The modern term is due to the fact that the sun will always pass this circle at six o’clock in the morning and in the evening, but the Sanskrit term is not at all related to it.

## 4. Comparing the descriptions of *GD1* and *GD2*

The first chapter in *GD1* is about the armillary sphere (1.3-1.15<sup>25</sup>, excluding introductory invocation), and likewise the *GD2* refers to it at the beginning (2-17<sup>26</sup>).

Both texts seem to describe the same and physically realizable armillary sphere. For instance, they both refer to the material of the miniature Earth<sup>27</sup>. The structure of the instrument, as well as the terminology used for describing the rings and their orientation are also identical. Later in the texts, the same terminology is used for introducing new arcs and segments<sup>28</sup> that form various figures, especially right triangles. Such visualization might have served as a grounding for computations appearing in both *Goladāpikās*, most of which are rules of three or Pythagorean theorems.

Meanwhile, *GD1* and *GD2* describe the same instrument in contrasting styles and arrangements.

The corresponding Sanskrit texts and translations are provided for reference in the appendix. For *GD1* (Appendix A) I have used the Sanskrit edition by Sarma [1956-57], but modified the translation to make it more literal. The Sanskrit text for *GD2* (Appendix B) shall be based on Sastri [1916] with amendments (based on my examination of manuscripts) given in the footnote. I have extracted the translations for *GD2* from my forthcoming doctoral dissertation which will provide its first complete English translation and a new critical edition of the text.

### 4.1. Description of the stellar sphere

There are two large differences between the two texts in the description of the stellar sphere: gradation and the order of rings.

*GD1* mentions the three orthogonal rings first, then describes how they are graduated and then introduces the ecliptic.

Here the celestial equator has 60 divisions.

Here the other two [circles] have 360 divisions. One should attach yet another circle called the ecliptic, likewise [having 360 divisions], passing through the east and west crosses, to the solstitial colure at 24 degrees north and south [respectively] from the [crosses at] the below and the top. (*GD1* 1.4d-1.6ab)

The auto-commentary explains the meanings of the gradations as follows:

... the celestial equator is marked with 60 lines. The use of marks is for knowing that it is the *Ghatikā*. ...the other two circles are marked with 360

---

<sup>25</sup>For *GD1* ‘1.3’ stands for the third verse of chapter one, ‘1.4’ for the fourth verse, and so on.

<sup>26</sup>For *GD2*, which has no chapter, the number indicates the verse number itself.

<sup>27</sup>‘clay and the like’ (*GD1* 1.10), ‘piece of wood or clay’ (*GD2* 6).

<sup>28</sup>For example, the sun’s declination is given as the separation between the celestial equator and the diurnal circle.

lines. The use of marks with these two is to know the units of 30 degrees<sup>2930</sup>.

The gradation for degrees in the solstitial colure could immediately be used in the next step for tilting the ecliptic 24 degrees against the celestial equator. Thus this passage, especially with the commentary, would have helped the reader assemble the rings, whether it be with his hands or in his mind.

In contrast, *GD2* mentions nothing about gradations on the rings. The inclination of the ecliptic is only mentioned as the ‘greatest declination’. Furthermore, the ecliptic is introduced after the solstitial colure and the celestial equator, without waiting for the third orthogonal ring (the equinoctial colure). This might be due to the fact that the ecliptic is far more important than the equinoctial colure. In fact the latter is never mentioned again once the description of the armillary sphere is completed<sup>31</sup>. The celestial equator is important because it is the reference for sidereal time<sup>32</sup> and the solstitial colure is the circle on which the ecliptic is separated from the celestial equator by its greatest declination, so there is good reason that these two come before the ecliptic.

In *GD1* the equinoctial colure plays a role in introducing the ecliptic: it produces two crosses in the east and west with the celestial equator, which are the points that the ecliptic has to pass through.

## 4.2. Introducing the latitude

*GD1* does not take into account the local latitude at the beginning, as if the observer were on the equator. It first describes the three orthogonal rings of the stellar sphere with their six conjunctions facing below, above and the four cardinal directions.

Here, a circle passing below, above, south and north is to be called the solstitial colure. There is also a circle inside it [attached to it at] the below and top, [passing through] the east and west, called the celestial equator. Outside them both horizontally should be another circle [producing] crosses in the four quarters. (*GD1* 1.3-1.4ab)

In this situation the ‘another circle’ (the equinoctial colure) is placed parallel to the horizon, and so is the polar axis which will pierce it at the north and south. Then the celestial sphere is introduced, aligned with the stellar sphere (Fig. 4). After that, the stellar sphere and the axis is tilted against the celestial sphere to represent the terrestrial latitude as in the following passage.

---

<sup>29</sup>Here a circle is divided into 12 signs each consisting of 30 degrees. Angles larger than 30 degrees in modern usage are always described using signs.

<sup>30</sup>*rekhāṇām śaṣṭyā aṅkitam ghaṭikāmaṇḍalam/ ghaṭikājñānārtham aṅkavidhiḥ// ...rekhāṇām śaṣṭyut-taraśatratrayeṇāṅkitam anyat maṇḍaladvayam/ triṃśāṃśakaparijñānārtham tayor aṅkavidhiḥ/*

<sup>31</sup>The word *viśuvat* which is used for the equinoctial colure alone and also as a collective name for the three orthogonal rings in the stellar sphere only appears in verses 4 and 5 of *GD2*. In *GD1* we can see it later in 2.18, but it is used to indicate the equinoctial point and not the circle itself.

<sup>32</sup>‘Here, the time in which a sixtieth of the celestial equator rotates is called a *nāḍikā*, not the sixtieth of a day, because a day is longer than a revolution of the [stellar] sphere.’ (*GD2* 9)

Thus should be the state of the sphere at a latitude-less location (equator). However for a given location, one should make two holes in the celestial sphere down and up from the south and north crosses [respectively] at the distance of the Sine of latitude and then make the axis of the celestial sphere pierce them. (*GD1* 1.11-1.12ab)

This is where the latitude is taken into account for the first time. The description of the six o'clock circle follows this in *GD1* 1.12cd-1.13ab.

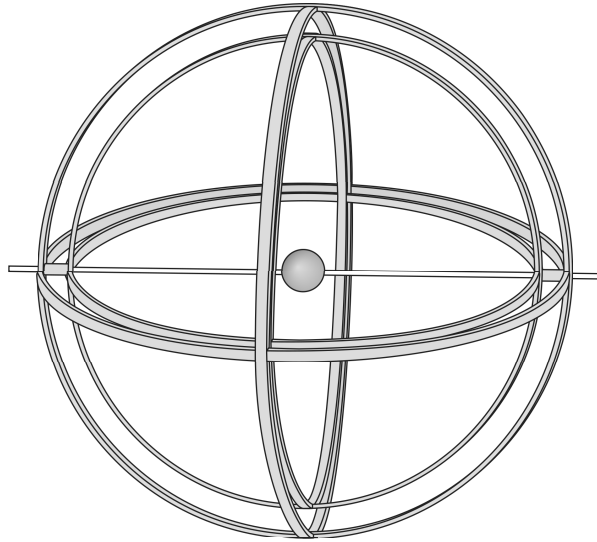


Figure 4: The celestial sphere put together outside the stellar sphere in exactly the same alignment.

By contrast, the celestial sphere in *GD2* is inclined in accordance with the latitude from the very beginning, as we can see from the position of the celestial equator in verse 2.

This circle going below, above, south and north is called the solstitial colure. The celestial equator is adhering at the tip of [an arc of] latitude north and south from below and above [respectively]. (*GD2* 2)

There is no reference to the state of the spheres at the terrestrial equator, and the word latitude does not appear again until the introduction of the six o'clock circle in *GD2* 14. Therefore, unlike *GD1*, the inclination of the axis is consistent throughout the explanation in *GD2*.

### 4.3. Function of the instrument and cosmology

*GD1* hardly ever explains the function or meaning of the rings. For example, the word ‘sun’ does not appear in *GD1* in chapter 1 nor in its commentary, despite the ecliptic

and diurnal circles being described. There is only some brief mention of the rotation of the spheres and their cause (a wind or pneumatic force called *pravaha*) in verse 14, after both the stellar sphere and celestial sphere have been described.

This stellar sphere continually rotates towards the west by the thrust of the *pravaha*. The celestial sphere should be completely still, for this is prepared in order to establish the directions and so forth. (*GD1* 1.14)

Meanwhile, *GD2* blends descriptions of rings with explanations on their functions or cosmological meanings. The sun is mentioned several times, where its motions conceived on the celestial circles as well as the resulting phenomena are explained:

The sun always moves eastward on the circle called the ecliptic. (*GD2* 4)

It is called the diurnal circle, the place of the revolution of the sun. Many of them exist, because for each day there is a difference in the motion of the sun. (*GD2* 10cd-11ab)

Therefore when the sun is to the north [of the celestial equator] the day is long and when to the south it is the night that is long. (*GD2* 16cd)

The rotation of the stellar sphere and the *pravaha* wind are mentioned right after the description of the stellar sphere, before the celestial sphere is introduced. By comparison with *GD1*, they are described in much greater detail.

The stellar sphere hurled by the *pravaha* wind goes clockwise around the Earth and rotates continuously towards the west in sixty *ghaṭikās*. (*GD2* 7)

The *pravaha* wind should have a constant movement towards the west above the Earth's surface at a distance of twelve *yojanas*. The wind of the Earth having a different movement is below it. (*GD2* 8)

Here, the time in which a sixtieth of the celestial equator rotates is called a *nāḍikā*, not the sixtieth of a day, because a day is longer than a revolution of the [stellar] sphere. (*GD2* 9)

Verses 7, 8 and 9 of *GD2* are identical with 2.2, 2.3 and 2.4 of *GD1* apart from a small paraphrasing in the last pair<sup>33</sup>. As we can see in this example, *GD2* guides the reader to cosmology from the very beginning, whereas *GD1* completely separates cosmological explanation from the description of the instrument in different chapters.

One manuscript<sup>34</sup> of *GD2* inserts quotations from the first verses of a chapter on *gola* (here in the sense of cosmology and geography) from the *Siddhāntaśekhara*<sup>35</sup> and the *Brāhmasphuṭasiddhānta*<sup>36</sup> right after the first invocation verse.

<sup>33</sup> *ghaṭikaṣaṣṭyaṁśasya bhramaṇe* in *GD1* 9 and *ghaṭikākhyasaṣṭibhāgabhramaṇe* in *GD1* 2.4, both meaning 'in which a sixtieth of the celestial equator rotates'.

<sup>34</sup> Indian Office 3530 of the British Library

<sup>35</sup> Verses 15.1-6

<sup>36</sup> Verse 21.1

The rotation of the planets and *nakṣatras* is not the same everywhere for the residents of the Earth. Because the knowledge about that is from the sphere, therefore I will explain the sphere. (*Brāhmasphuṭasiddhānta* 21.1)<sup>37</sup>

We can assume that the copier’s attention was drawn to cosmology from the very beginning of *GD2*.

#### 4.4. Forms of verb

Verbs conjugated in the optative<sup>38</sup>, which are frequently used in astronomical and mathematical texts to express prescription (Keller [2015]), are more frequently used in *GD1* than in *GD2*:

**One should attach** (*bādhnīyāt*) yet another circle called the ecliptic ... to the solstitial colure. (*GD1* 1.5bcd-1.6ab)

**one should make** (*kuryāt*) the axis of the sphere pierce them (*GD1* 1.12ab)

Meanwhile the author prefers nominal constructions that indicate a fixed situation in *GD2*:

The ecliptic **is adhering** (*lagnam*) at its greatest declination likewise from below and above. (*GD2* 3ab)

This axis of the sphere **goes through** (*yātas*) the crosses of the six o’clock circle and the prime meridian. (*GD2* 15ab)

This contrast gives the impression that *GD1* describes how to construct an armillary sphere while *GD2* describes the instrument in complete form from the beginning. However, there are two usages of the optative *kuryāt* in *GD2*. One is the description of the miniature Earth:

**One should make** (*kuryāt*) a uniformly round Earth located at the middle of the stellar sphere’s axis out of either a piece of wood or clay. (*GD2* 6abc)

It is interesting that this statement also refers to the material of the miniature Earth. This is the only reference to the material of any part in the armillary sphere that appears in *GD2*. We can only guess what Parameśvara’s intention was, but the reader’s attention would surely be drawn to the instrument.

The second *kuryāt* appears in the last verse concerning the armillary sphere in *GD2*:

---

<sup>37</sup>Translation from Ikeyama [2002]

<sup>38</sup>I have not included the optative of *as* (to be), *syāt*, which appears many times in both *GD1* and *GD2*. Although it may prescribe a certain situation, it does not directly ask the reader to take some action.

Or, having made the the celestial equator in the east-west direction, and having made another [circle?] according to it, **one should make** (*kuryāt*) an axis piercing the crosses of the six o'clock circle and the prime vertical. (*GD2* 17)

The meaning of the verse is ambiguous, but it most likely describes another ring, the representation of the celestial equator on the celestial sphere. Such a ring is not mentioned in *GD1*. In addition to the optative, the verse uses the expression 'or (*vā*)'. This implies that the ring is optional, and not necessarily included in the armillary sphere described in *GD2*, which appears to be complete from the beginning.

## 4.5. Describing the instrument and explaining the heavens

Let us summarize the differences between *GD1* and *GD2*.

*GD1* takes the form of a set of instructions to generate an armillary sphere. As we saw in the order of the equinoctial colure and the ecliptic, the rings are introduced so that the previously installed rings can help the reader to locate where to attach the new ones. The latitude is introduced by moving a part of the instrument. Verse 14, while being the only verse in chapter 1 to mention a cosmological element (the *pravaha* wind), can also be read as a guidance on the movement of the instrument – that the stellar sphere is constantly rotating while the celestial sphere is fixed.

Meanwhile, *GD2* focuses more on explaining the circles in the heavens and motions of objects, especially the sun, taking place on the circles. There are some phrases which enable us to imagine the armillary sphere, such as the description of the miniature Earth. Unlike *GD1* which seems to be instructing the reader on how to construct the instrument, the armillary sphere in *GD2* is described as if it were complete: rings are mentioned for their significance rather than for their role in construction and the latitude is introduced from the very beginning. The reader has to understand what each part of the instrument represents, rather than just its shape.

## 5. Conclusion

*GD1* was divided into four chapters, and we have seen that the first chapter was completely dedicated to the armillary sphere and left cosmological matters for the following chapters. The text proceeds as if one were building the instrument, although we cannot rule out the possibility that the reader might have simply followed the text with a finished armillary sphere on his side, or tried to imagine it in his head. I assume that target readers of *GD1* were not familiar with the armillary sphere and had to concentrate on the instrument at first.

On the other hand, my hypothesis with *GD2* is that Parameśvara supposed that the reader had better access (either physical or mental) to the instrument. Some crucial information about its form is missing, notably the gradation of rings and the specific inclination of the ecliptic against the celestial equator (24 degrees). This requires the

reader to have some knowledge beforehand. There are expressions which recall the instrument in between explanations of the heavens. Probably the armillary sphere was a medium for the reader to understand cosmology.

Some points that have been raised by focusing on the first parts of *GD1* and *GD2*, seem to agree with what we can see by looking at the entirety of the two treatises. *GD1* explains different topics step by step in a relatively organized manner, especially with the sectioning into four chapters. By contrast, *GD2* tends to jump between topics<sup>39</sup>. The explanation itself in *GD1* tends to be clearer, especially with the auto-commentary, while *GD2* often brings out new terms and concepts without explanation, implying that the reader must know them in advance.

Therefore I conclude that *GD1* and *GD2* were composed as different texts for different readers. They might have been parts of different curricula, *GD2* being more advanced. Considering the overlap it is unlikely that the reader of *GD1* would read *GD2* afterwards. To verify this, we would need to take a closer look at other texts in the same bundle or collection of manuscripts, as well as to carry on a thorough comparison of the remaining parts of both *Goladīpikās*. The question remains whether Parameśvara changed his text to fit another curriculum or composed two texts for two curricula. Whatever his intention, *GD1* and *GD2* spread independently and were being read independently.

Incidentally, we have also seen the difference between *GD1* and *GD2* in how they describe rings and locate them in relation to other rings in an armillary spheres. The rings represent celestial circles where arcs and segments are formed in order to compute positions and motions of heavenly objects. Therefore readers of different modes of descriptions might have interpreted and imagined the circles in distinct ways, and might have build different reasonings for the computations generated within them. The remaining parts of the two *Goladīpikās* must be examined from this viewpoint.

## Acknowledgements

My special gratitude goes to Richard Lynn Kremer and Matthieu Husson for their encouragement and helpful remarks. I would like to thank Agathe Keller and Michio Yano for their valuable suggestions and for correcting my English. I also thank the anonymous reviewers for their comments which helped me to improve the quality of this paper significantly.

---

<sup>39</sup>For instance, *GD2* 103-124 is on gnomons, but from *GD2* 125 the subject switches to planetary latitudes. Parameśvara goes back to gnomons in verse 209, this time with examples of computations. This continues until *GD2* 247, after which the topic turns to parallaxes, which is in fact strongly linked with planetary latitudes.



## A. Text and translation from *GD1* (1.3 - 1.15)

*adha-ūrdhvayāmyasaumyagam iha vṛttam dakṣiṇottarākhyam syāt /  
tanmadhye 'py adha-ūrdhvaṃ vṛttam pūrvāparam tu ghaṭikākhyam //1.3//*

Here, a circle passing below, above, south and north is to be called the solstitial colure. At its middle is a circle, also [passing] below and above, then [passing through] the east and west, called the celestial equator. //GD1 1.3//

*bahir anayos tiryak syāc caturāśāsvastikam param vṛttam /  
viṣuvatsamjñitam etat tritayaṃ kharasāṅkam atra ghaṭikākhyam //1.4//*

Outside them both horizontally should be another circle [producing] crosses in the four quarters. These three are known as the equinoctial [circles]. Here the celestial equator has 60 divisions.//GD1 1.4//

*kharasāgnyāṅkam ihānyad dvitayaṃ tadvat punaḥ param vṛttam /  
pūrvāparasvastikagam adha-ūrdhvābhyāṃ ca saumyadakṣiṇayoḥ //1.5//  
jīnabhāge badhnīyād apamākhyam dakṣiṇottare vṛtte /(1.6ab)*

Here the other two [circles] have 360 divisions. One should attach yet another circle called the ecliptic, likewise [having 360 divisions], passing the east and west crosses, to the solstitial colure at 24 degrees north and south [respectively] from the [crosses at] the below and the top. //GD1 1.5-1.6ab//

*ghaṭikākhyobhayapārśve 'bhīṣṭakrāntyantare tatas tadvat //1.6//  
svāhorātrākhyāni ca badhnīyān maṇḍalāny atulyāni /(7.ab)*

One should attach, on both sides of the celestial equator, at a given declination from it, likewise, circles called diurnal [circles] of unequal [sizes].//GD1 1.6cd-1.7ab//

*apamagapātadvayagam saumye yāmye tataś ca bhatritaye //1.7//  
paramakṣepāntaritam candrādeḥ kṣepamaṇḍalaṃ bhavati / (8.ab)*

The orbit of [each planet] beginning with the moon goes through the two nodes on the ecliptic and is separated by their maximum latitude north and south at [the two points] three signs from there.//GD1 1.7-1.8ab//

*viṣuvadbhavayāmyodaksvastikayor goladaṇḍakāḥ protaḥ //1.8//  
nakṣatragola eṣa syād iha bāhye khagolo 'sti /*

The axis of the sphere pierces the south and north crosses produced by the equinoctial [circles]. This is the stellar sphere. Here, outside, is the celestial sphere.//GD1 1.8cd-1.9ab//

*tasmīn viṣuvattritayaṃ prāgvat, kṣitijākhyam eṣu tiryakstham //1.9//  
samamaṇḍalaṃ tu pūrvāparaḡaṃ, yāmyottarākhyam aparaṃ syāt /*

Within it are the three equinoctial [circles] as before. Among them the [one] lying horizontally is called the horizon, the prime vertical passes through the east and the west and the other is to be called the prime meridian.//GD1 1.9cd-1.10ab//

*kalpyā bhūḥ samavṛttā mṛdādinā goladaṇḍamadhyagatā //1.10//*

A uniformly round Earth should be arranged with clay and the like at the middle of the sphere's axis.//GD1 1.10cd//

*golasthitir evaṃ syāt nirakṣadeśe hy abhāṣṭadeśe tu /  
adha ūrdhvaṃ ca khagole yāmyodaksvastikāt palajyānte //1.11//  
kṛtvā vedhadvitayaṃ tatprotam goladaṇḍakaṃ kuryāt /*

Thus should be the state of the sphere at a latitude-less location (equator). However for a given location, one should make two holes in the celestial sphere down and up from the south and north crosses [respectively] at the distance of the Sine of latitude<sup>4041</sup> and then make the axis of the sphere pierce them.//GD1 1.11-1.12ab//

*unmaṇḍalākhyavṛttaṃ golagadaṇḍāgrayugmakaprotam //1.12//  
kuryāt pūrvāparagasvastikagaṃ ceṣṭadeśagolo 'yam /*

A circle called the six o'clock [circle] fixed to the two tips of the axis going through the [stellar] sphere and passing the east and the west crosses should be made. This is the sphere at a given location.//GD1 1.12cd-1.13ab//

*śaradaṇḍike ca yojye daṇḍe golāntare sthiratvāya //1.13//*

<sup>40</sup>“Sine” is capitalized to indicate that it is not the modern sine defined in a circle of unit radius, but measured in a circle with a radius  $R$  other than 1. In modern notation  $\text{Sin}\theta = R\sin\theta$

<sup>41</sup>Sarma [1956-57] comments that “*Pala-jyā*, here, as also elsewhere below, means only the latitude, *akṣa*, and not sine latitude, *akṣa-jyā*”. However in verse 4.51 (p.117) he does translate it as “sin. latitude”. Moreover, whenever Parameśvara gives the value of a geographic latitude, it is always by its Sine (For example, verses 3.34 and 4.23 in *GD1*: translations in Sarma [1956-57] are respectively p.88 and p.107). For this reason, I have chosen to translate *palajyā* literally.

Two stalks of reed are attached to the axis between the spheres for stability.//1.13cd//

*bhramati hy aparābhimukhaṃ pravahākṣepāt sadā bhagolo 'yam /  
sthira eva khagolaḥ syād digādisiddhyai prakalpito hy eṣaḥ //1.14//*

This stellar sphere continually rotates towards the west by the thrust of the *pravaha*. The celestial sphere should be completely still, for this is prepared in order to establish the directions and so forth.//GD1 1.14//

*lambākṣajñānārthaṃ prakalpyate daṇḍanābhiharījānte /  
anyad dyuvṛttam anyair bhūjyākṣajyeha lambakaḥ krāntiḥ //1.15//*

In order to know the co-latitude and latitude, another diurnal circle is assumed by others at the end of the horizon which has the axis as center<sup>42</sup>. Here the Sine of Earth is the Sine of latitude and the declination is the co-latitude.//GD1 1.15//

## B. Text and translation from GD2 (2 - 17)

*adha-ūrdhvayāmyasaumyagam iha vṛttam dakṣiṇottarākhyam syāt /  
adha-ūrdhvābhyāṃ ghāṭikam akṣāgre saumyayāmyayor<sup>43</sup> lagnam //2//*

This circle going below, above, south and north is what is called the solstitial colure. The celestial equator is adhering at the tip of [an arc of] latitude north and south from below and above [respectively]. //GD2 2//

*tasyāpy adha-ūrdhvābhyāṃ tadvat paramāpame 'pamaṃ lagnam /  
ghāṭikamadhye tiryag raśanāvartasya<sup>44</sup> vṛttam aparaṃ syāt //3//*

The ecliptic is adhering at its greatest declination likewise from below and above. A girdle at the middle of the celestial equator, transverse to the rotation, is another circle. //GD2 3//

<sup>42</sup> *daṇḍanābhiharījānte* is one compound in the locative case in the singular form. Therefore *daṇḍanābhi* (having the axis as center) must be an adjective modifying *harija* (horizon) or *anta* (end). The latter is impossible as an interpretation, and even the former sounds strange; the axis indeed goes through the center of the circle of the horizon, but it makes no sense in the context. The translation by Sarma [1956-57], "...a diurnal circle ...with [a point on] the central axis as the centre and just touching the horizon ...", interprets this compound as an adjective of *dyuvṛttam* (diurnal circle), but then the compound must be in the adjective (*daṇḍanābhiharījāntam*).

<sup>43</sup> *yāmyasaumyayor* in Sastri [1916], which is only supported by C.224F. IO Sanskrit 3530 reads *saumyayor* and every other manuscript supports my amendment.

<sup>44</sup> *vṛttasya* in Sastri [1916], which is only supported by C.224F. Every other manuscript supports my amendment.

*etad viṣuvatsamjñam ghāṭikamapi dakṣiṇottaram ca tathā /  
apamaṇḍalākhyavṛtte pūrvābhimukho raviḥ sadā carati //4//*

This is known as the equinoctial colure [or equal division circle] (*viṣuvat*). The celestial equator and the solstitial colure are also [called] likewise<sup>45</sup>. The sun always moves eastward on the circle called the ecliptic. //GD2 4//

*ghāṭikamadhyagaviṣuvadyāmyottaravṛttayor mithoyogāt /  
svastikayugmaṇ yat syāt tatproto golamadhyagatadaṇḍaḥ //5//*

Since the equinoctial colure and the solstitial colure which are inside the celestial equator are connected to one another there is a pair of crosses. The axis passing the middle of the sphere pierces them. //GD2 5//

*samavṛttām api bhūmiṃ bhagoladaṇḍasya madhyagāṇ kuryāt /  
kāṣṭhena vā mṛdā vā prāṇinivāsādi kalpayet tasyām //6//*

One should make a uniformly round Earth located at the middle of the stellar sphere's axis out of either a piece of wood or clay. The dwelling of living beings and so forth are assumed to be within it. //GD2 6//

*pravahamarutprakṣipto bhagola urvīm pradakṣiṇīkṛtya /  
aparābhimukhaṃ ṣaṣṭyā ghaṭikābhir bhramati bhūyo 'pi //7//*

The stellar sphere hurled by the *pravaha* wind goes clockwise<sup>46</sup> around the Earth and rotates continuously towards the west in sixty *ghaṭikās*. //GD2 7//

*bhūprṣṭhād upari marud raviyojanasaṃmitāntare pravahaḥ /  
niyatagatir aparagaḥ syād bhūvāyur adhaś ca tasya bhinnagatiḥ //8//*

The *pravaha* wind should have a constant movement towards the west above the Earth's surface at a distance of twelve *yojanas*. The wind of the Earth having a different movement is below it. //GD2 8//

*ghāṭikaṣaṣṭyaṃśasya bhramaṇe kālo 'tra nāḍikety uditā /  
na tu divasaṣaṣṭibhāgo golabhramaṇād yato 'dhiko divasaḥ //9//*

<sup>45</sup>The term *viṣuvat*, literally “in the middle” or “central”, can stand for the equinoctial colure alone, but can also be used to indicate the set of three circles.

<sup>46</sup>*pradakṣiṇī*, literally “towards the right/south”, as seen from the north pole.

Here, the time in which a sixtieth of the celestial equator rotates is called a *nāḍikā*, not the sixtieth of a day, because a day<sup>47</sup> is longer than a revolution of the [stellar] sphere. //GD2 9//

*ghāṭikamaṇḍalapārśve ghāṭikavṛttānusāri yad vṛttam /  
sūryasya bhramaṇastham svāhorātrākhyavṛttam<sup>48</sup> uditam tat //10//*

On the side of the celestial equator is a circle that is a companion of the celestial equator. It is called the diurnal circle, the place of the revolution of the sun. //GD2 10//

*tāni bahūni bhavanti ca divase divase yato 'rkagatibhedah /  
nakṣatragola eṣa hi bāhye 'sya ca niścalaḥ khagolaḥ syāt //11//*

Many of them exist, because for each day there is a difference in the motion of the sun. This is the stellar sphere. The celestial sphere outside it should be [considered] immovable. //GD2 11//

*pūrvāparādha-ūrdhvagam uditam samamaṇḍalam khagolastham /  
yāmyottarādha-ūrdhvagam asmīn api<sup>49</sup> dakṣiṇottarākhyam syāt //12//*

The prime vertical located on the celestial sphere is said to go through the east, west, below and above. What is called the prime meridian on it (the celestial sphere) should go through the south, north, below and above. //GD2 12//

*pūrvāparayāmyodaggatam iha bhūpārśvasamsthitam kṣitijam /  
tasminn udayāstamayau sarveṣāṃ bhagrahāṇām<sup>50</sup> staḥ //13//*

Here, the horizon located on the side of the Earth goes through the east, west, south and north. The rising and setting of all the stars and planets takes place on it. //GD2 13//

*yāmye 'dhaś cordhvam udak kṣitijād akṣāṃśakāntare lagnam /  
prāgaparayos ca lagnam vidyād unmaṇḍalam khagolastham //14//*

---

<sup>47</sup>Average civil day.

<sup>48</sup>*svāhorātrārdhavṛttam* in Sastri [1916], which is only supported by C.224F. C.1024D, Burnell 107b and Burnell 17c read *svāhorātrākhyam*, others support my amendment.

<sup>49</sup>*api tasmin* in Sastri [1916], which is only supported by C.224F. Every other manuscript supports my amendment.

<sup>50</sup>*hi grahāṇām* in Sastri [1916], which is only supported by C.224F. Every other manuscript supports my amendment.

One should know that the six o'clock circle which is located on the celestial sphere is adhering at a distance in degrees of latitude below [the horizon] in the south and above the horizon in the north, and is adhering to the east and west. //GD2 14//

*unmaṇḍalayāmyodaksvastikayātaś ca goladaṇḍo 'yam /  
unmaṇḍalordhvabhāge bhramaṇaṃ golasya khāgnināḍībhīḥ //15//*

This axis of the sphere goes through the crosses of the six o'clock circle and the prime meridian. In the part above the six o'clock circle the revolution of the sphere takes thirty *nāḍīs*. //GD2 15//

*unmaṇḍalād adhaḥsthaṃ saumye yāmye tadūrdhvagaṃ kṣitijam /  
tasmāt saumyagate 'rke dinam adhikaṃ yāmyage niśā hy<sup>51</sup> adhikā //16//*

The horizon is situated below the six o'clock circle in the north and goes above it in the south. Therefore when the sun is to the north [of the celestial equator] the day is long and when to the south it is the night that is long. //GD2 16//

*kṛtvā vā prāgaparaṃ ghāṭikam anyac ca tadvaśāt kṛtvā /  
unmaṇḍalayāmyodaksvastikanisprotadaṇḍakaṃ kuryāt //17//*

Or, having made the celestial equator in the east-west direction, and having made another [circle] according to it, one should make an axis piercing the crosses of the six o'clock circle and the prime meridian. //GD2 17//

---

<sup>51</sup> *niśāpy* in Sastri [1916], which is only supported by 5867A and C.224F. Every other manuscript supports my amendment.

## References

- Setsuro Ikeyama. *The Brāhmasphuṭasiddhānta chapter 21 with the commentary of Pṛthūdakasvāmin*. Doctoral, Brown University, 2002.
- Arthur Berriedale Keith. *Catalogue of the Sanskrit and Prakrit manuscripts in the library of the India Office*, volume 2. Clarendon Press, Oxford, 1935.
- Agathe Keller. Ordering operations in square root extractions : Analyzing Some Early Medieval Sanskrit Mathematical Texts with the Help of Speech Act Theory. In Karine Chemla and Jacques Virbel, editors, *Texts, Textual Acts and the History of Science*, pages 183–218. Paris, 2015.
- Hendrick Kern. *The Āryabhaṭīya, a manual of astronomy, with the Commentary Bhatadīpikā of Paramādīçvara*. E.J. Brill, 1874.
- D. Udaya Kumar, G.V. Sreekumar, and U.A. Athvankar. Traditional writing system in Southern India — Palm leaf manuscripts. *Design Thoughts*, (July):2–7, 2009.
- T.S. Kuppanna Sastri. *Mahābhāskarīya of Bhāskarācārya: With the Bhāṣya of Govindasvāmin and the Super-commentary Siddhāntadīpikā of Parameśvara*. Government Oriental Manuscripts Library, Madras, 1957.
- Yukio Ohashi. Astronomical Instruments in Classical Siddhāntas. *Indian journal of history of science*, 29(2), 1994.
- David Pingree. *Census of the Exact Sciences in Sanskrit A-4*, volume 4 of A. America Philosophical Society, 1981.
- K.V. Sarma. *The Goladīpikā by Parameśvara : Edited with Introduction, Translation and Notes*. Number 32 in Adyar Library Pamphlet Series. The Adyar Library and Research Centre, Madras, 1956-57.
- K.V. Sarma. The Grahaṇāṣṭaka of Parameśvara : A Short Manual on Eclipses : Edition and Translation. *Journal of Oriental Research*, 28, 1960.
- K.V. Sarma. *Dr̥ggaṇita of Parameśvara*. Number 30 in Vishveshvaranand Indological Series. Vishveshvaranand Vedic Research Institute, Hoshiarpur, 1963.
- K.V. Sarma. *The Grahaṇamaṇḍana of Parameśvara*. Number 34 in Vishveshvaranand Indological Series. Vishveshvaranand Vedic Research Institute, Hoshiarpur, first edition, 1965.
- K.V. Sarma. *A History of the Kerala School of Hindu Astronomy (In Perspective)*. Number 55 in Vishveshvaranand Indological Series. Vishveshvaranand Institute, Hoshiarpur, 1972.
- T. Ganapati Sastri. *The Goladīpikā by Srī Parameswara*. Number 49 in Trivandrum Sanskrit Series. The Superintendent Government Press, Trivandrum, 1916.

Kripa Shankar Shukla. *The Sūrya-siddhānta with the commentary of Paramesvara*. Dept. of Mathematics and Astronomy, Lucknow University, 1957.





# Sphere confusion: a textual reconstruction of instruments and observational practice in first-millennium CE China

Daniel Patrick Morgan

► **To cite this version:**

Daniel Patrick Morgan. Sphere confusion: a textual reconstruction of instruments and observational practice in first-millennium CE China . Centaurus, Wiley, 2016, 58 (1-2), pp.87-103. <<http://onlinelibrary.wiley.com/doi/10.1111/1600-0498.12115/full>>. <halshs-01374805v2>

**HAL Id: halshs-01374805**

**<https://halshs.archives-ouvertes.fr/halshs-01374805v2>**

Submitted on 4 Oct 2016

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial - ShareAlike 4.0 International License

## **Sphere Confusion: a Textual Reconstruction of Astronomical Instruments and Observational Practice in First-millennium CE China**

Daniel Patrick MORGAN\*

**Abstract:** This article examines the case of an observational and demonstration-  
al armillary sphere confused, one for the other, by fifth-century historians of  
astronomy He Chengtian and Shen Yue. Seventh-century historian Li Chunfeng  
dismisses them as ignorant, supplying the reader with additional evidence. Using  
their respective histories and what sources for the history of early imperial ar-  
millary instruments survive independent thereof, this article tries to explain the  
mix-up by exploring the ambiguities of ‘observation’ (*guan*) as it was mediated  
through terminology, text, materiality and mathematics. Reconstructing the  
material features of the ‘sight’ (*yi*) and ‘effigy’ (*xiang*), the article will reflect  
upon the mathematics necessary for their operation. The ‘effigy’, as Li Chun-  
feng defines it, is a substitute for observation; the ‘sight’, however, is so medi-  
ated by the material and mathematical sphere as to confound Li’s distinction be-  
tween looking *through* and looking *at*. In the end, however, the difference is  
moot, since the observational model appears to have played a negligible role in  
the history of astronomy in first-millennium China, leaving us to wonder what  
instrument(s) *were* used for observation.

### **Introduction**

The most important thing to know about the Chinese armillary sphere is that it was made of money. You could use iron, or even wood, but to do it right you needed bronze, and bronze was the basis of the currency. It is for this most mundane of reasons—liquid capital—that the history of the armillary sphere in China was largely one of making do without. So too must the historian make do without, because, prior to the fifteenth-century reproduction of Guo Shoujing’s 郭守敬 (1231–1316) ‘simplified instrument’ at Purple Mountain Observatory, all we have to go on is texts and second-millennium illustrations—texts that reveal little about the instrument’s operation beyond words like ‘observation’ (*guan* 觀) and ‘watching’ (*hou* 候). Studies of more technically forthcoming traditions like Włodarczyk (1987) remind us that there is more to ‘observation’ than the tool, and that the tool, in this case, was the most unwieldy of the observer’s options. Absent any discussion of *practice*, the armillary sphere nevertheless enjoys a cult status among sinologists, Needham (1959, p. 339), for example, calling it ‘the indispensable instrument of all astronomers for the

---

\* The research leading to these results received funding from the European Research Council under the European Union’s Seventh Framework Program (FP7/2007–2013) / ERC Grant agreement n. 269804 and was conducted in the context of the project SAW: *Mathematical Sciences in the Ancient World* (SAW). I would like to thank Matthieu Husson and Rich Kremer for their invitation and critique of my original draft, and the editor and my anonymous reviewers, for highlighting what infelicities remained.

determination of celestial positions before the invention of the telescope’.

This paper offers a preliminary exploration of the practice(s) and physical realities of armillary-sphere ‘observation’ in first-millennium CE China. In the absence of the instrument, the question of practice is one that we must approach through text, for which we shall focus specifically on written traces of Zhang Heng 張衡 (78–139) and Kong Ting’s 孔挺 (fl. 323) ‘sphere sights’ (*hun yi* 渾儀). Our primary source in this regard will be the histories of astronomical instrumentation written by Li Chunfeng 李淳風 (602–670) and Shen Yue 沈約 (441–513) in their respective dynastic-history monographs on ‘heavenly patterns’ (*tianwen* 天文). Writing on the Jin (265–420) and Five Dynasties (502–618), in the case of Li Chunfeng, and the Liu-Song (420–479), for Shen Yue, their histories overlap as concerns the lead-up to the fifth century. Weaving lengthy descriptions and citations into a chronicle of ‘creations’ (*zao* 造) and ‘awakenings’ (*wu* 悟), these histories preserve most of what we know about astronomical instrumentation prior to the seventh century.<sup>1</sup> The one exception here is Zhang Heng’s treatise on constructing, measuring and extrapolating algorithms from a physical sphere, *The Sphere Heaven Sight* (*Hun-tian yi* 渾天儀), preserved in Li Xian’s 李賢 (654–684) commentary to the *Book of Later Han*.

We shall focus on the Zhang Heng and Kong Ting spheres to reveal some of the confusion surrounding this topic in early sources—a confusion of two physical instruments bespeaking a greater confusion about what it means to ‘observe’. In brief, the one disappeared from Luoyang in the fog of war, and when the other was captured a century later in Chang’an, fifth-century experts believed themselves to have recovered the wrong sphere. Their confusion is difficult for the seventh-century expert to understand, because the one sphere was built to look *through*, and the other, *at*. As different as that sounds, I will attempt to explain this confusion via the terminology, text, materiality and mathematics through which ‘observation’ is in this case mediated.

Note that, for the sake of concision, I shall translate reign-years into Julian years (e.g. ‘164’ for ‘Huandi, Prolongation of Brightness 7’) and reduce compound decimal length measures into the equivalent number of *chi* 尺 (e.g. ‘14.61 *chi*’ for ‘1 *zhang* 4 *chi* 6 *cun* 1 *fen*’), using 1 *chi* = 23.1–31.6 cm based on the inflationary historical rates in Qiu (1992). As to astronomical units, our subjects work in *du* 度 (‘measure/crossing’): a linear measure of the circumference of a great circle whereby one *du* equals the distance travelled by the mean sun in one day, and the ‘circuit of heaven’ (*zhou tian* 周天) thus equals the length in days of the tropical year, i.e.  $360^\circ \approx 365\frac{1}{4} du$  (Huang, 1992; Cullen, 1996, pp. 35–66).

## Lost & Found

The term that sinologists translate as ‘sphere’ is *hun* 渾/混, invoking the ‘confused’ and ‘undifferentiated’ state of matter at the beginning of time to describe the *tian* 天 ‘heaven(s)’. The rubric

‘sphere/confusion’ could not be more appropriate. The earliest reference to a *hun-tian* comes from Yang Xiong 揚雄 (53–18 BCE), the vagueness of which makes it difficult to distinguish the cosmology from the instrument (Cullen, 1996, pp. 53–59):

或問渾天，曰：「落下闔營之，鮮于妄人度之，耿中丞象之，幾乎幾乎！莫之能違也。」

Someone inquired about sphere heaven, [to which Yang Xiong responded]: ‘Luoxia Hong (fl. 104 BCE) worked it out, Xianyu Wangren (fl. 78–74 BCE) *du*-measured it, and Geng [Shouchang] the palace assistant (fl. 52 BCE) made an effigy of it. How exact it is! No one can contradict it (*Yangzi Fayan*, 7.2a–b).

It is only with the ‘Grand Clerk yellow-path bronze sight’ 太史黃道銅儀 of 103 that we see unequivocal evidence of something resembling an armillary sphere. Commissioned for the state observatory at the (late) behest of General Jia Kui 賈逵 (30–101), Cai Yong 蔡邕 (133–192) and Liu Hong’s 劉洪 (fl. 167–206) monograph in the *Book of Later Han* offers the following description of the device:

以角為十三度，亢十...凡三百六十五度四分度之一。冬至日在斗十九度四分度之一。史官以郭日月行，參弦望，雖密近而不為注日。儀，黃道與度轉運，難以候，是以少循其事。

With Horn<sup>L01</sup> as 13 *du*, Neck<sup>L02</sup> [as] 10, (see fig. 1)... it totalled to 365 *du* &  $\frac{1}{4}$  *du*. The winter solstice was at Dipper<sup>L08</sup> 19 *du* &  $\frac{1}{4}$  *du*. The Clerk’s Office perimtered (?) solar & lunar motion and checked quarter & full moons, and though it was tight & close (accurate), it was not used for noting the sun/days. As to the sight, the yellow path and *du* (equator ring) rotated; it was difficult to watch (*hou*) with, which is why [the order to use it] was rarely heeded (*Hou Han shu*, *zhi* 2, 3029–30).

The ‘rotating’ equator and ecliptic identify this as an armillary sphere, ‘watching’ suggests one made for looking *through*, but this is all we really know about the sphere prior to Zhang Heng.

As concerns instrumentation, Zhang Heng’s *Book of Later Han* biography attributes him with having ‘created [the] sphere heaven sight/s’ 作渾天儀 (*Hou Han shu*, 59.1898), which likely refers to the treatise by that name. Later sources like Li Chunfeng highlight a physical installation:

至桓帝延熹七年，太史令張衡，更以銅製，以四分為一度，周天一丈四尺六寸一分。亦於密室中，以漏水轉之。令司之者，閉戶而唱之，以告靈臺之觀天者，璇璣所加，某星始見，某星已中，某星今沒，皆如合符。

In 164, Prefect Grand Clerk Zhang Heng redesigned [a sphere] in bronze with 4 *fen* (9.4 mm) to the *du*, for a circuit of heaven of 14.61 *chi* (343.34 cm). It was placed in a sealed chamber and rotated by means of waterclock water. The person charged with its operation called it out from behind closed doors to announce to the observers of heaven of the Numinous Terrace (observatory) the add[ed hour] (?) of the ‘rotating mechanism’, that such-and-such star was first visible, that such-and-such star was already [culminated], and that such-and-such star was currently setting—all of



which were like matching [the two halves of] a tally (*Sui shu*, 19.516–17).<sup>2</sup>

Unmentioned in his biography, the device is attributed to a date twenty-five years after Zhang Heng's death in 139. Whatever that tells us, this sphere-clock turned *indoors*, separate from the activity of 'watching', which distinguishes it from the observational 'sight' of 103. Arai (1989, p. 325) labels this device a 'computer'.

Both the 103 and 164 spheres were installed at the Numinous Terrace observatory at Eastern Han (25–220) Luoyang. Excavated in 1974–1975, this 44,000 m<sup>2</sup> walled site revealed nothing but ruined foundations, floor tiles and the earthen terrace where the sphere once stood (*Kaogu* 1978.1, pp. 54–57). Much had happened in the meantime. In 189, Military Governor Dong Zhuo 董卓 (d. 192) sacked the city in a succession struggle between the palace and civil service. With Luoyang in flames, a child emperor was installed in Xuchang while real power devolved upon warlords fighting military rebellions, millenarian uprisings and one another in his name. In 220, the Han emperor abdicated to his generals, the Cao 曹 of Wei 魏, who abdicated to *their* generals, the Sima 司馬 of Jin 晉, in 265. The Cao and Sima clans re-established Luoyang as their capital over the Three Kingdoms (220–280) but only after massive reconstruction. Upon reunification, Jin Wudi 晉武帝 (r. 265–290) split the empire amongst his sons, who would go on to flood the central plains with mercenary steppe tribes in a new war for the imperial seat. So they fought, and so they eventually found themselves fighting rebellions within their armies until, in 311, an alliance of mercenary tribes sacked the capital and drove the Jin city by city into the south.

The heartland was lost, and so too in the fog of war and exodus had the spheres of the Luoyang observatory gone missing. A century later, in the 417 siege of the Qiang 羌 proto-Tibetan capital at Chang'an, General Liu Yu's 劉裕 (363–422) armies made an unexpected discovery amongst the city's ruins: a two-metre bronze sphere inscribed with astral symbols along its rings. The general transported his find to the new capital at Jiankang in 418 (where, with his armies, he would usurp the throne in 420). In 439, within the framework of Xu Yuan's 徐爰 (394–475) history project to legitimize the new dynasty, He Chengtian identified this instrument with Zhang Heng's water-driven 'sphere sight' of 164. In his 493 history, Shen Yue reiterates He's identification, noting that 'though the sight was visibly intact, it was [no longer] ornamented with the canon stars or seven luminaries' 儀狀雖舉，不綴經星七曜 (*Song shu*, 23.678).

This, according to Li Chunfeng's *Book of Sui* monograph, is what they were looking at (cf. fig. 2):

其制則有雙環規相並，間相去三寸許。正豎當子午。其子午之間，應南北極之衡，各合而為孔，以象南北樞。植榘於前後，以屬焉。又有單橫規，高下正當渾之半，皆周帛分為度數，署以維辰之位，以象地。又有單規，斜帶南北之中，與

春秋二分之日道相應。亦周币分為度數，而署以維辰，並相連著。屬榘植而不動。

**[Six-joint sight:]** Its construction featured a pair of ring-circles joined [parallel] to one another with a space of roughly 3 *cun* (9.09 cm) between them. It stood upright to serve as the *zi*.<sub>B1</sub>–*wu*.<sub>B7</sub> (the ‘00:00–12:00’, or meridian). Between *zi*.<sub>B1</sub> & *wu*.<sub>B7</sub>, accommodating the traverse (diameter) [between] south & north poles, [the parallel rings] each joined to form a hole in effigy of the southern & northern pivots [of the celestial sphere]. Lock pins in the front & back allowed joining [other rings] to it. **In addition**, it had a single horizontal circle at a height corresponding exactly with half the [vertical diameter of the] sphere, which was divided all around [its perimeter] into *du* numbers and inscribed with the positions of the corners & chronograms in effigy of the earth (the horizon). **In addition**, it had a single circle that belted at an incline midway between south & north (i.e. at an incline to the horizon circle and perpendicular to the N–S axis), corresponding to the path of the sun at spring & autumn equinox. It too was divided around its circumference into *du* numbers and inscribed with the corners & chronograms, the two of which were written together in a single [band]. It was held in place by a connecting bolt and did not move.

其裏又有雙規相並，如外雙規。內徑八尺，周二丈四尺，而屬雙軸。軸兩頭出規外各二寸許，合兩為一。內有孔，圓徑二寸許。南頭入地下，注於外雙規南樞孔中，以象南極。北頭出地上，入於外雙規北樞孔中，以象北極。其運動得東西轉，以象天行。

**[Four-direction displacement sight:]** Its interior had another pair of circles joined [parallel] to one another, like the outer double-circle (i.e. an internal meridian ring). [The double-circle’s] inner diameter was 8 *chi* (242.4 cm), its circumference was 24 *chi* (727.20 cm), and it was connected to the axle pair (i.e. it was fixed to and turned around the N–S axis ‘pivots’ of the six-joint sight). The two axle heads each protruded roughly 2 *cun* (6.06 cm) from the [four-displacement] circle, joining the two [parallel circles] as one. Inside of these were holes with a circular diameter of roughly 2 *cun*. The southern head went beneath the earth (horizon circle), where it was inserted into the southern pivot hole of the outer double circle in effigy of the south [celestial] pole. The northern head protruded from the earth, going into the northern pivot hole of the outer double circle in effigy of the north [celestial] pole. Its movement allows for east-west rotation in effigy of heaven’s motion.

其雙軸之間，則置衡，長八尺，通中有孔，圓徑一寸。當衡之半，兩邊有關，各注著雙軸。衡既隨天象東西轉運，又自於雙軸間得南北低仰。所以準驗辰曆，分考次度，其於揆測，唯所欲為之者也。

**[Sighting tube:]** Between its two axles was installed a traverse 8 *chi* (242.4 cm) in length, through the centre of which was a [sighting] hole 1 *cun* (3.03 cm) in diameter. Halfway down the traverse was, on either side, a [pivot] bolt, each of which were inserted & connected to [another] axle pair (at the midpoint of an unmentioned crossbar). The traverse could both rotate east–west to follow heavenly phenomena and achieve of itself north–south lowering & raising between the axle pair. This is how one levelled & verified the chronograms & *li* (time) and distinguished & exam-



ined the stations & *du* (space). In regards to observation & measurement, it did truly everything that one could desire (*Sui shu*, 17.517–18; cf. Maspero, 1939, pp. 322–23).

This, however, was not Zhang Heng's armillary sphere. In saying that it was, He Chengtian and Shen Yue were, in Li Chunfeng's words, 'both off by far' 皆失之遠矣 (*Sui shu*, 19.518).

## A Sphere for Calculation

Of everything that is wrong with He Chengtian and Shen Yue's identification of the preceding instrument, Li Chunfeng points to the most obvious: 'Inspection of the engraving [reveals that] this was constructed by Clerk's Office Assistant Kong Ting of Nan-yang in 323, under the rule of the [Xiongnu] imposter Liu Yao (r. 318–329) 檢其鑄題，是偽劉曜光初六年，史官丞南陽孔挺所造 (*Sui shu*, 19.518). There is also the fact that Kong Ting sphere was fitted with a *sighting tube* for use *outdoors*. On this point, Li insists on a rectification of names:

渾天儀者，其制有機有衡。既動靜兼狀，以效二儀之情，又周旋衡管，用考三光之分。所以揆正宿度，準步盈虛，來古之遺法也。

**The sphere heaven sight** is constructed with both engine (cage) and traverse. Not only in its at once moving & static state does it replicate the true situation of [yin & yang], the complete rotation of the transverse (sighting) tube allows examination of the fractions of the three lights (the sun, moon and stars). It is that by which one estimates & corrects the lodge *du* (widths) and levels & paces excess & void—a method handed down from antiquity (*Sui shu*, 19.517).

渾天象者，其制有機而無衡... 不如渾儀，別有衡管，測揆日月，分步星度者也。

**The sphere heaven effigy** is constructed with engine and no traverse... It is inferior to the sphere sight, which has in addition a traverse tube—the thing that [allows] the measure & estimation of sun & moon and the division & pacing of stars & *du* (*Sui shu*, 19.519).

By Li Chunfeng's definition, Zhang Heng's indoor sphere was an 'effigy', Kong Ting's outdoor sphere was a 'sight', and 'the sight & effigy are two [distinct] devices with nothing whatsoever to do with one another' 儀象二器，遠不相涉 (*Sui shu*, 19.519).

The terminology is however less clear than Li Chunfeng would make it. The term *yi* 儀 derives from the graduated sight/range-finder pegs of early missile weapons, which, extended to the sphere, came to stand for sighting pegs, graduated rings and the instrument itself, while *xiang* 象 refers to an 'effigy' or 'simulacrum' linking something in the world of man to a truth in the beyond (Li, 2014, pp. 171–77; Schafer, 1977, pp. 54–56). In second-millennium parlance, 'sight' refers to an armillary sphere, and

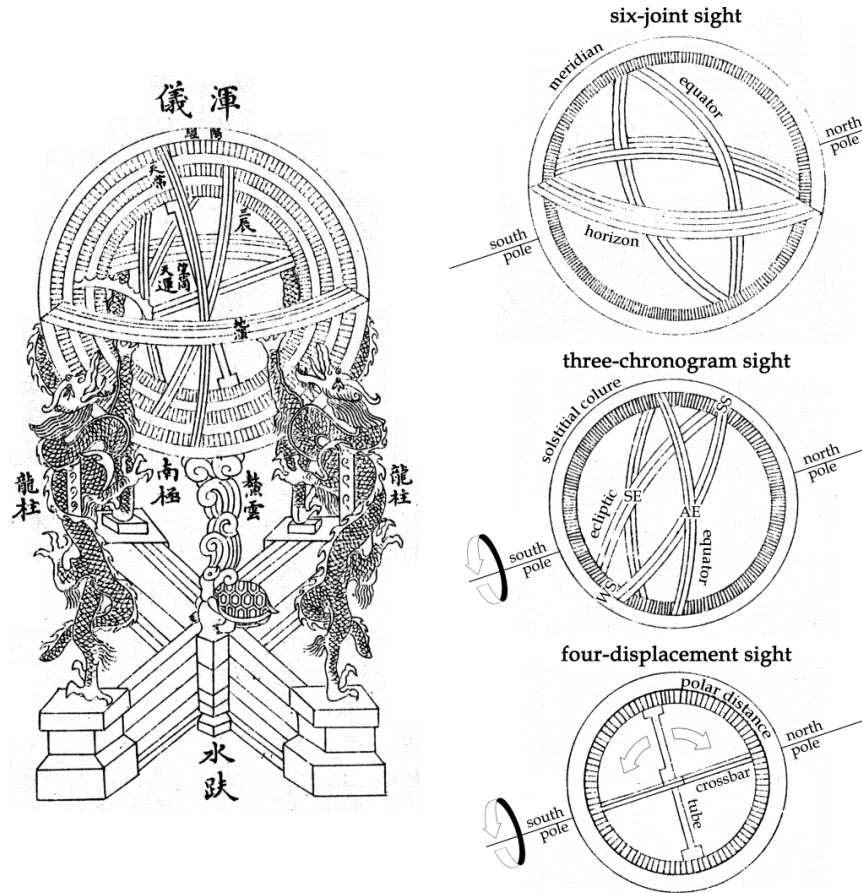


Fig. 2 Su Song's 蘇頌 (1020–1101) sphere heaven sight and component groups, from *Xin yixiang fayao*, A.9a, 11a, 13a, 14a. These illustrations are supplied for reference; note that Kong Ting's 323 'sight' is absent a 'three-chronogram sight'.

'effigy', a celestial globe, the differences being that between a hollow and solid sphere; for Li Chunfeng, however, the difference is between looking *through* and looking *at*, and the fact that the demonstrational sphere had *rings* (and that the observational sphere was an effigy) afforded a certain ambiguity as to which applies to a given sphere (Wang, 2015). Indeed, Qian Lezhi's 錢樂之 (fl. 436–443) non-observational 'sight' of 436 later presents Li Chunfeng with a conundrum:

至如斯制，以為渾儀，儀則內闕衡管。以為渾象，而地不在外。是參兩法，別為一體。就器用而求，猶渾象之流。

As to this construction, it could be taken for a sphere sight, but the sight is absent a traverse tube inside; it could be taken for a sphere effigy, but the earth is not on the outside. This references both models<sup>3</sup> but constitutes a separate form (*ti*). Gathering from the device's use, it would seem to belong to the current (school) of sphere effigies (*Sui shu*, 19.519).

Zhang Heng's reputation for having 'created [the] sphere heaven sight/s' would seem to suggest the label for his unnamed (and posthumous?) computer of 164, and *The Sphere Heaven Sight*, for

its part, deals with computation. At its core, the treatise is about measuring the ecliptic, which, without spherical trigonometry, means using a ruler:

是以作小渾，盡赤道黃道，乃各調賦三百六十五度四分之一，從冬至所在始起，令之相當值也。取北極及衡，各（誠）（鍼）琢之為軸，取薄竹篋，穿其兩端，令兩穿中間與渾半等，以貫之，令察之與渾相切摩也。乃從減半起，以為（百）八十二度八分之五，盡衡減之半焉。又中分其篋，拗去其半，令其半之際正直，與兩端減半相直，令篋半之際從冬至起，一度一移之，視篋之半際（夕）多（少）黃赤道幾也。其所多少，則進退之數也。從（此）（北）極數之，則（無）（去）極之度也。

For this, make a small sphere complete with red & yellow path, then allocate each with 365 *du* & 1/4 *du* and make sure to align their relative values starting from the position of winter solstice. Take the north pole and [the other end of the] transverse (i.e. the south pole) and stick each with a needle to form an axle. Take a thin bamboo strip and punch a hole at either end so that the distance between the two holes is exactly one half [of the circumference] of the sphere and that [the pins] may be run through them (affixing each end to opposite poles). Make sure to check that [the bamboo strip] rubs closely against [the surface of] the sphere. Then, starting from the diminished half-[way point] (the northern axis), make 182 *du* & 5/8 [*du*] [running] all the way down to the half-[way point] diminished at the transverse (the southern axis). Furthermore split the strip [along the] middle and remove its [one] half, making sure that the edge of its half (centreline) is true & straight and that it is aligned with the diminished half-[way points] (the poles) at both ends. Make sure to begin with the [centreline] edge of the bamboo strip at winter solstice and shift it one *du* at a time, looking at how much [is the north-polar distance of the ecliptic on] the half-edge of the bamboo strip and how many [*du* of longitude and RA have elapsed on] the yellow & red path. [The amount] by which [the latter] differ is the number of advance/retreat, while counting from the north pole [down the graduated bamboo strip] is (sic.) the *du* of polar distance (*Hou Han shu*, *zhi* 3, 3076 comm.).

Having thus determined the limits of ‘advance/retreat’ 進退 (the reduction to the equator) to be zero at the solstices and equinoxes and 3 *du* at the halfway points in between (the ‘establishments’ *li* 立), *The Sphere Heaven Sight* concludes with an algorithm for interpolating intermediate values by which to freely convert between ecliptic and equatorial ‘lodge-entry *du*’ (*ruxiu du* 入宿度). Deferring the reader to Western-language studies in Maspero (1939, pp. 337–52), Cullen (2000) and Lien (2012), one notes that the interest of this step function is in avoiding complex fractions by rounding quarter circuits ( $91 \frac{1}{5} du$ ) to 15- and 16-*du* blocks and ‘advance/retreat’ to  $\frac{1}{4} du$  intervals.<sup>16</sup>

The point here is that Zhang Heng’s ‘small sphere’, like his water-driven sphere of 164, is a material means to a computational end. These are spheres for looking *at*, and where the latter was in a ‘sealed chamber’, Zhang explains the former thus:

本當以銅儀日月度之，則可知也。以儀一歲乃竟，而中間又有陰雨，難卒成也。

What one should do is *du*-measure these [advance/retreat numbers] over days and months via the bronze sight—*then* could [they] be known—[but as] this would take a year at the sight to complete, and [as] there would furthermore be overcast & rainy [days] interspersed therein, it would be difficult to bring to successful completion (*Hou Han shu*, *zhi* 3, 3076 comm.).

In the end, the sphere was a substitute for observation, and the algorithm, a substitute for the sphere. Tellingly, in 721, the answer to Monk Yixing's 一行 (683–727) petition that '[we] must know the yellow-path advance/retreat [numbers]' 須知黃道進退 was that '[the clerk's] office does not have a/the yellow-path displacement sight [and thus] has no means of measure-watching [it]' 官無黃道游儀，無由測候 (*Jiu Tang shu*, 35.1293–94). Six centuries later, the physical sphere was apparently still the basis of coordinate conversion.

## A Sphere for Observation

Let us return to the observational sphere of 323 by point of comparison. As described above, the Kong Ting sphere was comprised of two of three component groups typical to later models (fig. 2). The first was the 'six-joint sight' (*liu he yi* 六合儀), a fixed outer cage 'joining' a horizon, meridian and equator ring at six points (and to a platform). Aligned at the horizon and celestial pole, the outer cage provided a fixed coordinate grid within which to turn interior rings. The second component group was the 'four-[directional] displacement sight' (*si you yi* 四遊儀), a meridian ring turning east–west around the polar axis and fitted with a sighting tube that pivoted north–south through its centre (Maspero, 1929, pp. 306–27).

The key to any precision instrument is graduation, without which a cage of rings is no more an armillary sphere than a metal slat a ruler. Shen Gua 沈括 (1031–1095) offers the following meditation on the subject:

度不可見，其可見者星也。日、月、五星之所由，有星焉。當度之畫者凡二十有八，而謂之舍。舍所以繫度，度所以生數也。度在天者也，為之璣衡，則度在器。度在器，則日月五星可搏乎器中，而天無所豫也。天無所豫，則在天者不為難知也。

*Du* cannot be seen; what can be seen are stars, and the course of the sun, moon, & five [planets] is replete with stars. Those [stars] that act as demarcations of *du*, they are twenty & eight in total, which we call lodges. Lodges are that by which *du* are measured out, and *du* are that by which numbers are born. *Du* are in heaven; but make a 'device-traverse' (sphere sight), and you have *du* on an apparatus. If you have *du* on an apparatus, then the sun, moon, & five [planets] can be modelled<sup>4</sup> within the apparatus, and heaven will have no play.<sup>5</sup> And if heaven has no play, then the things in heaven will not be difficult to know (*Song shi*, 48.954–55).

If the sphere sight were to be a microcosm of the sphere heaven, one would expect that it be graduated accordingly—into the *du* of the mean sun's daily progress over one tropical year as counted from twenty-eight unevenly spaced reference stars (fig. 1). The fact that lodge-entry *du* are indeed the *only* measures of RA and longitude attested in *li* 曆 mathematical astronomy makes it difficult to imagine the alternative.

Luckily, we need not rely on imagination. Li Chunfeng reports that Kong Ting's (fixed) equator ring featured 'du numbers and... the corners & chronograms', the latter a twenty-four point reference grid—twenty (stem and branch) 'chronograms' and four (trigram) 'corners'—counted 'leftward' (clockwise) from due north. Familiar from compass and divination boards, the 'corners & chronograms' scheme typically features in observational data and *li* procedure texts as an expansion of the standard duodenary (double-hour) civil day as counted from midnight. On a fixed equator ring, this would give the user a Mediterranean-looking 'hour angle' counted from the opposite (midnight) meridian line (fig. 1). Corroboration for the use of these 'added hours' (*jia shi* 加時) as spatial coordinates furthermore appears in a set of eclipse data presented as evidence in a debate of 226 (*Jin shu*, 17.500; cf. Qu, 1994).

For observational data to be of any use to *li* calculation, one needs lodge-entry *du*. For equatorial *du*, one would have had two options: (a) a sphere with a 'three-chronogram sight' (*san chen yi* 三辰儀), a moving equator and ecliptic ring, mounted on the polar axis, which allowed one to align the stars of the instrument with those of heaven as per the description of Shen Gua (fig. 2); (b) an algorithm for converting from the transit times and 'added hours' supplied by stationary instruments such as the gnomon and 'six-joint sight'. For ecliptic *du*, actors likewise speak of needing: (a) a 'three-chronogram sight' or (b) an algorithm, i.e. 'advance/retreat', for converting from equatorial *du*. Actors like Zhang Heng make it clear that the algorithmic solution was something of a 'plan b', but 'plan a', let us not forget, depends on the material availability of a giant instrument made of money, which should probably not be taken for granted.

What we read of known historical 'sights' confirms our suspicions. The last mention of the (unused) Luoyang observatory sphere of 103 comes in 178 (*Song shu*, 23.673), the instrument having likely been melted down between the sack of 189 and the loss of the city in 311. Judging from Shen Yue and Li Chunfeng's histories, the next observational armillary to grace a Chinese capital was the Xiongnu sphere of 323, which was captured and brought to Jiankang in 418. After that was an iron version of the same design made in 398 for the Xianbei Tuoba-Wei 拓跋魏 (386–535) court at Pingcheng, which was captured and moved to Chang'an by the Xianbei Yu-Zhou 宇文周 (557–581) in 577, later passing hands with the city to the Sui 隋 (581–605). Installed at the Chang'an observatory in 583, the Xianbei sphere would see official use there until replaced by Yixing's in 725 (Wu & Quan, 2008, pp. 433–40). In short, what observational spheres Chinese courts did possess prior to 725 were mainly barbarian hour-angle models *sans* lodges and *sans* ecliptic.

Availability, of course, depends as much on *allocation* as location. When we hear that ‘[the Clerk’s] Office does not have a/the yellow-path displacement sight’ in 721, for example, our source is referring to *the* sphere by that name finished by Li Chunfeng in 633. Financed in 627 to replace the Xianbei observatory sphere, whose ‘design & construction were loose & rough’ 法制疏略 (*Jiu Tang shu*, 35.1293), Li’s was the first observational model in 520 years to incorporate an ecliptic ring. Unfortunately,

其所造渾儀，太宗令置於凝暉閣以用測候，既在宮中，尋而失其所在。

[Tang] Taizong (r. 627–649) ordered the sphere sight that [Li Chunfeng] had constructed installed in the Pavilion of Congealed Light so as to [personally] use it for measuring & watching, and though it was right there in the palace, when [later] looked for, [they] had lost track of where it went (*Jiu Tang shu*, 35.1293).

Li’s was not the only priceless observational instrument to become a lawn ornament. What we know about the chain of custody for Kong Ting’s Xiongnu sphere, for example, is that General Liu Yu ‘donated it to the capital’ 獻于京師 (*Song shu*, 2.42), bringing it ‘to [a] royal palace’ 及王府 in Jiankang (*Yiwen leiju*, 1.6a–b), where, by the sixth century, it would be installed within the closed imperial park at Hualin 華林園 (*Sui shu*, 17.517). It is no wonder that He Chengtian, Shen Yue and other fifth-century writers managed to miss the ‘made in Chang’an’ label: they probably never saw the thing in person.

Where and when an observational armillary sphere *was* accessible, experts would have had to make do with a fixed equatorial ring. To work with the ecliptic, one would thus have had no other option but ‘advance/retreat’ unit conversion. Cited both north and south, *The Sphere Heaven Sight* clearly saw interstate circulation, as did the ‘advance/retreat numbers’ in the tables of 174 (*Hou Han shu*, *zhi* 3, 3074). The period likewise saw an explosion of ‘effigy’ production, by which one could reproduce Zhang Heng’s measurements (Wu & Quan, 2008, pp. 466–73). As to equatorial units, we do not know how actors converted from ‘added hours’ to equatorial lodge-*du*, as the Xiongnu and Xianbei spheres would have necessitated, but one imagines that it would have worked like this: (1) align an object at the centre of the sighting tube; (2) note the time or the ‘meridian star’ (*zhong xing* 中星) centred between the double meridian rings; (3) find the ‘added hour’ (the hour angle counted from the midnight meridian line) by noting where the double meridian circles of the ‘four-displacement sight’ intersect the graduated equatorial ring of the ‘six-joint sight’; (4) ‘add the hour’ to the lodge-entry *du* of the midnight meridian line as determined by either the time or the star opposite it on the meridian. The fact that the Kong Ting sphere’s equatorial ring featured ‘*du* numbers’, one notes, would have facilitated ‘adding the hour’ without converting from ‘corners & chronograms’.

This raises the question of *du*-graduation and its precision. Shen Gua, above, juxtaposes the celestial and material *du*, but he fails to mention the *mathematical du*, for which *li* experts used values like  $365\frac{385}{1539}$  and  $365\frac{455}{1843}$  *du* to the ‘circuit’. In practice, there must

have been some compromise—some ‘play’—between the material and mathematical *du*, the question being how much. Pan (1989, pp. 271–72) argues that, up until the thirteenth century, the Chinese armillary sphere was only ever graduated to the integer *du*, the trailing fractions of *shao* 少 (‘lesser’ = 1/4), *ban* 半 (‘half’) and *tai* 太 (‘greater’ = 3/4) seen in observational data being the product of estimation. Pan’s argument rests on three points. The first is that, in 1280, Guo Shoujing claims to have been the first to *really* empirically measure the twenty-eight lodges down to fractional widths. The second is the degree of precision witnessed in the observational record, where trailing fractions are rough and rare. The third is unequivocal documentation of 365-*du* observational spheres in late sources. The first two points are arguments from authority and absence, respectively, but the third gives us food for thought.

The 365-*du* sphere sight appears in four sources relating to three devices. First, Shen Gua complains in 1074 that the observatory’s observational sphere ‘could only be allocated 365 *du* with no way to possess the remainder part’ 但可賦三百六十五度，而不能具餘分 (*Song shi*, 48.959). His description matches that of a 365-*du* sphere sight constructed in 995 ‘on the basis of the method inherited from [Li] Chunfeng and Monk Yixing’ 本淳風及僧一行之遺法 (*Song shi*, 48.952). The *Old Book of Tang* indeed confirms that the Li Chunfeng sphere was graduated with ‘365 *du* in warp (RA) & weft (declination)’ 經緯三百六十五度 (*Jiu Tang shu*, 79.2718), but things get weirder when we turn to Yixing’s 725 design:

黃道單環：外一丈五尺四寸一分，橫八分，厚四分，直徑四尺八寸四分。日之所行，故名黃道。古人知有其事，竟無其器...臣今創置此環，置於赤道環內，仍開合使隨轉運，出入四十八度，而極晝兩方，東西列周天度數，南北列百刻，使見日知時，不有差謬。上列三百六十策，與用卦相準，度穿一穴，與赤道相交。

Yellow-path single ring: exterior (circumference) 15.41 *chi* (466.92 cm), traverse (width) 8 *fen* (2.42 cm), thickness 4 *fen* (1.21 cm), diameter 4.84 *chi* (146.65 cm). [This is] where the sun travels, thus is it named the yellow path. The ancients knew that there was such a thing and yet never possessed the apparatus... Your servant now creates & installs this ring, installing it within the red path ring and then opening & closing [it to] make [it] rotate accordingly (i.e. locking it to the rotating equator ring at the appropriate obliquity), emerging & entering 48 *du* (the difference in declination from winter to summer solstice). The solstices are drawn in two places; east–west are arrayed the *du*-numbers of the circuit of heaven, north–south are arrayed the 100 notches, making it so that one sees the sun and know the time without error or blunder, and atop are arrayed the 360 rods, levelled with the reigning hexagrams. At each *du* is drilled a hole (?) [where the ecliptic ring] crosses with the red path (thereby allowing for the repositioning of the ecliptic ring to accommodate precession) (*Jiu Tang shu*, 35.1297-98).

Curiously, the ecliptic ring is graduated with *du*, the 100 water-clock ‘notches’ (*ke* 刻) of the civil day, and the 360 ‘rods’ of *Book of Changes* numerology, thus providing the user with *du*, ‘hours’,

Year	Given name	Obs	Owner	Maker	City	Diameter		Circumference		$\pi$	Grad.	Unit size	
						<i>chi</i> (cm)		<i>chi</i> (cm)				<i>fen</i> (mm)	
103	Yellow-road bronze sight	X	state	–	Luoyang	8.0000	(188)	*25.1327	(590.6)	–	365¼ <i>du</i>	*6.88	(16.2)
< 139	Small sphere		priv.	Zhang Heng	Luoyang	–	–	–	–	–	365¼ <i>du</i>	–	–
?164	Sphere heaven sight		state	Zhang Heng	Luoyang	*4.6505	(109.3)	14.6100	(343.3)	–	*365¼ <i>du</i>	4.00	(9.4)
< 266	Sphere effigy		–	Wang Fan	Jianye	*3.4879	(83.7)	10.9575	(263.0)	–	*365¼ <i>du</i>	3.00	(7.2)
323	Sphere sight	X	st. / roy.	Kong Ting	Chang'an	8.0000	(195.2)	24.0000	(585.6)	*3	24 hrs ? <i>du</i>	100	(244)
398	Sphere sight (iron)	X	state	Chao & Xie	Pingcheng	"	"	"	"	"	"	"	"
436	Sphere heaven sight		state	Qian Lezhi	Jiankang	6.0825	(150.2)	18.2625	(451.1)	*3	*365¼ <i>du</i>	5.00	(12.4)
440	Small sphere heaven		state	Qian Lezhi	Jiankang	2.2000	(54.3)	6.6000	(163.0)	*3	–	2.00	(4.9)
633	Sphere sight	X	st. / roy.	Li Chunfeng	Chang'an	–	–	–	–	–	365 <i>du</i>	–	–
725	Yellow-road displ. sight	X	state	Yixing & Liang	Chang'an	4.5900	(139.1)	14.6100	(442.7)	*3.18	365¼ <i>du</i>	4.00	(12.1)
								14.5900	(442.1)		100 <i>ke</i>	14.59	(44.2)
								"	"		360 <i>ce</i>	4.05	(12.3)
995	Bronze watch/sphere sight	X	state	Han Xianfu	Bianjing	6.1300	(193.7)	18.3900	(581.1)	*3	365 <i>du</i>	5.04	(15.9)

Table 1 Ring dimensions and graduation of sphere instruments to 1000 CE as mentioned in this article. For more on these and other, more poorly-documented instruments from this period, see Wu & Quan (2008). Whether or not an instrument is observational (obs) is determined by context (attested use, presence of sighting tube vs. 'earth'-model, etc.). The asterisks indicate values calculated according to the data provided. Note that the entry for the 725 sphere includes the measurements of two separate rings.



and a functional equivalent of the  $360^\circ$  circle recently introduced from India.

Whatever Yixing and Li Chunfeng's choice of 'circuit', we do see documentation of the  $365\frac{1}{4}$ -*du* sphere prior to Shen Gua's call to action. As to observational spheres, we have the 'Grand Clerk yellow-path bronze sight' of 103 (above), but it is the 'effigy', oddly enough, where one finds consistent evidence of  $365\frac{1}{4}$ -*du* rings. *The Sphere Heaven Sight*, as we saw, has one 'allocate each with  $365\text{ du} \& \frac{1}{4}\text{ du}$ ', making for ' $182\text{ du} \& \frac{5}{8}\text{ du}$ ' per hemisphere. The Shen Yue and Li Chunfeng histories cite Wang Fan 王蕃 (228–266) describing historical 'effigies' of 2, 3, and 4 *fen* to the *du*, the circumference of which works out in each case to  $365\frac{1}{4}$ , e.g. '[I, Wang Fan,] have redesigned the sphere effigy taking 3 *fen* to the *du*, for a total circuit of heaven of  $1095\text{ fen} \& \frac{3}{4}\text{ fen}$  ( $365\frac{1}{4} \times 3\text{ fen}$ )' 更制渾象，以三分為一度，凡周天一丈九寸五分四分分之三也 (*Song shu*, 23.677; *Jin shu*, 11.288). Shen Yue and Li Chunfeng likewise attribute Qian Lezhi with demonstrational spheres at 2 and 5 *fen* to the *du* that work out to the same total (*Song shu*, 23.678–79; *Sui shu*, 19.519–20). It is difficult to know how actors could have worked to a precision of ' $\frac{5}{8}\text{ du}$ ' or ' $\frac{3}{4}\text{ fen}$ ', but the fact that contemporary *chi*-rules were graduated down to the *fen* does testify to the *capacity* for fractional graduation at a scale of at least 4 *fen* per *du* (Qiu, 1992).

If the potential for a  $365\frac{1}{4}$ -*du* sphere sight was there in the second century, why then would later constructions opt out? I think the answer lies in the way that the real-world practice of 'observation' was mediated by the material and arithmetic sphere. On the material end, there is always going to be 'play'. Whether or not one rounds the quarter *du*, the material 'circuit' will never meet the precision of its mathematical counterpart. Nor for that matter does *precision* translate into *accuracy*. Of the iron sphere of 398, for example, Yixing complains that 'the ring construction is crude & rough manner, and its *du* notches are uneven' 規制朴略，度刻不均，rendering an error of some  $\pm 2\frac{1}{2}\text{ du}$  when measuring lunar anomaly (*Jiu Tang shu*, 35.1295). Whatever the quality of construction, the fact that this and the Kong Ting sphere were war booty transported to new latitudes would have introduced further alignment errors (and damage). On the mathematical end, 'observation' was less spontaneous than our sources let on. For centuries, actors had developed 'effigies' and algorithms as a computational substitute for an ecliptic ring, and the rings they *did* build were graduated to unlikely integers, reminding us that the difference between a 365- and  $365\frac{1}{4}$ -*du* 'sight' is simply one of quotidian unit conversion. Either way, the absence of spherical trigonometry precludes corrections like refraction and parallax, otherwise necessary in, say, a Ptolemaic tradition to ensure an accuracy of anywhere near to  $\frac{1}{4}\text{ du}$  (Włodarczyk, 1987).

## Conclusion

From what we read about the material 'sphere heaven' we can infer something of how the 'observation' of the celestial sphere was

mediated by its material and mathematical counterpart. First-millennium sources tend to efface these processes of mediation, the inherency of which we recall when we turn to Ptolemaic writings, wherein ‘observation’ is mostly calculation. The difference, however, is less to do with ‘East vs West’ than the way that early Chinese observational practices are, in turn, mediated by our sources. Treatises like *The Sphere Heaven Sight* go into the details of practice—be it the extraction of a mathematical substitute for the material substitute for heaven—but the majority of what survives of such literature survives as excerpted in *histories*, the point of which is to provide names, dates and a narrative to what one (once) could read about somewhere else. Still, histories like Shen Yue and Li Chunfeng’s leave us just enough to reconstruct some of what ‘observation’ entailed. ‘Looking through’, for one, was necessarily mediated by algorithms for converting between ‘added hours’ and ‘lodge-entry *du*’ and between equatorial and ecliptic units, and so too was it mediated by material factors such as the precision and accuracy of graduation.

The most important material factor as concerns the history of the ‘sphere sight’, however, is its *absence*. However our sources philosophise about the object, the history of the observational armillary sphere in first-millennium China was one of want, waste, confusion and foreign production. Prior to 725, the only state observatories in possession of such ‘sights’ were those of Han-Wei-Jin Luoyang (103–189/311), Xiongnu-Qiang Chang’an (323–417), Xianbei Pingcheng (412–577) and Sui-Tang Chang’an (583 on), and those that did see use in Chinese hands were misaligned, ‘loose & rough’ and ‘difficult to watch with’. It would have been simpler and cheaper to refine observational practice at the *computational* end, which might explain the relative outpouring of demonstrational ‘computers’ by the likes of Zhang Heng, Wang Fan, Qian Lezhi and others in the intervening centuries. There was no shortage of armillary spheres, but the majority, as in the West, were made for looking *at*. This qualifies them as ‘effigies’ by Li Chunfeng’s definition, but others used these terms rather fluidly, leading one to wonder whether looking *at* is not incompatible with their idea of ‘observation’. Either way, He Chengtian and Shen Yue had ‘looked at’ neither of the spheres that they confused, for Zhang Heng’s had long since turned into cash, and Kong Ting’s, into an imperial lawn ornament.

Rather than leave things there, I would like to end on a question: What *did* actors rely upon for observational data all these centuries in the absence and dereliction of the ‘sphere sight’? And what was this perfect armillary sphere that the Shen Guas of the world are describing?

## Bibliography

- Arai Shinji 新井晉司 (1989) *Chō Kō Kontengi, Kontengi-chū saikō* 張衡『渾天儀』『渾天儀注』再考, in: Yamada Keiji 山田慶兒 (ed.) *Chūgoku kodai kagakushi ron* 中國古

- 代科學史論 (Kyōto: Kyōto daigaku jinbun kagaku kenkyūjo), pp. 317–36.
- Cullen, C. (1996) *Astronomy and Mathematics in Ancient China: The Zhou Bi Suan Jing* (Cambridge: Cambridge UP).
- Cullen, C. (2000) Seeing the Appearances: Ecliptic and Equator in the Eastern Han, *Ziran kexueshi yanjiu* 自然科學史研究 19(4), 352–82.
- Ho, P. Y. (1966) *The Astronomical Chapters of the Chin Shu* (Paris: Mouton).
- Hou Han shu* 後漢書 (Zhonghua shuju ed., 1965).
- Huang Yi-long 黃一農 (1992) Jixing yu gudu kao 極星與古度考, *Tsing Hua Journal of Chinese Studies* 22(2), 93–117.
- Jin shu* 晉書 (Zhonghua shuju ed., 1974).
- Jiu Tang shu* 舊唐書 (Zhonghua shuju ed., 1975).
- Li Zhichao 李志超 (2014) *Tianren gu yi: Zhongguo kexueshi lungang* 天人古義：中國科學史論綱 (3d ed. Zhengzhou: Daxiang chubanshe).
- Lien Y. E. (2012) Zhang Heng's Huntian yi zhu Revisited, *T'oung Pao* 98(1–3), 31–64.
- Maspero, H. (1939) Les instruments astronomique des Chinois au temps des Han, *Mélanges chinois et bouddhiques* 6, 183–370.
- Needham, J. (1959) *Science and Civilisation in China, vol.3: Mathematics and the Sciences of the Heavens and the Earth* (Cambridge: Cambridge University Press).
- Pan Nai 潘鼐 (1989) *Zhongguo hengxing guance shi* 中國恆星觀測史 (Shanghai: Xuelin chubanshe).
- Qiu Guangming 丘光明 (1992) *Zhongguo lidai du liang heng kao* 中國歷代度量衡考 (Beijing: Kexue chubanshe).
- Qu Anjing 曲安京 (1994) *Zhongguo gudai lifa zhong de jishi zhidu* 中國古代曆法中的計時制度, *Hanxue yanjiu* 漢學研究 12(2), 157–72.
- Schafer, E. H. (1977) *Pacing the Void: T'ang Approaches to the Stars* (Berkeley: University of California Press).
- Song shi* 宋史 (Zhonghua shuju ed., 1977).
- Song shu* 宋書 (Zhonghua shuju ed., 1974).
- Sui shu* 隋書 (Zhonghua shuju ed., 1973).
- Wang Yumin 王玉民 (2015) 'Hun-tian yi' kao 「渾天儀」考, *Zhongguo keji shuyu* 中國科技術語 2015.3, 39–42.
- Włodarczyk, J. (1987) Observing with the Armillary Astrolabe, *Journal for the History of Astronomy* xviii, 173–95.
- Wu Shouxian 吳守賢 and Quan Hejun 全和鈞 (2008) *Zhongguo gudai tianti celiangxue ji tianwen yiqi* 中國古代天體測量學及天文儀器 (Beijing: Zhongguo kexue jishu chubanshe).

*Xin yixiang fayao* 新儀象法要 (Siku quanshu ed., rpt. Taiwan shangwu yinshuguan, 1983–1986).

*Yangzi Fayan* 揚子法言 (Siku quanshu ed., rpt. Taiwan shangwu yinshuguan, 1983–1986).

*Yiwen leiju* 藝文類聚 (Siku quanshu ed., rpt. Taiwan shangwu yinshuguan, 1983–1986).

---

<sup>1</sup> For a translation of Li Chunfeng's *Book of Jin* monograph, see Ho (1966).

<sup>2</sup> Ge Hong 葛洪 (283–343) offers the same basic description of Zhang Heng's water-driven sphere as cited in *Jin shu*, 11.281–84; Ho (1966), pp. 55–56.

<sup>3</sup> The term *fa* 法 ('model') refers to a 'method', 'exemplar', 'norm', or 'law' for one to *fa* ('model') oneself upon to do something correctly. The 'sight' and the 'effigy' are thus different 'models' in the sense that 'road' and 'mountain' are different 'models' of bicycle adapted to different functions.

<sup>4</sup> The term *tuan* 搏 ('modelling') refers to the action of 'moulding', 'modelling' or 'kneading' as concerns, in particular, round forms and materials like clay and glutinous food objects. Shen Gua is clearly using the term in a more abstract sense of instrumental and mathematical reproduction.

<sup>5</sup> The term *yu* 豫 ('play') means 'comfort/relaxation', the 'play/sport' leading thereto, and, consequentially, 'laxity' or 'looseness' in relation to one's duties, thus encompassing nicely the semantic range of 'play' in a technical context.