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Universal Scaling Laws for Correlation Spreading in Quantum Systems with Short- and Long-Range Interactions

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We study the spreading of information in a wide class of quantum systems, with variable-range interactions. We show that, after a quench, it generally features a double structure, whose scaling laws are related to a set of universal microscopic exponents that we determine. When the system supports excitations with a finite maximum velocity, the spreading shows a twofold ballistic behavior. While the correlation edge spreads with a velocity equal to twice the maximum group velocity, the dominant correlation maxima propagate with a different velocity that we derive. When the maximum group velocity diverges, as realizable with long-range interactions, the correlation edge features a slower-than-ballistic motion. The motion of the maxima is, instead, either faster-than-ballistic, for gapless systems, or ballistic, for gapped systems. The phenomenology that we unveil here provides a unified framework, which encompasses existing experimental observations with ultracold atoms and ions. It also paves the way to simple extensions of those experiments to observe the structures we describe in their full generality.

Introduction. – The ability of a quantum system to establish long-distance correlations and entanglement, as well as mutual equilibrium between distant parts, is determined by the speed at which information can propagate throughout the system. For lattice models with short-range interactions, Lieb and Robinson have demonstrated the existence of a maximum propagation speed limit, even for non-relativistic theories. This bound sets a linear causality cone beyond which correlations decay exponentially [1]. In a large class of many-body systems the information is carried by quasi-particles and the cone velocity may be related to their maximum velocity [2, 3], whenever it exists. Ballistic propagation of quantum correlations has received experimental [4, 5] and numerical [6–8] assessment, with, however, a cone velocity that may significantly differ from that expected.

For long-range interactions, a different form of causality arise due to direct coupling between local observables at arbitrary long distances. Long-range interactions appear in a variety of contexts, including van der Waals interactions Rydberg atom gases [9–12], effective photon-photon interactions in nonlinear media [13], dipole-dipole interactions between polar molecules [14–16] and magnetic atoms [17–21], photon-mediated interactions in superconductors [22] and artificial ion crystal [23–27], and solid-state defects [28–30]. They can be modelled by couplings decaying algebraically, $1/R^\alpha$, with the distance R. For such systems, known extensions of the Lieb-Robinson (LR) bound in D dimensions include a logarithmic bound, $t^\star \sim \log(R)$, for $\alpha > D$ [31] and an algebraic bound, $t^\star \sim R^\beta$ with $\beta < 1$, for $\alpha > 2D$ [32]. In both cases, they are super-ballistic. No bound is known for $\alpha < D$, which in principle allows for instantaneous propagation.

Information spreading in systems with long-range interactions has been intensively studied, in a variety of models, both experimentally and numerically. One the one hand, there is a consensual agreement that long-range LR bounds are usually too loose to predict the actual dynamics. On the other hand, they appear to be strongly model dependent, and seemingly contradictory. For instance, for the long-range XY (LRXY) model in one dimension, super-ballistic propagation has been reported in ion chains [33] and numerical simulations within truncated Wigner approximation [34], even for $\alpha < 1$. Conversely, for the long-range transverse Ising (LRTI) model, experiments reported ballistic propagation irrespective to the value of α [35]. Moreover, time-dependent density matrix renormalization group (t-DMRG) and variational Monte-Carlo (t-VMC) numerical simulations indicate instantaneous, sub-ballistic, and ballistic spreading laws for increasing values of the exponent α [36–41].

Here, we provide a universal picture of correlation spreading in generic quantum lattice models with variable-range interactions. We show that, generally, the space-time correlation map features a twofold causality structure. While the outer structure determines the LR edge, the inner structure determines the propagation of local maxima. For short-range interactions, they are both determined by the many-body dispersion relation and can be associated, respectively, to the group and phase velocities. For intermediate long-range interactions, the inner structure is also completely determined by the dispersion relation and strongly depends on its

nature. It is super-ballistic for gapless systems and ballistic for gapped systems. In turn, the outer structure depends on the dispersion relation, but not on the gap, and depends on the observable. It is always sub-ballistic, except for observables with a strong infrared divergence. Our results shed new light on correlation spreading in quantum systems and provides a unified picture to interpret previous results present in the literature.

Time evolution of local observables. – Consider a quantum system defined on an hypercubic lattice of dimension D and governed by the Hamiltonian

$$\hat{H} = \sum_{\mathbf{R}} h(\mathbf{R})\, \hat{K}_1(\mathbf{R}) + \sum_{\mathbf{R},\mathbf{R}'} J(\mathbf{R},\mathbf{R}')\, \hat{K}_2(\mathbf{R},\mathbf{R}'), \quad (1)$$

where $\mathbf{R},\mathbf{R}' \in \mathbb{Z}^D$ span the lattice sites. The first term accounts for local interaction with an external field $h(\mathbf{R})$ via the one-site operators $\hat{K}_1(\mathbf{R})$. The second term describes off-diagonal coupling of amplitude $J(\mathbf{R},\mathbf{R}')$ via the two-site operators $\hat{K}_2(\mathbf{R},\mathbf{R}')$. The generic Hamiltonian (1) applies to a variety of systems. In the following, we consider both quantum particle or spin models. The Bose-Hubbard model is constructed using the particle operators $K_1(\mathbf{R}) \equiv \hat{n}_{\mathbf{R}}(\hat{n}_{\mathbf{R}}-1)$ and $K_2(\mathbf{R},\mathbf{R}') \equiv -\hat{a}^\dagger_{\mathbf{R}}\hat{a}_{\mathbf{R}'} - \hat{a}^\dagger_{\mathbf{R}'}\hat{a}_{\mathbf{R}}$, where $\hat{a}_{\mathbf{R}}$ and $\hat{n}_{\mathbf{R}} = \hat{a}^\dagger_{\mathbf{R}}\hat{a}_{\mathbf{R}}$ are, respectively, the annihilation and number operators in the lattice site $\mathbf{R}$. The amplitudes are, respectively, the two-body interaction strength, $h(\mathbf{R}) = U/2$, and the tunnel amplitude $J(\mathbf{R},\mathbf{R}') = J$. For spin models, the operators $\hat{K}_j$ represent spin operators, namely $\hat{K}_1(\mathbf{R}) \equiv \hat{S}^z_{\mathbf{R}}$ and $\hat{K}_2(\mathbf{R},\mathbf{R}') \equiv \hat{S}^x_{\mathbf{R}} \cdot \hat{S}^x_{\mathbf{R}'}$ (Ising model) or $\hat{K}_2(\mathbf{R},\mathbf{R}') \equiv \hat{S}^x_{\mathbf{R}} \cdot \hat{S}^x_{\mathbf{R}'} + \hat{S}^y_{\mathbf{R}} \cdot \hat{S}^y_{\mathbf{R}'}$ (XY model). Then $h(\mathbf{R})$ is the magnetic field and $J(\mathbf{R},\mathbf{R}')$ the spin-exchange amplitude.

The dynamics is induced by a quench protocol (see for instance Refs. [4, 5, 33, 35]): After preparing the system in the ground state of some initial (pre-quench) Hamiltonian $\hat{H}_0$, the Hamiltonian is abruptly changed to the final (post-quench) value $\hat{H}$ at time $t = 0$. The latter governs the resulting unitary evolution of the system. It may be studied by measuring either response functions, $C(\mathbf{R},t) = -i\langle[\hat{A}_X(t), \hat{B}_Y(0)]\rangle$, or equal-time correlation functions, $G(\mathbf{R},t) \equiv \langle\hat{A}_X(t)\hat{B}_Y(t)\rangle - \langle\hat{A}_X(t)\rangle\langle\hat{B}_Y(t)\rangle$, where $\hat{A}_X$ and $\hat{B}_Y$ are two local operators acting on the finite sublattices X and Y separated by the distance $\mathbf{R}$. Here, we focus on particle or spin correlation functions, which are accessible in present-day experiments [4, 5, 33, 35]. Our results are, however, straightforwardly extended to response functions.

The starting point of our analysis is the observation that the system's dynamics is driven by the many-body excitations of the post-quench Hamiltonian. In the most-relevant low-energy sector of the spectrum, they may be assumed to be quasi-particle excitations. Assuming discrete-space translation invariance within the lattice, they are plane waves with well-defined quasi-momentum $\mathbf{k}$ and energy $E_{\mathbf{k}}$. The correlation function may thus be generically written

$$G(\mathbf{R},t) = g(\mathbf{R}) - \int_{\mathcal{B}} \frac{d\mathbf{k}}{(2\pi)^D} \mathcal{F}(\mathbf{k})\, \frac{\mathrm{e}^{i(\mathbf{k}\cdot\mathbf{R}+2E_{\mathbf{k}}t)} + \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{R}-2E_{\mathbf{k}}t)}}{2} \quad (2)$$

where we set $\hbar$ to unity and the integral spans the first Brillouin zone $\mathcal{B}$ owing to lattice periodicity. In the remainder of this work, we disregard the quantity $g(\mathbf{R})$ since it does not contribute to the time evolution and we assume isotropic couplings. Equation (2) represents the motion of counter-propagating quasi-particle pairs, with velocities determined by the after-quench dispersion relation $E_{\mathbf{k}}$, and an amplitude $\mathcal{F}(\mathbf{k})$ that is constructed from the overlap of the initial state, the quasi-particle amplitudes, and the operators $\hat{A}_X$ and $\hat{B}_Y$. Formally, Eq. (2) can be derived explicitly in exactly-solvable models. An important example is that of quadratic theories, which can be diagonalized by means of Bogoliubov transformations. Many models can be mapped into this form, using canonical transformations, meanfield approaches or spin-wave theories (see for instance Refs. [7, 37, 39–43] for examples in the context of out-of-equilibrium dynamics). For non exactly-solvable models, one may resort on numerical calculations, e.g. exact diagonalization or tensor-network techniques [44, 45].

Short-range couplings. – To start with, it is instructive to consider the case where the quasi-particle group velocity is bounded. It applies to models with couplings restricted to the nearest-neighbor lattice sites or decaying sufficiently fast (e.g. exponentially or algebraically with a large-enough exponent α [37–40]). The integral in Eq. (2) is evaluated using the stationary-phase approximation. In the infinite time and distance limit along the line $R/t = \text{const}$, it is dominated by the momentum contributions with a stationary phase (sp), *i.e.* $\nabla_k(kR \mp 2E_k t) = 0$ or, equivalently,

$$2V_{\mathrm{g}}(k_{\mathrm{sp}}) = \pm R/t, \quad (3)$$

where $V_{\mathrm{g}} = \nabla_k E_k$ is the group velocity. Since the latter is bounded in the (finite) Brillouin zone, it has a maximum $V^\star_{\mathrm{g}}$ and Eq. (3) may have a solution k_{sp} or not. For $R/t < 2V^\star_{\mathrm{g}}$, Eq. (3) admits a solution and the correlation function reads [46]

$$G(R,t) \propto \frac{\mathcal{F}(k_{\mathrm{sp}})}{\left(|\nabla^2_k E_{k_{\mathrm{sp}}}|t\right)^{\frac{D}{2}}} \cos\left(k_{\mathrm{sp}}R - 2E_{k_{\mathrm{sp}}}t + \frac{\pi}{4}\right). \quad (4)$$

For $R/t > 2V^\star_{\mathrm{g}}$, $G(R,t)$ is vanishingly small. The correlations are thus activated ballistically at the time $t = R/2V^\star_{\mathrm{g}}$, which defines a linear LR edge characterized by the velocity $V_{\mathrm{LR}} = 2V^\star_{\mathrm{g}}$. This result is consistent with the Calabrese-Cardy picture [2]. Note that for $k^\star \neq 0$, the quantity $\mathcal{F}(k^\star)$ is in general finite while $|\nabla^2_k E_{k^\star}|$ vanishes. Hence the amplitude of $G(R,t)$ diverges on the cone, which indicates a sharp LR edge.