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## A SIMPLIFIED MODELLING FOR CRACKING IN PAVEMENTS

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### Abstract

The French pavement design method uses the 2D axi-symmetric structural analysis of Burmister's model [1]. This model calculates almost analytically and efficiently a multi-layer on a solid half space soil subjected to a circular load. However, to take into account pavement degradations as cracks in the structure, for concrete and bituminous pavements, the design method requires other models. They can be modelled by using 3D finite element methods, but such calculations can be very time-consuming. To make a proper analysis of crack initiation and propagation in pavement structures, we propose to use a simplified plate model linked to the Boussinesq model for the soil. This approach produces regular stress fields near cracks at the interface of two different layers. The resolution is carried out numerically under Matlab [2]. This model gives, very quickly, good approximations of the design stress fields near interfaces and cracks in comparison with finite element calculations. In the aim to simulate some fatigue problems, a method is presented to accelerate the calculations. It consists in writing the 3D problem in a form without dimension. Then, the evolution of the main fields can be obtained with a "table of results" that has been built.

### 1. Introduction

Under traffic load of heavy lorries, the role of the pavement layers is to decrease the stress fields that the soil cannot support. The French pavement design method uses the 2D axi-symmetric structural analysis of Burmister's model [1]. This model calculates almost analytically and effectively a multi-layer on a solid half space soil subjected to a circular load. However, to take into account visco-elastic behaviour of bituminous layers [3]; damage in bituminous material [4]; or cracks in the structure, for concrete and bituminous pavements, the design method requires other models.

The aim of this paper is to propose a modelling adapted to the study of cracks in pavements. For this purpose, one can model the 3D pavement structure by finite elements as [5]. As the mesh close to the cracks may be rather fine, even in 2D, finite element calculations are rather heavy and expensive. In the objective to build a software

for the evaluation of cracked pavements, and design solution for rehabilitation, we propose here to use a simplified model.

This model as well as the resolution of its equations are first described. Then, the validations are presented. As simplified calculations for the real 3D problem are still too time consuming, a method is presented to simulate some fatigue problem.

## 2. Simplified model

In the following, we denote the plane components of tensors by Greek indices  $(\alpha, \beta)$ .

Each pavement layer  $i$  is considered elastic, isotropic, with a thickness  $e_i$ , a Young modulus  $E_i$  and a Poisson's ratio  $\eta_i$ . The isotropic elastic soil is characterised by its Young modulus  $E_s$  and its Poisson's ratio  $\eta_s$ .

### 2.1 The M4-5n for pavement layers

The multi-particle model of multi-layer materials (M4) selected for this bending problem has 5 kinematic fields per layer  $i$  ( $i=1,n$  where  $n$  is the total number of layers). It is called M4-5n and belongs to the M4 family developed at ENPC for studying edge effects in composite structures [6]. Its displacement fields per layer  $i$  are: the average plane displacements  $U_\alpha^i(x, y)$ ; the average out of plane displacement  $U_3^i(x, y)$  and the average rotations  $\Phi_\alpha^i(x, y)$ . This model looks like a superposition of  $n$  Reissner's plates, connected by means of an elastic energy that depends on the inter-laminar stress fields. Its construction rests on a polynomial approximation per layer of degree 1 in  $z$  (vertical direction) for the in plane stress fields. According to the equilibrium equations, it gives, per layer, polynomials of degree 2 in  $z$  for shear stresses and degree 3 in  $z$  for the normal stress. The coefficients of these polynomial approximations are  $(x, y)$  fields per layer  $i$ . They are expressed with the use of the classical Reissner generalized stress fields. These polynomial approximations have the advantage to define the normal stress  $\nu^{i,i+1}(x, y)$  and the shear stresses  $\tau_\alpha^{i,i+1}(x, y)$  at the interface  $i,i+1$  (and similarly  $i-1,i$ ) between the layer  $i$  and  $i+1$ . Hellinger-Reissner's formulation [10] reduces the real 3D problem to the determination of  $(x,y)$  fields per layer  $i$  and interface  $i,i+1$  (and  $i-1,i$ ). M4 leads to finite values of the stress fields near cracks or edges.

On finite structures, M4-5n fields are very close to those obtained by 3D finite elements [8]. For infinite plane pavement layers resting on a soil, M4-5n gives good stress field approximations.

After combining the equations, the M4-5n pavement problem is summarised to solve the system of 5 differential equations of order 2 in  $(x, y)$  per layer  $i$  ( $i=1,n$ ) as follows :

$$e_i E_i \left( \frac{1}{1-\eta_i^2} U_{1,11}^i + \frac{1}{2(1+\eta_i)} U_{1,22}^i + \frac{1}{2(1-\eta_i)} U_{2,12}^i \right) = \tau_1^{i-1,1} - \tau_1^{i,i+1} \quad (1)$$

$$e_i E_i \left( \frac{1}{1-\eta_i^2} U_{2,11}^i + \frac{1}{2(1+\eta_i)} U_{2,22}^i + \frac{1}{2(1-\eta_i)} U_{1,12}^i \right) = \tau_2^{i-1,1} - \tau_2^{i,i+1} \quad (2)$$

$$\frac{2e_i^2 E_i}{1-\eta_i^2} \Phi_{1,11}^i + \frac{e_i^2 E_i}{1+\eta_i} \Phi_{1,22}^i + \frac{e_i^2 E_i}{1-\eta_i} \Phi_{2,12}^i - \frac{10 E_i}{1+\eta_i} (U_{3,1}^i + \Phi_1^i) = 10(\tau_1^{i-1,1} + \tau_1^{i,i+1}) \quad (3)$$

$$\frac{2e_i^2 E_i}{1+\eta_i} \Phi_{2,11}^i + \frac{e_i^2 E_i}{1-\eta_i^2} \Phi_{2,22}^i + \frac{e_i^2 E_i}{1-\eta_i} \Phi_{1,12}^i - \frac{10 E_i}{1+\eta_i} (U_{3,2}^i + \Phi_2^i) = 10(\tau_2^{i-1,1} + \tau_2^{i,i+1}) \quad (4)$$

$$U_{3,11}^i + U_{3,22}^i + \Phi_{1,1}^i + \Phi_{2,2}^i = \frac{12(1+\eta_i)}{5e_i E_i} (v^{i-1,i} - v^{i+1,i}) - \frac{(1+\eta_i)}{5E_i} (\tau_{1,1}^{i-1,1} + \tau_{2,2}^{i-1,1} + \tau_{1,1}^{i,i+1} + \tau_{2,2}^{i,i+1}) \quad (5)$$

As equilibrium equations, boundary conditions are written per layer  $i$  ( $i=1,n$ ). Let us note that the stresses at interfaces  $(0,1)$  and  $(n,n+1)$ , on the surface and the bottom of the pavement layers, are connected respectively to the boundary conditions known for the wheel load, and interaction with the soil (Cf. Figure 1).

### 2.2 Boussinesq's equations for the half space soil

Because too many unknown functions should be determined, a modelling by dividing the half space soil in several M4-5n layers, although giving good solutions cannot be considered. Also, with an aim of building a fast computation software, we choose to use an analytical model. The Boussinesq's equations for the elastic half space soil seem to be the best model to be used.

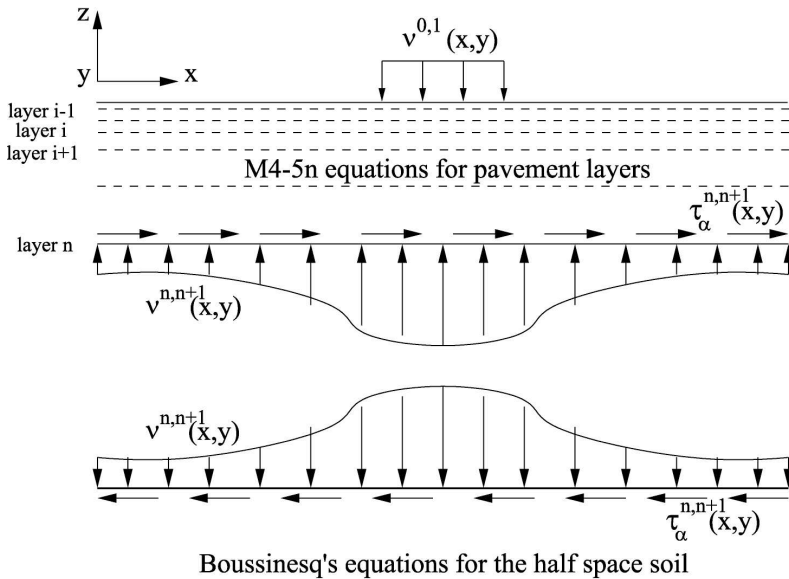


Figure 1 - Simplify modelling

Pavement layers are assumed to be bonded to the soil. The connection of the M4-5n and Boussinesq's model is then made using the continuity of pavement and soil fields at the interface  $n,n+1$  (Figure 1). Three more relationships between the surface displacement

field of the soil  $u_j^{\text{surface}}(x, y)$  ( $j \in \{1, 3\}$ ) of Boussinesq's model and the interface stress field  $\tau_\alpha^{n, n+1}(x, y)$  and  $v^{n, n+1}(x, y)$  of M4-5n can be written as follows:

$$u_1^{\text{surface}}(x, y) = \frac{(1 + \eta_s)}{\pi E_s} \times \left( \begin{aligned} & -\frac{1 - 2\eta_s}{2} \int_S \frac{v^{n, n+1}(\xi, \eta)(x - \xi)}{(x - \xi)^2 + (y - \eta)^2} d\xi d\eta \\ & + \int_S \tau_1^{n, n+1}(\xi, \eta) \frac{(x - \xi)^2 + (1 - \eta_s)(y - \eta)^2}{((x - \xi)^2 + (y - \eta)^2)^{3/2}} d\xi d\eta \\ & + \eta_s \int_S \tau_2^{n, n+1}(\xi, \eta) \frac{(x - \xi)(y - \eta)}{((x - \xi)^2 + (y - \eta)^2)^{3/2}} d\xi d\eta \end{aligned} \right) \quad (6)$$

$$u_2^{\text{surface}}(x, y) = \frac{(1 + \eta_s)}{\pi E_s} \times \left( \begin{aligned} & -\frac{1 - 2\eta_s}{2} \int_S \frac{v^{n, n+1}(\xi, \eta)(x - \xi)}{(x - \xi)^2 + (y - \eta)^2} d\xi d\eta \\ & + \eta_s \int_S \tau_1^{n, n+1}(\xi, \eta) \frac{(x - \xi)(y - \eta)}{((x - \xi)^2 + (y - \eta)^2)^{3/2}} d\xi d\eta \\ & + \int_S \tau_2^{n, n+1}(\xi, \eta) \frac{(1 - \eta_s)(x - \xi)^2 + (y - \eta)^2}{((x - \xi)^2 + (y - \eta)^2)^{3/2}} d\xi d\eta \end{aligned} \right) \quad (7)$$

$$u_3^{\text{surface}}(x, y) = \frac{(1 + \eta_s)}{\pi E_s} \times \left( \begin{aligned} & \int_S \frac{(1 - \eta_s)v^{n, n+1}(\xi, \eta)(x - \xi)}{(x - \xi)^2 + (y - \eta)^2} d\xi d\eta \\ & + \frac{(1 - 2\eta_s)}{2} \left( \begin{aligned} & \int_S \frac{\tau_1^{n, n+1}(\xi, \eta)(x - \xi)}{((x - \xi)^2 + (y - \eta)^2)^{3/2}} d\xi d\eta \\ & + \int_S \frac{\tau_2^{n, n+1}(\xi, \eta)(y - \eta)}{((x - \xi)^2 + (y - \eta)^2)^{3/2}} d\xi d\eta \end{aligned} \right) \end{aligned} \right) \quad (8)$$

### 2.3 Resolution

To solve the preceding equations, we divide the 2D resultant surface of the simplified model in  $N^2$  elementary surfaces along the x-axis 1 and the y-axis 2.

On these elementary surfaces, each Boussinesq's integral (6) (7) (8) can be evaluated as a sum of integrals. On those, considering that interface stress fields are constant, with a change of variable, these equations can be analytically integrated.

The M4-5n differential equations are solved with the finite difference method (order  $O(h^3)$  of the error).

The complete algorithm is then programmed in the Matlab software.

### 3. Validations

In the following, the load is uniformly distributed ( $q=0.662$  MPa). The half space soil

has for Poisson's ratio  $\eta_s=0.35$ . According to each case, the calculations are compared with the 3D or 2D finite element results of Cesar-LCPC

### 3.1 3D Case with no cracks

For the 3D case, we consider first one pavement layer with no cracks ( $e_c=0.08$  m,  $E_c=5400$  MPa and  $\nu_c=0.35$ ) bonded to the half space soil ( $E_s=50$ MPa). The side of the square load area is  $a=0.24$ m.

The 3D finite element mesh is made of elements with 20 nodes. It contains 16x16 elements in the plane and 18 elements in the vertical direction. The transverse dimensions are 2.4 m, that is to say, on each side, 10 times the dimension of the load. The depth of the soil is 6 m.

For the M4-5n-Boussinesq mesh, 10x10 plane elements are sufficient. Interface stress fields obtained from the M4-5n Boussinesq model are plotted together with the corresponding 3D finite element results (Figure 2).

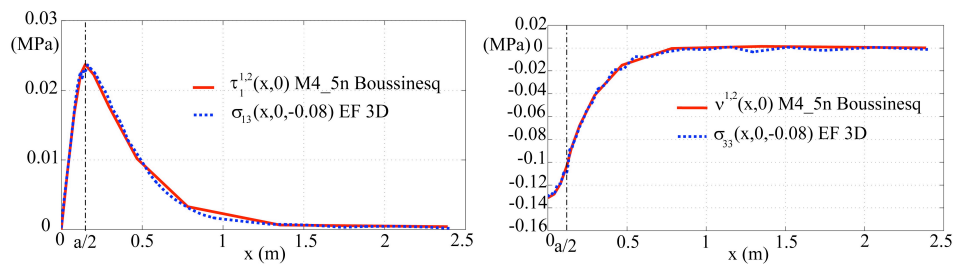


Figure 2 - 3D not cracked case: comparison between M4-5n Boussinesq shear and normal stresses and FEM ones at the interface between the layer and the soil

As can be seen, the M4-5n Boussinesq model matches the 3D finite element curve very well. Its solution is obtained about one hundred and a half times faster than the finite element calculation (1 minute for M4-5n Boussinesq and 2 hours 30' CPU's times for 3D finite element calculation ).

The 3D multi-layered case with no cracks has been also validated. Either, the pavement layer or the half space soil has been separated into two layers of the same modulus and compared successfully to the 3D case of one pavement layer on a soil.

### 3.2 Cases with vertical crack

Cracked cases for one and two pavement layers on a soil (2D and 3D) are solved. Being in linear elasticity, we can superimpose various loading cases. We consider the case of cracks that do not reach the soil. Thus to determine the interface stress state, it is only necessary to study the case where the load is at the edge of the crack (Figure 3).

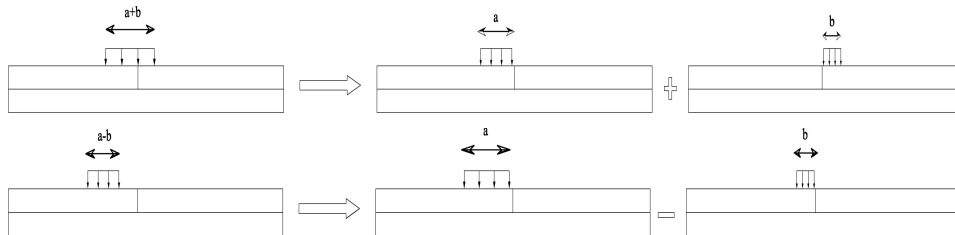


Figure 3 - Loading decomposition

Hereunder (Figure 4), we present the 2D plane strain validation of a classical pavement structure cracked along the vertical direction in its first layer. This structure is made of 2 pavement layers on a half space soil ( $e_1=0.08$  m,  $E_1=5400$  MPa,  $\eta_1=0.35$  and  $e_2=0.29$  m,  $E_2=9300$  MPa,  $\eta_2=0.35$ ,  $E_s=120$ MPa). The vertical crack (0.005m in width) is modelled as two free edge boundary conditions of the first layer. The width load is  $a=0.12$ m.

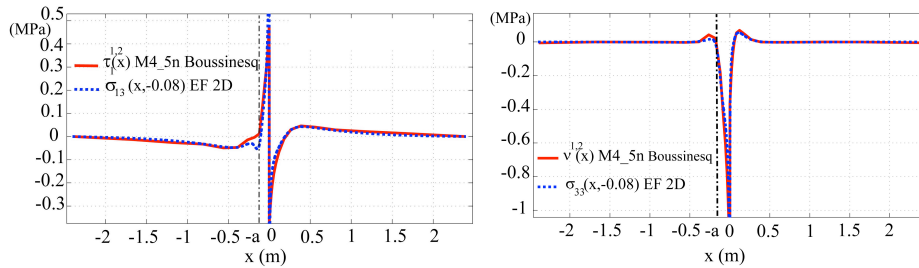


Figure 4 - 2D cracked case: comparison between the 1D M4-5n Boussinesq shear and normal stresses and 2D FEM ones at the interface between the two pavement layers

For this plane strain case, the 1D M4-5n Boussinesq model matches the 2D finite element curve near the crack very well. Its solution is obtained one thousand times faster than using finite elements.

The 3D cracked case has been simulated for the same data and a square load ( $a=0.12$ m). As known, stress values of these both calculations are different (Cf. figure 4 and 5).

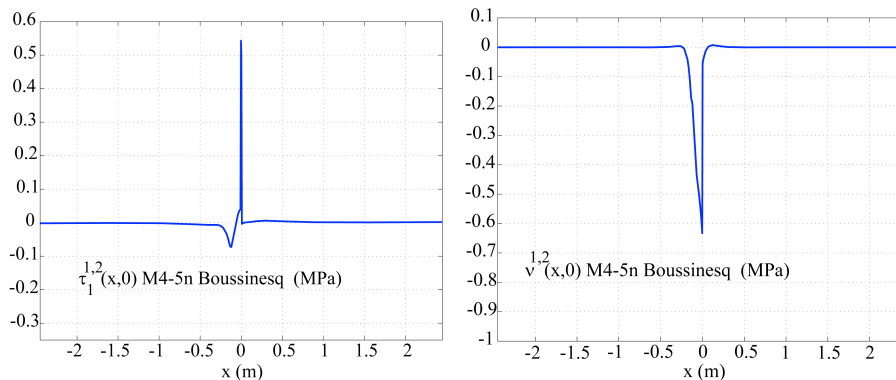


Figure 5 - 3D cracked case: 2D M4-5n Boussinesq shear and normal stresses at the interface between the two pavement layers

For fatigue calculations, the simulation of the 3D cracked problem could be too time-consuming (almost 15 minutes CPU per calculation for a mesh with 30x30 plane elements). In the following, we propose a simplified method to reduce the CPU-time.

#### 4. The non-dimensional problem of one 3D pavement layer with no cracks on a soil

Let us denote  $\bar{E} = E_c/E_s$  (ratio between Stiffness module of the pavement layer  $E_c$  and of the soil  $E_s$ ),  $\bar{H} = e_c/a$  (ratio between the layer thickness  $e_c$  and the value of the x-side

of the loaded area  $a$ ),  $\bar{X} \left( \frac{x}{a}, \frac{y}{a}, \frac{z}{a} \right)$  the vector of  $a$ -dimensional co-ordinates,  $q$  (value of

the load),  $b$  (y-side of the loaded area),  $\eta_c, \eta_s$  (Poisson's ratios of the layer and soil).

In the French design method, Poisson's ratios have fixed values (0.25 for concrete layers, 0.35 for bituminous layers and soils). The width  $b$  of the load is constant for each type of loading. Furthermore, all components are assumed to have a linear behaviour: solving the problem under a loading of 1 unit is sufficient to obtain the stress and strain fields for any value of  $q$ .

Thus, the 3D study of one pavement layer on a soil may be written in a form without dimension. The solutions depend only on two parameters  $\bar{E}, \bar{H}$  and on the vector of non-dimensional co-ordinates  $\bar{X}$ . For strategic values of  $\bar{X}$  (middle and edge of the load for instance), it is possible to list some components of this 3D fields in a "table of results".

To illustrate, we present the 3D numerical simulation of the in-plane stress  $\bar{\sigma}_{xx}(\bar{E}, \bar{H}, \bar{X})$  at the bottom of the pavement layer ( $\sigma_{xx} = q \bar{\sigma}_{xx}$ ) (Figure 5). We choose the pavement's modulus  $E_c$  between 2000MPa and 10000MPa, the soil's modulus  $E_s$  between 50MPa-200MPa, so that  $\bar{E}$  varies from 10 to 200. The ratio  $\bar{H} = e_c/a$  is chosen between 0.5 and 3.

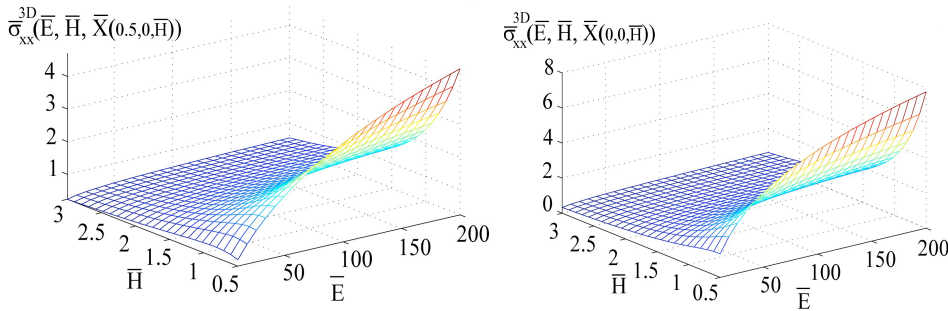


Figure 5 -  $\bar{\sigma}_{xx}(\bar{E}, \bar{H}, \bar{X})$  at the edge and the center of the load at the interface between the pavement layer and the soil

With those tables, for the one 3D pavement layer on a soil case, it seems possible to look at polynomial approximations of the main 3D fields according to  $\bar{E}$  and  $\bar{H}$ .

For the no crack case of multi pavement layers on a soil, the idea is to use classical laminated homogenization techniques of composite materials. It consists in replacing the stiffness of every layers as an equivalent one. Thus, the "tables of results" established for one pavement layer on a soil can be used to obtain the 3D stress and strain fields.

## 5. Conclusion

An elastic simplified model, the M4-5n-Boussinesq, is proposed for being used in a software for the 3D evaluation of cracks in pavements. The advantage of such a modelling is to reduce the 3D (respectively 2D) problem to 2D (1D) modelling. It produces regular fields near the edge or cracks. It also gives, quickly, very good

approximations of the design stress fields near interfaces and cracks in comparison with finite element calculations. A classical pavement structure made of 2 pavement layers on a half space soil cracked along the vertical direction of its first layer structure is presented. As it is known, 2D plane strain hypothesis can not be use for the modelling of stress fields at the interface. With other data, this modelling is useful for the mechanical study of adhesion between concrete and bituminous materials [9].

For the fatigue analysis of fields, because the 2D simplified modelling of the 3D problem could be to time-consuming, a quick method is proposed. It is based on the writing of 3D non-dimensional fields according to the main geometrical, loading and material parameters. For any elastic material case and strategic co-ordinates values, these 3D fields are calculated with the 2D M4-5n Boussinesq model, then, listed in a "table of results". For the study of structure made with several layers, the idea is to used laminated homogenisation techniques to replace each stiffness layer by an equivalent one.

With this method a lots of development can be done. First, because elastic evolution laws of the effective stiffness can be found for bituminous mixes [4], the no crack "table of results" presented in this paper could be used to simulate the local evolution of fields during the fatigue damage process. Next, for the study of bottom-up crack phenomena, non-dimensional 3D fields on a structure made of 2 pavement layers on a soil cracked on its second layer need to be found. Then, when the crack is growing, the structure can be modelled as an virtual structure made of one equivalent virgin layer and one equivalent crack layer on a soil. Then, with the use of previous non-dimensional 3D fields, the evolution of 3D stress and strain fields can be simulated.

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