The stability of group formation

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May 23, 2017

Abstract

In a large range of political and economic situations, the formation of coordinated groups is driven by two opposite forces: increasing returns to size on the one hand, the heterogeneity of preferences, which hampers coordination, on the other. An important question is whether competitive pressures, such as described by free mobility and free entry, lead to an efficient and stable organization of the society into possibly several self-sufficient groups. This paper discusses theoretical approaches to this question as well as recent empirical studies.

1 Introduction

The objective of this paper is to provide a selective survey of theoretical models of group formation, and to discuss some related empirical works. I focus on models that are based on the stability to coalition deviations, with a special attention to the context where there is a tradeoff between heterogeneity in individual preferences and increasing returns to coalitions.

In a large range of political and economic situations a group of individuals sharing common interests can pursue them more efficiently through a coordinated action. Returns to coordinated action explain why decisions are conducted within organized groups. Most often also, individuals differ in some aspects. The diversity in individual tastes hampers a full exploitation of coordination, and encourages the splitting of the society into smaller self sufficient groups. These two opposite forces -increasing returns to size and to coordination on the one hand and heterogeneity of preferences on the other- arise in a variety of contexts, including voting games, public goods choices, industrial organization, and jurisdictions boundaries.

The main issue addressed here is whether some form of competition helps the whole group to reach an efficient organization. Consider, for example, jurisdictions. When choosing its public goods levels and their financing, a jurisdiction is constrained by the ability of its citizens to move to another jurisdiction or to another country, that is, by the pressure exercised by free mobility (the 'vote with their feet' put forward by Tiebout (1956)). The jurisdictions' decisions are

[1]PSE-EHESS, 48 bd Jourdan, 75014 Paris, e-mail demange@pse.ens.fr. I have benefited from the comments of the editor of this special issue Philippe Solal and two anonymous referees.

also affected by the objections stemming from parts of their populations -their threats to secede, to recompose new jurisdictions, or to form unions- possibilities that we refer to free entry. Free mobility and free entry are competitive forces at work that shape the formation of jurisdictions and their decisions. Competition among jurisdictions is similar on many aspects to competition among firms under increasing returns to scale.

Cooperative game theory provides useful tools for conducting the analysis of such a competition. Free entry allows a new group to form if it is in the interest of its members. This is basically a no-blocking condition underlying the core notion. In the situations we are interested in, the splitting of the society into smaller self-sufficient groups may be an efficient outcome: the underlying game is not necessarily super-additive contrary to the standard set up.[2] This leads one to consider *coalition structures*, which specify a partition of the society into coalitions as well as the decision made by each one. According to the theory, the coalition structures that are stable (i.e. not blocked) are more likely to appear, thereby leading to predict which groups might form. Main issues are whether such stable structures exist, and how they depend on the primitives.

The building block of the theoretical works is not recent.[3] One reason to focus on this setting, apart from its importance and the insights drawn from the models, is that some recent empirical or experimental works investigate the stability of international arrangements, nations etc, or the validity of the concepts.

The paper is organized as follows. The next section describes coalitional games, presents some illustrations, and introduces basic properties that affect the incentives to form coalitions -namely, increasing returns to coalitions, super-additivity- and defines coalition structures. Section 3 considers the deviations of individuals or coalitions, giving rise to the free mobility and free entry conditions, and studies the stable coalition structures. Finally Section 4 presents some empirical approaches.

2 Coalitional Games

When studying the stability of groups formation, three key questions must be answered:

- How decisions are made within a group?
- What are the possible individuals' moves between groups?
- What are the possibilities for new groups to emerge?

The aim of this section is to introduce an abstract modeling flexible enough to account for the answers to the above questions and to analyze their impact on groups formation.

[2] Super-additivity may fail even under "technological" increasing returns to scale. This failure arises when individuals differ in their tastes but personalized prices or personalized taxes cannot be charged, as will be developed in Section 2.3.

[3] To name a few, see on public goods choices Wooders (1978), Guesnerie and Oddou (1981), Greenberg and Weber (1986), Alesina and Spolaore (1997), on industrial organization Demange and Henriet (1991), on a game on a network Demange (1994).

2.1 The model

Given a set of individuals, N, called the *society*, a non empty subset of N is called a *coalition*. The primitives are the set of feasible actions or decisions that each coalition can take when its members collaborate together, and the utility levels -also called payoffs- that each member derives from these actions.

The paper considers situations where there are no externalities across coalitions, meaning that the actions that are feasible for a coalition if it forms as well as the payoffs to its members are not affected by the organization of agents outside the coalition. Although restrictive,[4] the set of situations that can be analyzed through games without externalities (also called games without spillovers) is very rich, as will be illustrated in the next section. Furthermore, when the society benefits from splitting into smaller groups, insightful lessons can be drawn on the stable formation of groups.

The set of feasible actions for a coalition is thus described by a single set independently of the organization of outsiders. Let $A(S)$ denote the set of feasible actions for coalition S. The welfare of an individual depends on the decision made by the coalition to which he belongs: If S chooses a, a member of S, say i, obtains utility level $u_i(a)$. Therefore, each vector of utility levels $(u_i(a))_{i\in S}$ when a runs in $A(S)$ can be achieved by the coalition S to its members.

The description of the feasible sets depends on the problem at hand: it incorporates not only the technological constraints that each coalition faces if it forms, but also the various institutional or informational constraints that may restrict the way decisions are made. For example, a coalition of citizens can make decisions according to various voting rules, or may be more or less informed about the income of its citizens; both elements affect the sets of feasible actions and payoffs.

2.2 Illustrations

The two first illustrations are two simple classes of games. The third one deals with economies with production of differentiated products, either horizontal or vertical.

1. **Transferable utility games**. The proceed of a decision made by a coalition can be shared freely among its members: All transfers among the coalition's members are allowed and payoffs are linear in those transfers. The set of feasible payoffs to a coalition is therefore characterized by its maximal attainable aggregate payoff, called its *value*. Denoting this value by $v(S)$ for coalition S, the set of feasible payoffs to S are those whose sum is less than the value:

$$\{(x_i)_{i\in S} \text{ such that } \sum_{i\in S} x_i \leq v(S)\}$$

[4]Externalities across groups, negative or positive, have an important impact in some contexts, those concerning environmental agreements for example. Their analysis is still restricted to specific situations, due to conceptual and technical difficulties. See for example Yi (1997) for a non-cooperative approach to the formation of coalitions, Bloch (2005) for applications to industrial organization, and Currarini (2007) for an analysis of hierarchical structures.

where x_i denotes i's payoff. These games were introduced by Von Neumann and Morgenstern (1949).[5] As a simple example, consider a set of musicians. The cachet obtained by a performance of band S defines its value, which has to be shared between them.

2. **Hedonic games**. In hedonic games, transfers within a coalition are not possible and coalitions pick their decision according to a fixed rule. It follows that the feasible sets boil down to singletons and that the payoff to a player is entirely determined by the coalition to which he belongs : i's payoff writes as $u_i(a(S))$ where $a(S)$ is the action taken by S when it forms. These games were introduced by Dreze and Greenberg (1980).

   Hedonic games arise in various non economic contexts, in sport games for example if each player is concerned only with the composition of the team he joins (precluding cash transfers). They have been made popular recently to study the formation jurisdictions/nations. Let us mention two examples.

   The first example is the formation of nations as analyzed by Alesina and Spolaore (1997). Consider a horizontal differentiation model à la Hotelling in which individuals are all alike except for their location on a line. The location parameter can be interpreted as a geographical location or a taste parameter representing say political opinion. A 'country/coalition' is characterized by its borders populated with the individuals living within the interval.[6] Within a country, citizens vote on the location of the government using majority rule, and share equally the fixed cost c to finance the government. Under standard simplifying assumptions, preferences over the government's location are single-peaked so that the government is located at the median location of the citizens: A hedonic game is thus obtained.

   For instance, let us assume that an individual's utility is given by a benefit y of being a member of a country (identical for all) minus the sum of the tax and the distance between the individual's location and that of the government. Assume also that individuals are uniformly distributed over $[0, 1]$. Let us compute the payoffs to the citizens in country $[\theta_1, \theta_2]$ if it forms. The cost per citizen is $c/(\theta_2 - \theta_1)$, and the government is located at the middle $(\theta_1 + \theta_2)/2$. This yields the utility level

$$y - \frac{c}{(\theta_2 - \theta_1)} - |\theta - \frac{\theta_1 + \theta_2}{2}|$$

   to a citizen located at θ in the country $[\theta_1, \theta_2]$.

[5] They define the value $v(S)$ as the maxmin payoff to a coalition, computed as the maximum aggregate payoff the coalition can achieve whatever the actions of the outside members. Such a definition applies to games with externalities across groups by taking a pessimistic point of view but more optimistic views can be taken as well (see Aumann (1961)). See Chander and Tulkens (1995) for addressing environmental issues.

[6] If a country is formed with several disconnected intervals, each interval needs to have its own government. This allows one to restrict to intervals.

The second example is the model of growth' clubs between nations analyzed by Jaramillo, Kempf, and Moizeau (2005) and (2015). Here the players are the countries. Once a coalition is formed, each country within the coalition decides on its own contribution to a growth enhancing public good, and a hedonic game is obtained. The model provides an explanation for the persistent difference in growth rates across countries by the formation of coalitions of countries. They refer to 'clubs' instead of coalitions because they assume that a coalition of countries can exclude an outside country to join.[7] This assumption bears on the possible moves (not on the feasible sets) and its effect is studied in Section 3.

3. **Provision of differentiated goods**.

   Competition under horizontal and vertical differentiation, networks communications, choices of public goods by communities can be analyzed in this framework.[8]

   More specifically, let an economy with money and differentiated goods. The differentiated good can be private or public, divisible or indivisible, or be associated with a package of products such as the access through a network to various services. An 'offer' (which corresponds to an action in our abstract setting) specifies the characteristics of the differentiated good as well as the cost to each customer. The number of offers is not given *a priori*. Each individual is interested in one offer at most and the set of consumers who choose the same product is interpreted as a coalition. The aim of the stability analysis is to determine which products are offered and who buy them. This determination results from the diversity of preferences and the technology, in particular on the strength of the increasing returns to scale, if any. Consider first a horizontal differentiation model à la Hotelling (1929).

   (a) Horizontal differentiation. Each individual is interested in buying a unit of a product, say a car. His preferences are characterized by a 'location' parameter θ, which represents a geographical location or, more generally, is interpreted as a taste characteristic say over colors, or over movies (see the survey of Gabszewicz and Thisse (1986)). An offer is represented by a vector (q, p) where q is the characteristic ('location') of the product and p is its price. A consumer's payoff is decreasing in the distance between his location θ and that of the product q. For example, a consumer derives an intrinsic benefit v for buying a unit of the good (identical to all) and bears a cost made of the price p and the distance between his location θ and that of the product q:

$$u(q, p, \theta) = v - p - f(|q - \theta|) \tag{1}$$

[7] Buchanan (1965) introduced club goods as public goods that are excludable.

[8] In Demange and Henriet (1991), we present the model in terms of competition among firms under increasing returns to scale. Firms select the characteristics of their products, and individuals differ in their tastes or income. Under free entry, the firms make no profit and the equilibrium outcomes coincide with the stable coalition structures presented in the next section.

where f is a convex transportation cost.

Let S be the set of consumers who choose (q, p). The cost of producing $|S|$ units is given by $c(|S|)$ where $c(.)$ is a non-decreasing cost function. In case of a public good for example, c is constant, reflecting a fixed cost independent of the number of users. As each consumer pays p, S's overall budget writes

$$\pi(q, p, S) = p|S| - c(|S|).$$

The action (q, p) is said to be feasible for $|S|$ if $\pi(q, p, S) \geq 0$. This defines the set $A(S)$.

If the population is sufficiently disperse relative to the cost of production, there is scope for several offers differentiated according to their location, thereby attracting groups of consumers that are located in non-overlapping zones. But when average cost decreases with the number of consumers, $\frac{c(|S|)}{|S|}$ is decreasing, as in the case of a public good, there is a benefit in attracting large zones.

(b) Vertical differentiation. A similar model obtains in vertical differentiation models, in which products differ in quality instead of location and individuals differ in their income levels (Shaked and Sutton (1993)). Consider for example a public good model where individuals differ in their income only, denoted by θ. An offer $a = (q, t)$ specifies the quantity q of public good and t a proportional tax based on the (observed) income. A consumer derives an intrinsic benefit $v(q, \theta)$ which depends on the quantity q and his income θ and bears the cost linked to the tax. The payoff takes the form

$$u(q, t, \theta) = v(q, \theta) - t\theta. \tag{2}$$

The budget of coalition S writes

$$\pi(q, t, S) = t \sum_{i \in S} \theta_i - c(q) \tag{3}$$

where $c(q)$ is the cost function of producing q. The action (q, t) chosen by S is thus feasible if $\pi(q, t, S) \geq 0$. In some cases there are further constraints on feasibility, reflecting institutional or informational constraints, as for example if the level t is chosen by the median income in S.

If the distribution of income across population is sufficiently disperse relative to the cost of production, there is scope for several offers differentiated according to the amount of public good, each one attracting groups of individuals with different income levels. But this effect is surely limited due to the public good aspect.

Finally, observe that assuming a toll tax, instead of an income tax would change the budget of a coalition hence its feasibility. Thus, even in these very simplified models, the cost function and the financing tools have an impact on the feasible actions of each coalition, and ultimately on the stable organizations.

Illustrations in 3 allow for more flexibility in actions than hedonic games (illustration 2) and avoid the strong assumption of transferable utility (illustration 1).

2.3 Incentives to form coalitions

The incentives to form coalitions primarily depend on whether the members of a coalition derive some benefit by accepting new members, whether coalitions gain by splitting into smaller groups. We make these notions precise.

2.3.1 Increasing returns to coalitions

We start with notions of increasing returns.

Definition 1

Feasible sets are increasing if for any two coalitions S and T where S is contained in T, $A(S)$ is contained in $A(T)$: $A(S) \subset A(T)$ for $S \subset T$.

Returns to coalitions are increasing if for any two coalitions S and T where S is contained in T, for any a in $A(S)$ there is b in $A(T)$ such that

$$u_i(b) \geq u_i(a), \forall i \in S.$$

Feasible sets are increasing if any feasible action for a coalition remains feasible when new members join that coalition. For example, in the horizontally differentiated goods economy (illustration 3 (a) in 2.2), feasible sets are increasing if average cost is decreasing.

Returns to coalitions are increasing if the sets of payoffs are increasing with coalitions: whatever standing action, a coalition can, possibly by changing the current decision, accommodate any newcomer without hurting any of its current members. This is surely possible if feasible sets are increasing since the coalition can simply keep the same action: Increasing feasible sets imply increasing returns to coalitions.

The decision rule within a coalition has an impact on whether feasible sets (or returns to coalitions) are increasing. To give an example, consider illustration 3 (b) in 2.2: communities decide on their public good levels and individuals differ only with respect to their income level. If there are no institutional constraints on the financing of the public good, feasible sets are increasing. As can be seen from the budget equation (3), keeping the same public good and tax levels is feasible, and newcomers even allow for a decrease in the tax. If the jurisdiction instead finances the public good through a proportional tax chosen by its median voter,[9] the newcomers in a jurisdiction modify the median voter of the enlarged jurisdiction. In that case, not only feasible sets but also returns to coalitions may not be increasing. If, for some incumbent citizens, the gains from having more citizens financing the public good is offset by the losses due to a change in the median voter, the returns to coalitions are not increasing under the majority mechanism even though accepting newcomers would be Pareto improving through appropriate financing schemes.

[9] Under well known assumptions, the voter with median income is the majority voter.

2.3.2 Super-additivity, efficiency, and coalition structures

The benefits for disjoint groups to join crucially determine group formation. A game in which each coalition is always at least as efficient as any of its partition is called *super-additive.*

Formally, consider two disjoint coalitions S and T. Assume that for any a feasible for S, b feasible for T, there is c is feasible for the union $S \cup T$ under which all members are at least as well off:

$$\begin{cases} \text{for any } a \text{ in } A(S) \text{ and } b \text{ in } A(T), \text{ there is } c \text{ in } A(S \cup T) \text{ such that} \\ \quad \text{for each } i \in S : u_i(c) \geq u_i(a) \text{ and for each } i \in T : u_i(c) \geq u_i(b). \end{cases} \tag{4}$$

According to these conditions, the two coalitions S and T are not hurt by joining because the payoffs that they can achieve by acting separately can be achieved by their union. They even surely benefit from the merge if the inequalities in (4) are strict:[10]

$$\text{for each } i \in S : u_i(c) > u_i(a) \text{ and for each } i \in T : u_i(c) > u_i(a). \tag{5}$$

Definition 2 *The game is said to be super-additive if the conditions (4) hold for any two disjoint coalitions; it is said to be strictly super-additive if the strict inequalities (5) hold.*

Under super-additivity, the society is always at least as efficient as any of its partition, and under strict super-additivity it is the unique efficient organization. As for increasing returns, the decision process used by coalitions affects super-additivity. In a public goods economy, the game is super-additive when the good is financed through personalized prices (see Champsaur (1975), who shows the 'convexity' of the game, a much stronger property than super-additivity). But, the game is not necessarily super-additive if there are constraints on the financing of the good, for example if, as in illustration 3-(b), the good is financed by a tax on income (see Guesnerie Oddou (1981)). In the absence of super-additivity, the members of a group can all be made better off by being partitioned into smaller self-sufficient groups. A coalition structure, as introduced by Aumann and Dreze (1974), precisely takes into account the splitting possibilities.

Definition 3 *A coalition structure of N is a family $(a_\ell, S_\ell)_{\ell=1,\dots,L}$ where $(S_\ell)_{\ell=1,\dots,L}$ is a partition of N, and a_ℓ is feasible for S_ℓ, $\ell = 1, \dots, L$.*

Let $\ell(i)$ denote the coalition to which i belongs. i's utility level is equal to $u_i(a_{\ell(i)})$.

In the economies with differentiated goods (illustration 3), a coalition structure is described by the different offers and the set of consumers choosing each one.

[10] An intermediate requirement is that the players in the union of two disjoint coalitions can be made at least as well off and one strictly better off. It is not useful to consider these variants here.

In a hedonic game, a coalition structure is entirely described by its partition $(S_\ell)_{\ell=1,\dots,L}$.

The efficiency of a coalition structure is naturally defined by the Pareto optimality criterion:

Definition 4 *A coalition structure $(a_\ell, S_\ell)_{\ell=1,\dots,L}$ is (Pareto) optimal if no other coalition structure makes everybody better off: for no coalition structure $(a'_\ell, S'_\ell)_{\ell=1,\dots,L'}$ $u(a_{\ell'(i)}) > u(a_{\ell(i)})$ for each i.*

The Pareto optimality of a coalition structure implies *intra-group optimality*, according to which the action a_ℓ chosen by each coalition S_ℓ is Pareto optimal for its members among the alternatives in $A(S_\ell)$. But Pareto optimality imposes further conditions related to the partition itself when the game is not super-additive. For illustration, in a hedonic game, the structures, which are characterized by their partition, are all intra-group efficient since a coalition has a single possible decision. But a change in the partition may result in a Pareto improvement. More generally, in a non super-additive game, a structure (a, N) is intra-group Pareto optimal if a is Pareto efficient for the whole society but is not a Pareto optimal structure if the members of a group can all be made better off by being partitioned into a smaller self-sufficient groups choosing appropriate actions.

The main issue investigated in the next sections is whether some form of competition helps, or forces, the society to reach an efficient structure.

3 Stability

The section starts by examining the pressure exercised by individuals who are free to move and join whatever existing coalition. This gives rise to the free mobility conditions. We proceed with the pressure exercised by coalitions, which gives rise to the free entry conditions, and finally investigate the relationships between free mobility and free entry.

3.1 Stability under free mobility

There is free mobility if any person has the opportunity to leave the coalition to which she belongs and join any existing coalition or become isolated. In what follows, it is assumed that she decides to move or not according to the expected benefit of each alternative. The expected benefit from staying isolated is simply the maximum utility level over the individual's feasibility set, the *individual rationality level.* Let us denote it by $\overline{u}_i$ for i. The expected benefit from joining a coalition is affected by the impact that the individual expects to have on the action taken by that coalition.

Let us start with the situation where individual i expects to have no impact on the action of a coalition she joins. Specifically, assume that i expects the action a_k taken by S_k to be unchanged if she joins S_k and that $u_i(a_k)$ is well defined;[11] in that case, she expects the utility level $u_i(a_k)$ from joining S_k. To

[11]This assumption does not hold if a_k involves personalized components.

illustrate, consider a horizontal differentiated goods economy as in illustration 3-a in which action a_k specifies the characteristic and the price of the product chosen by S_k. A tentative newcomer is 'price and product taker'. Similarly, in the public good economy in illustration 3-b, citizens take the quantity of the public good and the tax level of any group as given. Such an assumption is standard in economies if coalitions are large.

Definition 5 *A coalition structure* $(a_\ell, S_\ell)_{\ell=1,\dots,L}$ *satisfies free mobility if*

$$\textit{for each } i,\ u_i(a_{\ell(i)}) \geq u_i(a_k) \ \textit{any } k = 1,\dots,L \ \textit{and } u_i(a_{\ell(i)}) \geq \overline{u}_i.$$

These conditions say that individuals do not expect to benefit from leaving the coalition to which they belong and joining another one or staying single.

Structures that satisfy free mobility typically exist, and in fact are numerous. Furthermore most of them are not Pareto optimal. To see this, consider a structure $(a,\ N)$ where the whole society forms. In that case, free mobility is satisfied if each individual achieves his rational utility level at least: $u_i(a) \geq \overline{u}_i$ for each i. In most problems of interest, there are many such actions. Moreover, they can be Pareto dominated by another feasible action for N, in which case the structure is not only inefficient but also intra–group inefficient.

When an individual has an impact on the decision taken by the coalition he contemplates joining, and is aware of this impact, free mobility should be defined in line of a Nash equilibrium. To illustrate, consider an hedonic game. A coalition structure is simply characterized by its partition $(S_\ell)_{\ell=1,\dots,L}$. Let i move from the coalition to which i belongs, $S_{\ell(i)}$, and join coalition S_k. Since the action taken by the enlarged coalition $S_k \cup i$ is unique, given by $a(S_k \cup i)$, i can anticipate without ambiguity the benefit from moving. This motivates the following definition.

Definition 6 *Consider a hedonic game. A partition* $(S_\ell)_{\ell=1,\dots,L}$ *of the society* N *is said to be Nash stable if no move is beneficial:*

$$\textit{for each } i,\ u_i(a(S_{\ell(i)})) \geq u_i(a(S_k \cup i)) \ \textit{for any } k = 1,...,L \ \textit{and } u_i(a(S_{\ell(i)})) \geq \overline{u}_i.$$

In games that are not hedonic, the reaction to the arrival of a new member may not be uniquely defined and individuals' incentives to move are affected by their conjectures about these reactions. This difficulty does not arise in a hedonic game.

For the same reasons as for free mobility, Nash stability does not imply Pareto optimality. The intuition for why single individual moves are far from ensuring Pareto optimality is clear: exploiting returns to collective action requires some coordination. We examine now coordinated moves.

3.2 Stability under free entry

Coordinated actions within a group are presumably the driving forces that explain the formation of groups and their decisions. This section assumes that any new coalition can form, a condition referred to as 'free entry', and destabilizes a coalition structure if it benefits each of its members. Formally

Definition 7 *A coalition structure* $(a_\ell, S_\ell)_{\ell=1,\dots,L}$

- is blocked by coalition S *if there is an action* b *feasible for* S *that makes every member of* S *better off :*

$$\textit{for some } b \textit{ in } A(S)\ u_i(b) > u_i(a_{\ell(i)}) \textit{ each } i \textit{ in } S,$$

- is stable under free entry if it is blocked by no coalition.

According to the blocking condition, S blocks a coalition structure if S can make each of its members better off. The stability to free entry requires that no coalition blocks. Observe that, even though there are no externalities between coalitions, the full organization of the society is relevant to assess its stability since the benefit from blocking is determined by the standing coalition structure.

The blocking condition was introduced in super-additive games to assess the stability of a decision a made by the whole society N; the set of stable decisions is called the *core* of the game. Stability under free entry thus extends the notion of the core to situations in which the society partitions itself.[12] The extension matters when the society may benefit from splitting. This follows from the following straightforward statement.

> *A coalition structure that is stable under free entry is Pareto optimal among all feasible coalition structures.*

As a result, when the game is strictly super-additive and the splitting of the society is always sub-optimal, the whole society must form to be stable, and the stable structures coincide with the core. When the game is not super-additive, the splitting of the society into smaller groups is triggered by efficiency forces. Several partitions may be associated to stable coalition structures, in which case stability does not pin down the organization of the society.

To illustrate, consider the model of formation of nations described in illustration 2, in which the location of a government within a nation is chosen according to majority rule. A new "nation" blocks if the majority winner within this new nation makes *each* of its members better off. As a result, a formation of nations that is immune to blocking is optimal (more precisely, the formation is 'constrained optimal', that is, optimal under the constraint that the government within a country is the median one). To better understand the blocking condition and the optimality consequences of free entry, let us consider an alternative coalition threat to blocking as considered by Alesina and Spolaore (1997). Assume that a new "nation" forms if the modification is approved by a *majority* in each of the existing countries affected by the change of the borders, so that, in contrast to blocking, *unanimity* within a destabilizing coalition is not required. Weakening the approval consditions makes secession easier and results in a sub-optimal formation with too many nations.[13]

[12] Gillies (1959) introduced the notion of the core. Stability under free entry exactly corresponds to the core of a certain game -called the super-additive cover- that takes into account the possibility of splitting for each coalition, see Aumann and Dreze (1974). I prefer to use a different term, as the term 'core' is usually associated to the stability of the whole N.

[13] Incidentally, the formation of a new jurisdiction (the threat) usually requires more support than a simple majority. Even though unanimity may not be required, the blocking condition is likely to be closer to normal practice than majority agreement.

Finally, it is important to note that stable outcomes may not exist, even in non pathological situations. The Condorcet paradox provides an example. There are three voters and three candidates. Preferences are $a \succ_1 b \succ_1 c$, $b \succ_2 c \succ_2 a$ and $c \succ_3 a \succ_3 b$. Under simple majority,[14] each candidate is feasible for doubletons because any two members can elect her by coordinating the votes in her favor. In particular $\{2, 3\}$ can impose c; $\{2, 3\}$ thus blocks a since both 2 and 3 prefer c to a; and similarly for other candidates given the described preferences: : no candidate is stable. In super-additive games with transferable utility, Bondareva (1963) and Shapley (1967), characterize the values for which a stable outcome exists. These conditions state that a set of linear inequalities must to be satisfied; they can be roughly interpreted as requiring the 'intermediate' coalitions not to be too strong, that is, to have low enough values.

We present in Sections 3.4 and 3.5 conditions under which stability is guaranteed.

3.3 Relationships between free mobility and free entry

The conditions of stability under free entry and free mobility a priori differ.

First, clearly, the stability under free mobility does not imply that under free entry, because free mobility only considers individuals' moves whereas free entry considers coalitions. A a result, a coalition structure that is stable under free mobility may not be Pareto optimal, in contrary to a stable one under free entry.

Second, though less clear, the stability under free entry does *not* imply stability under free mobility. The reason is that, under free mobility, an individual contemplates joining a standing coalition without evaluating the welfare of the insiders. An individual may therefore want to join a coalition S_k even if the members of S_k do not benefit from this newcomer. In that case, the enlarged coalition formed with the insiders and the newcomer i, $S_k \cup i$, does not block: the structure is unstable under free mobility but may be stable under free entry. This is due to the fact that free mobility does not consider the welfare of the members of the joined coalition. As Nash stability does not consider it either, a coalition structure stable under free entry is not necessarily Nash stable.

Under two circumstances, the welfare of a coalition accepting a newcomer is improved so that the stability under free entry implies stability under free mobility. The first circumstance arises when free mobility can be controlled and the second one is due to the monotony of the feasible sets.

Let us modify free mobility by assuming that a group can prevent newcomers to join. In a situation where a group can control the newcomers, it will prevent them to join whenever it is in the members' interest. If unanimity is required, an individual can join a group only if *each* member of that group agrees: free mobility is submitted to unanimous consent (Banerjee *et al.* (2001) refer to contractual individual stability). Individual's mobility is drastically

[14] The majority rule as well as weighted voting rules introduced by Shapley (1962) are represented by cooperative games. Research on these games is large, see e.g. Taylor and Zwicker (1999).

restricted:[15] assume that i benefits from joining S_k; she is allowed to do so only if no member of S_k is made worse off: $S_k \cup i$ 'weakly' blocks, in the sense that i is strictly better off and the other ones are not worse off. Under a condition referred to as 'smooth' payoffs,[16] the action can be slightly modified so that *every* member is strictly better off, hence blocks. This yields the following property.

> *Under smooth payoffs, a coalition structure that is stable under free entry is stable under free mobility submitted to unanimous consent.*

The possibility of forbidding entry can be reasonably assumed in a small population, as for example in partnerships (Farrell and Scotchmer (1988)) or in 'clubs'. In their analysis of growth clubs for example, Jaramillo, Kempf, and Moizeau (2005) or (2015) assume that a coalition of countries that form to contribute to a growth enhancing public good can exclude a country to join. Due to the differences in endowments across countries, a coalition may be hurt by a poorer country willing to join: in that case the coalition blocks the new entrant (so the structure may be stable under free entry) but is not stable under full free mobility.

A second circumstance where a coalition is not hurt by a newcomer is when feasible sets are increasing. In that case, stability under free entry implies stability under free mobility. The argument is by contradiction as follows. Consider a coalition structure that is not stable under free mobility: given the standing actions, individual i would be better off by joining the existing group S_k rather than stay a member of $S_{\ell(i)}$. The key point is that, because the feasible sets are increasing, S_k can keep the same action so that its members are equally well off under i's move; furthermore, under a smoothness assumption as explained above, the action can be slightly modified so that every member of $S_{k\cup i}$ is strictly better off: the coalition $S_{k\cup i}$ blocks. This proves that the coalition structure is not stable under free entry, as stated in the following property:

> *Assume that feasible sets are increasing. Under smooth payoffs, a coalition structure that is stable under free entry is stable under free mobility as well.*

The relationships between free mobility and free entry are not trivial when a group is unable to control its membership and feasible sets are not increasing, This occurs in a variety of situations. For example, exclusion policies based on identity are forbidden or are impossible to implement due to poor information on the newcomers' characteristics. In that case, the redistribution performed within a jurisdiction may prevent increasing returns to coalitions to be satisfied, as seen for the public goods economy of illustration 3-b. In such a context,

[15] For various definitions and discussions of free mobility in hedonic games see Banerjee *et al.* (2001) and Bogomolnaia and Jackson (2002). Immigration rules that place conditions on the acceptance of immigrants are a restriction to free mobility, see e.g. the analysis in Jehiel of Scotchmer (2001).

[16] For a formal definition see Hart and Mas-Colell (1996). These conditions are mild when the decisions entail a divisible good. Applied to N, they ensure that the weak and the strong Pareto optimal outcomes coincide.

the interaction between free mobility and free entry[17] generates stability issues similar to those in insurance markets where negative externality stems from adverse selection (Rothschild and Stiglitz (1976)).

3.4 Restrictions to free entry

Free entry assumes that all coalitions can form. In some contexts, the assumption is unrealistic. Limitations in the formation of coalitions are modeled here by defining the set of essential coalitions: a coalition is said *essential* if it can form and block. The more limited the set of essential coalitions, the more likely there are stable coalition structures. One question is to which extent the set of essential coalitions must be limited to *guarantee* the existence of stable structures, that is, for stable structures to exist *whatever* the preferences. As the focus of this section is stability under free entry, the term 'under free entry' is omitted.

In the following two examples, stability is guaranteed.

Marriage and assignment games. The society is partitioned into two sides, say men and women, buyers and sellers, colleges and students. Let us consider for example the marriage problem (Gale and Shapley (1962)). A marriage defines a set of couples, each individual having at most one spouse. The marriage is stable if (a) no new couple can form in which both the man and the woman prefer each other to their standing partner (no couple blocks) (b) no person would prefer to stay single (no individual blocks). Gale and Shapley define the deferred-acceptance mechanism and show that it converges to a stable marriage. Hence, when the singletons and the pairs consisting of one man and one woman are the essential coalitions, a stable outcome always exists.

Connected coalitions in a network. Let individuals be connected through a (social) network. The links in the network represent the possibility of exchanges or communication among the individuals, so that only the connected coalitions can form.[18] For example, if the network is a simple path, only intervals can form: a *consecutive* game is obtained as studied by Greenberg and Weber (1993).

The stability results are as follows. If the network is a tree, that is, a connected graph without cycles, stability is guaranteed (Demange (1994)). Furthermore, stable outcomes, called hierarchical, are easy to interpret and to compute (Demange (2004)). Such a result may provide a rationale for organizations such as tree-hierarchy structures.[19] If the network contains a cycle, stability is no longer guaranteed, i.e. there are preferences and values for the coalitions for which no stable structures exist.

[17] See Epple and Romer (1991) for such an analysis.

[18] Myerson (1977) introduces this modeling and studies allocation rules *à* la Shapley. Kalai, Postlewaite and Roberts (1978) studies an exchange economy where direct trades are conducted through the links of a network and perform comparative statics on the stable allocations when the network varies.

[19] For a recent treatment of organizations using coalition structures, see Morelli and Park (2016). Recent works study hierarchical outcomes and related concepts as an allocation mechanism, see e.g. Herings et al. (2008) to settings with externalities across coalitions. In the problem of 'sharing' a river, an ordering of the players is similar to that in a hierarchy, as studied by Ambec and Sprumont (2002).

Stability is thus guaranteed in marriage games and in tree structures but no other meaningful contexts are known. Instability is not pathological, as can be shown by the following observation, which extends Condorcet paradox. Let $\mathcal{C}$ be the set of essential coalitions. A *Condorcet triple* is defined as three coalitions that intersect each other and whose overall intersection is empty:

$$S_i, i = 1, 2, 3, \text{ with } S_i \cap S_j \neq \emptyset \text{ and } S_1 \cap S_2 \cap S_3 = \emptyset.$$

When the set $\mathcal{C}$ contains a Condorcet triple, it is easy to build a coalitional game that does not admit a stable outcome.[20] Therefore:

> *A set $\mathcal{C}$ of coalitions that contains a Condorcet triple does not guarantee stability.*

The absence of Condorcet triples is a strong restriction. Furthermore the restriction is only necessary for guaranteeing stability. Mathematical characterizations for the guarantee are provided by Kaneko and Wooders (1982) in terms of the extreme solutions to a system of linear equations (see also Le Breton, Owen, and Weber (1992)).

To summarize, the lesson that can be drawn from this analysis is that the blocking power of coalitions, or equivalently their possibility to form, must be severely restricted in order to guarantee stability. Other types of restriction to entry could be considered. For example, a new group can form only if it is a subset of an existing group (internal stability within a group) or if it is the union of two existing groups. We are not aware of any general result under such restrictions.

3.5 Restrictions on preferences: Intermediate preferences

In some situations, preferences' profiles can reasonably be assumed of a specific form, which might facilitate stability.[21] Intermediate preferences are such a meaningful restriction, introduced in social sciences by Kemeny and Snell (1962) and in economics by Grandmont (1978). Let us define them for preferences indexed by a scalar.

Definition 8 *Consider the family $\{u(a, \theta), \theta \in [\underline{\theta}, \overline{\theta}]\}$ of utility functions on A indexed by the parameter θ in the interval $[\underline{\theta}, \overline{\theta}]$. The family defines intermediate preferences if, for any a and b in A, the sets $\{\theta, u(a, \theta) > u(b, \theta)\}$and $\{\theta, u(a, \theta) \geq u(b, \theta)\}$ are intervals.*

The preferences in the family are thus related among each other while allowing heterogeneity in tastes.

To illustrate, consider a vertical differentiation model in which payoffs given by (2): $u(q, p, \theta) = v(q, \theta) - t$. Preferences are intermediate if $\frac{\partial^2 v}{\partial \theta \partial q}$ of constant

[20] For example, consider the transferable utility game with characteristic function V: $V(N) = 1$, $V(S) = b$ if S is a strict subset of N that contains at least one $S_i, i = 0, 1, 2$, and $V(S) = 0$ otherwise. For b smaller than 1 the game is super-additive but, for b strictly smaller than $2/3$, any feasible payoff is blocked by one of the S_i.

[21] A restriction to essential coalitions can be interpreted as a restriction of preferences, as giving a low enough values to the non essential coalitions.

sign (often called Spence-Mirlees condition).[22] Let us prove that a stable coalition structure exists. Since the characteristics are one-dimensional, individuals can be ordered according to their characteristics. Therefore, there is a coalition structure that is formed with "intervals" of individuals and such that no interval blocks, as follows from the analysis of consecutive games referred to in the previous section. Increasing returns to coalitions and intermediate preferences then imply that this coalition structure is blocked by no coalition at all (even the disconnected ones) hence is stable. This argument holds more generally and proves that a stable coalition structure exists when there are increasing returns to coalitions and preferences parameterized by a scalar are intermediate.

One issue is whether this positive result extends to intermediate preferences defined on a more general parameter space than a one-dimensional one. Intermediate preferences are easily defined when the characteristic belongs to multidimensional space or a tree (see respectively Grandmont (1978) and Demange (1994)). If the characteristic belongs to a tree, the previous line of argument applies, as shown in Demange (1994):[23] there is a coalition structure that is formed with connected subsets of individuals and such that no connected subset blocks. Increasing returns to coalitions and intermediate preferences then imply that this coalition structure is blocked by no coalition at all, as stated in the following result:

> *Let individuals' preferences be characterized by a parameter on a tree. Under intermediate preferences and increasing returns to coalitions, a coalition structure that is stable under free entry exists.*

When characteristics are multi-dimensional, even under intermediate preferences, a stable structure may not exist. Though there is a coalition structure formed with convex sets that is not blocked by any convex set, such a coalition may not be stable (see an example in Demange and Henriet (1991)). This is due to the fact that in a multi-dimensional space, there are Condorcet triples formed with convex sets.

4 Empirics

Recent papers building on the previous models conduct empirical works. My aim is not to describe these papers in detail -the reader is referred to the papers- but rather to show how the theoretical predictions fit the theoretical results.

Robett (2015) conducts laboratory experiments to study how agents partition themselves into groups in settings similar to those presented previously: there are benefits that are increasing with a coalition size but the preferences for a group policy are not perfectly aligned. Specifically, individuals' preferences are single-peaked (or intermediate), characterized by their location on an

[22]See other examples in Demange and Henriet (1991). Various restrictions on preferences over a one-dimensional set of alternatives have been introduced: single-peakedness, single-crossing, order restriction, and recently top-monotonicity, see Barbera and Moreno (2011).

[23]The argument extends when the characteristics belong to a median space, as I show in Demange (2012).

interval $[0,1]$. Specifically the payoff to an individual whose preferred policy is θ and member of S is

$$u(S,a,\theta) = |S| - \gamma(\theta - a)^2,$$

where a is the decision taken by S and γ is a parameter reflecting the trade-off between the group size and the policy.

Experiments are as follows. Once they know their θs, individuals form groups by choosing a location. Two types of experiments are conducted. In the Fixed Policy sessions, each location is associated with a fixed, posted policy. In the Voting sessions, the policy is the location of the median voter: the model is one of differentiated choices as in illustration 3. In both cases, a hedonic game is obtained. The parameters are chosen so that (a) the same set of Nash stable partitions exists in both the Fixed Policy and the Voting sessions and (b) there is a unique partition stable under free entry; two groups form, each comprised of the individuals whose ideal points fall within one-half of the [0,1] interval.

In all Nash stable partitions, the range of ideal points represented in each group is an interval. However, the specification of these ranges may be inefficient. (Recall that Nash stability considers the mobility of individuals who form correct expectations on the impact of their move, as seen in Section 3.1.)

The main results of the experiments are as follows: most subjects sort into a Nash stable partition but which partition is reached depends on the type of the session, that is on how the policy within a group is determined. In the Fixed Policy session, individuals fail to reach an efficient partition, specifically, they segregate too much and there are more than two groups. In the Voting session instead, they most often reach the stable structure. This difference can be due to two reasons. First, the Voting session allows groups to have more flexibility, as they can adjust their policy to changes in their membership composition. Second, voting may enhance coordination as individuals realize the importance of the membership size. These results suggest that the rule determining how group policies are chosen greatly affect whether agents reach optimal partitions and that some flexibility help them to coordinate more efficiently.

Melatos and Woodland (2007) investigate trade bloc formation in a general equilibrium model. They consider two types of trading arrangements -free trade and customs unions that choose common external tariffs- and rely on stability under free entry. The main question is how the stable agreements relate to the primitive characteristics, such as consumer preferences and commodity endowments (each agreement leads to a different equilibrium). As the economy has three goods and endowments and preferences are asymmetric, there are no explicit solutions for the stable structures. The authors rely on calibration and simulations. Interestingly, they find that, in a stable structure, customs unions are always unions of countries with adjacent elasticities of substitutions or adjacent endowments of their export goods. This is reminiscent of the intermediate preferences assumption presented in Section 3.5.

Desmet and al. (2011) present a model of nations where agents vote on the optimal level of public spending. The aim of the paper is to determine the likelihood of secessions or unions, with special attention to Eastern Europe.

The heterogeneity in preferences is interpreted as a cultural one. Though, to calibrate the model to Europe, genetic distances are used as a proxy for cultural heterogeneity (an hypothesis supported by some surveys). The paper shows that the model can account for the breakup of Yugoslavia and the dynamics of its disintegration.

Weese (2015) analyzes on real data the interaction between the national governmental transfers and the merger of municipalities in Japan. Subsidies that aim at correcting inequality between municipalities provide little incentives for small municipalities to merge, thereby precluding the exploitation of huge economies to scale in the provision of public services. In particular, Weese estimates a structural model in order to assess the inefficiency of the mergers. The basic model is in the style of the ones presented previously: there is a tradeoff between the heterogeneity in individual preferences, parameterized by a two-dimensional location, and there are efficiencies of scale in the production of public goods. A hedonic game is obtained by assuming that the selected policy maximizes a weighted sum of individual utilities in a probabilistic framework.[24] Finally, the coalitions' deviations are limited to mergers and splitting. A main conclusion is that some but not all efficiency gains were reached in the recent mergers of municipalities in Japan. To improve efficiency, Weese suggests to a counter-intuitive policy involving transfers to richer municipalities conditional on their participation in a merger.

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[24] A majority winner (nor any other reasonable deterministic voting outcome) is not well defined in that context due to the two dimensional characteristics.

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