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1 Shallow water sound source localization using the
2 iterative beamforming method in an image framework

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5 **Abstract**

6 Shallow water is a complicated sound propagation medium due to mul-
7 tiple reflections by water surface and bottom, imprecisely measured sound
8 speed, noisy environment, etc. Therefore, in order to localize a shallow wa-
9 ter sound source, classical signal processing techniques must be improved by
10 taking these complexities into account. In this work, the multiple reflections
11 and uncertain reflectivity of water bottom are explicitly modeled. In the pro-
12 posed model, a measured signal is a mixture of the direct propagation from
13 the source and the multiple reflections. Instead of solving the Helmholtz
14 equation with boundary conditions of reflections, each signal is interpreted
15 as a superposition of signals emitting from the physical source and its image
16 sources in a free space, which results in a fast computation of sound propa-
17 gation. Then, the source location, along with its amplitude, reflection paths
18 and power loss of bottom reflection, is estimated via the iterative beamform-
19 ing (IB) method, which alternatively estimates the source contributions and
20 performs beamforming on these estimates until convergence. This approach
21 does not need to compute the sound propagation for all the possible source
22 locations in a large space, which thus leads to a low computational cost. Fi-
23 nally, numerical simulations are introduced to illustrate the advantage of the
24 proposed model and the source estimation method. The sensitivity of the
25 proposed method with respect to model parameter uncertainties is also inves-
26 tigated via a full uncertainty quantification analysis. The localization error
27 of IB is proved to be acceptable in the given error range of sound speed and
28 water depth. Besides, the IB source estimate is more sensitive to the sound
29 speed while the matched-field processing methods have a stronger sensitivity
30 to the water depth: this result can guide the choice of source localization
31 method in the different cases of model parameter uncertainties.

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32 *Keywords:* sound source localization, shallow water, iterative
33 beamforming, multiple reflections, uncertainty quantification, acoustic
34 imaging.

35 1. Introduction

36 Sound source localization in shallow water is a challenging issue due to the
37 complexity of sound propagation environment and the difficulty of acoustical
38 measurements. Contrary to the ideal assumption in the classical source lo-
39 calization problem, experimental results obtained in shallow water acoustics
40 are always submitted to different kinds of uncertainties [1, 2]. Inhomoge-
41 neous and random sound speed profile [3–7], imprecise locations of sensors
42 [8–10], sound reflections by ships, water surface and bottom [11, 12], noisy
43 environment (due to shipping lanes for example) and presence of internal
44 waves [11, 13, 14] are all sources of uncertainties which may dramatically
45 affect the estimation result of sound source. Due to the complicated nature
46 of the underwater environment, until now no signal processing method for
47 source localization can be employed to eliminate all the bad effects from the
48 aforementioned uncertainties. In this study, a model quantifying the mul-
49 tiple reflections between the water surface and bottom is proposed. The
50 reflectivity of water bottom is assumed to be unknown.

51 By posing the boundary conditions at the water surface and bottom,
52 the depth-dependent function of sound wave in a Pekeris waveguide can be
53 analytically solved [2]. However, in order to compute the Helmholtz equation,
54 numerical methods, e.g., wavenumber integration [15] or normal modes [16],
55 have to be used. These approaches involve a low computational cost but
56 is not negligible when a very large number of sound propagation has to be
57 calculated, which is generally the case of source localization. Alternatively,
58 this paper considers an image source approach [2, 11, 17, 18], in which the
59 reflections between the two boundaries can be described by the images of
60 the physical source. In this case, the boundary conditions are not needed
61 and the sound propagation can be considered in a free-field. Furthermore,
62 the unknown loss of bottom reflection, which depends on the water bottom
63 properties, is also quantified in this image source model via the amplitudes of
64 the image sources. Finally, by identifying the multiple sources (the physical
65 source and its images) in the free-field, the sound source in the shallow water
66 environment can be localized. The reflectivity of water bottom can also be
67 decided via the estimated amplitudes of the sources.

68 Near-field acoustical holography (NAH), time reversal and beamforming
69 are the most widely used signal processing techniques dedicated to the sound
70 source localization problem. NAH [19] back-propagates the sound field from
71 the measurement plane to a surface near the sound source, which guarantees
72 a high resolution by taking evanescent waves into account. However, this
73 approach only works for the near-field source. The time reversal method
74 [20–24] inverses the measured signal in time and reinject it back into the
75 same medium. This approach is able to refocus the source and return a
76 super-resolution result in a medium with multiple reflections, scattering and
77 refractions [23]. However, time reversal is sensitive to the model uncertainty:
78 if the environmental parameters (e.g., sound speed profile) were imprecisely
79 known, it could not even return a source estimate. Classical beamforming
80 (CB) [25–27] estimates the direction of arrival of plane wave or the location
81 of point source via the delay of signal arrival. However, CB is still limited in
82 the frequency range and minimum resolvable source separation, particularly
83 for the underwater case which has a relatively small size of microphone ar-
84 ray and long sound propagation distance. Matched-field processing (MFP)
85 [3–5, 28, 29] is a generalized beamforming method which takes the sound
86 field complexity into account. Conventional and Capon’s [30] approaches
87 are the most widely used MFP methods, the former is more stable with re-
88 spect to the model parameter uncertainties while the latter returns a super-
89 resolution source localization. Furthermore, the MFP approaches can not
90 only localize the sound source but also estimate the parameters related to
91 the sound propagation environment. However, MFP needs to discretize the
92 space of source location and other unknown parameters which often involves
93 a large number of sound field computation. Iterative beamforming [10, 31]
94 is a maximum likelihood (ML) approach for multiple sources, based on the
95 expectation-maximization (EM) algorithm [32, 33]. This method assumes
96 that each acoustical measurement is a sum of latent signals emitted from the
97 multiple sources. Given an initial value of source parameters, IB iteratively
98 computes the latent signals (source contributions) using the current fit of
99 parameters and then performs beamforming on these source contributions to
100 estimate the corresponding source location and amplitude. The theory of the
101 EM algorithm guarantees the increase of likelihood function of model param-
102 eter after each iteration and the final result is a maximum likelihood estimate
103 (MLE) of the sound sources [32]. This parametric mechanism of IB is able
104 to avoid the limit of spatial resolution and augment the frequency range of
105 CB. In this paper, the IB approach is used to solve the source localization

106 problem in the framework of image sources.

107 Since the underwater environment is complicated and some of the envi-
108 ronmental parameters cannot be precisely measured, the sensitivity of the
109 source localization method with respect to these parameters must be investi-
110 gated. Kriging method [34, 35] is an efficient tool to build response surfaces
111 for surrogate modeling, which is extensively used in uncertainty quantifica-
112 tion (UQ) in mechanics [36, 37] and fluid dynamics [38–40]. In this work, a
113 response surface of the estimated sound source as a function of uncertain pa-
114 rameters (water depth and sound velocity) is built using the Kriging method.
115 Then, a quasi-Monte Carlo sampling of the response surface is used to ob-
116 tain the probability density function (PDF) of the source estimate and to
117 quantify the sensitivities of the source estimation methods with respect to
118 the uncertain environmental parameters.

119 In this paper, the problem of sound source localization in a shallow wa-
120 ter environment is investigated. The sound waves are emitted from a single
121 source and reflected by the water surface and bottom. The sound propa-
122 gation process is considered via the image source model. The single source
123 localization problem is then transferred to a multiple-source estimation prob-
124 lem in a free-field, which is solved via the IB method. The organization of
125 this paper is as follows. Section 2 introduces the proposed image source
126 model. In Section 3, the source estimation methods are briefly introduced
127 first; the algorithm for estimating a shallow water source using IB is then
128 explicitly given. Section 4 presents experimental results on simulated data.
129 In Section 5, the sensitivity of the source localization methods to the uncer-
130 tain parameters of sound speed and water depth is investigated. Finally, the
131 conclusions and perspectives are given in Section 6.

132 **2. Sound propagation in a shallow water environment**

133 *2.1. Model description*

134 In this section, the sound propagation model in a shallow water envi-
135 ronment is presented. It is assumed that the acoustic field is produced by
136 a sound source radiating continuous time signals at $\mathbf{r}_0 = (x_0, y_0, z_0)$. The
137 sound wave propagates through the underwater medium and is measured by
138 an array of sensors whose coordinates are $\mathbf{r}'_m = (x'_m, y'_m, z'_m)$, $m = 1, \dots, M$.
139 Figure 1 shows the problem in 3D space, in which the cross and circles repre-
140 sent the source and microphone locations respectively. By assuming that the

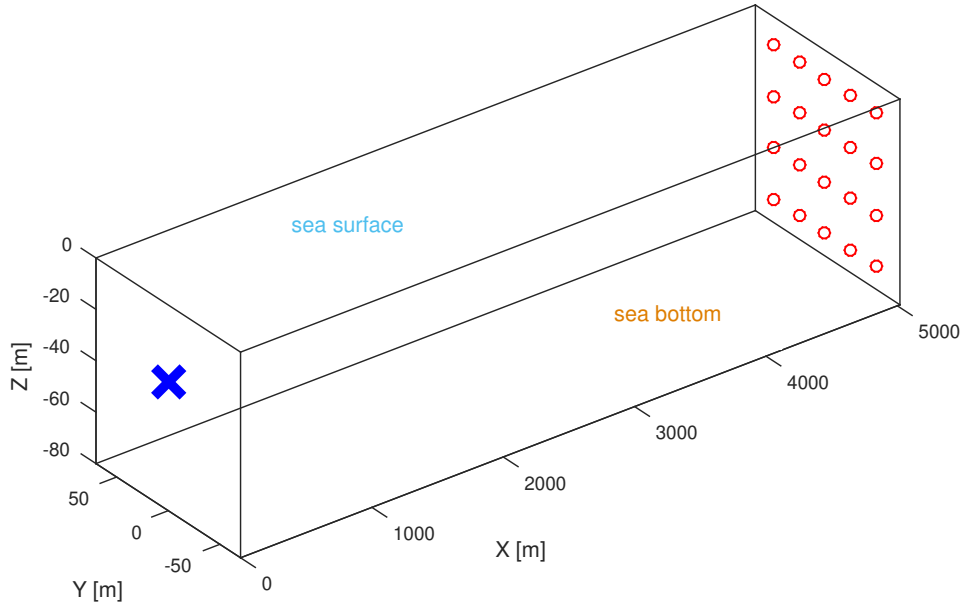


Figure 1: Description of the model in 3D space. The cross and circles represent the source and microphone locations, respectively.

141 medium is sourceless except at \mathbf{r}_0 , the sound pressure $\tilde{p}(\mathbf{r}, t)$ as a function of
 142 spatial coordinate $\mathbf{r} = (x, y, z)$ and time t is governed by the wave equation

$$\left(\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right) \tilde{p}(\mathbf{r}, t) = \delta(\mathbf{r} - \mathbf{r}_0) \tilde{W}(t), \quad (1)$$

143 in which v represents the sound speed, $\tilde{W}(t)$ is a deterministic function of
 144 source signal in the time domain, and δ is the Dirac delta function. Taking
 145 Fourier transform of the both sides of Eq. (1) with respect to t results in
 146 the following Helmholtz equation for the sound field $p(\mathbf{r}, f)$ in the frequency
 147 domain:

$$(\nabla^2 + k^2) p(\mathbf{r}, f) = \delta(\mathbf{r} - \mathbf{r}_0) W(f), \quad (2)$$

148 where $k = 2\pi f/v$ is the wavenumber, f is the frequency, and $W(f)$ is the
 149 Fourier transform of $\tilde{W}(t)$. The solution of the wave equation depends on
 150 the boundary condition. The water surface ($z = 0$) is assumed to be totally
 151 mirror reflective. The bottom ($z = d < 0$) is smooth and has a frequency-
 152 dependent reflectivity (denoted by $\alpha(f) \in [0, 1]$). Moreover, the sound waves
 153 propagating through the bottom are assumed to be fully absorbed by the

154 bottom, i.e., no sound wave reflects back to the considered shallow water
 155 region $\{\mathbf{r} : z \in [d, 0]\}$.

156 2.2. Image source model

157 In this work, the sound wave equation and source localization are consid-
 158 ered in an image source framework. The canonical (Pekeris) shallow water
 159 acoustic waveguide is considered. The sound speed field is assumed as a
 160 constant. The wavelength is much smaller than the water depth and wave
 161 propagation range such that the ray theory and the image method can be
 162 applied. The sound emitted from the source is reflected by the water surface
 163 and bottom. An acoustical measurement obtained by each microphone is
 164 a superposition of contributions from the direct propagation and the reflec-
 165 tions. Figure 2 demonstrates this multiple reflection problem: the solid lines
 166 display the direct propagation (P_0) and the other three paths of reflections
 167 (upper reflection P_1 , lower reflection P_2 and upper lower reflections P_3).

168 Instead of solving the wave equation Eq. (2) with the boundary condition,
 169 the sound pressure at each point \mathbf{r} is seen as a mixture of contributions
 170 propagating from the physical and image sources in a free-field:

$$\begin{aligned}
 p(\mathbf{r}, f) &= \sum_{s=0}^{\infty} (-1)^{n(s)} \alpha(f)^{n^-(s)} A_0 G(\mathbf{r}|\mathbf{r}_s, f) \\
 &= \sum_{s=0}^{\infty} (-1)^{n(s)} \alpha(f)^{n^-(s)} A_0 \frac{e^{jk\|\mathbf{r}-\mathbf{r}_s\|}}{4\pi\|\mathbf{r}-\mathbf{r}_s\|}, \quad (3)
 \end{aligned}$$

in which \mathbf{r}_s ($s \geq 1$) represents the locations of the image sources, A_0 is the amplitude of the physical source \mathbf{r}_0 , $n(s)$ stands for the number of reflections of s -th mode ($(s+1)$ -th term in Eq. (3)), and $n^-(s)$ is the corresponding number of bottom reflections. The coefficient $(-1)^{n(s)}$ corresponds to the sign change after each reflection due to the pressure-release boundary condition and $\alpha^{n^-(s)}$ means that the remaining power after each bottom reflection is $|\alpha A|^2$, where $\alpha \in [0, 1]$ is the reflectivity and A represents the amplitude of incident wave. Note that all the image sources are in the same line $\{\mathbf{r} : x = x_0, y = y_0\}$, as shown in Figure 2. The z -coordinates z_s of the image sources are obtained by successively imaging z_0 (symmetric with respect to the two

boundaries $z = 0$ and $z = d$):

$$z_{4i} = z_0 - 2id, \quad (4a)$$

$$z_{4i+1} = -z_0 - 2id, \quad (4b)$$

$$z_{4i+2} = -z_0 + 2(i+1)d, \quad (4c)$$

$$z_{4i+3} = z_0 + 2(i+1)d, \quad (4d)$$

171 in which $i \in \{0, 1, 2, \dots\}$. Figure 2 shows the locations of the first four
 172 sources, i.e., $i = 0$ in Eq. (4). It is remarkable that the image source locations
 173 \mathbf{r}_s , $s \in \{1, 2, 3, \dots\}$, do not depend on the observation point. Therefore, the
 174 sound pressure $p(\mathbf{r}, f)$ measured at any point \mathbf{r} can be seen as being generated
 175 by the image sources. Under the image source sorting method in Eq. (4), the
 176 corresponding number of reflections and number of bottom reflections are
 177 $n(s) = [(s+1)/2]$ and $n^-(s) = [(s+2)/4]$, in which $[\cdot]$ stands for the floor
 178 function. Therefore, the amplitudes of the image sources are

$$A_s = (-1)^{n(s)} \alpha^{n^-(s)} A_0 = (-1)^{\lfloor \frac{s+1}{2} \rfloor} \alpha^{\lfloor \frac{s+2}{4} \rfloor} A_0, \quad s = 1, 2, \dots \quad (5)$$

179 By considering the measurement noise and ignoring the terms larger than
 180 S (become negligible after several bottom reflections, depending on the de-
 181 sired precision and the reflectivity α), the measured sound pressure from
 182 m -th microphone is

$$p(\mathbf{r}'_m) = \sum_{s=0}^S A_s \frac{e^{jk\|\mathbf{r}'_m - \mathbf{r}_s\|}}{4\pi\|\mathbf{r}'_m - \mathbf{r}_s\|} + e_m, \quad m = 1, \dots, M, \quad (6)$$

183 in which the amplitudes A_s is obtained from Eq. (5) for $s \geq 1$ and the mea-
 184 surement error e_m is assumed to follow a 0-mean complex-valued Gaussian
 185 distribution [41].

186 In the framework of image method, the sound propagation can be analyt-
 187 ically computed, such that the source localization can be rapidly performed,
 188 which is crucial in the source localization problem. In Section 3, acoustic
 189 imaging techniques are employed to visualize all the (physical and image)
 190 sound sources. Since the presence of the two boundaries are known, the real
 191 (physical) sound source can be identified from its images.

192 3. Sound source localization using iterative beamforming

193 In this section, a shallow water sound source is localized. This single
 194 source localization problem with multiple reflections is transferred to a mul-

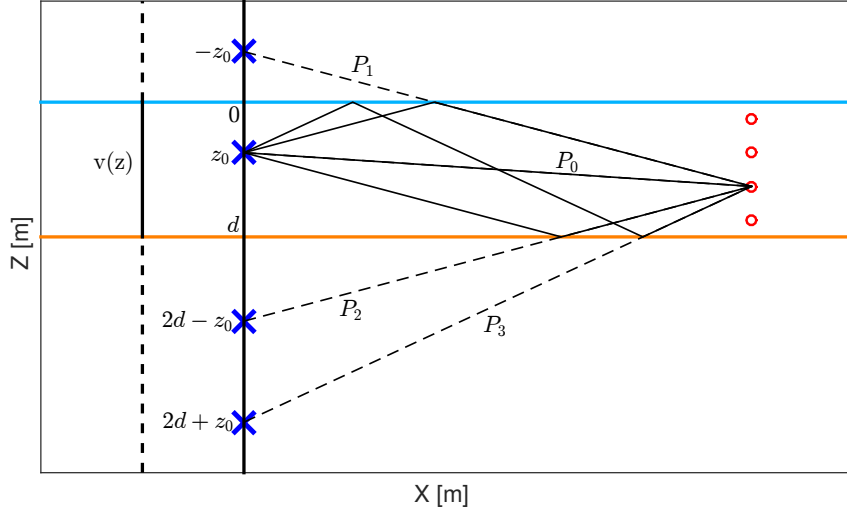


Figure 2: Sound propagation with multiple reflections on the water surface ($z = 0$) and bottom ($z = d$). The crosses and circles represent the (physical and image) sound sources and microphone locations, respectively. The solid and dashed lines stand for the physical and image sound propagation paths.

195 tiple source localization problem in a free-field using the image source ap-
 196 proach, as introduced in Section 2.2. The data used are T snapshots of
 197 sound pressure measurements in the frequency domain, denoted as $\mathbf{p} =$
 198 $(\mathbf{p}_1, \dots, \mathbf{p}_T)$. Each snapshot $\mathbf{p}_t = (p_{1t}, \dots, p_{Mt})^T$ includes the measure-
 199 ments from M microphones, in which $(\cdot)^T$ stands for the operation of vector
 200 transpose.

201 In Sections 3.1 and 3.2, two sound source localization methods (CB and
 202 IB) are introduced. The detailed source localization strategy is given in
 203 Section 3.3. Finally, for comparison in terms of computational cost, the
 204 MFP methods for the single source based on the image source method are
 205 introduced in Section 3.4.

206 3.1. Classical beamforming

207 The CB method [25, 26] is based on the assumption of single source. In
 208 this case, each snapshot of the measurement \mathbf{p}_t reads

$$\mathbf{p}_t = A_0 \mathbf{G}(\mathbf{r}_0) + \mathbf{e}_t, \quad (7)$$

209 in which $\mathbf{G}(\mathbf{r}_0) = (G(\mathbf{r}'_1 | \mathbf{r}_0), \dots, G(\mathbf{r}'_M | \mathbf{r}_0))^T$ is the Green's function vector
 210 describing sound propagation from the source \mathbf{r}_0 to the M microphones and

211 being computed from the Helmholtz equation (2), A_0 is the amplitude of the
 212 sound source, and $\mathbf{e}_t = (e_{1t}, \dots, e_{Mt})^T$ is the vector of measurement noises
 213 which follows an M -dimensional complex Gaussian distribution $\mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_M)$
 214 and \mathbf{I}_M is the M -dimensional identity matrix. Note that \mathbf{p}_t is the sound pres-
 215 sure in the frequency domain, which is obtained from a linear combination of
 216 measurements in the time domain (discrete Fourier transform). Therefore,
 217 the measurements noise vector can be reasonably assumed as Gaussian dis-
 218 tributed regardless of the distribution of the time domain noise according to
 219 the central limit theorem.

220 The CB approach localizes the sound source by “steering” the microphone
 221 array. The steering locations, which result in a maximum power, yield the
 222 beamforming estimate. “Steering the array” is performed by forming a linear
 223 combination of a measurement \mathbf{p}_t :

$$B_t = \mathbf{w}^H \mathbf{p}_t = \sum_{m=1}^M w_m p_{mt}, \quad (8)$$

224 in which $(\cdot)^H$ is the operation of conjugate transpose, $\mathbf{w} = (w_1, \dots, w_M)^T$ is
 225 called the steering vector. The CB method is to estimate the source location
 226 by maximizing $|B_t|^2$ with respect to \mathbf{w} , which finally reduces to maximizing
 227 the classical spatial spectrum

$$S(\mathbf{r}_0) = \frac{\mathbf{G}^H(\mathbf{r}_0) \mathbf{p}_t \mathbf{p}_t^H \mathbf{G}(\mathbf{r}_0)}{\mathbf{G}^H(\mathbf{r}_0) \mathbf{G}(\mathbf{r}_0)}. \quad (9)$$

228 Note that the CB estimate (maximizing Eq. (9)) is a MLE of the source
 229 location \mathbf{r}_0 .

230 In the case of multiple sources, CB solves the problem in the same way
 231 as the single source case. An \mathbf{r}_0 - $S(\mathbf{r}_0)$ color map for all \mathbf{r}_0 in the considered
 232 region may be employed: each local maximum in this color map corresponds
 233 to a source. However, this is not a parametric approach and thus has a spatial
 234 resolution limit, i.e., the sources cannot be separated when they are close to
 235 each other. The side lobes of beamformer may also be wrongly identified
 236 as a source. In Section 3.2, a generalized beamforming method addressing
 237 the case of multiple sources is introduced, which considers the parametric
 238 multiple-source model and is a MLE of the multiple sources.

239 3.2. Iterative beamforming

240 Multiple sound source identification using the iterative beamforming (IB)
 241 approach is introduced in Ref. [10] for deterministic amplitude sources and

242 in Ref. [31] for random amplitude sources. IB is based on a ML approach to
 243 deal with latent variables, known as EM algorithm [32, 33]. In this paper, the
 244 shallow water source and its images are viewed as multiple sources, which can
 245 thus be estimated using the IB method. Here the principle of this method is
 246 briefly reviewed; for an extensive presentation, the readers are invited to refer
 247 to Ref. [10]. The detailed algorithm for shallow water source localization is
 248 introduced in Section 3.3.

249 In the case of $S + 1$ sources, each snapshot of measurement \mathbf{p}_t has the
 250 expression

$$\mathbf{p}_t = \mathbf{G}(\mathbf{r})\mathbf{A} + \mathbf{e}_t. \quad (10)$$

251 In this equation $\mathbf{r} = (\mathbf{r}_0, \dots, \mathbf{r}_S)$ represents the locations of the sources,
 252 $\mathbf{G}(\mathbf{r}) = (G(\mathbf{r}'_m|\mathbf{r}_s))_{m=1, s=0}^{M, S}$ is the $M \times (S + 1)$ Green's function matrix, in
 253 which $G(\mathbf{r}'_m|\mathbf{r}_s)$ describes the sound propagation from s -th source to m -th
 254 microphone, $\mathbf{A} = (A_0, \dots, A_S)^T$ is the vector of source amplitudes, and
 255 $\mathbf{e}_t = (e_{1t}, \dots, e_{Mt})^T$, $t = 1, \dots, T$, are i.i.d. random vectors of measure-
 256 ment noises and follows a complex-valued Gaussian distribution $\mathbb{N}(\mathbf{0}, \sigma^2\mathbf{I}_M)$.
 257 The purpose of source identification is to estimate the source positions \mathbf{r} and
 258 amplitudes \mathbf{A} . The log-likelihood function of the measurements \mathbf{p} (after re-
 259 moving unnecessary terms which are independent of the unknown parameters
 260 and have no effect of the MLE) is

$$\log L(\mathbf{r}, \mathbf{A}|\mathbf{p}) = - \sum_{t=1}^T \|\mathbf{p}_t - \mathbf{G}(\mathbf{r})\mathbf{A}\|^2. \quad (11)$$

261 The MLE of \mathbf{r} and \mathbf{A} is obtained by maximizing Eq. (11). In the case of
 262 single source, i.e., $\mathbf{r} = \mathbf{r}_0$ and $\mathbf{A} = A_0$, this ML estimate is identical to the
 263 CB estimate (maximization of Eq. (9)).

264 Note that maximizing Eq. (11) is a $(4S + 4)$ -parameter optimization prob-
 265 lem, which is difficult to solve when S is large. However, this maximization
 266 problem can be largely simplified by introducing the latent variable, which is
 267 defined as an unknown information that could let the parameter estimation
 268 straightforward, should it be available. In this multiple-source identification
 269 problem, the contributions of the various sources to the measured pressures
 270 could be introduced as the latent variables, denoted as $\mathbf{c}_t = (\mathbf{c}_{0t}, \dots, \mathbf{c}_{St})$, in
 271 which

$$\mathbf{c}_{st} = \mathbf{G}_s(\mathbf{r}_s)A_s + \mathbf{e}_{st}, s = 0, \dots, S. \quad (12)$$

272 In this equation, $\mathbf{G}_s(\mathbf{r}_s)$ is the $(s + 1)$ -th column of the matrix $\mathbf{G}(\mathbf{r})$ and
 273 $\mathbf{G}_s(\mathbf{r}_s)A_s$ stands for the contribution from s -th source, \mathbf{e}_{st} ($s = 0, \dots, S$,

274 $t = 1, \dots, T$) are independent Gaussian random variables with mean $\mathbf{0}$ and
 275 covariance matrix $\Sigma_s = \frac{\sigma^2}{S+1} \mathbf{I}_M$. Therefore, the latent variables \mathbf{c}_{st} and the
 276 original measurements \mathbf{p}_t are related via $\mathbf{p}_t = \sum_{s=0}^S \mathbf{c}_{st}$ and the log-likelihood
 277 function of the latent variables (after removing unnecessary terms which are
 278 independent of the unknown parameters and have no effect of the MLE) is

$$\log L(\mathbf{r}, \mathbf{A} | \mathbf{c}) = - \sum_{t=1}^T \sum_{s=0}^S \|\mathbf{c}_{st} - \mathbf{G}_s(\mathbf{r}_s) A_s\|^2. \quad (13)$$

279 Since the contributions \mathbf{c}_{st} cannot be measured, it is impossible to directly
 280 maximize Eq. (13) with respect to each \mathbf{r}_s and A_s to separately estimate
 281 each source from the others using the corresponding source contribution.
 282 However, the EM algorithm [32] makes it possible to proceed with Eq. (13)
 283 by treating the missing data as random variables. The IB approach starts
 284 from an initial parameter $\Phi^0 = (\mathbf{r}^0, \mathbf{A}^0)$; given l -th parameter estimates, the
 285 $(l+1)$ -th iteration consists in the following steps [10]:

- 286 • compute the expected source contribution

$$\hat{\mathbf{c}}_{st}^l = \mathbb{E}(\mathbf{c}_{st} | \mathbf{p}_t, \mathbf{r}^l, \mathbf{A}^l) = \mathbf{G}_s^l A_s^l + \frac{1}{S+1} \left(\mathbf{p}_t - \sum_{s=0}^S \mathbf{G}_s^l A_s^l \right); \quad (14)$$

- 287 • decide the source location by performing a beamforming projection
 288 using the estimated source contribution:

$$\mathbf{r}_s^{l+1} = \arg \max_{\mathbf{r}_s} \frac{\mathbf{G}_s^H(\mathbf{r}_s) \hat{\mathbf{c}}_s^l (\hat{\mathbf{c}}_s^l)^H \mathbf{G}_s(\mathbf{r}_s)}{\mathbf{G}_s^H(\mathbf{r}_s) \mathbf{G}_s(\mathbf{r}_s)}, \quad (15)$$

289 in which $\hat{\mathbf{c}}_s^l = \frac{1}{T} \sum_{t=1}^T \hat{\mathbf{c}}_{st}^l$;

- 290 • estimate the source contribution via

$$A_s^{l+1} = \frac{\mathbf{G}_s^H(\mathbf{r}_s^{l+1}) \hat{\mathbf{c}}_s^l}{\mathbf{G}_s^H(\mathbf{r}_s^{l+1}) \mathbf{G}_s(\mathbf{r}_s^{l+1})}. \quad (16)$$

291 It is important to remark that the IB estimate is a MLE of the multiple-
 292 source parameters. The principle of EM algorithm guarantees the increase of

293 likelihood function after each iteration [32] and its convergence to a stationary
 294 point [33]. In order to obtain the estimate at the global maximum, a multiple
 295 initialization strategy could be employed: the above algorithm is performed
 296 with different initial parameters, and finally retain the estimate with highest
 297 likelihood.

298 3.3. Shallow water source localization using iterative beamforming

299 In this section, the sound source localization in a shallow water envi-
 300 ronment is considered. As introduced in Section 2.2, the problem of single
 301 source with multiple reflections is considered in a model of multiple sources
 302 in a free-field. In this case, the locations of the sound sources are denoted by
 303 $\mathbf{r} = (\mathbf{r}_0, \dots, \mathbf{r}_S)$, in which $\mathbf{r}_0 = (x_0, y_0, z_0)$ is the coordinate of the physical
 304 source. The image source coordinates are $\mathbf{r}_s = (x_s, y_s, z_s)$, $s \geq 1$, in which
 305 $x_s = x_0$, $y_s = y_0$, and z_s is obtained from Eq. (4). The amplitudes of the
 306 sources are represented by $\mathbf{A} = (A_0, \dots, A_S)^T$, in which A_0 is the amplitude
 307 of the physical source, and the amplitudes A_s ($s \geq 1$) of the image sources
 308 are obtained via Eq. (5).

Given an initial parameter of the physical source parameters, denoted
 as $\mathbf{r}_0^0 = (x_0^0, y_0^0, z_0^0)$ and A_0^0 , the corresponding initial location of each image
 sources is $\mathbf{r}_s^0 = (x_s^0, y_s^0, z_s^0)$, in which $x_s^0 = x_0^0$, $y_s^0 = y_0^0$, and z_s^0 is obtained by
 inserting z_0^0 back into Eq. (4). The initial amplitudes of the image sources
 are obtained by replacing A_0 in Eq. (5) by A_0^0 and giving an initial value
 of bottom reflectivity α^0 . Then, the source localization algorithm presented
 in Section 3.2 is proceeded with the initial parameters $\mathbf{r}^0 = (\mathbf{r}_0^0, \dots, \mathbf{r}_S^0)$
 and $\mathbf{A}^0 = (A_0^0, \dots, A_S^0)^T$. The returned source estimates are represented by
 $\mathbf{r}^* = (\mathbf{r}_0^*, \dots, \mathbf{r}_S^*)$ and $\mathbf{A}^* = (A_0^*, \dots, A_S^*)$. Then, each estimated image source
 $\mathbf{r}_s^* = (x_s^*, y_s^*, z_s^*)$, $s \geq 1$, is reflected back to the region $z = [d, 0]$ to obtain
 the corresponding physical source estimate, denoted by $\mathbf{r}_s^{**} = (x_s^*, y_s^*, z_s^{**})$, in
 which

$$z_s^{**} = z_s^* + 2 \left\lfloor \frac{s}{4} \right\rfloor d, \text{ if } \left\{ \frac{s}{4} \right\} = 0 \quad (17a)$$

$$z_s^{**} = -z_s^* - 2 \left\lfloor \frac{s}{4} \right\rfloor d, \text{ if } \left\{ \frac{s}{4} \right\} = \frac{1}{4} \quad (17b)$$

$$z_s^{**} = -z_s^* + 2 \left(\left\lfloor \frac{s}{4} \right\rfloor + 1 \right) d, \text{ if } \left\{ \frac{s}{4} \right\} = \frac{1}{2} \quad (17c)$$

$$z_s^{**} = z_s^* - 2 \left(\left\lfloor \frac{s}{4} \right\rfloor + 1 \right) d, \text{ if } \left\{ \frac{s}{4} \right\} = \frac{3}{4}. \quad (17d)$$

309 Here, $\{x\} = x - [x]$ is the sawtooth function. Then, the estimate of the
 310 physical source location can be obtained by

$$\hat{\mathbf{r}}_0 = \frac{1}{S+1} \left(\mathbf{r}_0^* + \sum_{s=1}^S \mathbf{r}_s^{**} \right). \quad (18)$$

311 The reflectivity could be also estimated by

$$\hat{\alpha} = \frac{1}{S-1} \sum_{s=2}^S \left(\frac{|A_s^*|}{|A_0^*|} \right)^{1/\lceil \frac{s+2}{4} \rceil}. \quad (19)$$

312 Finally, the amplitude of the physical source can be estimated by an average
 313 of the image sources considering the power losses of bottom reflections:

$$\hat{A}_0 = \frac{1}{S+1} \sum_{s=0}^S (-1)^{\lceil \frac{s+1}{2} \rceil} \hat{\alpha}^{-\lceil \frac{s+2}{4} \rceil} A_s^*. \quad (20)$$

314 The IB strategy for localizing the shallow water source is summarized in
 315 Algorithm 1.

Algorithm 1 Sound source localization in shallow water using IB

1. For $l = 0$, pick starting values for the source parameters (\mathbf{r}_0^0 and A_0^0) and generate the initial parameters of image sources (\mathbf{r}_s^0 and A_s^0 , $s = 0, \dots, S$) via Eqs. (4) and (5).
 2. For $l \geq 1$:
 - repeat**
 - estimate the source contributions $\hat{\mathbf{c}}_{st}^l$ via Eq. (14) for $s = 0, \dots, S$;
 - estimate the source locations \mathbf{r}_s^{l+1} via Eq. (15) for $s = 0, \dots, S$;
 - estimate the source amplitudes A_s^{l+1} via Eq. (16) for $s = 0, \dots, S$.
 - until** the relative increase of the measured data log-likelihood Eq. (11) is less than a given threshold κ .
 - return** location and amplitude estimates of the physical and image sources (\mathbf{r}_s^* and A_s^* , $s = 0, \dots, S$).
 3. The location and amplitude of the physical source and the reflectivity of water bottom are estimated via Eqs. (18), (20) and (19), respectively.
-

316 *3.4. Matched-field processing and computational costs*

317 In the underwater source localization problem, MFP [28] is probably the
 318 most widely used method, which generalizes the beamforming approach to
 319 estimate both source location and other unknown environmental paramete-
 320 ters. In this section, conventional and Capon's MFP methods are briefly
 321 introduced, their computational costs are quantified and compared with the
 322 IB method.

323 In this work, the MFP methods are based on the image source model.
 324 The Green's function at m -th microphone \mathbf{r}'_m is

$$G(\mathbf{r}'_m | \mathbf{r}_0, \alpha) = \sum_{s=0}^S (-1)^{n(s)} \alpha(f)^{n-(s)} \frac{e^{jk\|\mathbf{r}'_m - \mathbf{r}_s\|}}{4\pi\|\mathbf{r}'_m - \mathbf{r}_s\|}. \quad (21)$$

325 The conventional and Capon's MFP estimates for the source location \mathbf{r}_0 and
 326 the reflectivity α are respectively:

$$\{\hat{\mathbf{r}}_0, \hat{\alpha}\} = \arg \max_{\mathbf{r}_0, \alpha} \frac{\mathbf{p}_t^H \mathbf{G}(\mathbf{r}_0, \alpha) \mathbf{G}^H(\mathbf{r}_0, \alpha) \mathbf{p}_t}{|\mathbf{G}(\mathbf{r}_0, \alpha)|^2} \quad (22)$$

327 and

$$\{\hat{\mathbf{r}}_0, \hat{\alpha}\} = \arg \max_{\mathbf{r}_0, \alpha} \frac{1}{\mathbf{G}^H(\mathbf{r}_0, \alpha) \hat{\mathbf{K}}^{-1} \mathbf{G}(\mathbf{r}_0, \alpha)}, \quad (23)$$

328 in which $\mathbf{G}(\mathbf{r}_0, \alpha) = (G(\mathbf{r}'_1 | \mathbf{r}_0, \alpha), \dots, G(\mathbf{r}'_M | \mathbf{r}_0, \alpha))$.

329 Then, the computational costs of the MFP approaches are estimated.
 330 The 3D space and $\alpha \in [0, 1]$ are discretized; N_x, N_y, N_z and N_α denote
 331 the number of discrete points in x -, y -, z -axis and $\alpha \in [0, 1]$, respectively. In
 332 order to obtain the MFP estimate, the sound propagation has to be calculated
 333 $SN_x N_y N_z N_\alpha$ times, such that the computational cost is very high when the
 334 possible region of source is large and high estimation precision is requested.

335 By contrast, the computational cost of IB is relatively low, which mainly
 336 comes from the optimization step. For this approach, the sound propagation
 337 process needs to be calculated $\sum_{l=1}^L \sum_{s=1}^S N_{sl}$ times, in which N_{sl} is the number of
 338 calculation of Green's function in the optimization step for s -th source and
 339 l -th iteration, depending on the optimization method. Therefore, the compu-
 340 tational cost ratio between IB and MFP is approximately $N_x N_y N_z N_\alpha / (L \bar{N}_{sl})$,
 341 where \bar{N}_{sl} is the average of N_{sl} and in general is less than 100. In most cases,
 342 IB requires much less computational cost than the MFP approaches. Besides,
 343 IB is not limited to the estimation precision while high estimation accuracy
 344 of MFP results in a high computation cost.

345 4. Numerical experiments

346 In this section, numerical experiments are introduced to illustrate the
 347 proposed model and source estimation method. The experimental setup is
 348 displayed in Figure 1. The Pekeris shallow water waveguide is assumed with
 349 a sound speed $v = 1500$ m/s. The water depth is 100 m, i.e., $d = -100$ m.
 350 The sound source is located at $\mathbf{r}_0 = (0, 0, -32)$ m and its amplitude is
 351 $A_0 = 1$ at the frequency $f = 1500$ Hz. Note that in order to apply the
 352 image model, the wavelength must be much smaller than the water depth.
 353 Here, the wavelength is $\lambda = 1$ m, which is sufficiently small to guaran-
 354 tee the model accuracy. Besides, the source localization accuracy depends
 355 on the frequency. Refs. [10, 31] have discussed the influence of frequency
 356 on IB and MFP, therefore it is not the main concern of the present pa-
 357 per. In this experiment, five towed hydrophone arrays are used, each array
 358 has 7 sensors. More specifically, $M = 35$ sensors are placed on an x -plane
 359 $x'_m = 5000$ m; their y - and z -coordinates are $y'_m \in \{-100, -50, 0, 50, 100\}$ m
 360 and $z'_m \in \{-95, -80, \dots, -5\}$ m, respectively. The water surface is totally
 361 mirror reflective; the water bottom is smooth and has a reflectivity $\alpha = 0.1$.
 362 Generally speaking, an interference having an incident angle smaller than 75°
 363 can be ignored due to more bottom reflections (depending also on the ma-
 364 terial of water bottom and the microphone-source distance) [1]; in the data
 365 generation of this experiment, 34 modes (i.e., $S = 33$) are considered such
 366 that minimum incident angle is around 75° . Thus, the sound waves propa-
 367 gating from the source are within a cone whose apex angle is 30° . The sound
 368 propagation and measurement process are simulated via Eq. (3). The num-
 369 ber of snapshots is $T = 30$. The signal-to-noise ratio (SNR) is assumed to be
 370 10^4 , the standard deviation of measurement noise is thus $\sigma = 10^{-4} \frac{1}{M} \|\mathbf{GA}\|$.

371 First, CB is employed to estimate the sound source. By inserting the
 372 mean of T measured pressures in the frequency domain into Eq. (9) and
 373 plotting the \mathbf{r}_0 - $S(\mathbf{r}_0)$ color map for each \mathbf{r}_0 on the source plane $x = 0$, the CB
 374 sound field (sound pressure level whose reference pressure is the maximum
 375 value in the plane) of source plane can be constructed, as is shown in Figure 3
 376 (a). It is clear that CB has a problem of spatial resolution, i.e., the physical
 377 and the first image source cannot be separated. Actually, the resolution of
 378 CB can be theoretically estimated [27] by:

$$\text{Res} = \frac{|x'_m - x_0|\lambda}{D}, \quad (24)$$

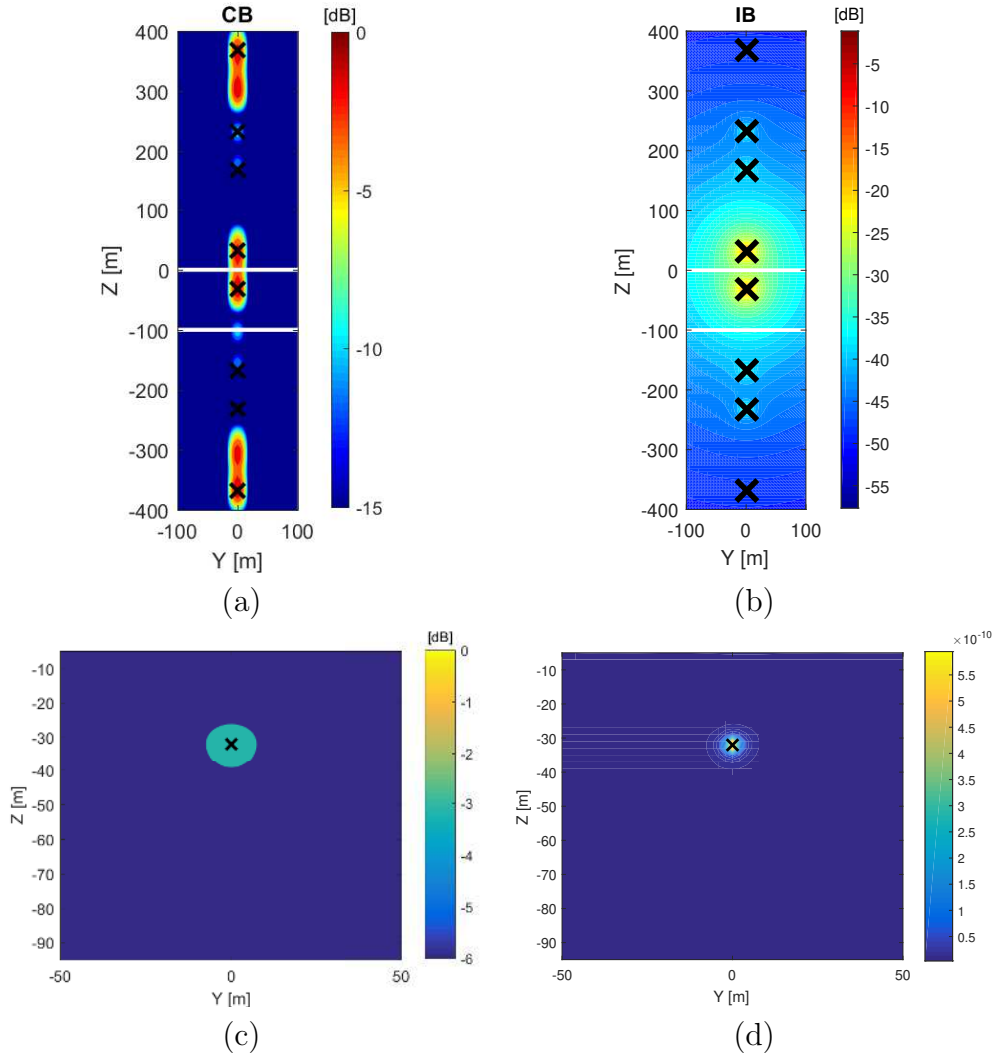


Figure 3: Sound source localization using (a) classical beamforming, (b) iterative beamforming, (c) conventional MFP, and (d) Capon's MFP. The distance between the source and the sensor plane is 5 km . The frequency of the sound source is 1500 Hz. In subfigures (a) and (b), the crosses stand for the physical and image source locations, and the two lines represent the water surface and bottom. (For interpretation of the references to color in this figure, the reader is referred to the web version of this paper.)

379 in which D is the size of the hydrophone array and λ is the wavelength. In
 380 this experiment, the size of the microphone array is 200 m in the y -direction
 381 and 90 m in the z -direction, thus the theoretical spatial resolution is $25 \text{ m} \times$

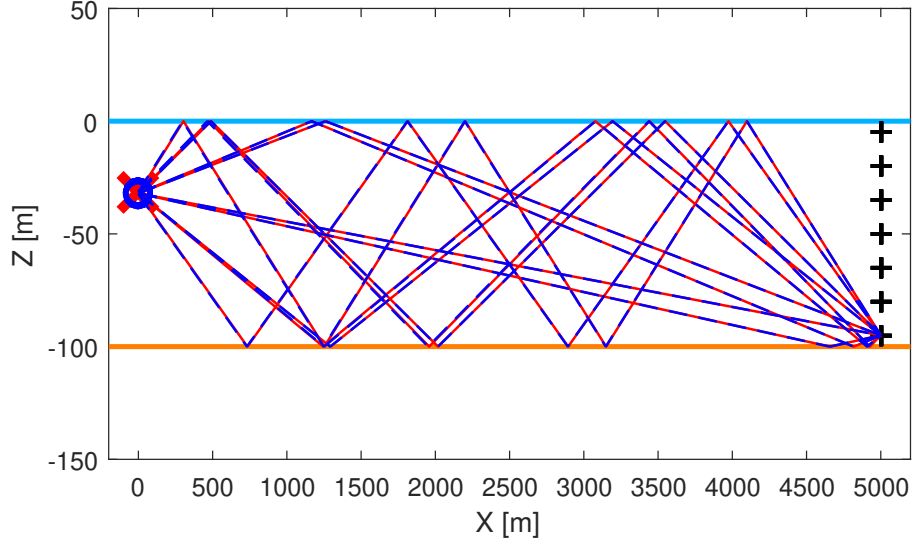


Figure 4: Actual (solid lines) and estimated (dash lines) paths of sound propagation from the source to each microphone. The cross and the circles stand for the actual source and the estimates of the image sources reflected back to the physical region.

382 56 m, which is consistent with the numerical result.

383 However, the above limitation can be overcome by IB [31]. Since the
 384 IB approach is parametric (the estimates of point sources are explicitly ob-
 385 tained), the reconstructed sound field is not limited by the minimum resolv-
 386 able source separation of CB. Here, in order to reduce the computational
 387 cost, the number of modes assumed in the IB estimation process is 10, i.e.,
 388 only the sources with powers larger than $|\alpha^3 A_0|^2$ are considered. Ten initial
 389 source location of \mathbf{r}_0 is randomly obtained, following a 3D uniform distribu-
 390 tion with support $\{(x, y, z) : x \in [-50, 50], y \in [-50, 50], z \in [-60, 0]\}$. The
 391 corresponding initial source amplitudes are randomly generated from a uni-
 392 form random variable in $[0.8, 1.2]$. Then, the IB estimates are obtained using
 393 the method introduced in Section 3.3: the estimate with highest likelihood
 394 is retained as the final source estimate. Figure 3 (b) shows the reconstructed
 395 sound pressure level on the source plane: for each discrete point \mathbf{r} on the
 396 source plane $z = 0$ ($\mathbf{r} \neq \mathbf{r}_s^*$), the reconstructed sound pressure is computed
 397 by

$$\mathbf{p}^*(\mathbf{r}) = \sum_{s=0}^S A_s^* |G(\mathbf{r}|\mathbf{r}_s^*)|. \quad (25)$$

398 This figure shows that IB avoids the limitation of spatial resolution of CB
 399 and can clearly separate all the sources. Besides, the source amplitude and
 400 the bottom reflectivity are estimated via Eqs. (20) and (19), being $\hat{A}_0 = 0.99$
 401 (the actual value is 1) and $\hat{\alpha} = 0.107$ (the actual value is 0.1). Figure 4
 402 displays the source estimates projected back to the physical region $z \in [d, 0]$
 403 on the X - Z plane via Eq. (17), denoted by the circles. These estimated
 404 physical sources are almost overlapped with the actual one, which illustrates
 405 the precision of the IB estimates. The sound propagation paths from the
 406 real source (solid lines) and the estimated sources (dashed lines, each of
 407 which is emitted from the corresponding projected image source) to each
 408 hydrophone are also shown in Figure 4. It is clear that the sound propagation
 409 paths can also be accurately identified by IB. Furthermore, the IB algorithm
 410 converges fast such that the computational cost is low. Actually, the number
 411 L of iteration steps is always less than 100 (and maximum 1000) while the
 412 threshold κ of relative likelihood function increment is 10^{-4} .

413 Finally, the conventional and Capon's MFP methods are used. The 3D
 414 coordinate of source and the reflectivity of water bottom are discretized by
 415 $\{(x, y, z) : x = -50 : 0.1 : 50, y = -50 : 0.1 : 50, z = -5 : -0.1 : -95\}$ m
 416 and $\alpha = 0 : 0.01 : 0.5$, respectively. The source estimates are obtained
 417 from Eqs. (22) and (23) and the reconstructed sound fields in the source
 418 plane $x = 0$ are shown in Figure 3 (c) and (d). Besides, the estimated
 419 reflectivity obtained from the both approaches are respectively $\hat{\alpha} = 0.1$ and
 420 $\hat{\alpha} = 0.09$. It is clear that the both MFP methods are also accurate in terms
 421 of source localization, although conventional MFP has a relatively large focal
 422 spot size. However, it is remarkable that the MFP methods require a much
 423 higher computational cost: in this example, the sound propagation process
 424 has to be computed 4.6×10^{11} times. By contrast, the IB approach needs to
 425 calculate the sound propagation maximum 10^6 times (with 10 initial values).
 426 Therefore, the computation speed of IB is approximately 4.6×10^5 times
 427 faster than the MFP approaches.

428 5. Sensitivity to environmental parameter uncertainties

429 In the above experiments, the sound speed and water depth are assumed
 430 to be precisely known in the source estimation process. In real applications,
 431 however, these environmental parameters often cannot be accurately mea-
 432 sured. For example, Ref. [3] proposes that the error ranges of shallow water
 433 sound speed and water depth are ± 2.5 m/s and ± 2.5 m. In this section, an

434 UQ analysis is presented. The source estimation method is proceeded with
 435 inaccurate information regarding the sound speed and water depth. Kriging
 436 is used to build a response surface (i.e., a meta-model of the true simulation
 437 output) of the predicted acoustic source localization. A quasi-Monte Carlo
 438 sampling of the response surface is then used to obtain the PDF of the source
 439 estimate, the sensitivities of the environmental parameters are also discussed.

440 5.1. Building a Kriging response surface

441 The Kriging method proposes a weighted linear estimator, i.e., given n
 442 values of the generic quantity of interest (QoI) $X(\omega)$ sampled over the un-
 443 certain space Ω , the estimate of the QoI $\hat{X}(\omega)$ at an unmeasured location ω
 444 is:

$$\hat{X}(\omega) = \sum_{s=1}^n \lambda_s(\omega) X(\omega_s), \quad (26)$$

445 where $\lambda_s(\omega)$ is the Kriging weight assigned to sample $X(\omega_s)$ and ω_s is the
 446 sample location. It is important to notice that the Kriging weights are func-
 447 tions of location at which an estimation is needed, since the s -th weight
 448 depends on the distance between the unmeasured sample and the s -th sam-
 449 ple location. The QoI $X(\omega)$ is arbitrarily decomposed into a residual $R(\omega)$
 450 and a trend component $\beta(\omega)$:

$$X(\omega) = R(\omega) + \beta(\omega). \quad (27)$$

451 The residual component can be treated as a stationary random field with 0-
 452 mean and covariance $\text{Cov}(R(\omega), R(\omega + h)) = C(h)$, where $C(h)$ is called the
 453 covariance function. The fundamental step of Kriging is to correctly select
 454 the covariance function defining the underlying Gaussian process. Here, the
 455 spline covariance function is used to build the response surface, i.e.,

$$C(h) = \begin{cases} 1 - 6(h\theta)^2 + 6(h\theta)^3, & h < \frac{1}{2\theta} \\ 2(1 - h\theta)^3, & \frac{1}{2\theta} \leq h < \frac{1}{\theta} \\ 0, & h \geq \frac{1}{\theta} \end{cases}, \quad (28)$$

456 in which θ can be obtained via MLE or empirical tuning.

457 The Kriging weights $\lambda_s(\omega)$ are determined by minimizing the variance of
 458 the estimator $\text{Var}(\hat{X}(\omega) - X(\omega))$ under the unbiasedness constraint $\mathbb{E}[\hat{X}(\omega) -$
 459 $X(\omega)] = 0$. As a matter of fact, Kriging estimator is a best linear unbi-
 460 ased predictor (BLUP). A number of variants of Kriging exists in literature,

461 among which the Ordinary Kriging approach is preferred, given that there
 462 is no a-priori assumption on trend and given the simplicity and robustness
 463 of Ordinary Kriging compared to other variants. In this approach, the trend
 464 part of the random field $X(\omega)$ is assumed to be constant but with unknown
 465 $\beta(\omega) = \mu_0$. Unbiasedness condition requires that the sum of Kriging weights
 466 is equal to 1. Then, by minimizing the error variance and adding a Lagrange
 467 parameter to respect the unbiasedness constraint, the well known Ordinary
 468 Kriging system is obtained:

$$\begin{cases} \sum_{j=1}^n \lambda_j(\omega)C(\omega_i - \omega_j) + \mu_0 = C(\omega_i - \omega) & i = 1, \dots, n \\ \sum_{j=1}^n \lambda_j(\omega) = 1 \end{cases} \quad (29)$$

469 To assess the reliability of the meta-model, a leave-one-out cross valida-
 470 tion strategy [42] is adopted. The cross-validation values are expressed here
 471 as a global relative L_2 -error:

$$CV_s = \frac{\sqrt{\int_{\Omega} (\hat{X} - \hat{X}^{-s})^2 d\omega}}{\sqrt{\int_{\Omega} \hat{X}^2 d\omega}} \quad (30)$$

472 where \hat{X}^{-s} is the Kriging response surface built without the s -th sample. A
 473 threshold ε on the computed cross-validation value CV_s for each sample is
 474 imposed to assess convergence.

475 5.2. Sensitivity estimators from the Kriging meta-model

476 The main statistics and the Sobol' variance-based sensitivity indices [43]
 477 are obtained through quasi Monte Carlo (Sobol sequences) estimators [44].
 478 The mean estimate $\mu(X)$ is calculated as

$$\mu(X) = \frac{1}{q_{MC}} \sum_{i=1}^{q_{MC}} \hat{X}_i, \quad (31)$$

479 where q_{MC} is the number of quasi Monte Carlo samples \hat{X}_i of the Kriging
 480 meta-model of $X(\Omega)$. Similarly, the variance $\sigma^2(X)$ for large q_{MC} can be
 481 obtained by

$$\sigma^2(X) = \frac{1}{q_{MC}} \sum_{i=1}^{q_{MC}} (\hat{X}_i - \mu(X))^2. \quad (32)$$

482 Finally, for the first order sensitivity indices, a common approach is to gen-
 483 erate two independent quasi Monte Carlo sample sets of Ω . The independent
 484 sets Ω_1 and Ω_2 are obtained from a $q_{MC} \times 2D$ matrix, which is a quasi Monte
 485 Carlo sample series in dimension $2D$, D is the dimension of the original prob-
 486 lem. The Sobol' indices $S_j = \sigma_j^2/\sigma^2$ for j -th dimension is derived from the
 487 estimate of the variance and the partial variance σ_j^2 :

$$\sigma_j^2 = \frac{1}{q_{MC}} \sum_{i=1}^{q_{MC}} \hat{X}_i(\Omega_2)(\hat{X}_i(\Omega_1^j) - \hat{X}_i(\Omega_1)), \quad (33)$$

488 where Ω_1^j is the first samples set Ω_1 where the j -th column has been replaced
 489 by the corresponding column of the second sample set Ω_2 . The residual
 490 variance can be estimated by the difference between the total variance and
 491 the sum of first order partial variances.

492 5.3. Uncertainty quantification of the source localization to the environmental 493 parameters

494 In this section, the UQ analysis of source localization with respect to un-
 495 certain environmental parameters is presented. As previously discussed, the
 496 shallow water sound speed and the water depth are assumed to be uncertain,
 497 the ranges of which are set to be $v = 1500 \pm 2.5$ m/s and $d = -100 \pm 2.5$ m,
 498 respectively. Here, both parameters are assumed as independent random
 499 variables. The UQ analysis of source localization using IB, conventional and
 500 Capon's MFP is done. The sound source is located at $(0, 0, -32)$ m and
 501 the frequency is 1500 Hz. The sensor locations are the same as in Sec-
 502 tion 4. In order to decrease the computational costs, the x -coordinate of
 503 sound source is assumed to be known for all the the three methods, thus
 504 only the y - and z -coordinates have to be estimated. Note that in this case
 505 the computational cost of MFP decrease (the region of possible source loca-
 506 tion becomes two-dimensional), but is still much heavier than IB: the com-
 507 putation speed of IB is approximately 460 times faster than the MFP ap-
 508 proaches. Besides, in order to eliminate the influence of random initialization
 509 of IB, the starting values for the source parameters are obtained from a grid
 510 $\{(y, z) : y = -100, -80, \dots, 100, z = 0, -10, \dots, -70\}$.

511 A regular grid with 25 samples of uncertain parameters ($v = 1500, 1500 \pm$
 512 $1.25, 1500 \pm 2.5$ m/s, $d = 100, 100 \pm 1.25, 1000 \pm 2.5$ m) is used to assess the
 513 reliability of the Kriging-based response surface. The number of samples of
 514 the UQ grid depends in general on the dimensionality of the UQ problem, on

515 the physical ranges of each UQ parameter and on the expected complexity of
 516 the response. In this case, a grid with 25 samples proved to be enough to get
 517 a reasonable convergence of the Kriging response surface. Further tests with
 518 additional samples are proved to be inefficient in terms of response surface
 519 reliability.

520 Figure 5 shows the computation results of IB with four different assumed
 521 parameters: (a) $v = 1500$ m/s, $d = -98.75$ m; (b) $v = 1500$ m/s, $d = -97.5$
 522 m; (c) $v = 1501.25$ m/s, $d = -100$ m; (b) $v = 1502.5$ m/s, $d = -100$
 523 m. In the latter two cases, the image sources with low strengths cannot be
 524 accurately estimated. However, the first two sources (the physical source and
 525 its first image which have the highest amplitudes) can be identified in all the
 526 cases. Since the main purpose of this work is to localize the physical source,
 527 these results are sufficient: the estimated source between the two boundaries
 528 can be identified as the estimate of the physical source. Here, the source
 529 location estimate of IB is the first mode of the multiple-source estimate. In
 530 the following, the estimation accuracy of all the three methods is quantified.

531 A response surface is built for the z -coordinate estimate of source through
 532 ordinary Kriging with spline covariance function and $\theta = 0.1$ (tuned to well
 533 fit the meta-model). Actually, the experimental results of IB show that the
 534 estimation error of y -coordinate is always much smaller than z -coordinate and
 535 relatively negligible (smaller than 0.1 m), thus the z -coordinate estimation
 536 error $|z_0 - z_0^*|$ can be seen as the source estimation error $\|\mathbf{r}_0 - \mathbf{r}_0^*\|$. Although
 537 the main interest of the UQ analysis is to quantify the error on the source
 538 localization, using it to build the response surface is not so efficient as it
 539 may appear, since the absolute value operator results in a response surface
 540 with sharp points, which can not be correctly fitted by a Kriging meta-model
 541 with a small sample size. Therefore, the response surface of the z -coordinate
 542 estimate is constructed and the error is then derived. The interested reader
 543 can see Ref. [40] for a discussion on the importance of the choice of the QoI in
 544 building the response surface and assessing its reliability. Furthermore, the
 545 leave-one-out cross validation is used to assess the reliability of the Kriging-
 546 based response surface, where the cross-validation value is $CV_s < 1\%$ for all
 547 s .

548 At first both uncertain parameters are assumed to be uniformly dis-
 549 tributed; then the results are compared to the truncated Gaussian distri-
 550 butions, which let the range of each parameter mentioned above be equal to
 551 six standard deviation centered at the mean value. The PDFs, along with
 552 the mean value $\mu(z)$ and the confidence interval $\mu(z) \pm \sigma(z)$ ($\sigma(z)$ is the

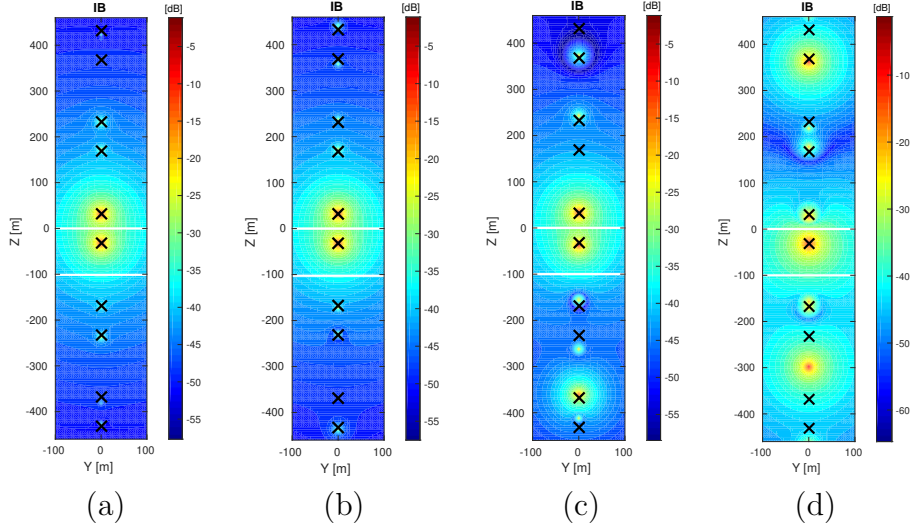


Figure 5: Reconstructed sound pressure levels using IB with imprecise information of water depth and velocity: (a) $d = -98.75$ m, $v = 1500$ m/s; (b) $d = -97.5$ m, $v = 1500$ m/s; (c) $d = -100$ m, $v = 1501.25$ m/s; (d) $d = -100$ m, $v = 1502.5$ m/s. The black crosses stand for the locations of (physical and image) sound sources. The white lines indicate the water surface and bottom. The source-microphone range is 5000 m and the frequency is $f = 1500$ Hz. (For interpretation of the references to color in this figure, the reader is referred to the web version of this paper.)

553 standard deviation), of the estimated z -coordinate using the three source
554 localization methods with uniform or Gaussian assumption of the uncertain
555 environmental parameters are shown in Figure 6. For the IB estimation, in
556 the case of uniform distribution, the mean value of the z -coordinate estimates
557 is $\mu(z_0^*) = -31.88$ m; by contrast, in the Gaussian case, the mean value is
558 $\mu(z_0^*) = -31.91$ m. In both cases the PDF is spread around the reference
559 value $z_0 = -32$ m and the confidence interval contains z_0 , meaning that the
560 IB method is able to retrieve the exact solution in most of the combina-
561 tions of the uncertain environmental parameters. Similarly, the conventional
562 MFP estimate has mean value $\mu(z_0^*) = -32.18$ m for uniform distribution
563 and $\mu(z_0^*) = -32.11$ m for Gaussian distribution, which imply that the exact
564 estimate can also be retrieved by the mean value. The mean value error of
565 conventional MFP is more or less same as IB but the variance is smaller,
566 which can be observed from Figure 6. The Capon's MFP estimate results in
567 $\mu(z_0^*) = -33.92$ m for uniform distribution and $\mu(z_0^*) = -33.61$ m for Gaus-
568 sian distribution, and the confidence interval is far away from z_0 , which leads

569 to a much larger localization error than the two methods. Table 1 displays
570 α -th percentile ($\alpha = 5\%, 10\%, 25\%, 50\%, 75\%, 90\%, 95\%$) of the distribution
571 of the z coordinate estimate obtained from the three methods under the uni-
572 form or Gaussian assumption for the uncertain environmental parameters.
573 It is interesting to notice that for IB the error is slightly larger far from the
574 nominal value for the uniform assumption, so that in the case of Gaussian
575 approximation for the environmental parameters the percentiles correspond-
576 ing to a given error of 1 m increase by about 15%: the z -coordinate estimate
577 at -33 m corresponds to the percentile being 10% for uniform assumption
578 and 25% for the Gaussian assumption.

579 To further investigate the error distribution against the uncertain en-
580 vironmental parameter and to compare it with the MFP approaches, the
581 response surface of $|\mathbf{r}_0^* - \mathbf{r}_0|$ is analysed for all the three source estimation
582 methods. Maps of the source estimation error $|\mathbf{r}_0^* - \mathbf{r}_0|$ for IB, conventional
583 and Capon's MFP are shown in Figure 7. As can be noticed, the conven-
584 tional MFP approach returns the most robust result while Capon's MFP is
585 most sensitive with respect to uncertain parameters. The IB estimates have
586 an intermediate error in terms of dispersion of the results, but it is also reli-
587 able enough since the absolute error never exceeds 2 m in the given range of
588 uncertain parameters. Here, we may conclude that IB is a good choice which
589 returns an acceptable source localization error in presence of environmental
590 parameter uncertainties with a relatively low computational cost.

591 Finally, Sobol' index calculation for IB shows a weak sensitivity of source
592 estimation to the shallow water depth ($S_d \approx 3\%$), a strong sensitivity to
593 the sound speed ($S_v \approx 92\%$) and a low effect of coupling between the two
594 uncertain environmental parameters ($S_{d-v} = \sigma_{d-v}^2 / \sigma^2 \approx 5\%$ where σ_{d-v}^2 is
595 the residual variance). On the contrary, the MFP methods are more sensitive
596 to the water depth and to the coupling of the two environmental parameters:
597 the Sobol' indices are $S_d \approx 67\%$, $S_v \approx 1\%$ for conventional MFP and $S_d \approx$
598 62% , $S_v \approx 2\%$ for Capon's MFP, respectively. The same conclusions can be
599 observed in Figure 7. It is remarkable that this result is able to guide the
600 choice of source localization method: when the uncertainty level of the water
601 depth is higher than the sound speed IB can more accurately estimate the
602 source, while in the opposite case the MFP methods are preferable in the
603 sense of estimation robustness.

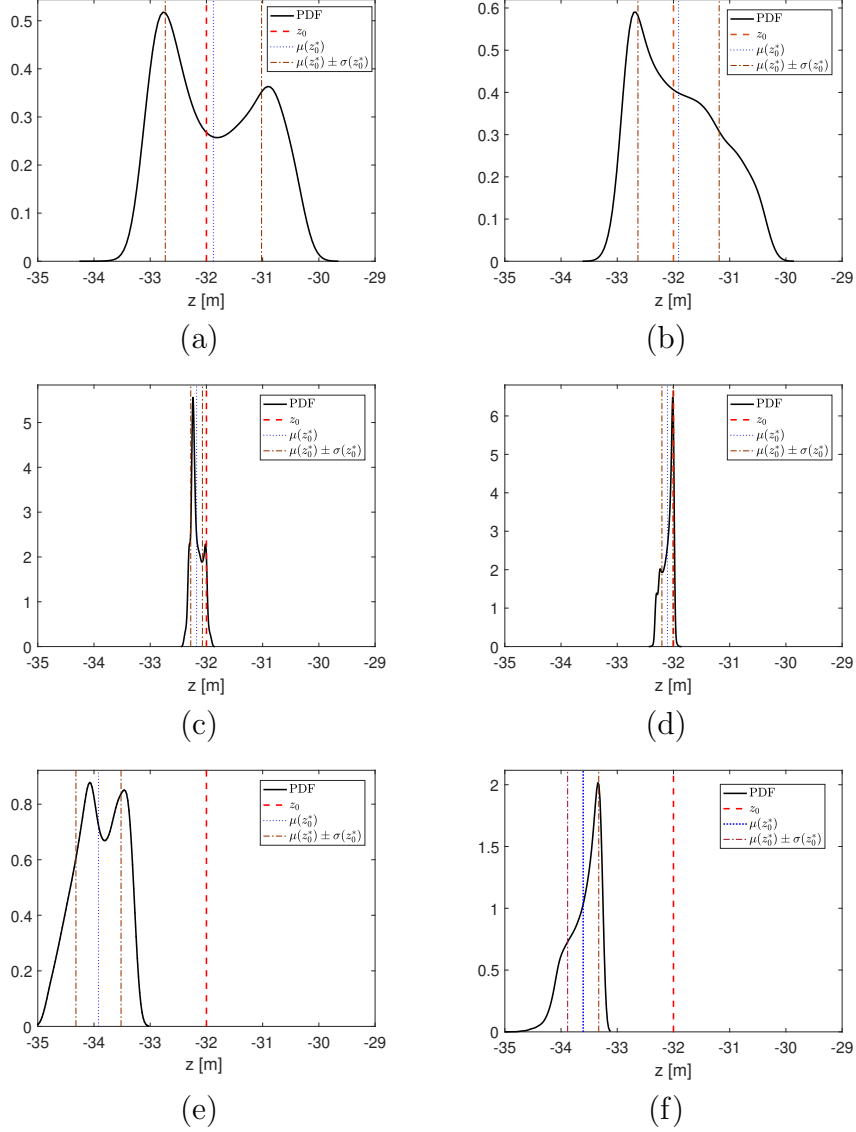


Figure 6: Probability density functions of the z -coordinate estimate using the IB (a,b), conventional MFP (c,d) and Capon's MFP (e,f) methods under the assumption of a uniform distribution (a,c,e) and a Gaussian distribution (b,d,f) for the uncertain environmental parameters.

604 6. Conclusions

605 In this paper, sound source localization in a shallow water environment is
 606 addressed. The multiple reflections on the water surface and bottom during

α	IB		Conventional MFP		Capon's MFP	
	Uniform	Gaussian	Uniform	Gaussian	Uniform	Gaussian
5%	-33.14	-32.97	-32.33	-32.30	-34.65	-34.14
10%	-33.00	-32.86	-32.31	-32.27	-34.50	-34.03
25%	-32.72	-32.59	-32.26	-32.19	-34.25	-33.82
50%	-32.03	-32.02	-32.20	-32.09	-33.93	-33.55
75%	-31.14	-31.38	-32.10	-32.03	-33.59	-33.38
90%	-30.72	-30.88	-32.02	-32.00	-33.42	-33.31
95%	-30.54	-30.65	-31.99	-31.99	-33.34	-33.29

Table 1: α -th percentile ($\alpha = 5\%, 10\%, 25\%, 50\%, 75\%, 90\%, 95\%$) of the distribution of the z -coordinate estimate obtained from the IB, conventional MFP and Capon's MFP methods under the uniform and Gaussian assumption for the uncertain environmental parameters.

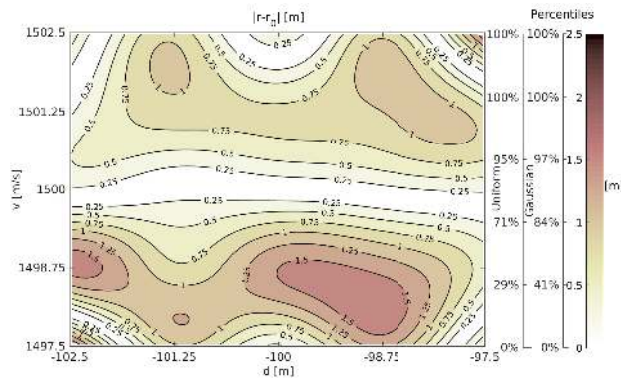
607 the sound propagation are considered. Instead of solving the wave equa-
608 tion with boundary conditions of reflection, an image source model including
609 multiple sound sources in a free-field is proposed. Therefore, the multiple-
610 reflection problem is transferred to a multiple source problem which results
611 in an analytical solution of sound propagation. Then, the multiple sources
612 are estimated using the iterative beamforming approach, which is a gener-
613 alization of classical beamforming to deal with the case of multiple sources.
614 The proposed source localization technique is a parametric method based
615 on the maximum likelihood, which avoids the limits of spatial aliasing and
616 resolution of classical beamforming.

617 The results obtained via numerical simulations show the interest of the
618 proposed model and the source estimation method. The experiment justifies
619 that classical beamforming is limited in the underwater source localization
620 problem due to a long microphone-source distance and a relatively small
621 size of microphones array. However, iterative beamforming is able to accu-
622 rately estimate the sound source and return a super-resolution reconstruction
623 of sound field. Furthermore, the proposed method has a much lower com-
624 putational cost than the matched-field processing methods. In this paper,
625 the uncertainty quantification study also justifies that the proposed iterative
626 beamforming method is stable with respect to the model uncertainties: when
627 the sound speed and water depth are imprecisely known (the error range is
628 ± 2.5 m for water depth and ± 2.5 m/s for sound speed), the sound source
629 can still be accurately localized.

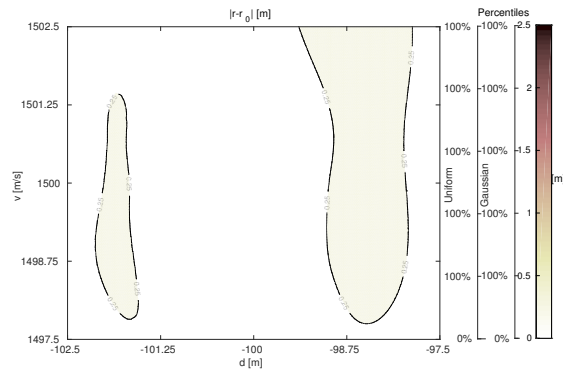
630 In this paper, only the case of homogeneous sound speed field is consid-
631 ered. However, the proposed image model and the source localization method
632 can be naturally generalized to the inhomogeneous case. Furthermore, this
633 work analyzes the sensitivities of the proposed method with respect to un-
634 certain sound speed and water depth. Other uncertainties, for example the
635 roughness of water bottom, can also be considered. It would be also inter-
636 esting to model the uncertain parameters as random variables, which may
637 further improve the accuracy of source localization.

638 **Acknowledgments**

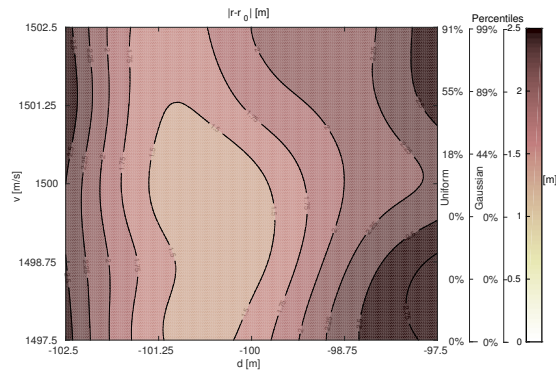
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(a)



(b)



(c)

Figure 7: Estimation error of sound source localization using (a) IB, (b) conventional MFP, and (c) Capon's MFP with imprecise information of water depth ($d \in [-102.5, -97.5]$) and sound velocity ($v \in [1497.5, 1502.5]$). Corresponding percentiles with respect to the uniform and Gaussian input distributions are given on the right side of the error map. (For interpretation of the references to color in this figure, the reader is referred to the web version of this paper.)

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