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# State Space Estimation Method for Robot Identification

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**Abstract:** In this paper, we study the identification of robot dynamic models. The usual technique, based on the Least-Squares method, is carefully detailed. A new procedure based on Kalman filtering and fixed interval smoothing is developed. This new technique is compared to usual one with simulated and experimental data. The obtained results show that the proposed technique is a credible alternative, especially if the system bandwidth is unknown.

**Keywords:** Robots identification; System identification; Closed-loop identification; Least-squares identification; Parameter identification; Kalman filters; State observers

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## 1. INTRODUCTION

The usual method for robot identification is based on the Least-Squares (LS) technique and the Inverse Dynamic Identification Model (IDIM). The IDIM indeed allows expressing the input torque as a linear function of the physical parameters thanks to the modified Denavit and Hartenberg (DHM) notation. Therefore, the IDIM-LS method is a really practical solution, which explains its success, see (Gautier, Janot & Vandanjon 2013) and the references given therein. However this method needs a well-tuned band pass filtering to get the derivatives of the joint positions. It requires a good a priori knowledge of the system to tune adequately the filters. That may be an issue for the early tests of a system, especially if there is no access to the key design parameters, such as a robot bought "off-the-shelf".

The goal of this article is twofold: first, to make clear the usual process of robot identification for people not related to this field; second, to show how this process can be improved. Robot identification may indeed be difficult for people coming from the general field of system identification, since the techniques rely on a priori knowledge of the system. For this work, the author designates by "a priori knowledge" the values of the parameters, which are known or guessed prior to the identification. In any case, the model structure is assumed to be known.

As it will be seen, the main part of the work consists in differentiating the position signal to construct the regressors (see Section 3 for a proper definition) for the LS method. In many fields, the problem of differentiating numerical signals was raised. In the domain of continuous-time system identification, it has been successfully dealt by different

techniques like the generalized Poisson moment functional (GPMF) in (Rao & Unbehauen 2006), the State Variable Filters (SVF) in (Mahata & Garnier 2006) or the Refined Instrumental Variable (RIV) in (Garnier et al. 2007). For further reading on the topic, see e.g. (Garnier, Mensler & Richard 2003). Nevertheless, those attractive methods require either the system to be linear in the states, in order to have a self-tuned filtering (RIV), or the user to provide the bandwidth for the filter (GPMF and SVF). As it will be seen, for a robot, the regressors are non-linear in the states. Hence, those techniques do not fulfil the requirements of our study. It would be worth to look at other fields to find a technique which does not require a priori knowledge of the system and which can handle non-linearities in the states.

The plan of this article is as follows. Firstly, the tools and methods considered are presented. Secondly, the results in simulation of numerical differentiation and parameters identification are summarized. Afterwards, the techniques are compared with experimental data. Then, two cases are considered: first, high precision position sensor is used; second, the precision is deteriorated. Finally, concluding remarks are expressed.

## 2. CLOSED-LOOP SYSTEM IDENTIFICATION

Traditionally, the closed-loop identification methods are divided in three main categories, see e.g. (Forssell & Ljung 1999). The first one, called direct approach, consists in identifying the open-loop system without taking into account the feedback loop. As it will be seen, it requires a careful process of the data to avoid biased estimation. The second category is the indirect approach. In this case, the knowledge of controller, or at least of the reference signal, is required to

identify the closed-loop system. The last category is the joint input-output approach, which consists in using open-loop techniques by considering at the same time the input and the output as an augmented output of the whole closed-loop system.

As it will be presented in the next section, robot identification usually relies on IDIM-LS and belongs to the direct approach. Recently, the Instrumental Variable method has proven to be interesting improvement, see e.g. (Janot, Vandanjon & Gautier 2014) or (Brunot et al. 2015). This last method identifies the open-loop system but it relies on the simulation of the whole closed-loop system. This article focuses on direct approach methods in order to deal with robots whose the controller may be unknown.

### 3. LEAST-SQUARES for ROBOT IDENTIFICATION

#### 3.1 Inverse Dynamic Model

If a robot with  $n$  moving links is considered, the  $(n \times 1)$  vector  $\boldsymbol{\tau}(t)$  contains the inputs of those links, which are the applied forces or torques. The signals  $\mathbf{q}(t)$ ,  $\dot{\mathbf{q}}(t)$  and  $\ddot{\mathbf{q}}(t)$  are respectively the  $(n \times 1)$  vectors of generalized joint positions, velocities and accelerations. With respect to the Newton's second law it comes out:

$$\mathbf{M}(\mathbf{q}(t))\ddot{\mathbf{q}}(t) = \boldsymbol{\tau}(t) - \mathbf{N}(\mathbf{q}(t), \dot{\mathbf{q}}(t)) \quad (1)$$

where,  $\mathbf{M}(\mathbf{q}(t))$  is the  $(n \times n)$  inertia matrix of the robot, and  $\mathbf{N}(\mathbf{q}(t), \dot{\mathbf{q}}(t))$  is the  $(n \times 1)$  vector modelling the disturbances or perturbations. Those perturbations contain the friction forces, gravity effects and other non-linearities depending on the studied robot. Experience has shown that those disturbances are, in the vast majority of cases, linear in the parameters, but not in the states. Therefore, it appears to be very convenient for the identification to consider the Inverse Dynamic Model (IDM). The IDM is described by (2), where: the input is the dependent (or observation) variable;  $\boldsymbol{\phi}$  is the  $(n \times n_\theta)$  matrix of regressors (or independent variables);  $\boldsymbol{\theta}$  is the  $(n_\theta \times 1)$  vector of dynamic parameters to be estimated.

$$\boldsymbol{\tau}(t) = \boldsymbol{\phi}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \ddot{\mathbf{q}}(t))\boldsymbol{\theta} \quad (2)$$

#### 3.2 Least-Squares Equation

The model described by (2) can straightforwardly be extended to the vector-matrix form:

$$\mathbf{u}_m = \begin{bmatrix} \boldsymbol{\tau}(t_1) \\ \vdots \\ \boldsymbol{\tau}(N_s) \end{bmatrix} = \mathbf{X}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\theta} + \mathbf{e}_{LS} \quad (3)$$

where,  $\mathbf{u}_m$  is a  $(N_t \times 1)$  vector constructed with the measured signals,  $\mathbf{X}$  is a  $(N_t \times n_\theta)$  matrix whose each column is called a regressor and  $\mathbf{e}_{LS}$  is a  $(N_t \times 1)$  vector of error terms, with  $N_t = N_s n$  and  $N_s$  the number of sampled points

considered. It is assumed that  $\mathbf{X}$  is full rank, i.e.  $\text{rank}(\mathbf{X}) = n_\theta$ , and that  $N_t \gg n_\theta$ , to have an over-determined system of equations.

From (3), the Least-Squares (LS) estimates and their associated covariance matrix are given by:

$$\hat{\boldsymbol{\theta}}_{LS} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{u}_m \quad (4)$$

$$\boldsymbol{\Sigma}_{LS} = \sigma^2 (\mathbf{X}^T \mathbf{X})^{-1} \quad (5)$$

$$\hat{\sigma}^2 = \frac{1}{N_t - n_\theta} \left\| \mathbf{u}_m - \mathbf{X} \hat{\boldsymbol{\theta}}_{LS} \right\|^2 \quad (6)$$

From a theoretical point of view, the LS estimates (4) are unbiased if the error has a zero mean and if the regressors are uncorrelated with the error, see relations (7).

$$E[\mathbf{e}_{LS}] = 0 \quad E[\mathbf{X}^T \mathbf{e}_{LS}] = E[\mathbf{X}^T] E[\mathbf{e}_{LS}] = 0 \quad (7)$$

The covariance matrix given by (5) assumes that  $\mathbf{X}$  is deterministic and that  $\mathbf{e}_{LS}$  is homoscedastic i.e.  $\text{var}(\mathbf{e}_{LS}(t)) = \sigma^2$ , for each  $t$ . It is assumed that those two assumptions hold. However, systems considered in this article operate in closed-loop. In that case, the assumption given by (7) does not hold (Van den Hof 1998). This partly explains why a tailor-made pre-filtering of the data is done in practice.

#### 3.3 States Estimation by Tailor-Made Filtering

To build the regressors matrix  $\mathbf{X}$ , the velocity and the acceleration are estimated from the measured position. As described in (Gautier 1997), the classical technique used in robots identification is divided in three sequential steps. Those steps are influenced by the sampling frequency, noted  $\omega_s$ . This frequency is usually chosen 100 times larger than the natural frequency of the highest mode which must be modelled,  $\omega_{dyn} = \omega_s / 100$ , in order to satisfy the Nyquist rule.

*Step 1.* The first step consists in reconstructing the missing data, or, more practically, to compute the derivatives of the measured position. It is usually done thanks to numerical differentiation (centred scheme). Prior to this, to avoid amplification of the noise at high frequency, a low-pass filtering is undertaken. This filter is applied forward and backward to avoid phase lag introduction. It is a Butterworth filter, whose order is  $n_d + 2$ . Where  $n_d$  is the desired derivative order, which is usually equal to two. The issue is to choose the cutting frequency of the filter,  $\omega_q$ , to have  $\hat{q}(t) = \dot{q}(t)$  and  $\hat{\hat{q}}(t) = \ddot{q}(t)$  over the frequency range of the system. The rule of thumb is to take it as  $2\omega_{dyn} \leq \omega_q \leq 10\omega_{dyn}$ . It obviously requires knowledge about the system.

*Step 2.* A filter is then applied to all signals. The objective is to remove high frequencies perturbations in the dependent variable measurements (generally, the input torque). To be

consistent, this filter is also applied to the independent variables. Its cut-off frequency,  $\omega_f$ , is chosen at about  $\omega_f \approx 5\omega_{dyn}$ .

*Step 3.* After the previous step, the signals do not contain any information above  $\omega_f$ . Therefore, they are re-sampled at a lower frequency (down-sampling). This frequency is usually taken equal to  $\omega_f$ .

In practice, three elements are worth noting. First, the filters frequencies may be defined taking into account the excitation signal spectrum instead of  $\omega_{dyn}$ . It allows being a little more "aggressive" on the filtering. The second element is that, with MATLAB®, the two last steps are performed simultaneously with the decimate function. The last element is that the described methodology is a rule of thumb. It only provides approximate relation or intervals. The choice relies on the practitioner skills. This is why another way is investigated for users without solid background in robotic identification.

#### 4. KALMAN OBSERVER and INTEGRATED RANDOM WALK

##### 4.1 The State Space Model: IRW

Many researches are on-going concerning the numerical differentiation issue; see e.g. (Dridi et al. 2010). Our goal is to suggest a practical and straightforward technique. Therefore, the study will focus on the well-known Kalman observer technique, in a discrete time framework. This technique is developed in (Young 2000) and (Norton 1975). Equation (8) defines the state vector of state space model, (9) is the state equation and (10) is the observation equation. Considering our robot velocity estimation,  $y$  would be the measured position of link  $j$ ,  $\mathbf{q}_j$ .

$$\mathbf{x}(k) = \begin{bmatrix} x(k) \\ \Delta x(k) \end{bmatrix} \quad (8)$$

$$\mathbf{x}(k) = \mathbf{A}\mathbf{x}(k-1) + \mathbf{D}\boldsymbol{\eta}(k-1) \quad (9)$$

$$y(k) = \mathbf{h}(k)\mathbf{x}(k) + e(k) \quad (10)$$

With,

$$\mathbf{A} = \begin{bmatrix} \alpha & \beta \\ 0 & \gamma \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \delta & 0 \\ 0 & \kappa \end{bmatrix}. \quad (11)$$

$\mathbf{h}$  is the row observation vector.  $\boldsymbol{\eta}$  is the state noise, assumed to be white and zero mean, with covariance matrix  $\mathbf{Q}_\eta$  (diagonal). The measurement noise  $e(k)$  is also zero mean and white. Its covariance is written  $\sigma^2$ . This model, developed in (Young 2011), is named Generalized Random Walk (GRW). Many variants exist depending on the choice of the hyper-parameters  $[\alpha \ \beta \ \gamma \ \delta \ \kappa \ \mathbf{Q}_{\eta_1} \ \mathbf{Q}_{\eta_2}]$ . For this study, only the *Integrated Random Walk* (IRW:  $\alpha = \beta = \gamma = \kappa = 1$ ,  $\delta = 0$  and  $\mathbf{h} = [1 \ 0]$ ) will be

considered. In that case, since  $\delta = 0$ , the term  $\mathbf{Q}_{\eta_1}$  has no influence. Therefore, it will be equal to  $\mathbf{Q}_{\eta_2}$  in order to preserve the definite-positive property of the covariance matrix. Finally, the only remaining hyper-parameter is  $\mathbf{Q}_{\eta_2}$ . As it will be seen later, its value may be estimated thanks to a Maximum Likelihood (ML) optimization.

##### 4.2 The Kalman and FIS Equations

From the model previously described, a specific Kalman filter is implemented. First of all, it is associated with a Fixed Interval Smoother (FIS) to take advantage of the off-line process. Secondly, the filter and smoother equations are modified to avoid the knowledge of the observation noise variance,  $\sigma^2$ . In a classical Kalman Filter, this information is required like the covariance of the state noise,  $\mathbf{Q}_\eta$ . Instead, all the equations are written as functions of the *Noise Variance Ratio* (NVR), which is defined by  $\mathbf{Q}_{nvr} = \mathbf{Q}_\eta / \sigma^2$ . The algorithm described in (Young 2000) and (Young 2011) is summarized below.

*Prediction step:*

$$\hat{\mathbf{x}}(k | k-1) = \mathbf{A}\hat{\mathbf{x}}(k-1 | k-1) \quad (12)$$

$$\mathbf{P}(k | k-1) = \mathbf{A}\mathbf{P}(k-1 | k-1)\mathbf{A}^T + \mathbf{D}\mathbf{Q}_{nvr}\mathbf{D}^T \quad (13)$$

*Correction step:*

$$\hat{\mathbf{x}}(k | k) = \hat{\mathbf{x}}(k | k-1) + \mathbf{g}(k)[y(k) - \mathbf{h}(k)\hat{\mathbf{x}}(k | k-1)] \quad (14)$$

$$\mathbf{g}(k) = \mathbf{P}(k | k-1)\mathbf{h}(k) \left[ \mathbf{1} + \mathbf{h}(k)\mathbf{P}(k | k-1)\mathbf{h}^T(k) \right]^{-1} \quad (15)$$

$$\mathbf{P}(k | k) = \mathbf{P}(k | k-1) - \mathbf{g}(k)\mathbf{h}(k)\mathbf{P}(k | k-1) \quad (16)$$

$$\mathbf{P}^*(k | k) = \hat{\sigma}^2 \mathbf{P}(k | k) \quad (17)$$

*Smoothing step:*

$$\hat{\mathbf{x}}(k | N) = \mathbf{A}^{-1} \left[ \hat{\mathbf{x}}(k+1 | N) + \mathbf{D}\mathbf{Q}_\eta \mathbf{D}^T \boldsymbol{\lambda}(k) \right] \quad (18)$$

$$\boldsymbol{\lambda}(k-1) = \begin{bmatrix} \mathbf{I} - \mathbf{P}^*(k | k) \frac{\mathbf{h}^T(k)\mathbf{h}(k)}{\hat{\sigma}^2} \\ \left( \mathbf{A}^T \boldsymbol{\lambda}(k) - \frac{\mathbf{h}^T(k)}{\hat{\sigma}^2} [y(k) - \mathbf{h}(k)\mathbf{A}\hat{\mathbf{x}}(k-1 | k-1)] \right) \end{bmatrix} \quad (19)$$

with  $\boldsymbol{\lambda}(N) = [0 \ 0]^T$

$$\mathbf{P}^*(k | N) = \mathbf{P}^*(k | k) + \mathbf{P}^*(k | k)\mathbf{A}^T \mathbf{P}^*(k+1 | k)^{-1} \left[ \mathbf{P}^*(k+1 | N) - \mathbf{P}^*(k+1 | k) \right] \mathbf{P}^*(k+1 | k)^{-1} \mathbf{A} \mathbf{P}^*(k | k) \quad (20)$$

The observation noise covariance,  $\sigma^2$ , is estimated at the end of the filtering process in order to obtain the state covariance matrix,  $\mathbf{P}^*$ , for the smoothing process. By defining  $n_x$  the size of the state vector ( $n_x = 2$  for the IRW), the estimation is given by:

$$\begin{aligned}\hat{\sigma}^2 &= \frac{1}{N - n_x} \sum_{k=n_x+1}^{N_x} \frac{(y(k) - \mathbf{h}(k)\hat{\mathbf{x}}(k|k-1))^2}{1 + \mathbf{h}(k)\mathbf{P}(k|k-1)\mathbf{h}^T(k)} \\ &= \frac{1}{N - n_x} \sum_{k=n_x+1}^{N_x} \frac{\varepsilon^2(k)}{v(k)}\end{aligned}\quad (21)$$

In the time domain, the first order derivative of the signal is then approximated as follows  $\frac{dx}{dt}(t_k) \approx \frac{\Delta x(k)}{t_{k+1} - t_k}$ , with  $\Delta x(k)$  the second term of the estimated state vector  $\hat{\mathbf{x}}(k|N)$ . Similarly,  $\mathbf{x}$  could be augmented with  $\Delta^2 x$  in order to estimate the second order derivative. From a practical point of view, this algorithm is implemented in the function *irwsm* of the CAPTAIN Toolbox, developed by a team of Lancaster University.

### 4.3 Hyper-Parameters Optimization

As it has been said, the user does not have to provide the observation noise covariance to *irwsm* contrary to a classical Kalman filter. It remains the issue of the hyper-parameters and more specifically of the NVR. Fortunately, the CAPTAIN Toolbox provides also a function called *irwsmopt* which estimates the hyper-parameters maximizing the likelihood of the prediction error,  $\varepsilon(k)$ , defined in (21). For further information, see e.g. (Durbin & Koopman 2012). This toolbox thus allows the user to process the data from a system without a priori knowledge about it. Obviously, it does not prevent him to be vigilant on the results.

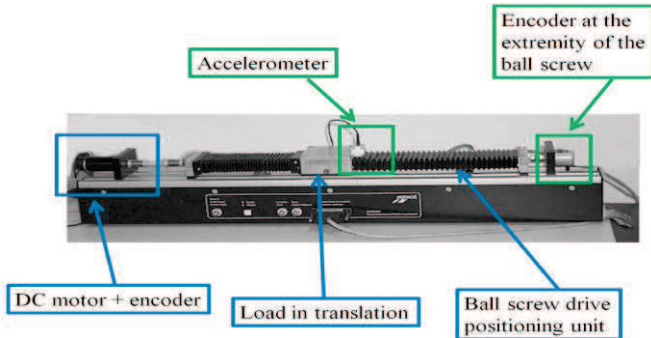


Fig. 1. EMPS prototype

## 5. SIMULATION RESULTS

### 5.1 Model Description

To evaluate the differentiation techniques, the rigid one-degree-of-freedom Electro-Mechanical Positioning System (EMPS) is studied. This experimental system (Fig. 1) is modelled by (22), where  $M$  is the inertia of the arm;  $F_v$  and  $F_c$  are respectively the viscous and Coulomb frictions;  $q$ ,  $\dot{q}$  and  $\ddot{q}$  are respectively the position, velocity and acceleration;  $\tau$  is the motor force. The system is driven by controller (23), where  $q_r$  is the reference trajectory and  $g_f$  is the electronic gain of the actuator. The bandwidth of the

electronic part of the system is larger than the one of the mechanical part. Therefore, it can accurately be approximated by a constant. By noting the differential operator  $p = \frac{d}{dt}$ , the closed-loop system can be represented by Fig. 2 where  $d(t) = F_v \dot{q}(t) + F_c \text{sign}(\dot{q}(t))$  is the nonlinear state dependant perturbation.

$$\tau(t) = M\ddot{q}(t) + F_v \dot{q}(t) + F_c \text{sign}(\dot{q}(t)) \quad (22)$$

$$\tau(t) = g_f k_v (k_p (q_r(t) - q(t)) - \dot{q}(t)) \quad (23)$$

In practice, the controller gains have been chosen equal to 160.2 (1/s) for  $k_p$  and 242.5 (V.s/m) for  $k_v$ . The actuator gain has previously been identified to 35.2 (N/V).

In first time, the study is done thanks to simulated data. The values of parameters are chosen close to those which have previously been estimated for other works (see "True Values" in Table 2). A measurement noise is added to the simulated output. To be realistic, the Signal to Noise Ratio (SNR) is taken equal to 100dB. Mathematically, the SNR is defined by (24), where  $en$  is the energy of the signal;  $q_{meas}$  and  $q_{nf}$  are respectively the measured and the noise-free signals. 100 Monte Carlo Simulations (MCS) are run to evaluate the effect of this noise over the estimates.

$$SNR_q = 10 \log_{10} \left( \frac{en_{q_{nf}}}{en_{q_{meas} - q_{nf}}} \right) \quad (24)$$

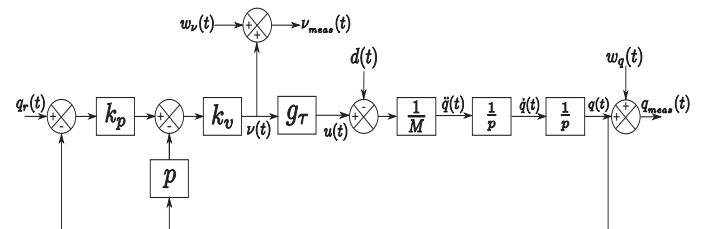


Fig. 2. Closed-loop block-diagram for the EMPS prototype

### 5.2 Signals and Parameters Estimation Results

From the noisy position signal generated by the simulator, three methods are compared. The first one is the classical approach, with Butterworth filters, described in Section 3.3 and will be named by "Classical". The second method is the *irwsm* implemented in the CAPTAIN Toolbox, described in Section 4.2, and will be referred as "IRWSM 1". The last one is a variant of the *irwsm* where the GRW model contains three states, which allows estimating directly the second derivative without calling the algorithm twice. This approach will be named "IRWSM 2".

Concerning the Classical method, (25) is the relation between the reference signal and the output, by neglecting the Coulomb friction. From this relation, the highest mode frequency is about 19 Hz. It should be noticed that this relation requires knowing the controller parameters and structure. The Butterworth filter is designed with a cut-off frequency equal to 40 Hz (i.e. twice the highest natural

mode). To get the acceleration, its order is fixed at 2+2=4. At last, the decimate frequency is chosen at 60 Hz.

$$q(t) = \frac{g_f k_p k_v}{Mp^2 + (F_v + g_f k_v)p + g_f k_p k_v} q_{ref}(t) \quad (25)$$

Table 1 summarizes the results by providing the mean of the 100 relative errors for each estimate. The relative error of the signal  $s(t)$  is given by (26), where  $s_{nf}(t)$  is the noise free component of  $s(t)$  generated by the simulator. IRWSM 1 and 2 give very good results for the velocity and the acceleration, since the relative errors are very small and less than those of the classical approach. The methods seem to be equivalently effective in order to estimate the position.

$$\text{RelErr}(s) = 100 \frac{\|s_{nf} - \hat{s}\|}{\|s_{nf}\|} \quad (26)$$

To conclude this simulation part, a LS estimation of the parameters was undertaken to compare their performances. Table 2 presents the results. The performances of the IRWSM methods seem to be better than those of the Classical. Actually, their mean estimated parameters are closer to the real one. That must be confirmed, or not, with the experimentation.

**Table 1. Mean relative errors of the estimated signals for 100 Monte Carlo Simulations**

Signal	Method		
	Classical	IRWSM 1	IRWSM 2
$q$	$6.38 \cdot 10^{-5} \%$	$5.78 \cdot 10^{-5} \%$	$5.10 \cdot 10^{-5} \%$
$\dot{q}$	$5.49 \cdot 10^{-1} \%$	$1.20 \cdot 10^{-3} \%$	$3.20 \cdot 10^{-3} \%$
$\ddot{q}$	$1.15 \cdot 10^0 \%$	$1.34 \cdot 10^{-1} \%$	$2.64 \cdot 10^{-1} \%$

**Table 2. Mean estimated parameters (relative error) for 100 Monte Carlo Simulations**

Parameter	True Values	Method		
		Classical	IRWSM 1	IRWSM 2
$M$ (kg)	96.00	96.11 (0.12%)	95.99 (0.01%)	95.88 (0.13%)
$F_v$ (N/(m/s))	205.00	190.35 (7.15%)	205.36 (0.18%)	205.12 (0.06%)
$F_c$ (N)	20.00	22.44 (12.2%)	19.71 (1.45%)	19.91 (0.47%)

## 6. EXPERIMENTAL RESULTS

### 6.1 Robot Identification with Good a priori Knowledge and High Precision Sensor

The model of the experimental setup is the same than the one previously considered for the simulation. Concerning the controller, its structure and its parameters are exactly the same. The sampling frequency of 1 kHz is also identical. Table 3 summarizes the results of the identification from the experimental data. The IRWSM methods almost estimate the

same parameters. For the experimental results, the relative error (26) is defined with respect to the measured signal instead of the noise free one. The relative errors can be considered as equivalent for the three methods.

**Table 3. Experimental with high precision sensor- Estimated parameters (relative standard deviation)**

Parameter	Method		
	Classical	IRWSM 1	IRWSM 2
$M$ (kg)	95.12 (0.11%)	94.87 (0.04%)	94.48 (0.06%)
$F_v$ (N/(m/s))	203.54 (0.56%)	212.96 (0.21%)	212.95 (0.28%)
$F_c$ (N)	20.39 (0.49%)	19.67 (0.20%)	19.67 (0.27%)
Relative Error	4.04%	4.53%	4.72%

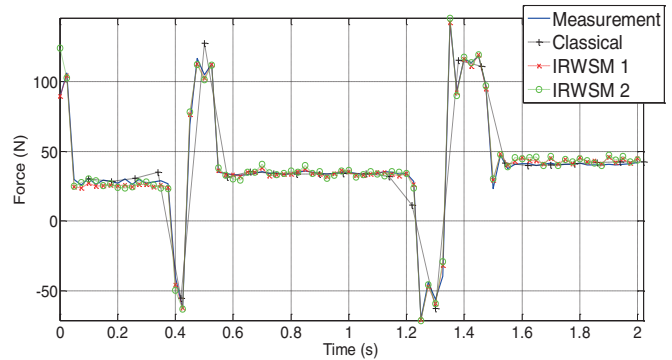


Fig. 3. Estimated signals

Fig. 3 illustrates the estimated signals thanks to the three methods. There is no clear difference between the IRWSM 1 and the IRWSM 2 estimates, which explains why the relative errors of both methods are equivalent. Thus, it is still difficult to discriminate those methods. To do so, a case more representative of an industrial robot will be considered. That is to say that the sensor (the encoder) will be less precise:  $\pm 0.1$  deg of precision against  $\pm 0.01$  deg previously. Furthermore, we will not assume good a priori knowledge on the system. The filtering cut-off frequencies will be multiplied by three.

### 6.2 Robot Identification with Poor a priori Knowledge and Low Precision Sensor

The results of the identification with a low precision sensor and poor a priori knowledge are summarized in Table 4. This table includes the values and the relative standard deviation of the estimated parameters as well as the relative errors. The difficulty of the IRWSM 1 method is flagrant. Its estimated values are far from those previously estimated and its error is large. On the contrary, the Classical and IRWSM 2 methods find estimates relatively close to the previous results (i.e. with high precision sensor). Compared to high precision case, the Classical method performances are slightly worsened with respect to the mass estimation and the error.

One fact is worth noting about this low precision case. In this case, the *irwsmopt* algorithm indeed tends to catch all the dynamic of the noisy signal (large NVR). In other words, it gives too much importance to the covariance of the state

noise compared to the one of the measurement noise. A careful visual inspection of the signals, prior to the identification, by the user is therefore required. For the present case, a NVR equal to  $10^{-5}$  has proved to be an appropriate choice, as it can be seen in Fig. 4. That figure illustrates the estimated velocity and acceleration by IRWSM 1, with low precision sensor, thanks to the *irwsmopt* function (blue) and with the SNR manually found (red). This value of NVR was fixed for both IRWSM methods presented in this section.

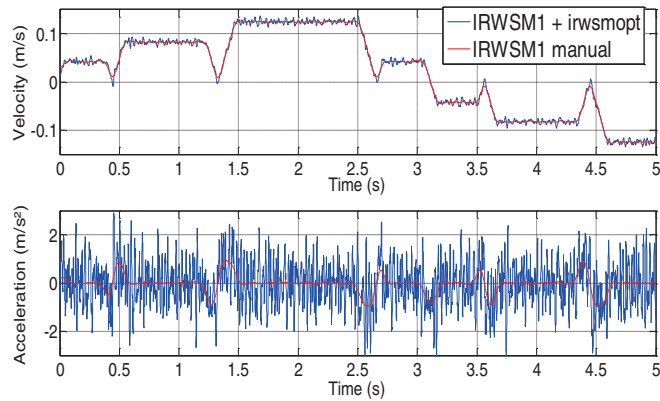


Fig. 4. Impact of the NVR on the estimated signals

Finally, this more realistic experiment did not totally confirm the simulation results. If the IRWSM 2 method provides better estimates than the classical one, it requires the user intervention to set the NVR. Nonetheless, IRWSM methods proved to be able to provide a valuable estimation of the parameters. This estimation can be used as a first step for the design of pre-filters for the Classical method. In practice, the IRWSM 2 solution should be preferred.

**Table 4. Experimental with low precision sensor – Estimated parameters (relative standard deviation)**

Parameter	Method		
	Classical	IRWSM 1	IRWSM 2
$M$ (kg)	89.52 (0.16%)	104.26 (0.01%)	95.99 (0.03%)
$F_v$ (N/(m/s))	204.50 (0.17%)	209.88 (0.23%)	213.34 (0.08%)
$F_c$ (N)	20.29 (0.16%)	19.98 (0.02%)	19.63 (0.08%)
Relative Error	18.00%	32.67%	8.95%

## 7. CONCLUSION

In this paper the usual robot identification methodology is presented. It is based on the well-known Least-Squares method but it requires a careful tailor-made pre-filtering to deal with closed-loop issues. This tailor-made pre-filtering process is summarized and a new pre-filtering methodology is developed. That one is based on a combination of a Kalman filter and a fixed interval smoother. The obtained results suggest that the new method is a suitable alternative when the system bandwidth is not known prior to the identification. Future work will focus on more complex industrial robots with multiple degrees of freedom.

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