



# Imperfect communication in markets : a big world problem

Alan Kirman

## ► To cite this version:

Alan Kirman. Imperfect communication in markets : a big world problem. [Research Report] Institut de mathématiques économiques (IME). 1980, 9 p., bibliographie. hal-01533674

**HAL Id: hal-01533674**

**<https://hal.science/hal-01533674>**

Submitted on 6 Jun 2017

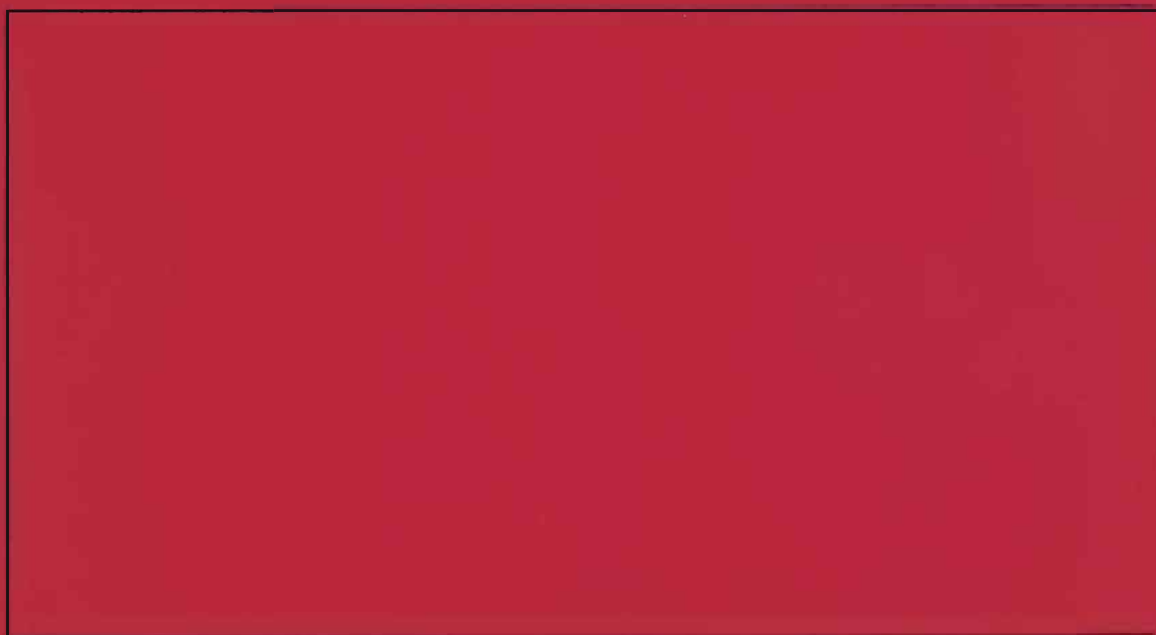
**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# I.M.E.

EQUIPE DE RECHERCHE ASSOCIEE AU C.N.R.S.

DOCUMENT DE TRAVAIL



**INSTITUT DE MATHEMATIQUES ECONOMIQUES**

UNIVERSITE DE DIJON

FACULTE DE SCIENCE ECONOMIQUE ET DE GESTION

4, BOULEVARD GABRIEL — 21000 DIJON

N°44

IMPERFECT COMMUNICATION IN MARKETS  
A BIG WORLD PROBLEM

Alan KIRMAN

April 1980

Cette étude a fait l'objet d'une communication au  
Xème Colloque annuel de l'Institut de Mathématiques  
Economiques, le 29 novembre 1978.  
L'auteur est professeur à l'Université d'Aix-Marseille  
et chercheur au Groupe de Recherche en Analyse de  
Système et Calcul Economique (G.R.A.S.C.E.).

#### TRAVAUX DEJA PUBLIES

- N°24 Pietro BALESTRA: Determinant and Inverse of a Sum of Matrices with Applications in Economics and Statistics (avril 1978)
- N°25 Bernard FUSTIER: Etude empirique sur la notion de région homogène (avril 1978)
- N°26 Claude PONSARD: On the Imprecision of Consumer's Spatial Preferences (avril 1978)
- N°27 Roland LANTNER: L'apport de la théorie des graphes aux représentations de l'espace économique (avril 1978)
- N°28 Emmanuel JOLLES: La théorie des sous-ensembles flous au service de la décision: deux exemples d'application (mai 1978)
- N°29 Michel PREVOT: Algorithme pour la résolution des systèmes flous (mai 1978)
- N°30 Bernard FUSTIER: Contribution à l'analyse spatiale de l'attraction imprécise (juin 1978)
- N°31 TRAN QUI Phuoc: Régionalisation de l'économie française par une méthode de taxinomie numérique floue (juin 1978)
- N°32 Louis De MESNARD: La dominance régionale et son imprécision, traitement dans le type général de structure (juin 1978)
- N°33 Max PINHAS: Investissement et taux d'intérêt. Un modèle stochastique d'analyse conjoncturelle (octobre 1978)
- N°34 Bernard FUSTIER, Bernard ROUGET: La nouvelle théorie du consommateur est-elle testable? (janvier 1979)
- N°35 Didier DUBOIS: Notes sur l'intérêt des sous-ensembles flous en analyse de l'attraction de points de vente (février 1979)
- N°36 Heinz SCHLEICHER, Equity Analysis of Public Investments: Pure and Mixed Game-Theoretic Solutions (April 1979)
- N°37 Jean JASKOLD GABSZEWICZ : Théories de la concurrence imparfaite : illustrations récentes de thèmes anciens ( juin 1979).
- N°38 Bernard FUSTIER : Contribution à l'étude d'un caractère statistique flou (janvier 1980).
- N°39 Pietro BALESTRA : Modèles de régression avec variables muettes explicatives (janvier 1980).
- N°40 Jean-Jacques LAFFONT: Théorie des incitations - Un exemple introductif (février 1980)
- N°41 Claude PONSARD : L'équilibre spatial du consommateur dans un contexte imprécis (février 1980)
- N°42 Jean-Marie HURIOT: Rente foncière et modèles de production (Avril 1980)
- N°43 Claude PONSARD: Fuzzy Economic Space (April 1980)
- N°44 Alan KIRMAN: Imperfect Communication in Markets. A Big World Problem. (April 1980)

This paper examines the problem of markets where not all individuals are in contact with each other. The nature of the communication structure in a market is rarely made explicit in standard economic analysis although a structure is implicitly assumed in the description of the model. To take as an example the general equilibrium model of exchange, it is clear that in such a model each individual must receive price signals. Hence a structure which would correspond to the Walrasian description would be one in which each agent was in contact with a central auctioneer. Thus viewing the agents as nodes in a graph and denoting contact or communication between two individuals by an arc, the Walrasian model would correspond to a star-shaped graph. In the analysis of the core it is assumed that any coalition of agents can form. If then we were to impose the condition that coalitions must consist of agents, all of whom know each other, then allowing all coalitions to form is equivalent to assuming that the graph is complete. In fact the communication pattern existing between the individuals in a market might be very different from those mentioned. If we specify the way in which agents' actions are constrained by the communication network, then we could proceed to examine the effects of different structures and arrive at deterministic statements about the market outcomes associated with those structures. Suppose however that we do not know the exact structure, but for example by sampling we are able to make statements as to the probability that individuals are in communication with each other. For simplicity we will assume that the probability  $p_{ij}$  that any two individuals  $i$  and  $j$  know each other does not depend on who these individuals are, thus we assume,

$$(1) \quad p_{ij} = p > 0 \quad \text{for all } i, j \quad i \neq j.$$

---

(1) This is a belated revision of a paper presented at the 1974 European Meeting of the Econometric Society in Grenoble. I had hoped to prepare a version with much stronger results in the second part. However to do so seems to require the use of much more powerful machinery and hence make the paper less appealing to a general audience and this would frustrate its original purpose. Many friends and colleagues have offered suggestions, but the only two that I can recall are Jerry Green who originally provoked my interest in this sort of problem and Claude Oddou who tried to make me make the statement of the Theorem a little more precise. It is not his fault if he has not been successful.

This is not essential for our results and we could also assume that  $\min_{i,j} p_{ij} > 0$  that is in a finite economy that everyone knows everyone else with a positive probability.

We shall be concerned with what happens as markets become large, that is, as more individuals are added. In this connection we will make the much more restrictive assumption, that individuals know each other with the same probability regardless of the total number of people in the economy. Thus we assume in effect that people know some constant proportion of the agents in an economy. One could obtain weaker versions of the results by assuming that  $p_n$ , the probability that two people know each other, in an economy with  $n$  individuals went to zero reasonably slowly. This is made precise in the paper of Hoivik and Gleditsch [1970], for the moment assume

$$(2) \quad p_n = p > 0 \quad n = 1, 2, \dots$$

To illustrate these ideas we consider first a simple and naive partial equilibrium model with a group of sellers of a single product. Each seller views himself as being in competition with certain other sellers. In this case  $p$  expresses the (symmetric) probability that two sellers view themselves as competitors. The sellers behave as follows. Each one sets a price and in the next period he adjusts his price to correspond to the lowest price set by his competitors. If the communication network is a strongly connected graph, that is if there is a path from every node to every other node, then after a finite number of periods there will only be one price in the market. We might then ask two questions. What is the probability that there will only be one price? How long will it take for this price to obtain everywhere? The answer to the first question is given by a standard graph theoretic result, Gilbert, [1959]. For a fixed  $p$  as defined previously, the probability that the graph  $\Gamma$  will be strongly connected depends on the number of nodes in  $\Gamma$ . Defining  $\Gamma_n(p)$  as a graph with  $n$  nodes with probability  $p$  of any two nodes having an arc between them, then

$$\text{Prob} (\Gamma_n(p) \text{ is strongly connected}) = 1 - nq^{n-1}$$

where  $q = 1 - p$ .

Thus as  $n$  becomes large the single price becomes a certainty.

The answer to the second question depends on the maximum distance or diameter of the graph. Let  $d_{ij}$  be the distance between two agents, i.e., the length, number of arcs, in the shortest path between nodes  $i$  and  $j$ . Then we define the diameter  $D(\Gamma)$  by

$$D(\Gamma) = \max_{i,j} d_{ij} \quad i,j \in A$$

where  $A$  is the set of nodes of  $\Gamma$ .

$D(\Gamma)$  is conventionally defined as infinite if there are two nodes which are not linked by a path.

A remarkable results of Hoivik and Gleditsch [1970] provides the answer required. They showed that for any fixed  $p > 0$  using the definition of  $\Gamma_n(p)$  as before,

$$\lim_{n \rightarrow \infty} \text{Prob} (D(\Gamma_n(p)) \leq 2) = 1$$

Phrased in simple terms, given our assumption about a fixed  $p$ , then in a large enough society any two individuals will almost certainly have a common friend. Indeed we should perhaps remark when meeting a stranger who knows somebody known to us, "It's a big world" rather than the more conventional observation. Thus large graphs are very strongly linked.

We can use this result for example to draw conclusions about the connection between the core and the Walrasian allocations in economies where only probabilistic statements can be made about the communication network. The second part of the paper is devoted to proving a limit theorem in probability terms about this relation.

### A probabilistic limit theorem on the Core

We will consider the simplest version of such a limit theorem in order to illustrate the use of the notion of imperfect communication that we have introduced. The deterministic theorem we use is the modern version of Edgeworth's result and is due to Debreu and Scarf [1963]. We will adopt the notation and presentation used by Hildenbrand [1978].

We will show that in an economy in which people know each other with probability  $p$  and can only form coalitions in which each pair of members has a common acquaintance, if the economy is large enough an allocation that is not Walrasian will almost certainly not be in the core.

First we start with some definitions and results in the standard deterministic framework.

Consider  $\mathcal{E}$  an exchange economy i.e.  $\mathcal{E} : A \rightarrow \mathcal{P} \times \mathbb{R}_+^l$  then a feasible allocation  $f$  for  $\mathcal{E}$  is  $f : A \rightarrow \mathbb{R}_+^l$  with  $\sum_{a \in A} e(a) = \sum_{a \in A} f(a)$ .<sup>1)</sup>

Definition 1:

Let  $f$  be an allocation for the economy  $\mathcal{E} : A \rightarrow \mathcal{P} \times \mathbb{R}_+^l$ . The coalition  $S$  can improve upon the allocation  $f$  if there exists an allocation  $g$  for  $\mathcal{E}$  such that

$$(i) \quad g(a) \succ_a f(a) \quad \text{for all } a \in S$$

and

$$(ii) \quad \sum_{a \in S} g(a) = \sum_{a \in S} e(a) .$$

The set of all feasible allocations for the economy  $\mathcal{E}$  that no coalition can improve upon is called the core of the economy denoted by  $C(\mathcal{E})$ .

To this we add

Definition 2:

An allocation  $f$  for the economy  $\mathcal{E} : A \rightarrow \mathcal{P} \times \mathbb{R}_+^l$  is called a Walras allocation if it is feasible and if there exists a price vector  $p \in \mathbb{R}^l$  such that

$$\left\{ x \in \mathbb{R}_+^l \mid p \cdot x \leq p \cdot e(a) \right\} \text{ is a maximal element for } \succeq_a \text{ in the budget set}$$

for every agent  $a$  in  $A$ .

The set of all Walras allocations of the economy  $\mathcal{E}$  is denoted by  $W(\mathcal{E})$ .



Now we turn to the familiar result that as an economy becomes large its core "shrinks" to the set of Walras allocations.

Consider a sequence of "replica" economies, or more formally:

Let  $\mathcal{E} : A \rightarrow \mathcal{P}_X R_+^\ell$  be an exchange economy. Then its n fold replica  $\mathcal{E}_n$  is an economy in which each agent  $\underline{a}$  is "replicated" n times i.e.

$$\mathcal{E}_n : A \times \{1, \dots, n\} \rightarrow \mathcal{P}_X R_+^\ell$$

when  $\chi_{(a,i)} = \chi_a$  and  $e(a,i) = e(a)$   $a \in A$   $1 \leq i \leq n$ .

Now consider  $\delta(C(\mathcal{E}), W(\mathcal{E}))$  where  $\delta$  is the "distance" between the Core and the set of Walras allocations defined as follows.

Let  $\delta$  be the smallest number such that for every allocation  $\underline{f}$  in  $C(\mathcal{E})$  there exists a Walrus allocation  $\underline{f}^*$  of  $\mathcal{E}$  with,

$$|\underline{f}(a) - \underline{f}^*(a)| < \delta \quad \text{for all } a \text{ in } A.$$

It is well known, see Debreu and Scarf [1963], that if we replicate an economy with "nice" preferences and initial endowments then  $\delta(C(\mathcal{E}_n), W(\mathcal{E}_n)) \xrightarrow{n \rightarrow \infty} 0$ . To establish a result in our stochastic framework we will first need two deterministic results showing that if we only allow "large" coalitions to form, the redefined core still shrinks to the set of Walras allocations. Then since we know that large coalitions have a high probability of forming we can prove the desired result.

Given a number  $N$  define the N Core of an economy  $\mathcal{E}$ ,  $C_N(\mathcal{E})$  as the set of allocations which cannot be improved upon by any coalition  $C$  with  $\# C > N$ . Clearly  $C(\mathcal{E}) \subset C_N(\mathcal{E})$ . We state the following:

Theorem 1 (Limit Theorem for the N Core)

Given  $N$  and  $\mathcal{E}$  an economy with monotonic strongly convex preferences and strictly positive endowments then

$$\delta(C_N(\mathcal{E}_n), W(\mathcal{E}_n)) \rightarrow 0.$$

where  $\mathcal{E}_n$  is the n fold replica of  $\mathcal{E}$ .

First we will establish the "equal treatment" property, that is agents of the same type receive the same allocations in the N Core of  $\mathcal{E}_n$  for  $n$  large enough. This enables us to concentrate solely on the allocations received by one member of each type, that is allocations for the economy  $\mathcal{E}$ . Let the number of agents in that economy i.e.  $\#A = r$ .

Lemma (Equal Treatment for the N Core).

Given  $N$ , let  $f \in C_N(\mathcal{E}_n)$  with  $n \geq \frac{N}{r} + 1$  then  $f(a,i) = f(a,j)$  for all  $a \in A$  and  $1 \leq i \leq n$  and  $1 \leq j \leq n$ .

Proof.

If the Lemma does not hold then there is  $f \in C_N(\mathcal{E}_n)$ ,  $a \in A$  say  $a^*$  and  $i \neq j$  such that  $f(a^*,i) \neq f(a^*,j)$ . Now without loss of generality we can order commodity bundles for each type according to the common preference  $\succsim_a$  and assume that

$$f(a,1) \succsim_a f(a,i) \quad 1 \leq i \leq n.$$

Now by strong convexity we have

$$\begin{aligned} \frac{n-1}{n} f(a^*,n) + \frac{1}{n} f(a^*,1) &\succ_{a^*} f(a^*,n), \\ \text{and} \\ \frac{n-1}{n} f(a,i) + \frac{1}{n} f(a,1) &\succ_a f(a,i) \quad 2 \leq i \leq n \\ &\text{and for all } a \in A. \end{aligned}$$

Thus the coalition  $S$  consisting of  $n-1$  agents  $(a,i) \quad 2 \leq i \leq n$  of each type  $a$  can improve upon  $f$ . Since by monotonicity  $(a^*,n)$  can give a small amount of some commodity to the other members of  $S$  and make them all strictly better off. Since  $(n-1)r \geq N$ ,  $f$  is not in  $C_N(\mathcal{E}_n)$ .

Q.E.D.

Proof of Theorem 1

With the Lemma the Theorem follows directly from Hansen [1969] Theorem 2, when he proved that any "equal treatment allocation"  $f_n$  i.e.  $n$  fold replica of  $f$  for  $\mathcal{E}$  where  $f$  was not in  $W(\mathcal{E})$  would be improved upon for  $n$  large by a coalition with  $(n-1)r + 1$  agents.

Q.E.D.

Now consider that in the exchange economy  $\mathcal{E}$  there is a fixed probability  $p$  of any pair of agents knowing each other. Thus with an economy is associated a communication network which is probabilistic and as previously discussed is represented by a stochastic graph.

Now taking a sequence of replica economies and the associated communication networks, let us allow only coalitions with diameter less than or equal to 2 to form. Thus coalitions can only form if every pair of individuals in the economy either knows each other or has a mutual acquaintance in the coalition. The core will now consist of those allocations which cannot be improved upon by such coalitions. However since the coalitions are now probabilistic, allocations belong to the core with a certain probability.

Now define the Epsilon Core of an economy  $\mathcal{E}$  in our sequence.

Definition:

The Epsilon Core  $C_\epsilon(\mathcal{E}_n)$  consists of those allocations with probability less than  $1 - \epsilon$  of being improved upon.

Thus for an allocation to belong to the epsilon core the probability that none of the coalitions which can improve upon it can form must be greater than  $\epsilon$ .

The careful reader will ask precisely how this probability is defined. Recall that if an allocation is not in the core then it is improved upon by at least one coalition  $S$ . Consider now all the graphs on  $A$  in which at least one of the sub graphs corresponding to an improving coalition has diameter 2. Now each of these graphs has a probability of forming, easily calculated by counting the number of arcs say  $S$  in the graph with  $N$  possible arcs and is given by  $p^S (1-p)^{N-S}$ .

Now the probability we need is the probability that at least one of these graphs will form. All this is rather complicated and we will adopt a simplistic approach which will suffice for our purposes. Consider an allocation  $f$  which is not in  $W(\mathcal{E})$ , then for some  $n$  and thereafter  $f_n$  will not be in  $C_n(\mathcal{E}_n)$  by Theorem 1. Now if we choose  $N$  appropriately we know that a large coalition will improve upon  $f_n$ . But by Hovik and Gleditsch's results we can choose  $N$  such that the probability of a coalition with  $N$

numbers forming will be greater than  $1-\xi$  for a given  $\xi$ . This then will be the strategy of the proof of our main

### Theorem 2 (Probabilistic Limit Theorem on the Core)

Let  $\xi$  be an economy with strongly convex, monotonic preferences and  $\xi_n$  the associated sequence of replica economies then for  $\xi > 0$

$$S(C_\xi(\xi_n), W(\xi_n)) \xrightarrow{n \rightarrow \infty} 0 \quad (1)$$

### Proof

By the result of Hoivik and Gleditsch for any  $\xi$  there exists  $N_\xi$  such that

$$\text{Prob } (D(\Gamma_{N_\xi}(p)) \leq Z) \geq 1 - \xi.$$

Now we use Theorem 1 setting  $N = N_\xi$ . This guarantees that for  $n$  large enough a coalition larger than  $N_\xi$  will improve upon  $f_n$  if  $f$  is not in  $W(\xi)$ . Since the probability that this coalition will form, by the above remark is greater than  $1 - \xi$ ,  $f_n$  is not in  $C_\xi(\xi_n)$  for all  $n > N_\xi$ .

Q.E.D.

### Conclusion

The aim of this paper has been to illustrate an approach to the handling of the problem of imperfect communication in markets. The idea of using stochastic graphs, while widespread in sociology and seeming to be a natural one for handling many problems in economics, does not seem to have been employed. Since the first version of this paper was written Myerson [1976] has used deterministic graphs to analyse coalition structures in cooperative games. It would seem a natural extension to allow for some indeterminacy in the formation of coalitions by using stochastic graphs. The simple result in the second part of this paper can be greatly strengthened and generalised as suggested at the outset, but this will be the subject of another paper.

(1) Where  $C_\xi(\xi_n)$  is the epsilon Core as defined above, given  $p$  the probability that any two agents are in communication.

What is more important is to draw attention to the fact that once we break away from the "starlike" structures of a classical economy with central auctioneer, who trades with whom and who passes information to whom becomes very significant. An obvious example is the formation of expectations in an inflationary situation. Furthermore the fact that such economies are not completely determinate suggests that economists could usefully profit in their analysis from an approach already adopted by sociologists and epidemiologists.

#### References:

- Gilbert E.N., (1959) "Random Graphs", Ann.Math. Statistics 30, 1141-1144.
- Hildenbrand W., (1978), "Core of an Economy", Working Paper IP6, Institute of Business and Economic Research, University of California, Bekerley. To appear as Chapter 18 in Handbook of Mathematical Economics edited by Arrow and Intriligator.
- Hansen T., (1969), "A Note on the Limit of the Core of an Exchange Economy". International Economic Review, Vol. 10. pp. 479.
- Myerson R.B., (1976), "Graphs and Cooperation in Games". Discussion Paper No 246. Northwestern University, Evanston, Illinois.