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LAND RENT, PRODUCTION AND LAND USE

Jean-Marie HURIOT

Juillet 1986

LAND RENT, PRODUCTION AND LAND USE

1. Introduction

Let us consider land rent as a monetary surplus generated by a productive activity. Land is allocated to the activity which yields the highest rent, as a result of a bidding process. This principle which THÜNEN (1826) and later LÖSCH (1940) applied to agricultural location, leads to the famous theory of concentric circles which is still at the heart of present day urban structure theories. These theories concentrate on utility and demand rather than on the conditions of production.

In a different approach, the aim of this paper is to generalize some aspects of the allocation of land to productive activities. For different activities, we shall define different vectors of technical coefficients, showing different degrees of intensity in the use of land, labor and capital. Then, we shall try to determine how price-, cost- and distribution parameters influence the relative locations of these activities.

This work is related to papers by SCOTT (1976, 1979, 1980) and by BARNES and SHEPPARD (1984). We shall use rent equations which are formally connected with SRAFFA's equations of production (SRAFFA, 1960), yet we shall not concentrate on distribution-price relations, as SRAFFA's followers do (for example, ABRAHAM-FROIS and BERREBI, 1976 and 1980), but shall seek merely to highlight spatial patterns. The interrelated endogenous variables of neo-Ricardian economics become here exogenous parameters. On the contrary, distances and locations, which are simply ignored by the latter school, now become endogenous variables. More precisely, we need the following assumptions.

Space is like a homogeneous THÜNEN's plain with a central market where all the product is sold and/or distributed. Land is privately owned and the landowner and the land user are different.

Production is described by fixed coefficients for the use of land, labor, capital and transport. There is only circulating capital. Production of one commodity can be achieved by different techniques, or different combinations of factors, but only one technique is used at any given place.

The value of the product is distributed in the form of wages, profits and land rent, relations between costs, production and distribution being viewed in the classical framework of advance economics (SCHUMPETER, 1954). At the beginning of the production period, producers must advance capital outlay, including the cost of transporting this capital from the central market where it is bought. At the end of the production period the producer must pay the cost of transporting his product to the market. It will be assumed that the product is sold. After the capital has been reconstituted, the net income from the sale is distributed, i.e. wages, profits and rents are paid.

Land rent is both differential and residual. It is paid after the institutional wage rate and the normal profit rate have been deducted. Land rent also plays an active role in determining locations according to THUNEN's principle.

Differential rents stem from differences in locations, i.e. from differences in transport costs to the center, and from differences in the techniques employed. The analysis uses comparative statics. Every commodity produced by a given technique generates a rent depending on the technical coefficients, the market prices, the distribution variables and the cost of transport to the center. The equilibrium allocation of land is characterized by spatial variables, namely the limits of rings corresponding to the different commodities or techniques. The values of these variables are obtained by solving a system of rent equations, from which we can see how location depends on technical price and distribution parameters.

While section 2 presents the structure of two very simple models, section 3 gives some elements of analysis, with theoretical exercices illustrated by imaginary numerical examples.

2. Two models of land allocation

These models have been kept as simple as possible. The first one considers only one commodity produced by two alternative techniques. The second one deals with two commodities, each one being produced in this case by only one technique.

2.1. One commodity , two techniques

Only one commodity -say corn- is produced and used as capital in production and as consumer commodity. Wages, profits and rents are paid in terms of this commodity. There is no price involved, because only corn is circulating. The commodity can be produced by one of two fixed techniques, identified by the superscript $k(k = 1,2)$. One unit of land produces x^k units of commodity by technique k (x^k is the inverse of the quantity of land used per unit of commodity). The production of one unit of commodity uses a^k units of capital (the commodity itself) and l^k units of labor. w is the wage rate and r is the profit rate, both of which are uniform. The cost of transport per unit of distance and per unit of commodity is the transport rate, τ . The land rent, ρ, depends on the technique, k, and on the distance to the center, δ ; we shall call this rent $\rho^k(\delta)$.

In a one-commodity-economy, there is no exchange, and the center cannot be a market. It retains its attractive power if we assume that it is the sole distribution place, meaning that the payment of wages, profits and rents are made at that place. When one unit of commodity is produced, the quantity a^k is kept as capital and the quantity $1 - a^k = t^k$ has to be transported to the distribution center ; we have $0 < t^k < 1$. t^k is a coefficient of production. Let us distinguish the two techniques in assuming that technique one is more land-intensive (or that it makes the land less productive), and that it is less labor- and capital-intensive than technique two. This means that :

$$x^1 < x^2 ,$$

(1) $$a^1 < a^2 , \text{ hence } t^1 > t^2$$

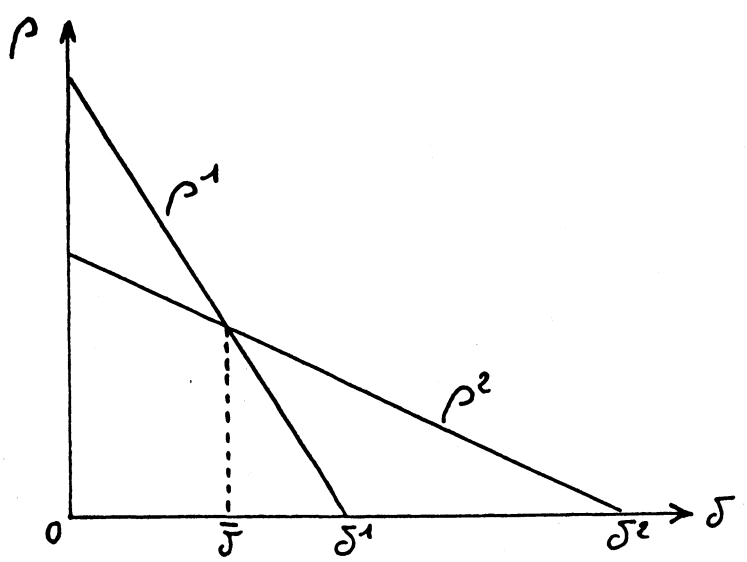
$$l^1 < l^2$$

Finally, we have to specify the spatial variables. For a given technique, the land rent is a decreasing function of distance. So, we can define the distance δ^k such that $\rho^k(\delta^k) = 0$.
Since each rent function is linear, there exists a unique value of $\overline{\delta}$ such that

$$\rho^1(\overline{\delta}) = \rho^2(\overline{\delta}).$$

$\overline{\delta}$ is the boundary between the two technique rings. δ^1, δ^2 and $\overline{\delta}$ determine the relative circular locations of the techniques, as shown on figure 1.

Figure 1. The spatial variables



According to the definition, the expression of the rent per unit of land, when technique k is used, is :

(2) $$\rho^k(\delta) = x^k - a^k x^k(1 + r) - l^k x^k w - t^k x^k \tau\delta \ , \ (k = 1,2)$$

where the negative terms measure the amounts of commodity used to reconstitute capital ($a^k x^k$), to pay profit ($a^k x^k r$), wages ($l^k x^k w$) and transport costs ($t^k x^k \tau\delta$).

At distance δ^k, we have $\rho^k(\delta^k) = 0$, and, from **(2)** :

$$x^k - a^k x^k(1 + r) - l^k x^k w - t^k x^k \tau\delta^k = 0 \qquad (k = 1,2), \text{ or more simply :}$$

(3) $$1 - a^k(1 + r) - l^k w - t^k \tau\delta^k = 0 \qquad (k = 1,2).$$

This system can be solved in δ^1, δ^2. Then we can write the rent functions in terms of these values :

(4) $$\rho^k(\delta) = t^k x^k \tau(\delta^k - \delta).$$

According to the definition of $\overline{\delta}$, we have :

$$t^1 x^1 \tau(\delta^1 - \overline{\delta}) = t^2 x^2 \tau(\delta^2 - \overline{\delta}) \ , \text{ thus}$$

(5) $$t^1 x^1(\delta^1 - \overline{\delta}) = t^2 x^2(\delta^2 - \overline{\delta}),$$

which we can solve in $\overline{\delta}$.
From **(3)**, we find immediately :

(6) $$\delta^k = \frac{1 - a^k(1 + r) - l^k w}{t^k \tau} \ , \ (k = 1,2) \ .$$

For given techniques, the precise value of δ^k depends on the values taken by r, w and τ.
From **(5)** and **(6)**, we have :

$$t^1 x^1 \left(\frac{1 - a^1(1 + r) - l^1 w}{t^1 \tau} - \overline{\delta}\right) = t^2 x^2 \left(\frac{1 - a^2(1 + r) - l^2 w}{t^2 \tau} - \overline{\delta}\right) ,$$

thus : $\frac{x^1}{\tau}(1 - a^1(1 + r) - l^1 w) - t^1 x^1 \bar{\delta} = \frac{x^2}{\tau}(1 - a^2(1 + r) - l^2 w) - t^2 x^2 \bar{\delta}$,

and $$\bar{\delta} = \frac{x^2(1 - a^2(1 + r) - l^2 w) - x^1(1 - a^1(1 + r) - l^1 w)}{\tau(t^2 x^2 - t^1 x^1)} ,$$

which, after some arrangements, gives :

$$\bar{\delta} = \frac{x^2(1 - a^2) - x^1(1 - a^1) + (a^1 x^1 - a^2 x^2) r + (l^1 x^1 - l^2 x^2) w}{\tau(t^2 x^2 - t^1 x^1)} .$$

Remembering that $t^k = 1 - a^k$, and putting :

$a^k x^k = \alpha^k$, the quantity of capital per unit of land, using technique k, and

$l^k x^k = \lambda^k$, the quantity of labor per unit of land, using technique k.

Then we obtain :

(7) $$\bar{\delta} = \frac{t^2 x^2 - t^1 x^1 + (\alpha^1 - \alpha^2) r + (\lambda^1 - \lambda^2) w}{\tau(t^2 x^2 - t^1 x^1)} ,$$

where, from the inequalities **(1)** and the definitions of α^k and λ^k,

(8) $x^1 < x^2$, $\alpha^1 < \alpha^2$, $\lambda^1 < \lambda^2$.

$\bar{\delta}$ depends on the values taken by r, w and τ, for a given technique.

2.2. Two commodities, no technical alternative

In our second model, we have two commodities respectively identified by the subscript i (i = 1, 2), each of them being produced by only one technique. These two commodities have to be distinguished by their conditions of production. But technical coefficients in physical terms are not directly comparable for two different commodities when units are not. For this reason, we define the unit of commodity i as the quantity of that commodity produced on one unit of land, so that $x_1 = x_2 = 1$. We assume that the production of one unit of commodity i uses labor in quantity l_i, and the other commodity -as intermediate input, i.e. as capital- in quantity a_i. Note that l_i and a_i could also be defined as quantities used per unit of land ; a_i could also measure the indirect use of land, i.e. the quantity of land used to produce the input needed for the production of one unit of commodity i, i.e. for one unit of land devoted to commodity i. We assume that one of the production processes is more labor-intensive, while the other one is more capital-intensive, so that :

(9) $$l_1 < l_2 \quad \text{and} \quad a_2 < a_1 \quad .$$

Commodities 1 and 2 are evaluated by their delivery prices p_1 and p_2 to the central market. Since we are only interested here in relative prices, we can assume that $p_2 = 1$, so that

$$\frac{p_1}{p_2} = p_1 = p \ .$$

The other preceding notations hold.
The rent function is now expressed in value terms. For one unit of land devoted to commodity 1, the value of the product is p. The capital advance increased by the corresponding profit is $a_1(1 + r)$; the expenditure on labor is $l_1 w$; the transport cost is $\tau\delta$ for the output and $a_1\tau\delta$ for the input. This transport cost is also an advance

which generates a profit $a_1\tau\delta r$. It is easy to state the same items for the production of commodity 2. Then we have :

$$\textbf{(10)} \quad \begin{cases} \rho_1(\delta) = p - a_1(1 + r) - l_1 w - [1 + a_1(1 + r)]\tau\delta, \\ \rho_2(\delta) = 1 - pa_2(1 + r) - l_2 w - [1 + a_2(1 + r)]\tau\delta. \end{cases}$$

If $\rho_1(\delta_1) = \rho_2(\delta_2) = 0$, we have the new system :

$$\textbf{(11)} \quad \begin{cases} p - a_1(1 + r) - l_1 w - [1 + a_1(1 + r)]\tau\delta_1 = 0 \\ 1 - pa_2(1 + r) - l_2 w - [1 + a_2(1 + r)]\tau\delta_2 = 0 \ , \end{cases}$$

which is solved in δ_1 and δ_2 .
The rent functions become :

$$\textbf{(12)} \quad \begin{cases} \rho_1(\delta) = [1 + a_1(1 + r)]\tau(\delta_1 - \delta) \\ \rho_2(\delta) = [1 + a_2(1 + r)]\tau(\delta_2 - \delta). \end{cases}$$

From the definition of $\overline{\delta}$:

$$\textbf{(13)} \qquad [1 + a_1(1 + r)]\tau(\delta_1 - \overline{\delta}) = [1 + a_2(1 + r)]\tau(\delta_2 - \overline{\delta})$$

which gives the value of $\overline{\delta}$.
From **(11)**, we find directly :

$$\textbf{(14)} \quad \begin{cases} \delta_1 = \dfrac{p - a_1(1 + r) - l_1 w}{\tau[1 + a_1(1 + r)]} \\ \delta_2 = \dfrac{1 - pa_2(1 + r) - l_2 w}{\tau[1 + a_2(1 + r)]} \ . \end{cases}$$

By substituting **(14)** in **(13)** and solving in $\overline{\delta}$, we obtain :

$$(15) \qquad \bar{\delta} = \frac{p - 1 + (a_2 p - a_1)(1 + r) + (l_2 - l_1)w}{\tau(a_1 - a_2)(1 + r)} .$$

3. The spatial patterns of production

From the preceding models we have derived expressions of the spatial variables in terms of price, distribution and transport cost parameters. These results can be used in two ways.
First, we can specify in terms of the parameters above the conditions for concentric locations. This is a generalization of the Löschian conditions (LÖSCH, 1940). Secondly, we can show how the spatial pattern can change with the values of the parameters (comparative statics).

3.1. The conditions for concentric locations

For which possible combinations of values of the parameters are the two activities actually present, the one at the center, and the other one at the periphery (the spatial pattern we term concentric locations) ?

3.1.1. The concentric locations of the two techniques

When both techniques are used, their spatial order depends on the order of the slopes of the corresponding rent curves. If technique 2 is used at the center and technique 1 at the periphery, the slope of the second curve must be steeper than the slope of the first one (see figure 1). Then, from **(4)** :

$$(16) \qquad t^2 x^2 > t^1 x^1 ,$$

which means that the amount of commodity to be transported per unit of land is greater with the second technique than with the first one. This is a necessary condition. If the spatial order is reversed so is this condition.
Suppose now that condition **(16)** is realized. The sufficient conditions for concentric locations must be stated. We have two sets of simultaneous conditions.

1. Technique 2 is actually used at the center. This is true if and only if

(17) $\rho^2(0) > \rho^1(0)$, and

(18) $\rho^2(0) > 0$.

The first inequality gives, from **(2)** and the definitions of α^k and λ^k :

$$x^2 - \alpha^2(1 + r) - \lambda^2 w > x^1 - \alpha^1(1 + r) - \lambda^1 w, \text{ or}$$

$$(\alpha^1 - \alpha^2)r + (\lambda^1 - \lambda^2)w + x^2 - x^1 + \alpha^1 - \alpha^2 > 0,$$

with $x^2 - x^1 + \alpha^1 - \alpha^2 = x^2 - x^1 + a^1x^1 - a^2x^2$

$$= (1 - a^2)x^2 - (1 - a^1)x^1 = t^2x^2 - t^1a^1 .$$

Thus, we have :

(19) $$\begin{cases} (\alpha^1 - \alpha^2)r + (\lambda^1 - \lambda^2)w + t^2x^2 - t^1x^1 > 0, \text{ or} \\ f_1(r, w) > 0 . \end{cases}$$

The second inequality, **(18)**, is equivalent to

$$x^2 - \alpha^2(1 + r) - \lambda^2 w > 0 , \quad \text{and, if } x^2 > 0,$$

(20) $$\begin{cases} 1 - a^2r - l^2w + 1 - a^2 > 0, \text{ or, in other terms :} \\ f'_1(r, w) > 0 . \end{cases}$$

(19) and **(20)** define the set of vectors (r, w) such that technique 2 is actually used at the center.

2. Technique 1 is actually used at the periphery. This means that (see figure 1) :

(21) $\delta^1 > \delta^2$, and

(22) $\delta^1 > 0$.

From **(6)** and **(21)**, we have :

$$\frac{1 - a^1(1 + r) - l^1 w}{t^1 \tau} > \frac{1 - a^2(1 + r) - l^2 w}{t^2 \tau} \text{ , thus}$$

$$t^2 - t^2 a^1(1 + r) - t^2 l^1 w > t^1 - t^1 a^2(1 + r) - t^1 l^2 w, \text{ and}$$

$$(t^1 a^2 - t^2 a^1) r - (t^2 l^1 - t^1 l^2) w - t^1 + t^2 + (t^1 a^2 - t^2 a^1) > 0, \text{ where}$$

$$t^1 a^2 - t^2 a^1 = (1 - a^1) a^2 - (1 - a^2) a^1 = a^2 - a^1. \text{ Then,}$$

$$(a^2 - a^1) r + (t^1 l^2 - t^2 l^1) w + t^2 - t^1 + a^2 - a^1 > 0, \text{ where}$$

$$t^2 - t^1 + a^2 - a^1 = (1 - a^2) + a^2 - (1 - a^1) - a^1 = 0 \text{ ; then}$$

(23) $$\begin{cases} (a^2 - a^1) r + (t^1 l^2 - t^2 l^1) w > 0, \text{ or, in other terms} \\ f_2(r, w) > 0 . \end{cases}$$

Note that, from **(1)**, $a^2 - a^1 > 0$, and $t^1 l^2 - t^2 l^1 > 0$, so that **(23)** is **always satisfied** for positive values of r and w.
(22) is equivalent to :

$$\frac{1 - a^1(1 + r) - l^1 w}{t^1 \tau} > 0, \text{ or simply}$$

(24) $$\begin{cases} - a^1 r - l^1 w + 1 - a^1 > 0, \text{ or in other terms} \\ f_2'(r, w) > 0, \end{cases}$$

which defines the set of vectors (r, w) such that technique 1 is actually used at the periphery.
To sum up, when the technical conditions imply inequality **(16)** -the second rent curve is steeper than the first one- the concentric locations are obtained if and only if the vector (r, w) belongs to the set defined by inequations **(19)**, **(20)** and **(24)** for positive values of r and w. Thus we have related spatial patterns to distribution parameters.

Since the aforementioned conditions have no obvious meaning, we shall now propose a numerical example using the data of table 1.

Table 1. Data for the first model

	a^k	$t^k = 1 - a^k$	l^k	τ	x^k	$\alpha^k = a^k x^k$	$\lambda^k = l^k x^k$
Technique 1	0.2	0.8	0.1	0.1	20	4	2
Technique 2	0.4	0.6	0.1	0.1	30	12	3

Condition **(16)** holds. Conditions **(19)**, **(20)** and **(24)** become respectively :

(19 e) $f_1(r, w) = -8r - w + 2 > 0$, thus

$r < -0.125\,w + 0.25$.

(20 e) $f_1'(r, w) = -12r - 3w + 18 > 0$, thus

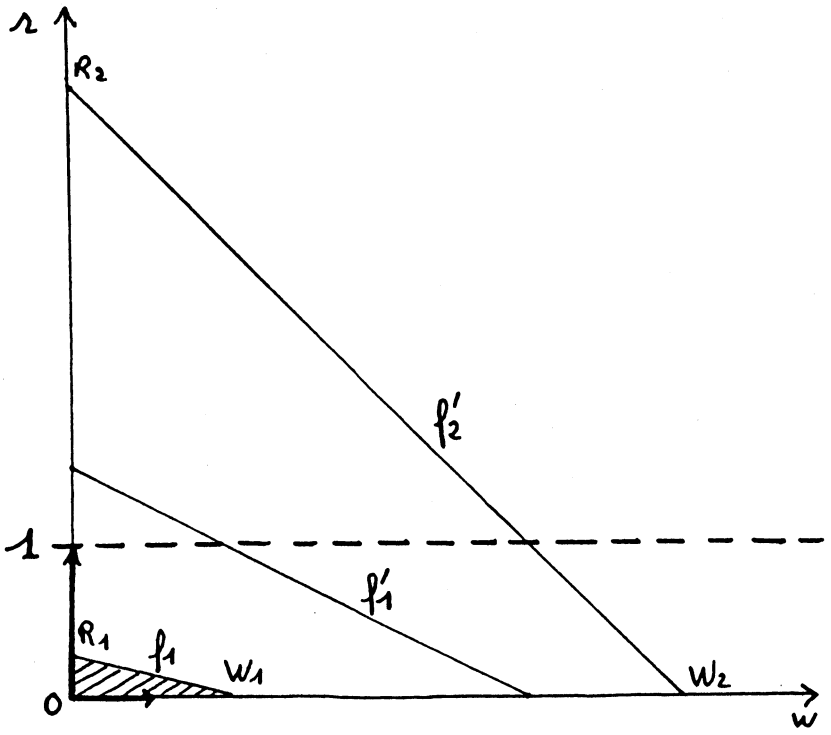
$r < -0.25\,w + 1.5$.

(24 e) $f_2'(r, w) = -0.2r - 0.1w + 0.8 > 0$, thus

$r < -0.5w + 4$.

In figure 2, the set of vectors (r, w) corresponding to concentric locations is represented by the small shaded triangle R_1OW_1. Outside this triangle, but inside the triangle R_2OW_2, condition **(19)** does not hold, which means that $\rho^1(0) > \rho^2(0)$. Condition **(24)** holds, indicating that technique 1 is used everywhere. In the real world, r will probably be small, and we can consider that $r < 1$.

Figure 2. Conditions for concentric locations. First model .



If, instead of inequality **(16)**, we have

(16') $\quad t^1x^2 > t^2x^2$,

then the rent curve associated with technique 1 is steeper than the one associated with technique 2. In these circumstances, the sole possible concentric pattern shows technique 1 at the center and technique 2 at the periphery. The choice of technique 1 at the center is made if and only if :

(17') $\rho^1(0) > \rho^2(0)$, and

(18') $\rho^1(0) > 0$.

(17') leads to

(19') $(\alpha^2 - \alpha^1)r + (\lambda^2 - \lambda^1)w + t^1x^1 - t^2x^2 > 0$.

Now, from **(8)** : $\alpha^2 - \alpha^1 > 0$, $\lambda^2 - \lambda^1 > 0$ and from **(16')**, $t^1x^1 - t^2x^2 > 0$, hence inequality **(19')** always holds for positive values of r and w. $\rho^1(\delta)$ is linear and decreasing, thus condition **(18')** is equivalent to condition **(22)** : $\delta^1 > 0$, which led to **(24)** : $- a^1r - l^1w + 1 - a^1 > 0$.

The choice of technique 2 at the periphery is made if and only if

(21') $\delta^2 > \delta^1$, and

(22') $\delta^2 > 0$.

(21') leads to

(23') $(a^1 - a^2)r + (t^2l^1 - t^1l^2)w > 0$,

which is never realized since the coefficients of r and w are both negative. As it is impossible to produce anywhere using technique 2, it is therefore impossible to obtain concentric locations. Technique 1 is employed only if inequality **(24)** holds.

3.1.2. The second model

As for the first model, let us first compare the slopes of the rent curves, namely, from **(12)**, $[1 + a_1(1 + r)]\tau$ and $[1 + a_2(1 + r)]\tau$. Since $a_1 > a_2$, the first slope is in any case the steepest one. So, in any concentric pattern the first commodity will be more central than the second one. The necessary and sufficient conditions are derived as in the first model.

For the first commodity to be produced at the center, we must have :

(25) $\rho_1(0) > \rho_2(0)$, and

(26) $\rho_1(0) > 0$.

Combining **(25)** and **(10)** gives :

$$p - a_1(1 + r) - l_1 w > 1 - pa_2(1 + r) - l_2 w, \text{ and thus :}$$

(27) $$\begin{cases} (1 + a_2)p - a_1 r + (l_2 - l_1)w + a_2 pr - 1 - a_1 > 0, & \text{or} \\ g_1(p, r, w) > 0 \end{cases}$$

From **(14)** and **(26)**, we obtain :

(28) $$\begin{cases} p - a_1 r - l_1 w - a_1 > 0, \text{ or} \\ g_1'(p, r, w) > 0 \end{cases}$$

(27) and **(28)** define the set of vectors (p, r, w) corresponding to the production of the first commodity at the center.
For the second commodity to be produced at the periphery, we must have :

(29) $\delta_2 > \delta_1$, and

(30) $\delta_2 > 0$.

(29) is equivalent to :

$$\frac{1 - pa_2(1 + r) - l_2 w}{\tau\,[1 + a_2(1 + r)]} > \frac{p - a_1(1 + r) - l_1 w}{\tau\,[1 + a_1(1 + r)]} .$$

After multiplying the first term by $[1 + a_1(1 + r)]$ and the second one by $[1 + a_2(1 + r)]$, rearranging and symplifying, we obtain condition

(31) $g_2(p, r, w) > 0$, with

$$g_2 = (1 + 2a_2 + a_1a_2)p + (2a_1 + 2a_1a_2)r + (l_1 - l_2 + a_2l_1 - a_1l_2)w$$

$$- (2a_2 + 2a_1a_2)pr + (a_2l_1 - a_1l_2)wr + a_1a_2r^2 - a_1a_2pr^2 + 1 + 2a_1 + a_1a_2 .$$

From **(14)** and **(30)**, we have :

(32) $$\begin{cases} - a_2p - l_2w - a_2pr + 1 > 0, \text{ or, in other terms} \\ g'_2(p, r, w) > 0 . \end{cases}$$

(31) and **(32)** define the set of vectors (p, r, w) such that the second commodity is produced at the periphery. Thus **(27)**, **(28)**, **(31)** and **(32)** together determine the set of vectors (p, r, w) corresponding to concentric locations. These conditions define a three-dimensional set and the equations of the bounds are very complicated. We will specify technical data (see table 2) and successively fix one of the three parameters so as to show two-dimensional sections of the concentric locations set.

Table 2. Data for the second model

	a_i	l_i	τ
Commodity 1	0.2	0.1	0.1
Commodity 2	0.4	0.2	0.1

Let us fix r = 0.1 . The four conditions are :

$g_1 = 1.22\ p + 0.1\ w - 1.44 > 0 \quad \Rightarrow \quad p > -\ 0.08\ w + 1.18$.

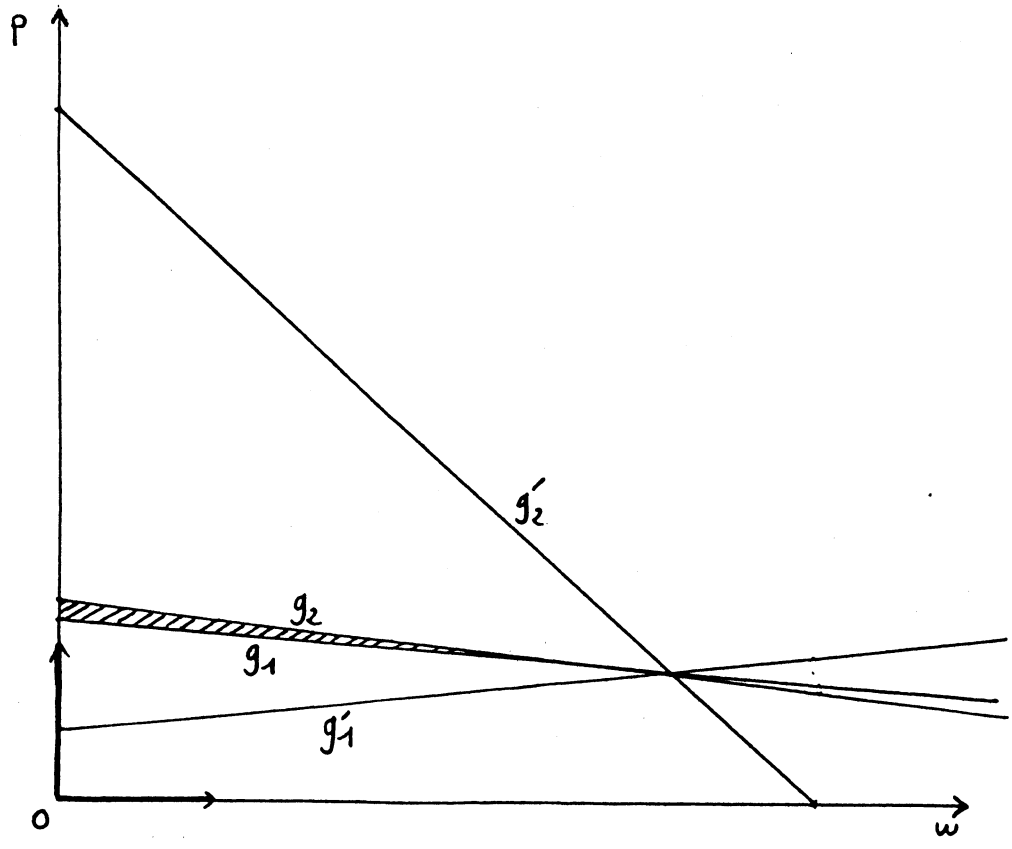
$g'_1 = p - 0.1\ w - 0.44 > 0 \quad \Rightarrow \quad p > \ 0.1\ w + 0.44$.

$g_2 = -\ 1.5\ p - 0.17\ w + 1.98 > 0 \quad \Rightarrow \quad p > -\ 0.11\ w + 1.31$.

$g'_2 = -\ 0.22\ p - 0.2\ w + 1 > 0 \quad \Rightarrow \quad p < -\ 0.91\ w + 4.55$.

Then figure 3 shows what happens for different vectors (p, w). Only in the shaded zone do the concentric locations hold. Outside this area, only one of the two commodities can be produced ; however this is impossible since each of the commodities needs the other as input.

Figure 3. The conditions for concentric locations of commodities, with a fixed r



If we fix w = 1, the conditions become :

$$g_1 = 1.2\ p - 0.4\ r + 0.2\ pr - 1.3 > 0 \quad \Rightarrow \quad p > \frac{4\ r + 13}{2\ r + 12}\ .$$

$$g'_1 = p - 0.4\ r - 0.5 > 0 \quad \Rightarrow \quad p > 0.4\ r + 0.5\ .$$

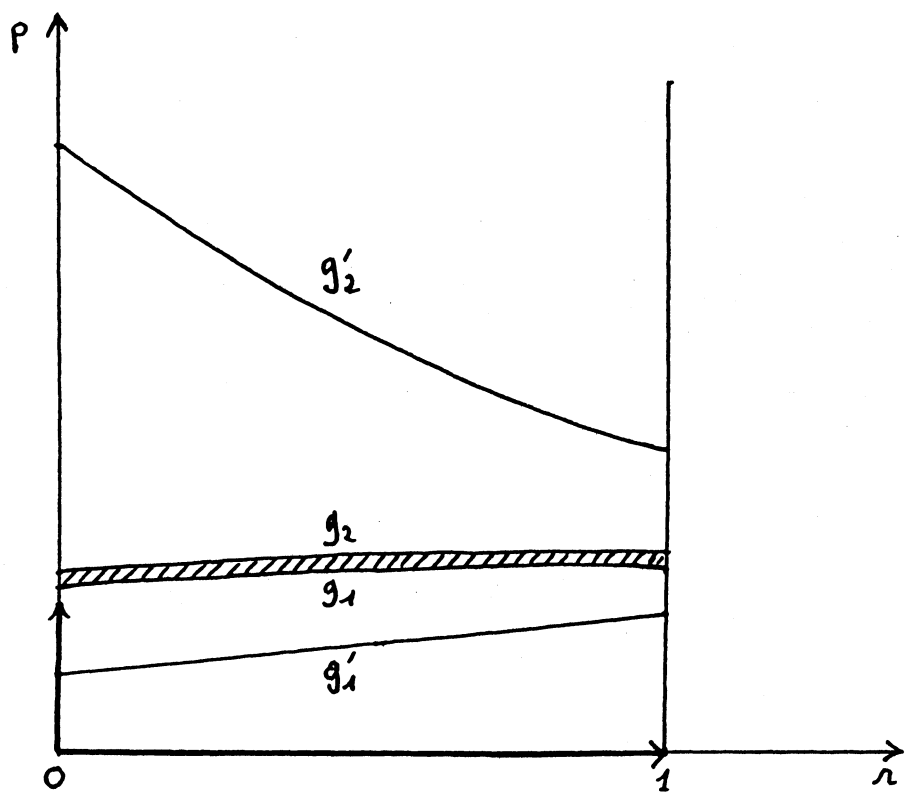
$$g_2 = -\ 1.48\ p + 0.9\ r - 0.56\ pr + 0.08\ r^2 - 0.08\ pr^2 + 1.74 > 0$$

$$\Rightarrow \quad p < \frac{1.74 + 0.9\ r + 0.08\ r^2}{1.48 + 0.56\ r + 0.08\ r^2}\ .$$

$$g'_2 = -\ 0.2\ p - 0.2\ pr + 0.8 > 0 \quad \Rightarrow \quad p < \frac{4}{1 + r}\ .$$

The ad hoc set is shaded in figure 4 (with r varying between 0 and 1).

Figure 4. The conditions for concentric locations of commodities, with a fixed w



Finally, let us fix p = 1.10 . Then we have :

$$g_1 = 0.18\ r + 0.1\ w + 0.08 > 0 \qquad \Rightarrow w > 1.8\ r + 0.8\ .$$

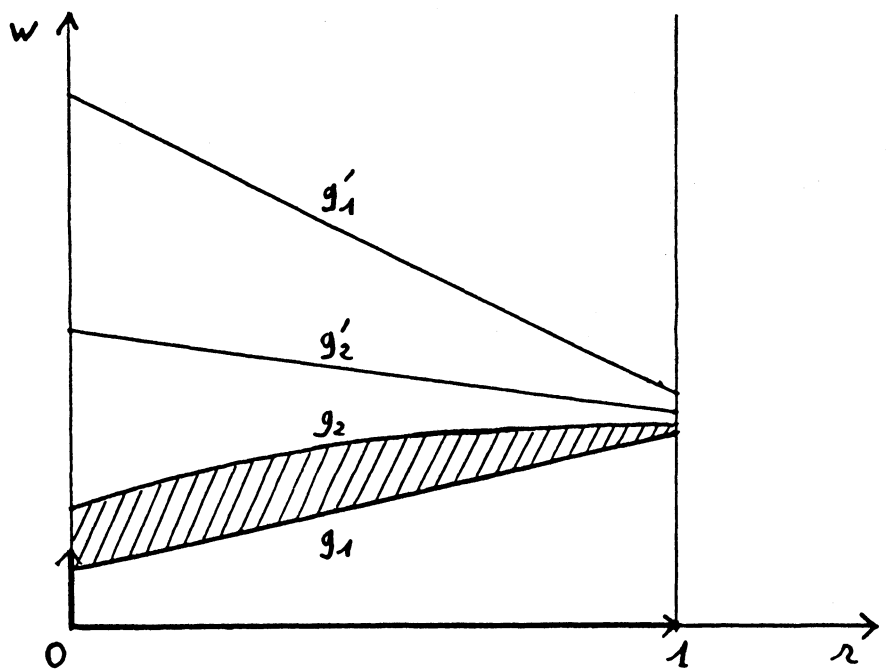
$$g'_1 = -\ 0.4\ r - 0.1\ w + 0.7 > 0 \qquad \Rightarrow w < -\ 0.4\ r + 7\ .$$

$$g_2 = 0.34\ r - 0.16\ w - 0.06\ wr - 0.008\ r^2 > 0 \Rightarrow w < \frac{0.25 + 0.34\ r - 0.008\ r^2}{0.16 + 0.06\ r}\ .$$

$$g'_2 = -0.22\ r - 0.2\ w + 0.78 > 0 \qquad \Rightarrow w < -\ 1.1\ r + 3.9\ .$$

These conditions define the shaded set in figure 5 (with r varying between 0 and 1).

Figure 5. The conditions for concentric locations of commodities, with a fixed p



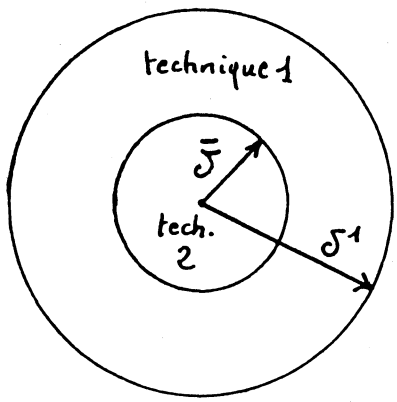
3.2. The changes of locations

Suppose that the preceding conditions for concentric locations are fulfilled. How does the spatial pattern of activities change in response to variations in the parameters p, r, w and τ ? Once again, the two models are examined.

3.2.1. The first model

In the case of inequality **(16)**, i.e. when the second rent curve is steeper than the first one, the boundaries of the rings corresponding to the two techniques are determined by δ^1 and $\bar{\delta}$, as defined by **(6)** and **(7)**, and as shown on figure 6.

Figure 6. The technique-rings when condition (16) is realized



It must be remembered that

(6) $$\text{for } k = 1 \;:\; \delta^1 = \frac{1 - a^1(1 + r) - l^1 w}{t^1 \tau} .$$

$$(7) \qquad \bar{\delta} = \frac{t^2x^2 - t^1x^1 + (\alpha^1 - \alpha^2)r + (\lambda^1 - \lambda^2)w}{\tau(t^2x^2 - t^1x^1)} .$$

The problem of the changes in locations can be studied theoretically by means of the derivatives of δ^1 and $\bar{\delta}$, and numerically with an example and graphs of δ^1 and $\bar{\delta}$. The example is based on the data already used for model 1 (see table 1).

<u>The effect of variations in the rate of profit</u>

We have : $\dfrac{\partial\bar{\delta}}{\partial r} = \dfrac{\alpha^1 - \alpha^2}{\tau(t^2x^2 - t^1x^1)} < 0,$

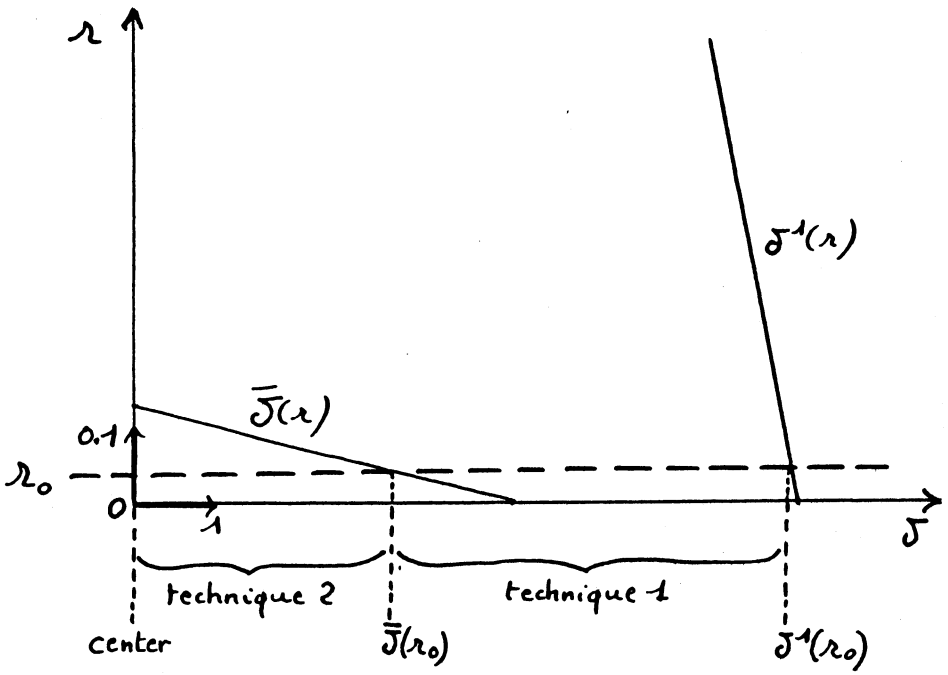
$$\frac{\partial\delta^1}{\partial r} = - \frac{a^1}{t^1\tau} < 0 .$$

The inner ring, where technique 2 is used, becomes smaller, which means that there is a substitution of techniques on the land located near the initial boundary $\bar{\delta}$. δ^1 is decreasing but we cannot ascertain whether the outer ring becomes smaller or larger. The direction in which this area varies is given by the sign of the derivative of $(\delta^1)^2 - (\bar{\delta})^2$ for $\bar{\delta} > 0$. The general case leads to very complicated formulae, and consequently this problem shall be treated only for the numerical example.
Let us fix w = 1 (which is compatible with concentric locations). We have

$$\bar{\delta} = - 40r + 5$$

$$\delta^1 = - 2.5r + 8.75 .$$

Figure 7. The boundaries of the technique-rings as functions of r



We can see from figure 7 how $\overline{\delta}$ and δ^1 vary with r : for example, when w increases from a very low value, the area S_2, allocated to technique 2, decreases and disappears when r = 0.125 . For $\overline{\delta} > 0$, i.e. for $0 < r < 0.125$, the derivative of $(\delta^1)^2 - (\overline{\delta})^2$ is always positive, and the area S_1, allocated to technique 1, increases. When $r > 0.125$, it is clear that this area decreases when r increases. When both techniques are used, <u>an increase in the rate of profit leads to a relative spatial expansion of the most capital saving technique</u>.

<u>The effect of variations in the wage rate</u>

We have : $\dfrac{\partial\overline{\delta}}{\partial w} = \dfrac{\lambda^1 - \lambda^2}{\tau(t^2x^2 - t^1x^1)} < 0$, and $\dfrac{\partial\delta^1}{\partial w} = \dfrac{l^1}{t^1\tau} < 0$.

This case is analogous to the preceding one. In the example, let us fix $r = 0.10$, so that :

$$\bar{\delta} = -5w + 6 \text{ , and}$$

$$\delta^1 = -1.6w + 9.33 \text{ .}$$

Figure 8. The boundaries of the technique-rings as functions of w

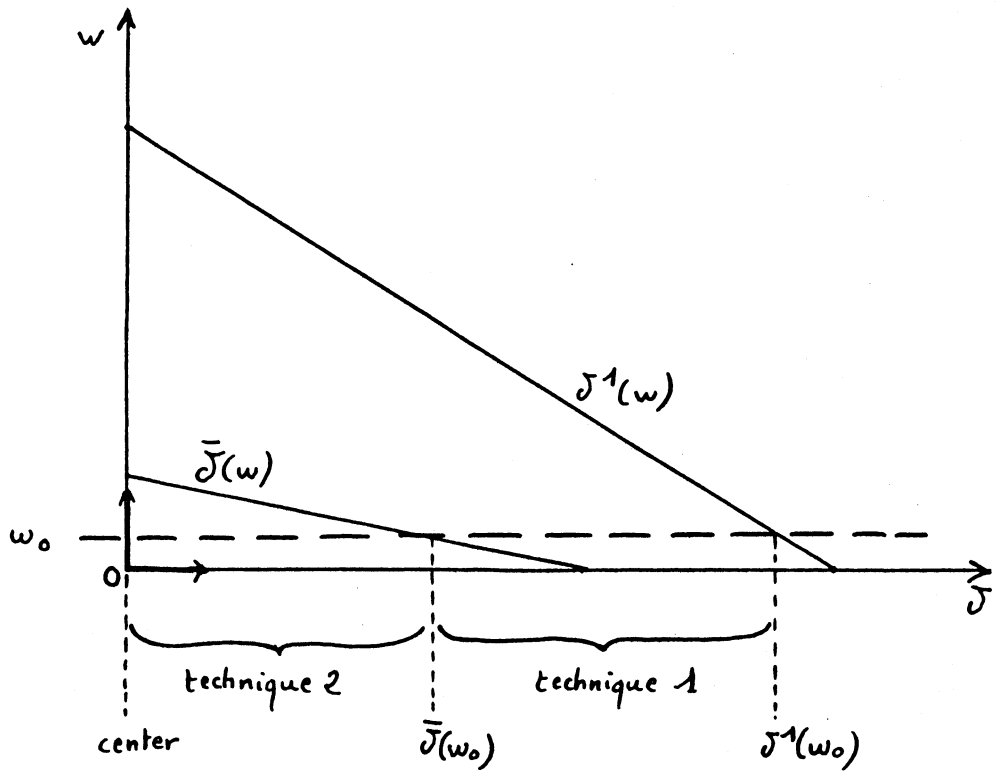


Figure 8 shows how $\bar{\delta}$ and δ^1 vary with a change in w. When w increases the surface S_2 decreases. When w varies from 0 to 1.2 (field of concentric locations), the study of the derivative of $(\delta^1)^2 - (\bar{\delta})^2$ shows that the area S_1 begins to increase and then decreases. But the ratio of the two areas, $\frac{S_1}{S_2}$, appears to be increasing :

$$\frac{S_1}{S_2} = \frac{\Pi(\delta^1)^2 - \Pi(\overline{\delta})^2}{\Pi(\overline{\delta})^2} = \left(\frac{\overline{\delta}^1}{\overline{\delta}}\right)^2 - 1,$$ of which the derivative is always positive. Thus, even if S_1 decreases, it will do so less than S_2, with the result that the change is relatively beneficial to technique 1. Again, an increase in the wage rate leads to a relative spatial expansion of the more labor-saving technique.

We can extend the preceding analysis to the transport rate. Here the result will be slightly different.

The effect of variations in the transport rate.

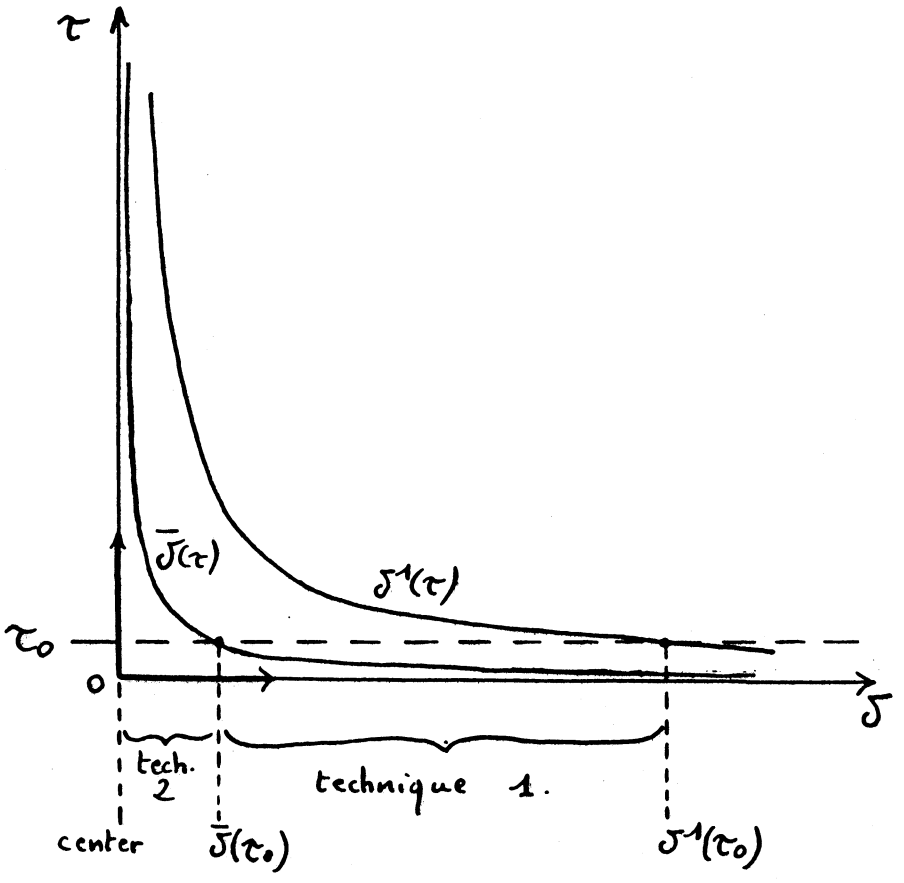
$$\frac{\partial\overline{\delta}}{\partial\tau} = -\frac{\overline{\delta}}{\tau} < 0, \text{ and } \frac{\partial\delta^1}{\partial\tau} = -\frac{\delta^1}{\tau} < 0 .$$

In the example, if we fix w = 1 and r = 0.1, we have :

$$\overline{\delta} = \frac{0.1}{\tau} \text{ and } \delta^1 = \frac{0.85}{\tau} \qquad \text{(see figure 9).}$$

It is easy to verify that the derivative of $(\delta^1)^2 - (\overline{\delta})^2$ is always negative, meaning that S_1 varies in the same direction as S_2. Clearly, the derivative of $\left(\frac{\delta^1}{\overline{\delta}}\right)^2 - 1$ is zero. An increase in the transport rate results in the two rings decreasing at the same rate, despite the fact that one technique is more transport-using than the other one.

Figure 9. The boundaries of the technique-rings as functions of τ



We have shown through an imaginary example how variations in the distribution parameters affect the relative locations of techniques. The effect is always comparatively beneficial to the technique which relies less on the factor for which the income rate increases. As a consequence, the order of the rents can change with r or w. This is a new illustration of the variability of the rent order which we have demonstrated elsewhere (HURIOT, 1981).

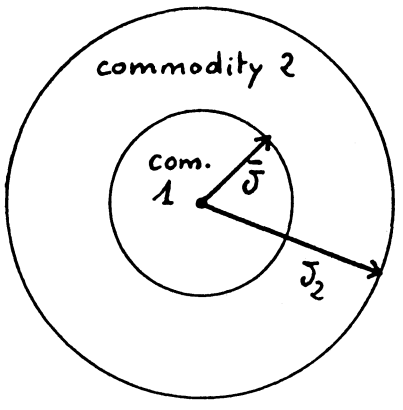
Moreover, let us imagine that we start with technique 1 alone, and that a new technique, 2, becomes available. Then this second technique can appear at the center, if and only if it is more transport-using per unit

of land than the first technique (condition 16). If it is land-saving, but capital- and labor-using, it can appear if and only if the wage rate and profit rate are sufficiently low, and it can spread towards the periphery when these rates decrease.

3.2.2. The second model

Again, the conditions of concentric locations are supposed to hold, with the first commodity at the center and the second at the periphery. The boundaries of the corresponding rings are $\overline{\delta}$ and δ_2 (see figure 10).

Figure 10. The commodity-rings with concentric locations



As for the two-techniques-model, we have to study how the parameters influence $\overline{\delta}$ and δ_2. Remember that :

$$\overline{\delta} = \frac{p - 1 + (a_2 p - a_1)(1 + r) + (l_2 - l_1)w}{\tau(a_1 - a_2)(1 + r)} \tag{15}$$

(14) $$\delta_2 = \frac{1 - a_2 p(1 + r) - l_2 w}{\tau[1 + a_2(1 + r)]} .$$

We use the same example as in section (3.1.2.).

<u>The effect of variations in the relative price</u>

From **(15)** and **(9)**, we have :

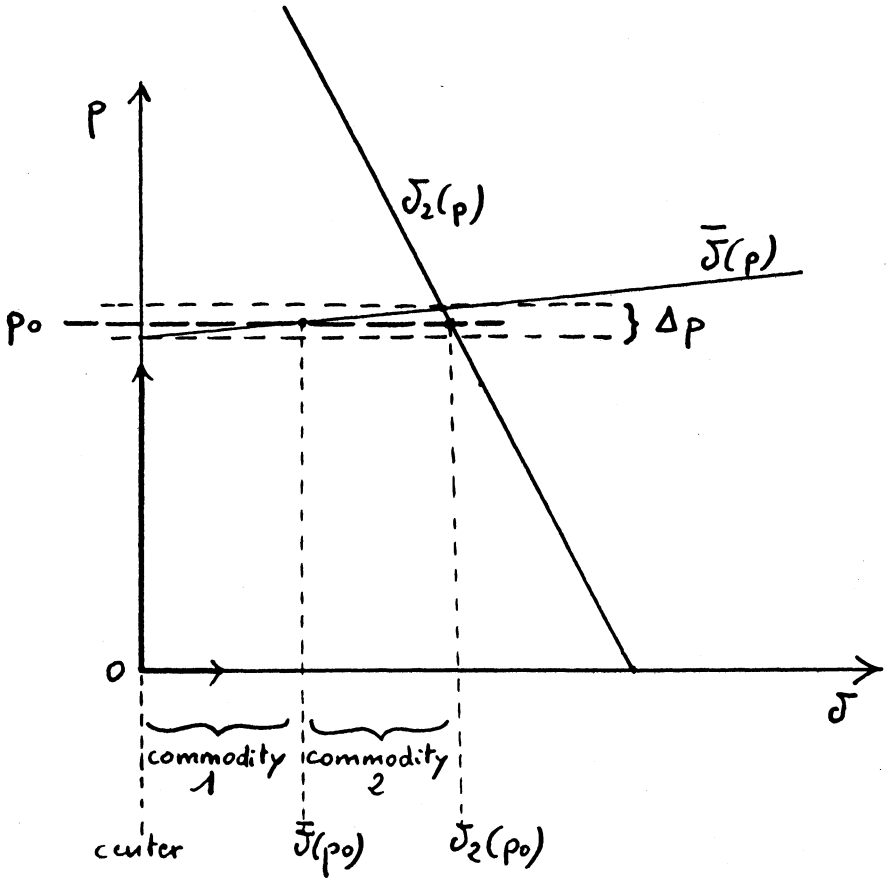
$$\frac{\partial \bar{\delta}}{\partial p} = \frac{1 + a_2(1 + r)}{\tau(a_1 - a_2)(1 + r)} > 0 \text{ , and}$$

$$\frac{\partial \delta_2}{\partial p} = \frac{- a_2(1 + r)}{\tau[1 + a_2(1 + r)]} < 0 ,$$

which imply an increase in the inner ring area and a decrease in the outer ring area. This means that <u>the commodity whose relative price rises expands in space</u> (commodity 1).
With our data, if we fix $r = 0.1$, $w = 1$ and $\tau = 0.1$:

$$\bar{\delta} = 55.45\ p - 60.9 \text{ and } \delta_2 = - 1.8p + 6.55 .$$

Figure 11. The boundaries of the commodity-rings as functions of p



These functions are shown on figure 11, where only the interval Δ_p corresponds to concentric locations.

<u>The effects of variations in the wage rate</u>

$$\frac{\partial\bar{\delta}}{\partial w} = \frac{l_2 - l_1}{\tau(a_1 - a_2)(1 + r)} > 0, \text{ and } \frac{\partial\delta_2}{\partial w} = \frac{-l_2}{\tau[1 + a_2(1 + r)]} < 0 ,$$

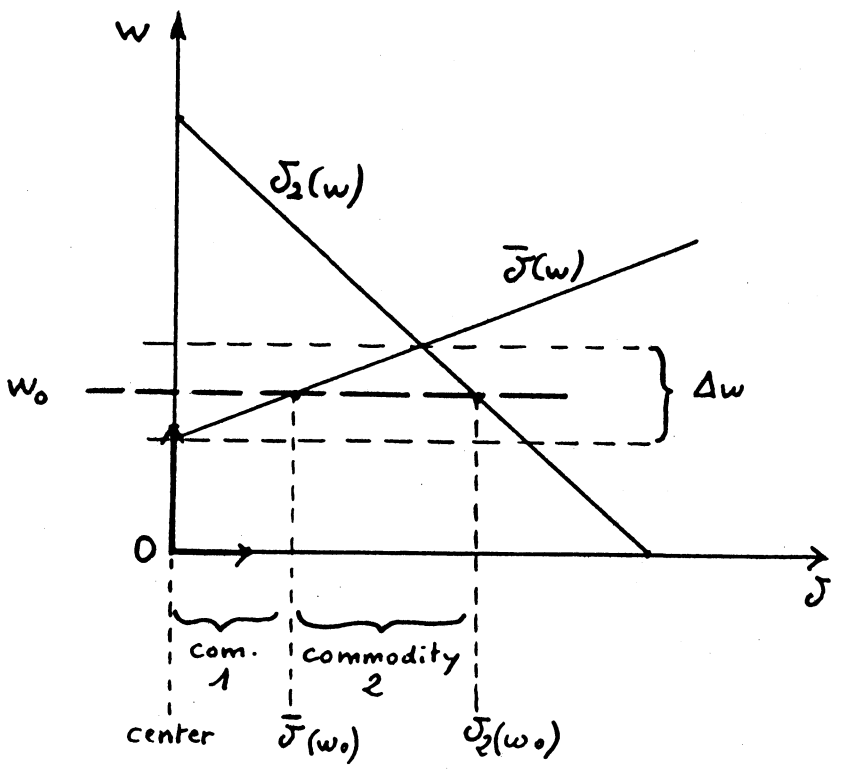
and we observe a spatial expansion of the labor-saving commodity when the wage rate increases.

In the example, if we fix p = 1.1., r = 0.1 and τ = 0.1 :

$$\bar{\delta} = 4.54w - 4.45 \text{ and } \delta_2 = -1.64w + 6.21 .$$

We can see these functions in figure 12, where the interval Δ_w corresponds to the concentric locations.

Figure 12. The boundaries of the commodity-rings as functions of w



The effect of variations in the profit rate

$\frac{\partial\bar{\delta}}{\partial r}$ has the sign of its numerator :

$$\bar{n} = (a_2p - a_1)[\tau(a_1 - a_2)(1 + r)] - [p - 1 + (a_2p - a_1)(1 + r) + (l_2 - l_1)w]\tau(a_1-a_2)$$
$$= \tau(a_1 - a_2)[1 - p - (l_2 - l_1)w] .$$

The sign of this expression is undetermined unless we add a new assumption. $\bar{n} > 0$ if and only if :

$$1 - p - (l_2 - l_1)w > 0 \text{ , i.e.,}$$

(33) $$p < 1 - (l_2 - l_1)w \text{ , where } l_2 - l_1 > 0 .$$

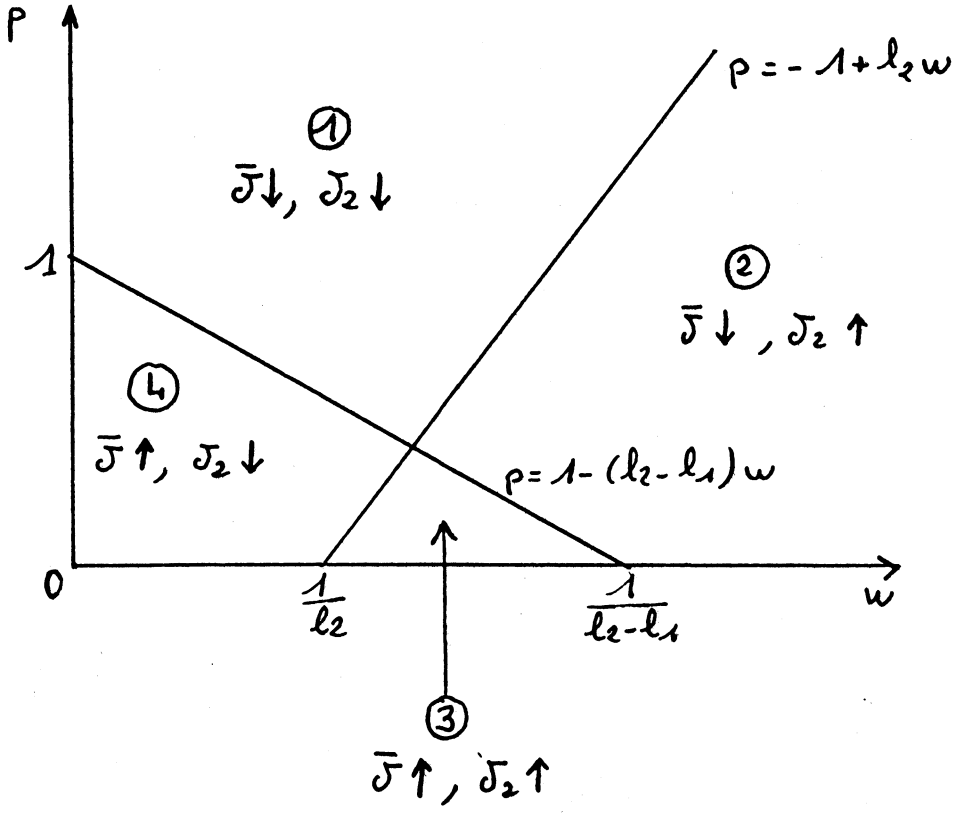
$\frac{\partial\delta_2}{\partial r}$ has the sign of its numerator :

$$n_2 = - a_2p\tau[1 + a_2(1 + r)] - [1 - a_2p(1 + r) - l_2w]\tau a_2$$
$$= - a_2p\tau - \tau a_2 + \tau a_2 l_2 w$$
$$= a_2\tau[- p + l_2w - 1], \text{ which is positive if and only if}$$

(34) $$p < l_2w - 1 .$$

Figure 13 shows the different possibilities of variations of $\bar{\delta}$ and δ_2 depending on the values of p and w.

Figure 13. Variations of $\bar{\delta}$ and δ_2 as functions of r, depending on p and w

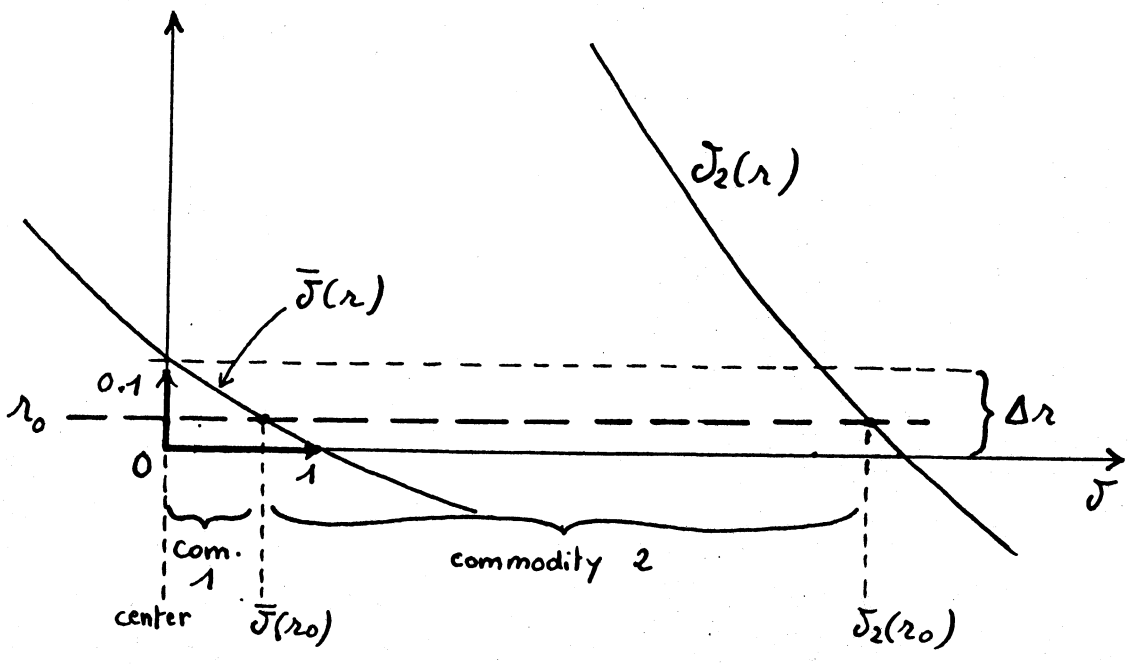


Thus the effect of the profit rate is undetermined. The expected effect, namely the expansion of the capital-saving production to the detriment of the other one with a rise in the rate of profit, is garanteed only for the subset of values of (p, w) corresponding to zone (2) in figure 13. We can have an <u>unexpected effect</u> which can be explained by the application of the profit rate not only to the expenditure on direct inputs, but also to the expenditure on transport. If we return to the example and fix p = 1.1., w = 1 and τ = 0.1, we have :

$$\bar{\delta} = \frac{1 - 9r}{1 + r}, \text{ and } \delta_2 = \frac{58 - 22r}{12 + 2r}$$

Both derivatives are negative. The functions are represented in figure 14, where the interval Δ_r corresponds to the concentric locations of commodities.

Figure 14. The boundaries of the commodity-rings as functions of r



<u>The effect of variations in the transport-rate</u>

Let us calculate the derivatives :

$$\frac{\partial\bar{\delta}}{\partial\tau} = \frac{-\bar{\delta}}{\tau} < 0 \text{ , and}$$

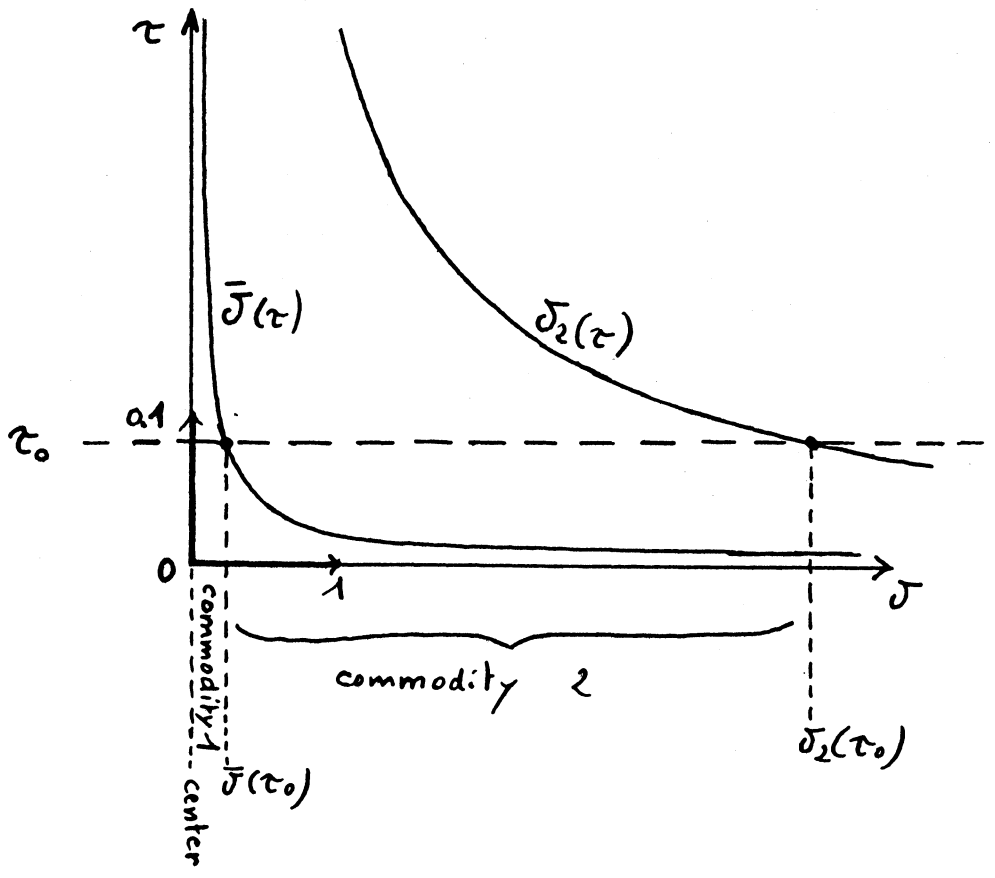
$$\frac{\partial\delta_2}{\partial\tau} = \frac{-\delta_2}{\tau} < 0 \text{ .}$$

In the example, with p = 1.1, w = 1, r = 0.1 ,

$$\bar{\delta} = \frac{0.009}{\tau} \text{ , and } \delta_2 = \frac{0.46}{\tau} \quad .$$

We can easily verify that the derivative of $(\delta_2)^2 - (\bar{\delta})^2$ is always negative : an increase of the transport rate creates difficulties for both commodities, and the areas of the two rings diminish. The derivative of $\left(\frac{\delta 2}{\bar{\delta}}\right)^2 - 1$ is zero, so that the two areas diminish at the same rate when τ increases (see figure 15).

Figure 15. The boundaries of the commodity-rings as functions of τ



A special case : Suppose that we add $a_2 = 0$ to the set of our assumptions. Then the second commodity is produced only with labor and land, but it is used to produce the first one. We keep the relation $l_2 > l_1$. Thus the second commodity can be produced even if the first one cannot. This can be interpreted as a primitive agrarian economy without capital, analogous to that of SMITH (1776) and RICARDO (1817). We can use the preceding analysis to state conditions of appearance and spatial diffusion of a new process using the agricultural commodity to produce a luxury commodity. These conditions relate only to conditions of production, to prices and distribution. So we could "spatialize" a Ricardian economy.

4. Conclusions

We have examined some features of productive land use, namely the conditions for concentric locations of techniques or of commodities. We have emphasized the role of conditions of production, through technical coefficients, and the role of relative prices, of wage rate and profit rate, in the framework of advance economics. We have generalized the Löschian conditions of circle formation and we have stated the effect of price and distribution on the pattern of THÜNEN's rings. In some cases, the results confirm what common sense would expect : we see a relative spatial expansion of the more factor-saving activity in response to the rising cost of this factor. But this simple general result takes several forms. Moreover, in the case of the profit rate, our expectations are not confirmed : indeed here we can even see the opposite effect. This shows the usefulness of this kind of analysis. Nevertheless, all that we have done is very limited. We have neglected the demand side and a more complete model would be very helpful if we are to understand the land allocation process more fully.

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