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To cite this version:

Louis de Mesnard. Controlled-by-owner firms, mobility of capital and microeconomic profit rate maximization. [Research Report] Laboratoire d'analyse et de techniques économiques(LATEC). 1999, 37 p., Figure, ref. bib. : 26 ref. hal-01527141

HAL Id: hal-01527141

https://hal.science/hal-01527141v1

Submitted on 24 May 2017

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Pôle d'Economie et de Gestion

2, boulevard Gabriel - 21000 DIJON - Tél. 03 80 39 54 30 - Fax 03 80 39 54 43

ISSN : 1260-8556

n° 9901

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mars 1999

Controlled-by-owner Firms, Mobility of Capital and Microeconomic Profit Rate Maximization

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ABSTRACT. When they actively control the firm, owners select the firm that has the best profit rate if the hypothesis of mobility of capital is adopted: controlled-by-owner firms are profit-rate-maximizing when sleeping-owner firms are pure-profit-maximizing. Both types are compared in monopoly, in perfect competition, in classical or in mixed duopoly. Always, controlled-by-owner firms have a lower output than comparable sleeping-owner firms. It only takes a fixed coefficient of equity capital to do that price plays no role for controlled-by-owner firms in perfect competition; in duopoly, it only takes a similar condition plus a linear demand to do that reaction functions vanish.

JEL classification. L21, D21, D24, D41, D42, D43, L13.

KEYWORDS. Profit Rate, Firm, Control, Coordination, Objective.

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I. Introduction

Traditionally, the firm is assumed to maximize its pure profit. Some authors have proposed some alternative theories as Baumol's theory (1959) of maximization of the size or maximization of the revenue of the firm, the behaviorist theory of the firm (Cyert and March 1963), the theory of separation between ownership and control (Scherer 1980), the maximization of consumer surplus, the maximization of added-value for labor managed firms (Sertel, 1982); if it is a non-profit organization, like a mutual benefit society, a hospital or a charity organization, the organization may want to equilibrate its accounts; one may quote also the maximization of consumer surplus and the maximization of the profit by unit. However, pure profit maximization remains widely adopted, because it seems to be more natural and intuitive, and more justified: as the distributed profit is in the revenue of the owner, and as the owner is also a consumer, he wishes that the firm maximizes its profit. Some papers (Hart, 1979; Makowski, 1983) have tried also to prove that there is unanimity between owners under some conditions about the firm's objective.

We name *objective function of the firm* the quantity that the firm maximizes. Slade (1994) uses the term "objective function" as the function that is maximized at the global level. For her, in the case of monopoly, both functions coincide but in any other cases, they may diverge: in competition, firms maximize profit but the market acts as a social welfare maximizing agent. The global objective function is called *ficticious-objective function* by Slade and she studies the conditions of existence of such a function for an oligopoly. In the following we only use the objective function of the firm that we call shortly *the objective function.*

In the usual context, the capital plays a very minor role because it is fixed in the short term and cannot participate to the short run objective function of the firm. The capital as fund K serves to pay the real capital denoted K_r and all the charges paid by advance by the firm. The real capital is a more popular concept inside microeconomic theory, for example in the production function. The real capital K_r (machines, real estates, and so on) is usually considered in place of the capital as fund. Real capital has an incidence on production through production functions as $f(K_r, L)$ or through fixed costs. It must be distinguished from capital as fund that is used to buy real capital but also other factors (as L in the above production function).

Remember that the link between real capital and capital as fund is variable: for example, the real capital can be fixed whereas the capital as fund is variable if the prices of inputs vary. In the following, the word capital refers to the fund of capital, equity capital plus debts; when it is question of the real capital it is explicitly indicated.

The status of the opportunity cost of the equity capital is also debatable when the capital as fund (equity capital plus debts) is considered as variable, in the three cases of mobility of capital:

- Either the long run is considered, and the capital is obviously variable. An additional phenomenon could play a role: some firms may enter into the sector and the price will fall down because of the increasing competition. Here, the number of firms is assumed to remain fixed.

- Either the capital does not correspond to sunk costs: it is free *ex ante* and *ex post* because it can be resold at any moment for the same amount and without any additional costs or losses. This idea is close to those of Baumol, Panzar and Willig (1982): when there are no sunk costs, hit-and-run raids are possible and investments can be set and removed with no cost, so the amount of capital engaged can be considered as free always. In this case, the long run is similar to the short run. Even in the long run, new firms may not be able to enter the sector if the price yet fixed at its minimum by the installed firms. Note that sunk costs are not fixed costs.

- Either a given capital yet installed corresponds to sunk costs and is not free *ex post*, but it can be considered nevertheless as free *ex ante*, before all decisions of investing are done. In this case the capital as fund is considered as variable even if it is not the long run.

If the capital is variable, the opportunity cost of the equity capital plays a role and the concept of profit considered must be the *economic profit* or *pure profit*. Denote Π_g as the profit of production or gross profit with $\Pi_g = R - C_g$, where R is the firm's revenue and C_g is the cost of production (g stands for "gross"). The *accounting profit* is equal to the gross profit minus the real financial cost $t_d\, K_d$: $\Pi_a = \Pi_g - t_d\, K_d$, where K_d is the debt and t_d is the interest rate on the debt. The economic profit is equal to the accounting profit minus the opportunity cost of the equity capital: $\Pi = \Pi_a - t_e\, K_e$, where K_e stands for the equity capital and t_e stands for

the opportunity cost of one unit of equity capital of equity capital. Only opportunity costs on equity capital are considered and not other sources of opportunity costs. If the opportunity cost of one unit of capital as fund is assumed to be equal to the interest rate t, what requires the financial market to be perfect, then the cost of capital K is equal to $t\,K$. The economic profit is then $\Pi = \Pi_g - t\,K$ and traditionally this quantity is assumed to be maximized by the producing firm (again, *ex ante* or always if no sunk costs).

Note that the pure profit is not a clean and quiet, good old concept: it is problematical because is no so easy to compute. Knowing the actual structure of the debts of the firm, t_d can be computed. Even if in practice the mean interest rate of the firm's debt can be complicated to compute, there is no theoretical obstacle to do it. So, the problem lies on t_e: it can be difficult to calculate the exact opportunity cost of equity $t_e\,K_e$, not because it is complicated to know the amount K_e of capital as fund engaged, but because the opportunity cost by unit of equity, t_e, is really a problematical idea. Does it is assumed to be equal to the real mean interest rate paid by the firm on all its debt ($t_e = t_d$)? Does it is assumed to be equal to the interest rate of the market ($t_e = t = t_d$, if the financial market is perfect, considering that this capital can be lend on the financial market at the rate t)? Does it is equal to the real cost on the market of such an amount of capital (if the financial market is imperfect, and the value of t_e when lending must be distinguished of the value of t_e when borrowing)? Etc. In this last case, the evaluation could be impossible because the real cost of a credit can only be known if the credit is really supplied or demanded, not if it is only potentially supplied or demanded as with virtual transactions conducted only to calculate an opportunity cost. Remember that not only real transactions can be charged with transaction costs but also virtual transactions. The cost of the debt is seen from the borrower's viewpoint (the firm knows how the debt will cost) when the opportunity cost of equity is seen from the moneylender's viewpoint (the firm has a fund and tries to determine how much it can yield); the difference is all the more important that there are transaction costs. Moreover, the alternative standard investment must be well defined, perfectly known and unique. So, the evaluation of the opportunity cost is always speculative and based on hypotheses that can be contested. These critics about opportunity costs are very general when the market is not perfect, what corresponds to the reality. When you try to know the opportunity cost of an apartment that you have, you are embarrassed: either you try to compare it with other apartments around to know its rental value, either you try to rent it, just to see how it could be worth. However, in both cases, the result is not necessarily equal to the

real value that you could have found in a real process. In any case, the process is costly (transaction costs) and the value found itself depends on the time spent to calculate it.

The paper will prove that, in the case of mobility of capital, the pure profit maximization is not the objective function of the a controlled-by-owner firm because the firm maximizes a ratio pure profit / equity capital, where both pure profit and equity capital are variable. Then, the two functions, profit rate maximization (for controlled-by-owner firms) and pure profit maximization (for sleeping-owner firms), will be compared for their consequences on the economics of the firm (output, coordination, etc.).

II. The firm's objective function for controlled-by-owner firms, in case of mobility of capital

I consider the case of mobility of capital, i.e., in the long term or *ex ante*, before all investments are done, or when it faces to no sunk costs. Pure profit maximization seems more natural and intuitive for economists. The main argument really discriminant between pure profit maximization and other objectives is the owner's behavior argument. The profit distributed by firms is either in the utility of the owner, either in its budget constraint. So, when maximizing its utility, he wishes that the firm maximizes the profit because and, as the owner's behavior influences the firm's behavior, the firm will maximize the profit. This is obviously true in the simple case when the producer is an individual entrepreneur because the profit is also the revenue of the consumer: as the profit allows the producer to buy the goods that he will consume, he might maximize its profit. For a salaried employee this is false because its revenue comes from its wages, and not from the profit, except if the system of remuneration links the salary to the output or if the salaried employee is also an owner. In other cases, it could be more difficult.

A. One firm

Mas-Colell, Whinston and Green (1995, pp. 152-154) study the case of a unique firm, even multiproduct, owned by simple consumers. Adapted to our notations (Π simply replaces **p y**, where **y** is the vector of net output), their formula for one consumer i is:

$$\max U^i(\mathbf{x}^i), \text{ s.t. } \mathbf{p}\,\mathbf{x}^i \leq w^i + \theta^i\,\Pi \tag{1}$$

where $\mathbf{x}^i$ stands for the consumption vector of commodities by i, **p** for the fixed price vector, w^i for the wage received by i, Π for the firm's pure profit and $\theta^i = \frac{\Pi^i}{\Pi}$ for the share of the distributed pure profit of the firm that i receives, with $\Sigma_i\,\theta^i = 1$ (the agent has provided to the firm an equity capital equal to $\theta^i\,K_e$, when the total equity capital is equal to K_e); note that the question of profits that are not distributed is left aside. The agent tries to increase its budget and prefers to receive the maximal Stock Exchange revenue $D^i = \theta^i\,\Pi$, what implies that the unique firm maximizes its pure profit Π and the discussion could seem to be closed. Note that Mas-Colell, Whinston and Green (1995, pp. 153) also indicate that, for one firm and many goods, if prices depend on the production of the firm, the result can be undetermined because the function to be maximized by the firm changes with the preferences of consumers regarding to the goods. However, this case is not essential in practice because each owner buys a very small part of the production of the firm. Anyway, if one owner has a significant power on the firm, either by its shares or by its consumption, he can influence the firm's objective. See also Hart (1979) and Makowski (1983) for incomplete markets.

However, as the profit is distributed proportionally to the invested capital in a first approximation -- other forms of payment are neglected here -- one have $\theta^i = \frac{\Pi^i}{\Pi} = \frac{K_e^i}{K_e}$, but one cannot accept the hypothesis of fixity of coefficients θ^i. If the θ^i are assumed to be fixed this implies that K_e^i varies as K_e, that is to say the agent i subscribes to all new issues of firm's shares, what is very unrealistic. So, considering that both Π and K_e can vary simultaneously, the second influencing the first, the above program can be written as:

$$\max U^i(\mathbf{x}^i), \text{ s.t. } \mathbf{p}\,\mathbf{x}^i \leq w^i + K_e^i\,\pi \tag{2}$$

where $\pi = \frac{\Pi}{K_e}$ stands for the profit rate of the firm (K_e and π are controlled by the firm, when K_e^i is controlled by owner). Knowing its capital K_e^i, the agent i wishes that the firm maximizes π. Note that if the owner is a sleeping partner, its wishes cannot influence the firm and consequently this one maximizes its pure profit classically, because this objective corresponds to the maximization of the net value of the firm.

Remarks.

1) The profit rate can be written $\pi = \frac{\Pi}{K_e} = \frac{\Pi_a}{K_e} - t_e = \pi_f - t_e$. $\pi_f = \frac{\Pi_a}{K_e}$ stands for the *financial profit rate* which consists into comparing the accounting profit $\Pi_a = \Pi_g - t_d\, K_d$ to the equity capital: it is also referred as ***return on equity***, what is familiar to shareholders who use it to measure profitability. Even if the expression of the profit rate incorporates t_e, the opportunity cost of the equity capital plays no role in the maximization of π as long as t_e is fixed: knowing the difficulty to determine t_e, this is a great advantage on the pure profit maximization.

2) The profit rate fails to separate industrial decisions (determining the optimal output) to financial decisions (determining the optimal financial structure, i.e. the ratio of debts $\alpha = \frac{K_d}{K_d + K_e}$), what is often considered as a disadvantage in a theoretical viewpoint. However, the result is clear: the agent maximizes a ratio with the equity capital only in the denominator and not with all the capital. Moreover, the maximization of π determines the optimal output Q^*, and so the optimal equity capital $K_e(Q^*)$, *via* the function of debt $K_e(Q)$, but it determines also the optimal debt $K_d(Q^*)$, *via* the function of debt $K_d(Q)$ and the whole capital $K(Q^*) = K_e(Q^*) + K_d(Q^*)$. In other terms, the maximization of π determines at the same time the optimal output and the optimal financial structure. If the amount of debt is exogenously given, there is no more difficulty: simply the function of debt is a treated as a constant.

However, from the firm's viewpoint, these ratios fail to discriminate between the production decision (what level of output and how much capital as fund?) and the financing decision (what sharing between equity and debts?). The accounting profit is calculated by subtracting the cost of debts but not the cost of equities, so it depends of the main financial

decision: what is the source of financing for the firm? Taking into account the "lever effect" (the interest rate is lower than the profitability of the firm), the firm has interest to get into debt because this increases the dividend by share, what increases the satisfaction of shareholders and the value of the share. However, if all firms and markets are competitive, the profitability of firms becomes equal to the interest rate (no firms have interest to enter the industry) and the lever effect plays no role; this is the Modigliani-Miller theorem: it proves that the value of the firm is independent of the interest rate for competitive economies (Modigliani and Miller 1958). Jensen and Meckling (1976) prove that there is an optimal financial structure because of agency cost: agency costs spent by moneylenders to control the firm may prevent it to reduce the part of shareholders; also bankrupt costs imply an optimal financial structure. So, a large part of the financial structure of the firm is exogenously determined: when trying to calculate the optimal output level, the manager also decides about financing, what assumes an *ex ante* link between production and financing; clearly it is a function of which form is partly exogenous for most firms. Figure 1 shows that the ratio of debts, α, is near zero for a small amount of output: small or new firms cannot obtain loans from banks because the risk seems high (this is why special Stock Exchange Markets have been created in some countries for the risky capitals). When the scale of production increases a little, firms can obtain loans from banks because it can prove its profitability, and the function increases. Later the function becomes more endogenously determined and the form of the function becomes less clear: firms can either go on the Stock Exchange Market, either borrow more and more, depending on their financial policy. However, as the firm grows, its size increases and the conditions for a competitive economy become less valid: unspecification and instability increase. That this link may be complicated is not the problem: the problem is that the link can be exogenously determined for small or new firms and it can be unstable for large firms.

Figure 1. Financial structure: ratio of debts by total capital

As an illustration, a recent strategy of some companies in the Stock Market consists for a firm into buying back its own shares, to reduce the equity capital and so increase the price of the share (or to prevent this price to decrease). When a firm buys back its own shares, it

modifies the form of the function $\alpha(Q)$, or simply it shifts the function and the optimal output is modified, or it tries to reduce the exogenous parameter α: in this last case, the output is not modified, only the profit rate is increased (but the pure profit is also not modified).

3) What is here the justification of the use the pure profit? In money terms, the firm distributes the accounting profit $\Pi_a = \Pi_g - t_d K_d$. The opportunity cost of equity capital, $t_e K_e$ is distributed (the pure profit is always distributed): it is not retained by the firm even it is an opportunity cost, because it must pay the equity capital (in the other case, the owners would support a real cost on their capital because it would be never paid). So the agent receives the dividend $D_a^i = \theta^i \Pi_a$ and not $D^i = \theta^i \Pi$. So, if the owner is assumed to have no opportunity cost on its capital, he should wish the firm to maximize directly $\pi_f = \frac{\Pi_a}{K_e}$ instead of π. The owner maximizes π if he is assumed to support an opportunity cost on its capital, $t_e \theta^i K_e$ (by simplification, the same price t_e is used here for the firm and for the agent), and the "pure" dividend is considered: $D^i = D_a^i - t_e \theta^i K_e = \theta^i (\Pi_a - t K_e) = \theta^i \Pi$. ■

If there is only a unique owner, pure profit maximization and profit rate maximization coincide. In other cases, if the firm is profitable, each agent will maximize K_e^i naturally (under an eventual constraint $K_e^i \leq \check{K}_e^i$) but he wishes that the firm maximizes π: it is only a wish, because, as there is only one firm, this firm has no incentives to do that.

B. Many firms

There should be many firms to be sure that the wish is transformed into an obligation for firms, by selecting the firm that has the best profitability. This is why it is necessary to introduce many firms in a more general approach, extrapolated from the former to introduce the question of allocation of capital. Consider many firms, each denoted j, eventually multiproduct. Π^j is the profit of j, $\theta^{ij} = \frac{K_e^{ij}}{K_e^j}$ is the share of the profit of firm j that i receives,

with $\Sigma_i \theta^{ij} = 1$ (where K_e^j is the total equity capital invested inside firm j, and K_e^{ij} is the equity capital invested by i in j). One have:

$$\max U^i(\mathbf{x}^i), \text{ s.t. } \mathbf{p}^j \mathbf{x}^i \leq w^i + \Sigma_j \theta^{ij} \Pi^j \tag{3}$$

As above, the agent tends to prefer a budget as larger as possible, so he prefers that firms maximize their Stock Exchange revenue $\Pi^i = \Sigma_j \Pi^{ij} = \Sigma_j \theta^{ij} \Pi^j$, the sum of the profits that he receives. Note that this sum is not the simple sum of firm's profits: even if the shares θ^{ij} are fixed and for example equal to 1 (only one shareholder for each firm), it remains a sum of profits that is not *the profit* of a particular firm: some may become decreasing so long as some are increasing. As above, the shares are determined by the agent and are linked to the capital that can be invested in each firm. One can write:

$$\Pi^i = \Sigma_j K_e^{ij} \frac{\Pi^j}{K^j} = \Sigma_j K_e^{ij} \pi^j = \Sigma_j \lambda^{ij} K_e^i \pi^j \tag{4}$$

where $\pi^j = \frac{\Pi^j}{K_e^j}$ stands for the profit rate of firm j and $\lambda^{ij} = \frac{K_e^{ij}}{K_e^i}$ stands for the share of the capital of i that is invested in firm j. If there is at least one firm with a profitability higher than the interest rate, the agent i will select the firm j (or the firms if placed equal first) that has the best profitability (i.e., the best profit rate π^j) to maximize its Π^i: to do this, he puts $\lambda^{ij} = 1$ and $\lambda^{ij'} = 0$ for all j'. If all agents do the same thing (and they did if they are assumed to be similar to i), firm j must maximize its profit rate. Note that this result is obtained as the agent's capital, K_e^i, is fixed. To summarize, in a perfect world, the optimal behavior of the agent consists into ordering firms by decreasing order of profitability and investing on firms up to the point where the profitability becomes equal to the interest rate, depending on the possibilities authorized by the competition between owners, eventually by borrowing and lending the necessary money to the financial market. This proves that the owner must maximize the profit rate of firms. This is why I introduce in this paper the profit rate, $\pi = \frac{\Pi}{K_e}$, as an objective of the firm.

Remarks.

1) The comparison of two pure profits has no sense, when this comparison is correct for two profit rates: profit rates are allowing to take into account the alternative possibilities of

investment (Cf. the theory of investment), when pure profits cannot. However, let's take an example. A pure profit of 1 M$ for the construction of a plant with a capital of 20 M$ cannot be compared to the pure profit of $1000 to install a small itinerant pizzeria with a capital of $10000, when a profit rate of 5% can be compared always to a profit rate of 10%. The same difficulty exists inside investment theory: two intern rates of return (rate that cancels out the net actual value) can be compared always when two actual net values can be compared only if the two projects are similar in size.

2) Additionally, the capital invested by an agent can be considered also as variable. If the amount that the agent can invest exceeds the capacity of one firm j', one can have $\theta^{ij'} = 1$ and the agent becomes the unique owner of this firm; but one can also have $\theta^{ij'} < 1$ if there is a competition between shareholders (even if the capital of the agent is lower than the capacity of firm j', he can be obliged to find another firm is there is a competition); in both cases, the agent will be obliged to limit its investment. Either he can search another firm to invest the rest of its capital but at a lower profitability (what is acceptable only is the profitability of this second firm is itself higher than the interest rate of the market), either he can lend the exceeding money to the financial market. As the agent also can borrow the quantity of money that he wants to invest in firms if the financial market is efficient, the equity capital becomes completely variable. ■

C. *Ex post*

Three situations have been considered: the long run, no sunk costs and *ex ante*. In the two first cases, all have been said, but in the third case, what happens *ex post*? The reasoning could be more complex. When shares have been issued, the agents have computed their investment by selecting the most profitable firm, obliging the firm to adopt profit rate maximization. However, what happens after the creation of the firm, when K_e is now considered as fixed by everybody? Obviously, if K_e is fixed, the agent wishes the firm to maximize the profit; both objectives of the firm (maximizing the pure profit, $\Pi = \Pi_a - t_e\, K_e$ and maximizing the profit rate, $\pi = \frac{\Pi}{K_e} = \frac{\Pi_a}{K_e} - t_e$), become equivalent to the maximization of the accounting profit Π_a. So, there is no betrayal after the initial issue of shares if the firm maximizes the pure profit

instead of the profit rate. Remember that in any case, if the firm's results are disappointing, the agent can sell its shares to another shareholder at a lower price even if in the firm's viewpoint the investment is done irrevocably; if the "game" has only one period, the firm can ignore the agent's wishes, but if it is repeated, the firm must respect its commitment to preserve the trust of shareholders for future issues.

But this is too simple. *Ex ante*, or even *ex post* if no sunk costs, as the firm can choose or adapt its production capacity to what is exactly required, the firm is always on its Viner's "long term" cost function (even it is not the long term), because the Viner's "long-term" cost is always lower or equal to the short-term cost for a same level of output. So, it is false to say that all discussions about the opportunity cost of K_e – and its associated pure profit – are closed when K_e is fixed: the cost function changes from the Viner's "long term" cost function to the short term cost function C_g^{ST}. So, the *ex ante* maximization program, $\max \pi$, will determine $K(Q^*) = K_e(Q^*) + K_d(Q^*)$, the level of capital as fund that is required, where Q^* is the *ex ante* output that is solution of the program. Then, $K(Q^*)$ will be fixed in the *ex post* short term and appears as a constraint for the *ex post* short term maximization program:

$$\max \Pi_g^{ST}(Q), \text{ s.t. } Q \in [Q_{\min}, Q_{\max}] \tag{5}$$

where $\Pi_g^{ST}(Q) = R - C_g^{ST}$ is the short term gross profit. Note that alternately, one can introduce into the gross profit, at each period, the losses caused by the potential transfer of the assets corresponding to the sunk costs: $\Pi_g - s\,K^0$, where K^0 is the capital as fund formerly installed and s is the cost by unit of capital caused by its transfer.

Remark. A link between the output and K is assumed. The form $Q \to K$, i.e., $K(Q)$, is considered here, but the inverse function $K \to Q$, i.e., $Q(K)$, could be also, so the implicit form $f(K, Q) = 0$ could be more satisfactory. Note that $K \geq C_g$. The hypothesis of variability of capital *ex ante* is not so new. For example, see what Tirole (1989, pp. 214-217) calls the "Stackelberg-Spence-Dixit model", with an explicit function $\Pi(K_r)$, but it is the real capital that is variable here, not the capital as fund. ■

The bounds $Q_{\min}$ and $Q_{\max}$ depend on $K(Q^*)$, with logically $Q_{\min} \leq Q^* \leq Q_{\max}$. The short term solution, denoted Q^{**}, can be different to the *ex ante* solution Q^* but it is not sure that $Q^{**} > Q^*$ or $Q^{**} < Q^*$ at this step. Note that the output Q^* is virtual, the output Q^{**} is

realized. Even if the amount of capital is fixed when the firm maximizes the gross profit in the *ex post* short term, the initial profit rate maximization has determined the scale of production. To summarize, the scale of production is determined *ex ante* by "$\max \pi$" for a controlled-by-owner firm and by "$\max \Pi$" if the firm has sleeping owners; *ex post* "$\max \Pi_g$" determines the following optimal outputs inside the limits imposed by $K(Q^*)$, i.e., by the function of capital $K(Q)$ and the *ex ante* optimal output. As the profit rate maximization theory determines the level of capital and not the real output, it can be said that it constitutes a microeconomic theory of demand of capital as fund.

In the following of this paper, I will show that the behavior of firms is largely modified if firms are assumed to maximize the profit rate instead of the pure profit. The two functions "$\max \pi$" and "$\max \Pi$" will be compared: profit rate maximization differs of pure profit maximization for its consequences on the output of the firm and on coordination between firms, in perfect and imperfect competition.

III. The economics of the profit-rate-maximizing firm

Two extreme cases will be studies: monopoly obviously but also perfect competition because firms do not interact one with other in this case. The profit-rate-maximizing firm will be always compared to the pure-profit-maximizing firm, with a variable capital as fund, in the long run, *ex ante* or when no sunk costs. According to the above developments, the first case, "$\max \Pi/K_e$", corresponds to a firm controlled actively by its owners, the second, "$\max \Pi$", that is the more familiar, to a firm with sleeping owners.

A. The reference case, monopoly

First, we shall study a monopoly as a formal general case for the calculation of the firm's equilibrium; other cases, as perfect competition, will be derived from it. As said above, even a unique monopoly must maximize its profit rate as soon as there are many owners (if the owner

is alone, both objectives coincide). In this paper, for simplicity, only the case of the monoproduct firm will be studied.

Proposition 1. The controlled-by-owner monopoly produces up to the point where marginal accounting profit is equal to average accounting profit multiplied by the relative elasticity of equity capital; or the firm produces up to the point where the relative elasticity of the accounting profit is equal to the relative elasticity of the equity capital:

$$\Pi'_a = e_{K_e/Q}\,\bar{\Pi}_a \Leftrightarrow e_{\Pi_a/Q} = e_{K_e/Q} \tag{6}$$

in which $e_{\Pi_a/Q}$ is the relative elasticity of the equity capital to the output, $e_{K_e/Q}$ is the relative elasticity of the equity capital to the output and $\bar{\Pi}_a = \dfrac{\Pi_a}{Q}$ is the average accounting profit.

Proof. It is immediate by deriving the profit rate $\pi = \dfrac{\Pi}{K_e} = \dfrac{\Pi_a}{K_e} - t_e$ with respect to Q :

$$\Pi'_a = \Pi_a \frac{K'_e}{K_e} \Leftrightarrow \Pi'_a = e_{K_e/Q}\,\bar{\Pi}_a \ \blacksquare \tag{7}$$

Note. It is equivalent to say that $\Pi' = \Pi \dfrac{K'_e}{K_e} \Leftrightarrow \Pi' = e_{K_e/Q}\,\bar{\Pi} \Leftrightarrow e_{\Pi/Q} = e_{K_e/Q}$, but the above formulation is adopted because the accounting profit is more familiar to firms.

We have also at optimum:

$$\Pi' = \pi\, K'_e \Leftrightarrow R' = C' + \pi\, K'_e \tag{8}$$

where $C = C_g + t_d\, K_d + t_e\, K_e$ is the total cost (but remember that C_g is formally similar to a long term function of cost because the capital is variable). The left member of this expression means that the pure profit of an extra unit of output is equal to the return of an extra unit of equity capital at the rate π (that is the value taken by the profit rate at the optimum). The right member of this expression also means: the firm equalizes its marginal receipt to its marginal cost augmented with the return of an extra unit of equity capital at the rate π. Whereas the optimum of the sleeping-owner monopoly (pure profit maximizing) is given by $\Pi' = 0 \Leftrightarrow R' = C'$: the firm equalizes its marginal receipt with its marginal cost, what is very familiar. This correctly corresponds to $e_{K_e/Q} = 0$ when applied on the above results.

Remark. A particular case of profit rate monopoly is interesting for its simplicity: the fixed coefficient of equity capital: the controlled-by-owner monopoly, with a fixed coefficient of capital $K_e = k\,Q$ with $k > 0$, produces up to the point where marginal accounting profit is equal to average accounting profit (or to the point where average accounting profit is at a maximum), $\Pi'_a = \bar{\Pi}_a$, because in this case $e_{K_e/Q} = 1$. So, with a fixed coefficient of equity capital k, a controlled-by-owner monopoly produces and invests so that the pure profit contributed by an extra unit is greater than or at least equal to the average pure profit yielded by other units. Instead of comparing the pure profit brought by the last unit of output, the nth, to zero, the firm compares the pure profit brought by the last unit of output to the average pure profit brought by former units, n - 1, n - 2, ... Note that this optimum is similar to the optimum of a monopoly that maximizes the *average* pure profit. Note that this is a very special case, far from an ordinary case of coefficient capital, because only the equity capital is assumed to be linearly linked to the output, not the debts or the whole capital (or the real capital). More realistically, it could be assumed that the whole capital is linearly linked to the output, $K(Q) = k\,Q$, what implies $K_e(Q) = [1 - \alpha(Q)]\,K(Q) = [1 - \alpha(Q)]\,k\,Q$: the results obtained with a fixed coefficient of equity capital do not remain valid. ■

We may compare the optimum of profit rate for a controlled-by-owner monopoly with the classical microeconomic optimum (of pure profit, for a sleeping-owner monopoly). Denoting Q^{Π} as the classical pure profit equilibrium and Q^{π} as the rate of profit equilibrium, Q^{Π} is before Q^{π} as shown by the figure 2, for simplicity with a fixed coefficient of equity capital:

Figure 2. Monopoly (here, with fixed coefficient of equity capital)

Theorem 1. Consider the more probable case $K'_e(Q^{\pi}) > 0$, in which Q^{π} is the optimal quantity that maximizes the rate of profit π. A controlled-by-owner monopoly (profit rate maximization) has a lower optimal output level than a sleeping-owner monopoly (pure profit maximization) when the financial profit rate is higher than t_e at the optimum of profit rate:

$$\pi(Q^{\pi}) > 0 \Leftrightarrow \pi_f(Q^{\pi}) > t_e \Rightarrow (\max \pi) \text{ occurs before } (\max \Pi)$$

If $\pi_f(Q^\pi) < t_e$, then (max π) occurs after (max Π): it is less probable because in this case the firm leaves the sector. All results are reversed if $K'_e(Q^\pi) < 0$.

Proof. The first order condition of optimality to maximize the pure profit Π is $\Pi'(Q) = 0$. The first order condition of optimality to maximize the profit rate π is $\Pi'(Q) - \pi(Q)\, K'_e(Q) = 0$.

Consider the curve $f_\pi(Q) = \Pi'(Q) - \pi(Q)\, K'_e(Q)$. Write Π' as a function of f_π: $\Pi'(Q) = f_\pi(Q) + \pi(Q)\, K'_e(Q)$; assume that Π' is decreasing. Denote Q_Π as the point of maximization of pure profit, such as $\Pi'(Q^\Pi) = 0$ and denote Q_π as the solution of the maximization of the profit rate, such that $f_\pi(Q^\pi) = 0$.

1) Suppose that $K'_e(Q_\pi) > 0$ and $\pi(Q_\pi) > 0$. Then, $\Pi'(Q_\pi) = \pi(Q_\pi)\, K'_e(Q_\pi) > 0 = f_\pi(Q_\pi)$. Near Q^π the curve f_π is under curve Π'. Thus, Q_Π must be greater than Q_π. At the limit, if Π' is vertical near Q_Π, Π' will cut the x-axis at Q_π and then $Q_\Pi = Q_\pi$. Remark that it is sufficient that $K'_e(Q_\pi) > 0$. See figure 3.

Figure 3. Mathematical comparison of optima

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2) If $\pi(Q_\pi) < 0$ and $K'_e(Q_\pi) > 0$, the results are reverted and near to Q_π, curve f_π is over curve Π'. Thus, Q_Π must be smaller than Q_π.■

Following this theorem, the monopolistic firm produces less, so at a higher price, when it maximizes the profit rate. Thus the monopoly is harsher with profit rate maximization. So monopoly is reinforced: the "max π" firm has a more monopolistic behavior than the "max Π" firm. This favors the "max π" theory.

It would be necessary to study second order conditions. However, the mathematical aspects (convexity, etc.) are similar in pure profit and in profit rate maximization, so this step will be left aside.

Now, we will prove that coordination among firms could fail when firms are controlled by owners (i.e., profit rate maximization) instead of having sleeping owners (i.e., pure profit

maximization). Some cases of coordination failures have been studied in the literature (Heller, 1986) (Bagwell and Ramey, 1994).

B. Perfect competition

Consider a single firm in a market in perfect competition.

Proposition 2. In perfect competition the optimum of a controlled-by-owner firm is given by:

$$\bar{p} - C' = e_{K_e/Q} \left(\bar{p} - \bar{C}\right) \tag{9}$$

Proof. We apply above results, with a price independent of Q, i.e., $p = \bar{p}$.

For sleeping-owners firms, $e_{K_e/Q} = 0$ and $\bar{p} - C'$. As seen above, the output of controlled-by-owner firm is smaller than for sleeping-owner firms. So, there could be more firms in the first case (if the global supply does not change from one objective to the second); thus interaction among firms could be lower and competition greater (even if the concept of competition becomes particular).

Proposition 3. In perfect competition, it is sufficient that there is a fixed coefficient of equity capital (or, more generally, $e_{K_e/Q} = 1$), to do that price plays no role in the equilibrium of the controlled-by-owner firm. See figure 5.

Proof. We have: $e_{K_e/Q} = 1$. Thus $C' = \bar{C}$ at optimum: the firm produces up to the point where the average cost $\bar{C}$ is minimum. ■

The following conclusions are deduced from proposition 3 for controlled-by-owner firms with coefficient of equity capital.

1) The optimum does not depend on price; thus, there is not coordination in the market. Each firm is alone, doing nothing with the price. Price is no more a signal. We need no more additional theories to explain that the output is independent to the price (like *cost plus pricing*, U-curves of cost, etc.). As a corollary, the output level does not vary if price varies (as long as

the profit remains positive: when it becomes negative, the firm leaves the sector as usual). Consequently, output level does not matter with taxation or other manipulations of prices.

2) The individual supply curve is vertical, even in perfect competition. The global supply curve is completely inelastic and the market is in global equilibrium, following a classical pattern, as shown in figure 4.

Figure 4. Global equilibrium in short-run perfect competition

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This proves that a global equilibrium is compatible with a lack of coordination by price, unlike good sense tends to believe. It must be noted that, unlike in the classical pure profit maximization (where price is determined by the intercept of the global curve of demand with the global curve of supply), it is not the global equilibrium that determines individual equilibrium of the firm, but it is the individual equilibrium that determines global equilibrium by adding up: price is only a consequence of the global equilibrium, without an impact on individual level (except for the amount of firm's earnings but not for its optimal output). Moreover, if the demand curve is also completely inelastic, there may be no equilibrium; if it is only very inelastic, the price may be very variable to slipping of demand (without any effect on the optimal firm's output, obviously).

Remarks.

1) As said before, in the *ex post* short run, profit rate maximization or pure profit $\Pi^{ST} = \Pi_g^{ST} - t\,\bar{K}$·e both identical to gross profit maximization (see figure 5):

and $\pi^{ST} = \dfrac{\Pi_g^{ST}}{\bar{K}}$, so $\max \Pi^{ST} \Leftrightarrow \max \pi^{ST} \Leftrightarrow \max \Pi_g^{ST}$, where $\Pi_g^{ST} = R - C_g^{ST}$, so at the optimum, $R' = C_g^{ST}$ classically. Here the *ex post* short term equilibrium is higher the *ex ante* profit rate equilibrium but it is lower than the *ex ante* pure profit equilibrium: $Q_\pi < Q^{**} < Q_\Pi$.

However, remember that the main decision is taken *ex ante*: how much capital to invest.

Figure 5. Equilibrium of the controlled-by-owner firm in perfect competition with coefficient of equity capital

2) Only in the case of capital coefficient, the optimum coincides with the classical point of firm's long-term equilibrium which would be situated at the minimum of $\bar{C}$. Even if the number of firms remains fixed *ex ante*, before all irreversible decisions or *ex post* but with no sunk costs, the number of firms is variable in the long-run and $\bar{p}$ decreases as new firms enter the sector; the movement stops at $\bar{p}^{LT}$ which is the entry point in the sector, corresponding to the minimum of $\bar{C}$. Note that both objective functions are exactly equivalent concerning the condition of entry into the sector, that is when deciding whether the firm should or should not produce: $\Pi \geq 0$ and$\pi \geq 0 \Leftrightarrow \pi_f \geq t_e$ (this second formulation with π_f seems perhaps more natural and intuitive). Also, note that as $\Pi = \Pi_g - t\,K$ and $\pi = \dfrac{\Pi_g - t\,K}{(1-\alpha)\,K}$, the entry condition in the sector is not dependent of α. ■

C. Optimal input combination and efficiency in perfect competition

Now, let's see what happens at the side of inputs. Consider n factors, used in quantity q^i at a fixed price $\bar{p}^i$. The function of production is $Q(q^1,...,q^n)$. The costs of production are $C_g(q^1,...,q^n) = \bar{\mathbf{p}}\,\mathbf{q} = \sum_{i=1}^{n} \bar{p}^i\,q^i$. Remind that the capital $K(q^1,...,q^n)$ is not considered as a factor, even if the cost of the capital (interest of debts and opportunity cost of equity) is a part of the total cost (at the interest rate t).

Classically, for a sleeping-owner firm, a "max Π" firm, each factor is paid at its *net marginal productivity* $\bar{p}\,Q^{i\prime} - t_d\,K_d^{i\prime} - t_e\,K_e^{i\prime}$:

$$\frac{\partial \Pi}{\partial q^i} = 0 \Leftrightarrow \bar{p}_{\Pi}^{i} = \bar{p}\,Q^{i\prime} - t_d\,K_d^{i\prime} - t_e\,K_e^{i\prime} \tag{10}$$

where $Q^{i\prime} = \dfrac{\partial Q}{\partial q^i}$ stands for the marginal productivity of i, $K_d^{i\prime} = \dfrac{\partial K_d}{\partial q^i}$ stands for the marginal amount of debts required by i, $K_e^{i\prime} = \dfrac{\partial K_e}{\partial q^i}$ stands for the marginal amount of equity capital required by i and $\Pi = \bar{p}\, Q - \sum_{i=1}^{n} \bar{p}^i\, q^i - t_d\, K_d - t_e\, K_e$.

Thus the sleeping-owner firm pays its factors at their gross marginal productivity in value, minus the marginal amount of capital required by this factor valorized at the rates t_d *and* t_e. If Q and K are homogenous functions of degree 1, so $\sum_{i=1}^{n} q^i\, Q^{i\prime} = Q$, $\sum_{i=1}^{n} q^i\, K_d^{i\prime} = K_d$ and $\sum_{i=1}^{n} q^i\, K_e^{i\prime} = K_e$, the *net product* $Q - t_d\, \dfrac{K_d}{\bar{p}} - t_e\, \dfrac{K_e}{\bar{p}}$ is exhausted:

$$\sum_{i=1}^{n} q^i\, \bar{p}^i = \sum_{i=1}^{n} q^i \left(\bar{p}\, Q^{i\prime} - t_d\, K_d^{i\prime} - t_e\, K_e^{i\prime} \right) = \bar{p}\, Q - t_d\, K_d - t_e\, K_e \tag{11}$$

In the other hand, for a controlled-by-owner firm, a "max π" firm, each factor is paid at its *modified net marginal productivity* $\bar{p}\, Q^{i\prime} - \pi\, K_e^{i\prime}$:

$$\frac{\partial \pi}{\partial q^i} = 0 \Leftrightarrow \bar{p}_\pi^i = \bar{p}\, Q^{i\prime} - t_d\, K_d^{i\prime} - t_e\, K_e^{i\prime} - \pi\, K_e^{i\prime} = \bar{p}_\Pi^i - \pi\, K_e^{i\prime} \tag{12}$$

with $\pi = \dfrac{\bar{p}\, Q - \sum_{i=1}^{n} \bar{p}^i\, q^i - t_d\, K_d - t_e\, K_e}{K_e}$.

Thus the controlled-by-owner firm pays its factors at their marginal productivity in value, minus the marginal amount of equity capital required by this factor valorized at the rate π.

<u>Proposition</u> 4. If $\pi > 0 \Leftrightarrow \pi_f > t_e$ (normal situation), then the remuneration of each factor is lower for a controlled-by-owner firm than for a sleeping-owner firm: each factor is no longer paid at its net marginal productivity. Therefore, less of each factor is used when the firm is controlled by owners (and $Q^\pi < Q^\Pi$).

<u>Proof</u>. We provide a graphical proof with figure 6, denoting q_π^i as the quantity of factor i used when "max π" and q_Π^i as the quantity of factor i used when "max Π". ■

Figure 6. Optimal quantities of factors

Corollary. In the case where the functions Q and K are homogenous functions with degree 1, if $\pi > 0$, the *net modified product* $\hat{Q} = Q - \frac{t_d}{\bar{p}} K_d - \frac{t_e}{\bar{p}} K_e - \frac{\pi}{\bar{p}} K_e$ is completely exhausted:

$$\sum_{i=1}^{n} q^i \bar{p}^i = \sum_{i=1}^{n} q^i \left(\bar{p}\, Q^{i\prime} - t_d\, K_d^{i\prime} - t_e\, K_e^{i\prime} - \pi\, K_e^{i\prime} \right) = \bar{p}\, Q - t_d\, K_d - t_e\, K_e - \pi\, K_e \tag{13}$$

The criterion of efficiency is changed for a controlled-by-owner firm: an efficient system is no more a system where "marginal cost equals marginal revenue", it becomes a system where "the average profit is maximum" (or more generally "marginal profit equals average profit multiplied by elasticity of capital"). These developments prove that it is possible to build a general equilibrium, beyond Borisov's work (1993). In all sectors of economic life, the new criterion of efficiency leads to a limited service but also fewer expenditures.

D. Duopoly

Let's take a look at the case of duopoly (competition in quantities) as the most simple form of oligopoly; the result will be generalized to oligopoly later. The demand for the sector with two firms 1 and 2 is $D(Q^1 + Q^2)$. The inverse demand $p = D^{-1}(Q^1 + Q^2)$ is assumed to be decreasing. The total revenues of each firm are $R^i(Q^1, Q^2) = Q^i\, D^{-1}(Q^1 + Q^2)$. The production costs of each firm are $C_g^i(Q^i)$. The total capital (equity plus debt) of each firm is $K_d^i(Q) + K_e^i(Q)$. The pure profits of each firm are $\Pi^i(Q^1, Q^2) = R^i(Q^1, Q^2) - C_g^i(Q^i) - t_d\, K_d^i(Q^i) - t_e\, K_e^i(Q^i)$. The profit rates are:

$$\pi^i(Q^1, Q^2) = \frac{\Pi^i(Q^1, Q^2)}{K_e^i(Q^i)} \tag{14}$$

For the classical maximization of pure profit, we solve the equations $\frac{\partial\, \Pi^i(Q^1, Q^2)}{\partial\, Q^i} = 0$. We obtain the following system:

$$\left\{ \begin{array}{l} \dfrac{\partial \Pi^1(Q^1,Q^2)}{\partial Q^1} = 0 \\ \dfrac{\partial \Pi^2(Q^1,Q^2)}{\partial Q^2} = 0 \end{array} \right\} \tag{15}$$

$$\Leftrightarrow \left\{ \begin{array}{l} \dfrac{\partial D^{-1}(Q^1+Q^2)}{\partial Q^1} Q^1 + D^{-1}(Q^1+Q^2) - C_g^{1\prime}(Q^1) - t_d\, K_d^{1\prime}(Q^1) - t_e\, K_e^{1\prime}(Q^1) = 0 \\ \dfrac{\partial D^{-1}(Q^1+Q^2)}{\partial Q^2} Q^2 + D^{-1}(Q^1+Q^2) - C_g^{2\prime}(Q^2) - t_d\, K_d^{2\prime}(Q^2) - t_e\, K_e^{2\prime}(Q^2) = 0 \end{array} \right\} \tag{16}$$

Generally, except special cases, the system has two reaction functions: the solution is found by the intersect of two curves (see the example later for a very simple linear demand function). These reaction functions allow coordination between firms, by way of conjectural variations denoted $v^i = \dfrac{\partial Q^j}{\partial Q^i}$. There are the questions of uniqueness (Gaudet and Salant, 1991), existence or convergence of equilibrium (Novshek, 1985), and discontinuity in reaction functions involving a non-existent equilibrium (Roberts and Sonnenchein, 1977). We do not enter the debate about the interest of reaction functions and that they are essentially concerned with competition in price (with an auctioneer). However, it must be noticed that profit rate maximization is compatible obviously with a model including capacity constraints (as seen above, it is possible to maximize profit rate with bounds on the capital).

Now, assume that each firm maximizes its profit rate. The necessary conditions of optimality are:

$$\frac{\partial \pi^i(Q^1,Q^2)}{\partial Q^i} = 0 \text{ for all } i \tag{17}$$

$$\Rightarrow \left\{ \begin{array}{l} \dfrac{\partial \Pi^1(Q^1,Q^2)}{\partial Q^1} = e_{K_e^1/Q^1} \dfrac{\Pi^1(Q^1,Q^2)}{Q^1} \\ \dfrac{\partial \Pi^2(Q^1,Q^2)}{\partial Q^2} = e_{K_e^2/Q^2} \dfrac{\Pi^2(Q^1,Q^2)}{Q^2} \end{array} \right\} \tag{18}$$

with $e_{K_e^i/Q^i} = \dfrac{\dfrac{d\,K_e^i(Q^i)}{K_e^i(Q^i)}}{\dfrac{d\,Q^i}{Q^i}}$.

It is easy to find counter-examples in which the reaction functions vanish with the profit rate maximization, while they do not vanish with the maximization of pure profit. Reaction functions "vanish" if the first equation excludes Q^2, and the second equation excludes Q^1.

Proposition 5. In duopoly, between controlled-by-owner firms, the reaction functions vanish if the inverse function of demand is linear, $D^{-1}(Q^1, Q^2) = d - a\,Q^1 - b\,Q^2$, and if the elasticity of equity capital with respect to the output level is equal to 1 for each firm, $e_{K_e^i/Q^i} = 1$ for all i (this occurs for a coefficient of capital: $K_e^i(Q^i) = k^i\,Q^i$ for any i).

Note that only the condition on the equity capital is required, not on debts.

Proof. With these conditions, the system of reaction functions becomes:

$$\left\{\begin{array}{c} a\,Q^1 + b\,v^1\,Q^1 + C_g^{1\prime}(Q^1) + t_d\,K_d^{1\prime}(Q^1) + t_e\,K_e^{1\prime}(Q^1) = \\ \bar{C}_g^1(Q^1) + t_d\,\bar{K}_d^1(Q^1) + t_e\,\bar{K}_e^1(Q^1) \\ \\ a\,v^2\,Q^2 + b\,Q^2 + C_g^{2\prime}(Q^2) + t_d\,K_d^{2\prime}(Q^2) + t_e\,K_e^{2\prime}(Q^2) = \\ \bar{C}_g^2(Q^2) + t_d\,\bar{K}_d^2(Q^2) + t_e\,\bar{K}_e^2(Q^2) \end{array}\right\} \tag{19}$$

and if there is a true coefficient of capital ($K_e^i(Q^i) = k^i\,Q^i$ for any i):

$$\left\{\begin{array}{l} a\,Q^1 + b\,v^1\,Q^1 + C_g^{1\prime}(Q^1) + t_d\,K_d^{1\prime}(Q^1) = \bar{C}_g^1(Q^1) + t_d\,\bar{K}_d^1(Q^1) \\ \\ a\,v^2\,Q^2 + b\,Q^2 + C_g^{2\prime}(Q^2) + t_d\,K_d^{2\prime}(Q^2) = \bar{C}_g^2(Q^2) + t_d\,\bar{K}_d^2(Q^2) \end{array}\right\} \tag{20}$$

As each equation is independent to the other, there are no more reaction functions. Thus, Q^1 does not depend on Q^2 and reciprocally. ■

In fact, reaction functions continue to exist: they are vertical and with no interest because each firm does not take into account the other firm to determine its own output (see figure 7). This is an interesting case because all ordinary models of duopoly have a reaction function, even though it is known that duopoly does not always have a stable solution (and even if dynamics may be complicated or chaotic (Rand, 1978) (Piatecki, 1994)) or even this solution can be non-unique or cannot exist. Here are some counter-examples of non-existence when maximizing profit rate.

Figure 7. Reaction functions of a profit-rate-maximizing duopoly with linear demand and coefficient of equity capital

Note that for a pure-profit-maximizing duopoly, the reaction functions do not vanish, even if demand is linear with a coefficient of equity capital:

$$\left\{ \begin{array}{l} -a\,Q^1 - b\,v^1\,Q^1 + d - a\,Q^1 - b\,Q^2 - C_g^{1\prime}(Q^1) - t_d\,K_d^{1\prime}(Q^1) - t_e\,k^1 = 0 \\ -a\,v^2\,Q^2 - b\,Q^2 + d - a\,Q^1 - b\,Q^2 - C_g^{2\prime}(Q^2) - t_d\,K_d^{2\prime}(Q^2) - t_e\,k^2 = 0 \end{array} \right\} \tag{21}$$

When reaction functions are canceled, that is when the solution is not found by the intersect of two functions, each firm acts independently of the other to find the optimal solution (even it has an opinion about the other firm). In a dynamic perspective, each firm never observes the other firm and there is no iterative adjustment ("immediate" solution). Finally, there is no coordination among firms and no game. We can say that duopoly is degenerated. Nash equilibrium exists anyway but it is special. As the slope of reaction functions is vertical it does not matter to know if firms are strategic substitutes or strategic complements to the mind of (Bulow, Geanakoplos and Klemperer, 1985).

The monopoly can be mixed, that is it can a duopoly where each firm may have a different objective, depending on it is controlled or not by owners. In itself, the idea of a mixed duopoly or oligopoly, with one profit-maximizing firm and one revenue-maximzing firm for example, is not new (De Fraja and Delbono, 1990), (Barros, 1994 and 1995). Barros insists on contracts and incentives on managers. Other types of mixed duopoly can be considered. For example, following Baumol one a firm may want to maximize its revenue or want to equilibrate its budget when the other is profit rate-rate maximizing, or want to maximize the consumer surplus. For example, one firm will be controlled by owners (profit-rate-maximizing, assume that it is firm 1) and the second will have sleeping owners (pure-profit-maximizing, assume that it is firm 2):

$$\left\{\begin{array}{c} \dfrac{\partial D^{-1}(Q^1+Q^2)}{\partial Q^1} Q^1 + D^{-1}(Q^1+Q^2) - C_g^{1\prime}(Q^1) - t_d\, K_d^{1\prime}(Q^1) - t_e\, K_e^{1\prime}(Q^1) \\ = e_{K_e^1/Q^1} \dfrac{\Pi^1(Q^1,Q^2)}{Q^1} \\ \dfrac{\partial D^{-1}(Q^1+Q^2)}{\partial Q^2} Q^2 + D^{-1}(Q^1+Q^2) - C_g^{2\prime}(Q^2) - t_d\, K_d^{2\prime}(Q^2) - t_e\, K_e^{2\prime}(Q^2) = 0 \end{array}\right\} \quad (22)$$

Following theorem 1, the controlled-by-owner firm 1 has a lower output than if is a sleeping-owner firm (the left side of the first equation is decreasing by respect to Q^1, the left side of the first equation is decreasing by respect to Q^2, the right side of the first equation is positive when the right side of the second equation is zero). So, if firm 1 is controlled-by-owner instead of sleeping-owner when firm 2 remains sleeping-owner in any case, firm 1 has a lower output and firm 2 has a higher output (see figure 8).

Figure 8. Reaction functions of a mixed duopoly

If the demand is linear and $e_{K_e^i/Qi} = 1$, one have:

$$\left\{\begin{array}{c} a\,Q^1 + b\,v^1\,Q^1 + C_g^{1\prime}(Q^1) + t_d\, K_d^{1\prime}(Q^1) + t_e\, K_e^{1\prime}(Q^1) \\ = \bar{C}_g^1(Q^1) + t_d\, \bar{K}_d^1(Q^1) + t_e\, \bar{K}_e^1(Q^1) \\ a\,Q^1 + (a\,v^2 + 2\,b)\,Q^2 - d + C_g^{2\prime}(Q^2) + t_d\, K_d^{2\prime}(Q^2) + t_e\, K_e^{2\prime}(Q^2) = 0 \end{array}\right\} \quad (23)$$

and the reaction function of firm 1 is vanishing (i.e., it is vertical) but the reaction function of firm 2 remains. It is true even if both firms have a coefficient of equity capital:

$$\left\{\begin{array}{c} a\,Q^1 + b\,v^1\,Q^1 + C_g^{1\prime}(Q^1) + t_d\, K_d^{1\prime}(Q^1) = \\ \bar{C}_g^1(Q^1) + t_d\, K_d^1(Q^1) \\ a\,Q^1 + (a\,v^2 + 2\,b)\,Q^2 - d + C_g^{2\prime}(Q^2) + t_d\, K_d^{2\prime}(Q^2) + t_e\, k^2 = 0 \end{array}\right\} \quad (24)$$

One question remains. Does collusion is possible with profit rate maximization? If yes, a certain form of coordination remains possible. Classically, the collusion equilibrium is found by the maximization of the sum of the pure profit functions: max $\Pi(Q^1+Q^2)$, where,

$$\Pi(Q^1+Q^2)=\Pi^1(Q^1)+\Pi^2(Q^2)=$$

$$D^{-1}(Q^1+Q^2)\,(Q^1+Q^2)-C_g^1(Q^1)-C_g^2(Q^2)-t_d\left[K_d^1(Q^1)+K_d^2(Q^2)\right]-t_e\,[K_e^1(Q^1)+K_e^2(Q^2)] \tag{25}$$

and then one solve $\dfrac{\partial\Pi(Q^1+Q^2)}{\partial Q^i}=0$ for all i.

With the profit rate maximization, one calculate max $\pi(Q^1+Q^2)$ with,

$$\pi(Q^1+Q^2)=$$

$$\frac{D^{-1}(Q^1+Q^2)\,(Q^1+Q^2)-C_g^1(Q^1)-C_g^2(Q^2)-t_d\left[K_d^1(Q^1)+K_d^2(Q^2)\right]-t_e\,[K_e^1(Q^1)+K_e^2(Q^2)]}{K_e^1(Q^1)+K_e^2(Q^2)} \tag{26}$$

and one solve $\dfrac{\partial\pi(Q^1+Q^2)}{\partial Q^i}=0$ for all i.

Mixed collusion, i.e., collusion between a controlled-by-owner firm and a sleeping-owner firm is not possible. Assume that firm 1 is controlled-by-owner and firm 2 is sleeping-owner:

$$\Pi(Q^1+Q^2)=$$

$$D^{-1}(Q^1+Q^2)\,(Q^1+Q^2)-C_g^1(Q^1)-C_g^2(Q^2)-t_d\left[K_d^1(Q^1)+K_d^2(Q^2)\right]-t_e\left[K_e^1(Q^1)+K_e^2(Q^2\right.$$

$$\pi(Q^1+Q^2)=\frac{\Pi(Q^1+Q^2)}{K_e^1(Q^1)}$$

Firm 1 will try to maximize $\pi(Q^1+Q^2)$ when firm 2 will try to maximize $\Pi(Q^1+Q^2)$.

IV. Conclusion

In this paper, the usual theory of the firm is reexamined by considering the capital as variable, either in the long run, or *ex ante*, or even *ex post* if no sunk costs. After reminding in introduction that pure profit maximization is unrealistic because it is very difficult to evaluate the exact opportunity cost of equities, in the first part of this paper it is shown that the owner wishes the firms to maximize their profit rate instead of their pure profit. So, in these

situations, the firm's output, and above all the scale of production *via* the firm's demand of capital, depends on a profit rate maximization program, and not on a pure profit maximization program, when the short run with a fixed capital remains as usual, gross profit maximizing.

In a second part, the two objective functions, profit rate maximization for a firm controlled by its owners and pure profit maximization for a firm with sleeping owners, were compared for their consequences on the main models of industrial organization theory: monopoly, perfect competition, classical or mixed duopoly. Even if the long term conditions of entry into the sector are similar for both objectives, the optimal output level is always lower for controlled-by-owner firms than comparable for sleeping-owner firms. This has obvious consequences on the results of the models of industrial organization from the moment that the hypothesis of mobility of capital is adopted.

If a modified definition of marginal productivity is adopted by subtracting the marginal amount of equity capital required by the factor valorized at the equilibrium profit rate, then factors remain paid at their modified marginal productivity. Even if fewer factors are used, the net modified product remains exhausted for homogenous functions with degree 1. The duopoly can be mixed, with one firm controlled by owners and another firm with sleeping owners. However collusion remains always possible.

We have proved that coordination among controlled-by-owner firms may fail by contrast to sleeping-owner firms. It only takes a fixed coefficient of equity capital to do that price plays no role for controlled-by-owner firms in perfect competition: in this case, the price signal plays no role and firms are not coordinated even if they are smaller than in the classical model; there remains a global equilibrium but with an inelastic supply curve. In imperfect competition, for a duopoly in quantities between controlled-by-owner firms, it only takes a fixed coefficient of equity capital and a linear demand function to do that reaction functions vanish: in this case, controlled-by-owner firms do not consider the other firms to determine their optimal output.

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Fig i

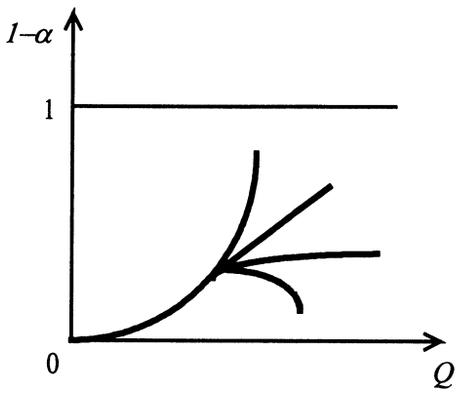

Figure 1. Financial structure: ratio of debts by total capital

Fig ii

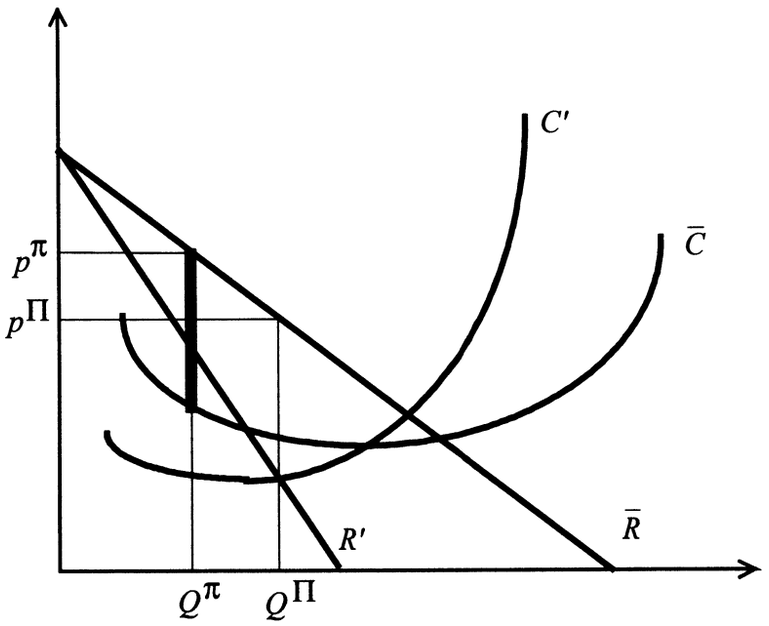

Figure 2. Monopoly (here, with fixed coefficient of equity capital)

Fig iii

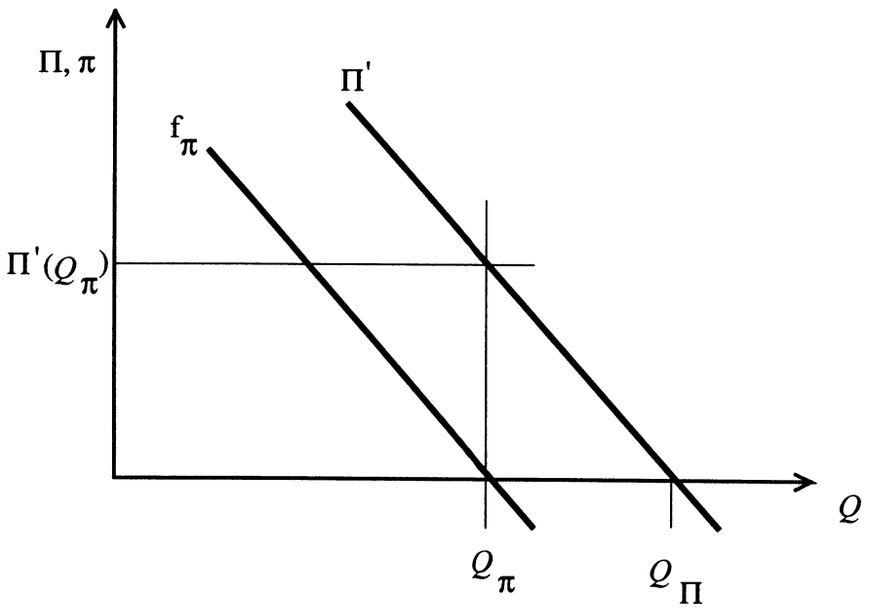

Figure 3. Mathematical comparison of optima

Fig iv

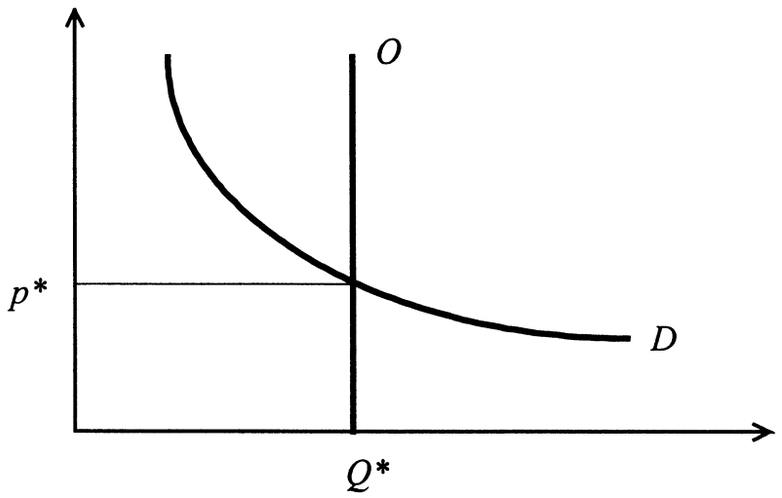

Figure 4. Global equilibrium in short-run perfect competition

Fig v

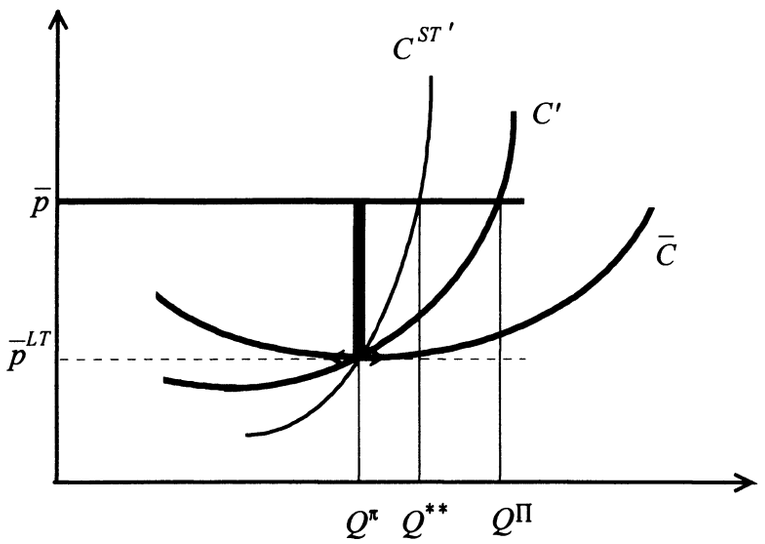

Figure 5. Equilibrium of the controlled-by-owner firm in perfect competition with coefficient of equity capital

Fig vi

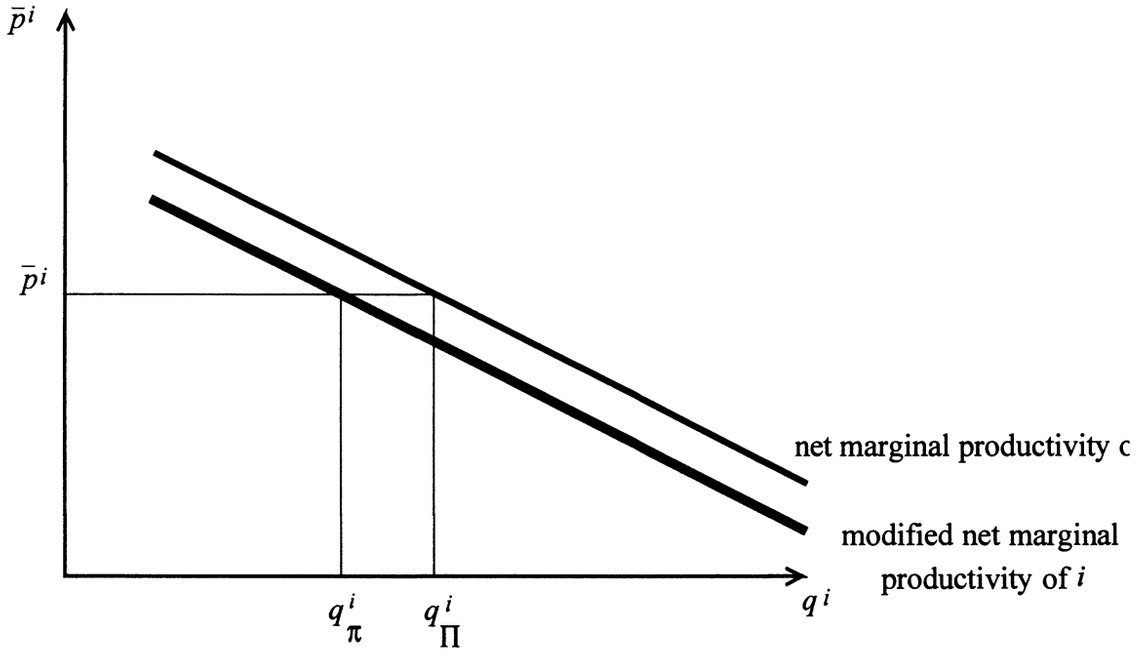

Figure 6. Optimal quantities of factors

Fig vii

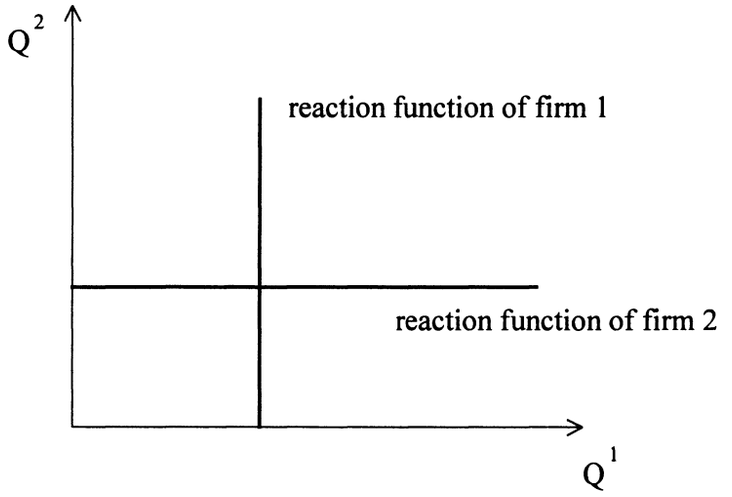

Figure 7. Reaction functions of a profit-rate-maximizing duopoly with linear demand and coefficient of equity capital

Fig viii

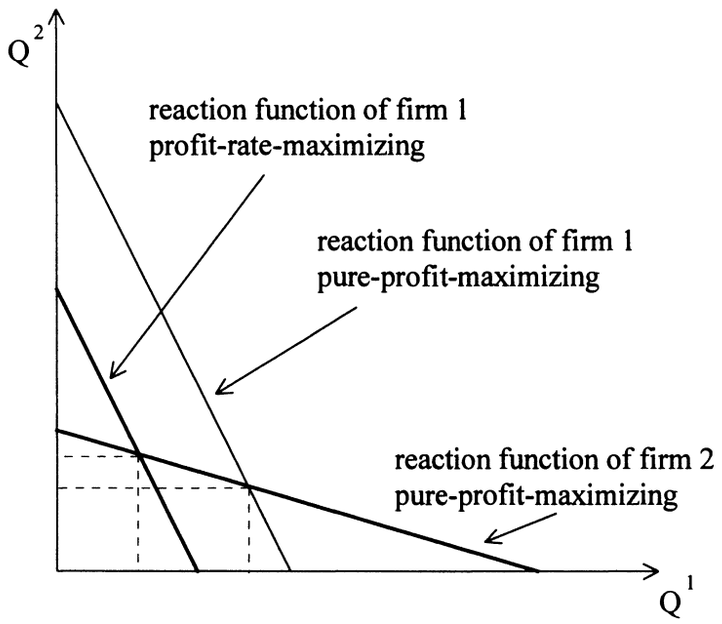

Figure 8. Reaction functions of a mixed duopoly