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# A Stochastic Programming Approach for the Design of Multi-Storey Recycling Facility

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# Abstract

A rapid increase in urban population creates major challenges related to urban sprawl, pollution and waste generation, unsustainable production and consumption patterns. These challenges become even more crucial in the case of land-constrained urban territories, such as Singapore and Hong Kong, and require the development of decision-making methodologies for flexible long-term land use planning.

The paper explores the possible relocation of decentralized companies with similar work processes to relocate towards centralized Multi-Storey Factories (MSF) for a higher density of land use. The developed decision-making methodologies aim, on the one hand, to maximize land savings and, on the other hand, to decrease each company's operational budget evaluated under uncertainties in future operational conditions, such as transportation costs. The optimization problem addressed has been formulated as a two-stage stochastic problem and tested for the application case of Multi-Storey Recycling Facility (MSRF). Optimization under uncertainty shows a 16.46% increase in estimated land savings in comparison with the solution obtained under deterministic conditions.

# Keywords

Land use, facility relocation, decision-making process, heuristic approach, uncertainty

# 1. Introduction

According to data from the United Nations, urbanization and growth of the world's population will result in an estimated increase of 2.5 billion people to the urban population by 2050, with roughly 90% of this increase coming from Asia and Africa [1]. Forecasts show that by 2050, 66% of the world's population will come from urban areas. This rapid increase in urban population poses great challenges for future urban development and requires adapted urban planning policies. This requires for proper urban planning policies, as uncontrolled urban expansion could result in detrimental effects such as urban sprawl, pollution as well as unsustainable production and consumption patterns.

Current urban development policies are highly reliant on governmental driven deployment preventing an unplanned growth. This unplanned growth becomes more crucial especially in fast growing and densely populated territories, leading to disastrous consequences, e.g., very rapid market fluctuations followed by private sector short-term responses may pose long-term repercussions for an entire city [2], [3]. The fast expansion of cities, especially in Asian and African countries, result in the fragmentation of land use patterns within urban areas and reduction of cities efficiencies to provide services to infrastructures [4], [5]. Land-constrained countries and territories, such as Singapore, Hong Kong and Maldives, have even less room for error when it comes to urban and long term land use planning. As a consequence, these territories attach particular importance to the continuous review of urban strategies to reshape previous mistakes [2] and to promote urban renewal and regeneration projects for the improvement of older urban areas [6], [7].

One of the currently considered solutions to make better use of scarce urban land is to shift towards Multi-Storey Factories (MSF) allowing a higher density of land use [8]. Such ideas have opened up the possibility of relocating existing companies with similar work processes into centralized MSF by freeing up the original land space that they occupy for other uses. However, such urban strategy based on existing companies' relocation could interfere with these companies' individual objectives. Currently available research done in the field of Capacitated Centralized Facility Location Problem (CCFLP), e.g., healthcare [9] and supply chain logistics [10], typically assume that existing decentralized facilities will always relocate to a centralized facility if it leads to an increase in overall welfare. In practice, however, companies that operate as individual entities may decide not to relocate to a centralized facility if it results in an individual welfare loss.

To address these issues, the paper proposes a decision-making framework for optimal placement of existing companies with similar work processes to move into centralized facilities by taking into account urban territory welfare and companies' individual performance objectives. The focus is on the economic component of the relocation decision, such as the relocation costs and lease conditions, and associated uncertainties. An alternative approach has been proposed to solve the stochastic problem that differs from the Sample Average Approximation (SAA) method used here. The approach does not require solving the problem using a random sample from the set of all possible scenarios, thereby eliminating convergence issues inherent in sampling methods to solve stochastic programming problems. In addition, a heuristic approach has been proposed and tested to incorporate uncertainties into the optimization model and to increase computational efficiency of the algorithm.

The breakdown of the remaining sections is as follows. Section 2 presents a two-stage stochastic formulation of the CCFLP and Section 3 introduces the SAA and heuristic optimization methodologies used to solve the CCFLP problem. Section 4 describes an application study and provides the optimization results and analysis. Section 5 concludes this study and discusses future research directions.

#### 2. Problem definition

The CCLFP is formulated as a Mixed-Integer Liner Programming (MILP) problem. Let I denote the set of potential moving companies and J denote the set of potential centralized facility locations. Each centralized facility  $j \in J$  is associated with an annual operating budget  $B_i$  and a fixed repayment  $R_i$  incurring if the facility is opened. Each centralized facility j contains a certain number of floors  $n_i$ , and a certain amount of space  $Q_i$  in each floor. The total capacity of each centralized facility j is  $n_i Q_j$ . Each company  $i \in I$  is associated with a transportation cost at its original location  $c_i \hat{d}_i$  and a transportation cost after moving into the centralized facility  $c_i d_{ij}$ , where  $c_i$  is the transportation cost per unit distance of company i,  $\hat{d}_i$  and  $d_{ii}$  are the annual total transportation distances to its customers covered by company *i* at original location and after moving to a facility *j*, respectively. Each company *i* also has an amount of land used  $q_i$  and original unit annual rental cost  $\hat{r}_i$ , thus the original total annual rental costs is  $q_i \hat{r}_i$ . A company's decision-making process is defined as follows: company *i* will be willing to relocate only if the total rental and transportation costs after moving to centralized facility j is lower than company i's original rental and transportation costs, i.e., if  $(c_i d_{ii} + q_i r_i) < (c_i \hat{d}_i + q_i \hat{r}_i)$ , where  $r_i$  is the rental cost charged by centralized facility *j*. In practice, future conditions of a company are not known with certainty, and can be affected by several factors, such as changes in population size and economic situations. Also, transportation costs are not always fixed and dependent on exogenous factors such as travel times and fuel costs. The uncertainty in these factors would affect the transportation costs per unit distance  $c_i$  of each company *i*.

To incorporate the uncertainty in the transportation costs  $c_i$  of each company *i*, a two-stage stochastic programming model has been formulated, in which  $\tilde{c}_i$  denotes the uncertain  $c_i$ . The first stage aims to decide whether to open up a centralized facility at each individual potential location  $j \in J$  at the corresponding rental cost  $r_j$ . In the second stage, after the evaluation of each company's transportation costs, different companies will make the allocation decisions, i.e., whether to move and which centralized facility to allocate. In the scenario-based formulation of CCFLP, let  $\xi$ denote a vector of transportation costs per unit distance for each company  $i \in I$ , while  $\xi^s$  stands for a particular realisation of the uncertain parameters for scenario  $s \in S$ , where *S* is the set of all scenarios. The binary decision variable  $\tilde{x}_{ij}$  specifies whether or not company *i* moves to centralized facility *j* for the particular realization  $\xi$ . The binary decision variable  $y_j$  specifies whether centralized facility *j* is opened or not. In this view, a two-stage stochastic model for the CCFLP is formulated as follows:

P1-1:  

$$\max \quad \varphi(\widetilde{\mathbf{x}}) = E[F(\mathbf{y}, \mathbf{r}, \boldsymbol{\xi})] - \sum_{j \in J} Q_j y_j$$
(1)  
s.t.  $r_j \leq U y_j, \ j \in J; \ y_j \in \{0,1\}, \ j \in J; \ r_j \geq 0, \ j \in J$ 

P1-2: 
$$F(\mathbf{y}, \mathbf{r}, \xi^s) = \max \sum_{j \in J} \sum_{i \in I} q_i \tilde{x}_{ij}$$
(2)

s.t. 
$$\sum_{j \in J} \left( B_j y_j + R_j y_j - \sum_{i \in I} q_i \tilde{x}_{ij} r_j \right) \le L$$
(3)

$$\sum_{i \in I} q_i \tilde{x}_{ij} \le n_j Q_j, \qquad j \in J$$
<sup>(4)</sup>

$$\left(\tilde{c}_{i}d_{ij}+q_{i}r_{j}\right)-\left(\tilde{c}_{i}\hat{d}_{i}+q_{i}\hat{r}_{i}\right)< M\left(1-\tilde{x}_{ij}\right), \qquad i\in I, j\in J$$
<sup>(5)</sup>

$$\tilde{x}_{ij} \le y_j, \sum_{j \in J} \tilde{x}_{ij} \le 1, \tilde{x}_{ij} \in \{0, 1\} \ i \in I, j \in J$$
(6)

Subjected to a binary constraint for  $y_j$  and a non-negative constraint for  $r_j$ , Eq. (1) is to maximize the total land area saved, i.e., all the land occupied by relocated companies minus the space of selected facilities. It is also ensured that the rental cost for a non-selected site is zero, and U is a large enough number to enforce this condition. The optimal value  $F(y, r, \xi^s)$  of the 2<sup>nd</sup> stage problem is a function of the 1<sup>st</sup> stage decision variables y and r and the uncertain parameters realization. Its expectation is taken with respect to the known probability distribution of  $\xi$ .

Eq. (3) indicates that the annual total amount spent on operating the facilities does not exceed its allowable loss *L*. The non-linear term  $\tilde{x}_{ij}r_j$  can be linearized with additional constraints and decision variables to the model. Eq. (4) limits the number of companies that can relocate to centralized facility *j* due to its capacity. Eq. (5) ensures that a company will only shift if the total costs after relocation is less than that before moving. In this case *M* is a large enough number introduced to fulfil this inequality. Eq. (6) ensures that  $x_{ij}$  would be 0 if centralized facility *j* is not operating; each waste sorting company *i* will only be assigned to one centralized facility; there is a binary constraint for variable  $\tilde{x}_{ij}$ .

Some difficulties exist in solving the above models. For one, solving the model would involve computing the expected value of the linear programming function  $F(y, r, \xi)$ . For continuous distributions, computing the expectation of this function requires calculating multiple integrals which could be computationally infeasible. Furthermore,  $F(y, r, \xi)$  is known to be a convex non-linear function of y and r and thus solving the above models would involve maximizing non-linear functions, as well as highly increasing computational complexity.

#### 3. Optimization methodology

## 3.1.Sample Average Approximation (SAA) Method

The SAA method is introduced to address the aforementioned difficulties in solving the stochastic model. The basic idea behind the SAA method is that the expected value of the objective function in the stochastic model is estimated using sample average estimation derived from a random sample. In this view, K independent and identically distributed (*i.i.d*) scenarios of the random vector  $\boldsymbol{\xi}$ , denoted by  $\xi^1, \ldots, \xi^K$ , are generated according to their probability distributions. The expectation  $E[F(y, r, \xi^s)]$  can then be approximated using the sample average function  $\frac{1}{\kappa} \sum_{k=1}^{\kappa} F(y, r, \xi^k)$ . The SAA problem of P2 is formulated as follows:

P2:

$$\hat{\varphi}_{K} = max \quad \frac{1}{K} \sum_{k=1\dots K} \sum_{j \in J} \sum_{i \in I} q_{i} x_{ij}^{k} - \sum_{j \in J} Q_{j} y_{j}$$

$$\tag{7}$$

s.t.  $r_j \leq Uy_j, \ j \in J; \ y_j \in \{0,1\}, \ j \in J; r_j \geq 0, \ j \in J$ 

$$\sum_{j \in J} \left( B_j y_j + R_j y_j - \sum_{i \in I} q_i x_{ij}^k r_j \right) \le L, \qquad k = 1, \dots, K$$
<sup>(8)</sup>

$$\sum_{i \in I} q_i x_{ij}^k \le n_j Q_j, \qquad j \in J, k = 1, \dots, K$$
<sup>(9)</sup>

$$(c_i^k d_{ij} + q_i r_j) - (c_i^k \hat{d}_i + q_i \hat{r}_i) < M(1 - x_{ij}^k), \qquad i \in I, j \in J, k = 1, \dots, K$$
<sup>(10)</sup>

$$x_{ij}^{k} \le y_{j}, \sum_{j \in J} x_{ij}^{k} \le 1, x_{ij}^{k} \in \{0, 1\} \quad i \in I, j \in J, k = 1, \dots, K$$
(11)

#### **3.2.Heuristics** Approach

To solve the above two-stage stochastic model more efficiently, we develop a simple heuristic when the constraint (10) is replaced with probability  $P\left(\left(\tilde{c}_i d_{ij} + q_i r_j\right) - \left(\tilde{c}_i d_i + q_i \hat{r}_i\right) > 0\right)$ . In this case,  $\tilde{x}_{ij}$  becomes a random variable where  $\tilde{x}_{ij}$  equals  $\bar{x}_{ij}$  with probability  $p_{ij}$ , and equals 0 with probability  $1 - p_{ij}$ . We define  $p_{ij}$  as:

$$p_{ij} = \begin{cases} P\left(\tilde{c}_i \leq \frac{q_i(\hat{r}_i - r_j)}{d_{ij} - \hat{d}_i}\right), & \text{if } d_{ij} > \hat{d}_i \\ P\left(\tilde{c}_i > \frac{q_i(\hat{r}_i - r_j)}{d_{ij} - \hat{d}_i}\right), & \text{otherwise} \end{cases}$$

$$(12)$$

 $\bar{x}_{ij}$  is defined as a binary variable equal to 1 if waste sorting company *i* moves to potential site *j* and 0 otherwise. Since the land used by company *i* (i.e.,  $q_i$ ) is known with certainty, an approximate problem of P2, i.e., model P3 can be expressed as:

P3:  

$$\hat{\varphi}_{h} = max \sum_{j \in J} \sum_{i \in I} q_{i}E[\tilde{x}_{ij}] - \sum_{j \in J} Q_{j}y_{j} = max \sum_{j \in J} \sum_{i \in I} q_{i}\bar{x}_{ij}p_{ij} - \sum_{j \in J} Q_{j}y_{j} \qquad (13)$$
s.t.  $r_{i} \leq Uy_{i}, j \in I : y_{i} \in \{0,1\}, j \in I : r_{i} \geq 0, j \in I \qquad (14)$ 

$$r_j \le U y_j, \ j \in J \ ; \ y_j \in \{0,1\}, \ j \in J; \ r_j \ge 0, \ j \in J$$
 (14)

$$\sum_{j \in J} \left( B_j y_j + R_j y_j - \sum_{i \in I} q_i \bar{x}_{ij} r_j \right) \le L$$
(15)

$$\sum_{i \in I} q_i \bar{x}_{ij} \le n_j Q_j, \bar{x}_{ij} \le y_j, \sum_{j \in J} \bar{x}_{ij} \le 1, \bar{x}_{ij} \in \{0,1\}, \quad i \in I, j \in J$$
(16)

Problem P3 reduces the scale of the problem compared with that of the SAA model P2, as it does not have to consider k = 1, ..., K scenarios for (2) - (6). One challenge of solving P3, however, is to evaluate the term  $p_{ij}$ , which is a function of decision variable  $r_j$ . We find that if the distributions of uncertain variables are known, P3 can be greatly simplified. For example, assuming that  $\tilde{c}_i$  follows a uniform distribution between  $a_i$  and  $b_i$ , P3 can be re-written as:

P4: 
$$\hat{\varphi}_{h} = max \sum_{j \in J} \sum_{i \in I} q_{i} \bar{x}_{ij} w_{ij} - \sum_{j \in J} Q_{j} y_{j}$$
s.t. (14) - (16) (17)

where 
$$w_{ij} = \begin{cases} D\left(\frac{q_i(\hat{r}_i - r_j)}{d_{ij} - \hat{d}_i}\right) = \frac{1}{b_i - a_i} \left(\frac{q_i(\hat{r}_i - r_j)}{d_{ij} - \hat{d}_i} - a_i\right), & \text{if } d_{ij} > \hat{d}_i \\ 1 - D\left(\frac{q_i(\hat{r}_i - r_j)}{d_{ij} - \hat{d}_i}\right) = \frac{1}{b_i - a_i} \left(b_i - \frac{q_i(\hat{r}_i - r_j)}{d_{ij} - \hat{d}_i}\right), & \text{otherwise} \end{cases}$$
 (18)

Where  $D(\cdot)$  is the cumulative distribution function of  $U(a_i, b_i)$ . Other distributions such as normal distribution can also be considered by changing  $w_{ij}$  according to different cumulative distribution functions. Therefore, this heuristic approach can be applied to many cases if the distributions of uncertain variables are known.

#### 4. Application

The above models and the heuristic approach were programmed in Matlab. CPLEX is used to solve models. The computational experiments are conducted on a PC with Intel(R) Core(TM) i5-3570 CPU, 3.40 GHz processor and 8.0 GB RAM running on Windows 7 OS. Five potential and identical MSRF sites located around Singapore are considered (see Figure 1 and Table 1). Twenty selected waste sorting companies are assigned with unique parameters related to annual total transportation distance  $\hat{d}_i$ , amount of the land used  $q_i$ , original unit rental land cost  $\hat{\tau}_i$  and unit transportation cost  $c_i$ . The urban planner's allowable loss *L* is 500,000 \$. The transportation costs  $\tilde{c}_i$  are modelled as a uniform distribution in the interval between  $a_i$  and  $b_i$  defined for each companies *i*. To test the effectiveness of a stochastic approach on non-uniform distributions,  $\tilde{c}_i$  was also assumed to follow a normal distribution with mean  $\mu_i$  and standard deviation  $\sigma_i$  for each company *i*. To make two distributions comparable, we let  $a_i + b_i = 2\mu_i$  for each company and assign an identical value to all standard deviations. The data set used in this paper is available upon request.

The stopping criteria were either if the solver terminates due to an out-of-memory error or the optimality gap relative to the best upper bound is smaller than 0.01%. For the SAA solutions obtained in the out-of-memory cases, we introduce a performance measure  $GAP = (\varphi_K^{LR} - \hat{\varphi}_K)/\hat{\varphi}_K$ , where  $\hat{\varphi}_K$  is the SAA solution with sample size *K* and  $\varphi_K^{LR}$  is its linear relaxation with the binary decision variables  $\boldsymbol{x}$  relaxed. The linear relaxation solution  $\varphi_K^{LR}$  can be considered as an upper bound of the candidate solution  $\hat{\varphi}_K$  for sample size *K*.





Table 1. Parameters of MSRF for multiple facilities case study.

| Parameters                          | Value      |
|-------------------------------------|------------|
| B (Budget/\$)                       | 10,000,000 |
| <i>R</i> (Fixed cost repayment/\$)  | 2,000,000  |
| Q (Floor space of MSRF/sq ft)       | 150,000    |
| <i>n</i> (Number of floors of MSRF) | 5          |

Figure 1. Map of Singapore showing locations of the potential MSRF sites across Singapore.

Table 2. Results from deterministic, SAA and heuristics for multiple centralized facilities case study.

| Method                  | $\widehat{\boldsymbol{\varphi}}$ (sqft) | (total land saved)/(total<br>land used) | Potential Sites<br>Chosen                     | Rent for Chosen Site <i>j</i> (\$/sqft/mth) | Average Number<br>of Companies | CPU Time (s) | GAP (%) |
|-------------------------|---|---|---|---|--------------------------------|--------------|---------|
| Deterministic           | 1,199,763                               | 1.8661                                  | <i>y</i> <sub>1</sub> , <i>y</i> <sub>4</sub> | $r_1 = 1.4305; r_4 = 1.1957$                | 15.0                           | -            | -       |
| SAA: $K = 10$ (Uniform) | 1,199,649                               | 1.8656                                  | <i>y</i> <sub>1</sub> , <i>y</i> <sub>4</sub> | $r_1 = 1.5757; r_4 = 1.0398$                | 14.6                           | 67626 16     | -       |
| SAA: $K = 10$ (Normal)  | 1,199,631                               | 1.8655                                  | <i>y</i> <sub>1</sub> , <i>y</i> <sub>4</sub> | $r_1 = 1.5962; r_4 = 1.0165$                | 14.2                           | 07030.40     | -       |
| SAA: $K = 20$ (Uniform) | 1,199,180                               | 1.8635                                  | <i>y</i> <sub>1</sub> , <i>y</i> <sub>4</sub> | $r_1 = 1.5787; r_4 = 1.0354$                | 14.5                           | 142410 42(1) | 16.13   |
| SAA: $K = 20$ (Normal)  | 1,197,222                               | 1.8548                                  | <i>y</i> <sub>1</sub> , <i>y</i> <sub>4</sub> | $r_1 = 1.5795; r_4 = 1.0568$                | 14.5                           | 142410.45    | 16.33   |
| SAA: $K = 30$ (Uniform) | 1,193,987                               | 1.8406                                  | <i>y</i> <sub>1</sub> , <i>y</i> <sub>4</sub> | $r_1 = 1.5384; r_4 = 1.0870$                | 14.7                           | 126410 67(1) | 16.64   |
| SAA: $K = 30$ (Normal)  | 1,197,222                               | ,222 1.8548                             |   | $r_1 = 1.5761; r_4 = 1.0475$                | 14.7                           | 120410.07    | 16.33   |
| SAA: $K = 50$ (Uniform) | 1,193,987                               | 1.8406                                  | $y_4$   | $r_4 = 1.2790$                              | 7.54                           | 26320 43(1)  | 132.17  |
| SAA: $K = 50$ (Normal)  | 1,179,782                               | 2 1.7797 y <sub>1</sub> , y             |   | $r_1 = 1.4599; r_4 = 1.3132$                | 14.9                           | 20320.43     | 18.04   |
| Heuristics              | 1,207,879                               | 1.9027                                  | $y_1, y_2, y_4$                               | $r_1 = 2.2241; r_2 = 1.3969; r_4 = 0.9939$  | 20.0                           | 76566.40     | -       |

<sup>(1)</sup> Out-of-memory

Table 3. Results of out-of-sample tests using uniform and normal distribution for different methods for multiple centralized facilities case study (K' = 1000).

| Land                            | Uniform                            |                                    |                                    |                                    |                                  | Normal                             |                                    |                                    |                                    |                                    |                                  |                                    |
|---------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|----------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|----------------------------------|------------------------------------|
| Savings                         | Deterministic                      | SAA<br>( <b>K</b> = 10)            | SAA<br>( <b>K</b> = 20)            | SAA<br>( <b>K</b> = 30)            | SAA<br>( <b>K</b> = 50)          | Heuristics                         | Deterministic                      | SAA<br>( <b>K</b> = 10)            | SAA<br>( <b>K</b> = 20)            | SAA<br>( <b>K</b> = 30)            | SAA<br>( <b>K</b> = 50)          | Heuristics                         |
| Average<br>$CV = \frac{SD}{CV}$ | 1,195,565<br>2.44·10 <sup>-3</sup> | 1,199,542<br>3.86·10 <sup>-4</sup> | 1,199,542<br>3.49·10 <sup>-4</sup> | 1,198,327<br>1.39·10 <sup>-3</sup> | 599,982<br>1.47·10 <sup>-5</sup> | 1,392,342<br>3.61·10 <sup>-3</sup> | 1,195,521<br>2.47·10 <sup>-3</sup> | 1,199,699<br>8.08·10 <sup>-5</sup> | 1,199,698<br>8.03·10 <sup>-5</sup> | 1,197,890<br>1.19·10 <sup>-3</sup> | 599,979<br>4.43·10 <sup>-6</sup> | 1,392,560<br>2.18·10 <sup>-3</sup> |
| Mean<br>Minimum<br>Maximum      | 1,192,833<br>1,199,763             | 1,194,732<br>1,199,719             | 1,194,732<br>1,199,719             | 1,192,833<br>1,199,719             | 599,933<br>599,999               | 1,314,162<br>1,392,693             | 1,192,833<br>1,199,763             | 1,198,796<br>1,199,719             | 1,198,796<br>1,199,719             | 1,192,833<br>1,199,719             | 599,979<br>599,999               | 1,312,233<br>1,392,693             |
| 95% CI                          | [1195384,<br>1195682]              | [1199502<br>,<br>1199560]          | [1199516,<br>1199568]              | [1198224,<br>1198431]              | [599982,<br>599982]              | [1392029,<br>1392654]              | [1195338,<br>1195704]              | [1199693,<br>1199705]              | [1199692,<br>1199704]              | [1197801,<br>1197978]              | [599979,<br>599980]              | [1392371,<br>1392749]              |
| Difference<br>(+/- %)           | -                                  | + 0.33                             | + 0.33                             | + 0.23                             | - 49.81                          | + 16.46                            | -                                  | +0.35                              | +0.35                              | +0.20                              | -49.81                           | +16.48                             |

From Table 2 it can be observed that when K > 10, the optimal SAA solutions cannot be obtained due to the out-ofmemory issues. Thus the solution gap increases exponentially from 16.13% to 132.17% as sample size *K* increases from 20 to 50. For the same reason, the average, minimum, maximum and confidence interval of the estimated objective value  $\hat{\varphi}_{K'}$ , as well as the ratio of saved land, all fall as the sample size increases as shown in Table 3.

In contrast, the heuristic-derived solutions outperform the SAA solutions in terms of the aforementioned indicators, since only poor suboptimum SAA solutions can be obtained in the presence of out-of-memory errors. Therefore, compared with the SAA method, the heuristics is able to not only capture the effects of uncertain factors, but also remain computationally solvable within acceptable time limits. Furthermore, the out-of-sample results using the heuristics have a higher estimated objective value than that obtained by using the SAA solutions, as show in Table 3. These findings suggest that for the multiple facilities case with known distributions of uncertain factors, the proposed heuristics is very effective and efficient in tackling the stochastic CCFLPs.

Finally, the deterministic solutions are compared. The estimated land savings provided by the SAA solution is around 0.2 - 0.3% higher than that provided by the deterministic solution (excluding the SAA solution when K = 50). As for the heuristics solution, the estimated land savings is 16.46% higher than that provided by the deterministic solution. This result shows that the stochastic models can increase the land savings achieved compared to the deterministic model, highlighting the importance of considering uncertainties.

### 5. Discussion and conclusion

This paper introduces a Capacitated Centralized Facility Location Problem (CCFLP) for relocation of decentralized facilities. The model considers the realistic behavior of each individual company when the relocation decision depends on operational expenses and uncertain transportation costs. The optimization problem is formulated in terms of a two-stage stochastic programming model aiming at maximizing the total land savings from the relocation under operational uncertainties and other constraints. A heuristics approach is proposed to solve the stochastic model with known distributions of uncertain factors, and its performance is compared to the Sample Average Approximation (SAA) method. Both algorithms have been applied to solve a multiple Multi-Storey Recycling Facility (MSRFs) case of Singapore. The proposed heuristic approach provides a noticeable improvement to the solution quality over the SAA method with reasonable computational effort. The solutions from the deterministic model and the stochastic model have been compared, showing that more lands could be saved by further considering uncertainties in the CCFLP.

There are several possible avenues for future research on the CCFLP, e.g., extending the heuristic approach to general distribution cases, testing various other algorithms for comparison. Besides, the trade-offs between economic feasibility and land savings can be considered in the model.

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