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Internal-time in Quantum Mechanics

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Abstract

This paper aims to construct a falsifiable model of an internal dimension of time that fits the observed results, that maximizes simplicity for explaining different quantum phenomena such as superposition, measurement and entanglement, that does not contradict the established principles and laws of the quantum mechanics and that predicts new testable features or phenomena.

Key words: entanglement, superposition, measurement.

1. Introduction

Conventionally, some of the remarkable features of quantum mechanics such as superposition, measurement and particle-wave duality are illustrated by the double-slit thought experiment. When electrons are sent one at a time through a double-slit plate (hereafter called slit A and slit B), single random impacts are observed on a screen behind the plate as expected out of individual particles. However, when the electrons are allowed to build up one by one, the cumulative effect of a great number of impacts on the screen reveals an interference pattern characteristic of waves arriving at the screen from the two slits. Meanwhile, the interference pattern is made up of individual particle-like impacts. This phenomenon illustrates a particle-wave duality nature of the electrons.

Moreover, when the electrons are made to build up one by one while detectors D_A and D_B are placed at slits A and B respectively to find out through which slit each electron went, the interference pattern disappears and the electrons behave solely as particles. It seems thus impossible to observe interference and to simultaneously know through which slit the particle has passed. Furthermore, the distribution of the interference pattern formed on the screen cannot be understood merely in a probabilistic manner by simply saying that half of the electrons pass through slit A and the other half through slit B. The best explanation that can be made is that the same electron passes simultaneously through both slits [1]. This seemingly paradoxical statement is conform with the experimental data and is best illustrated by a state-vector.

In quantum mechanics, for a given observable (e.g. position) having different possible values, the quantum system (e.g. the position of the particle) can be defined by a state-vector (noted $|\psi\rangle$) as a linear combination of different possible sub-states (noted $|\psi_i\rangle$). In the case of the double-slit experiment the state vector of an electron passing through slit A may be denoted as $|A\rangle$, similarly, the state vector of an electron passing through slit B is denoted $|B\rangle$. A particle passing through both slits A and B at the same time is said to be in a superposition state and its state-vector is denoted $|\psi\rangle = a|A\rangle + b|B\rangle$, where “a” and “b” are called the probability amplitudes. The mod-square of “a” represents the probability of the particle to be measured

by the D_A detector at the slit A and likewise the mod-square of “b” represents the probability of the particle to be measured by the D_B detector at the slit B.

When no detectors are present at the slits, the state of the electron is said to be a wave of probabilities defined by the superposition state $a|A\rangle + b|B\rangle$ in which the electron passes through slit A and slit B at the same time. A calculation assuming wave-like behaviour of electrons explains pretty well the interference pattern created on the screen by their cumulative impacts. Nevertheless, when detectors D_A and D_B measure from which slit the electron passes, the superposition state simply disappears and creates a so-called measurement problem.

Conventionally, when no detectors are present, the state-vector $|\psi\rangle$ is said to evolve per a deterministic continuous unitary evolution U whereas, when detectors D_A and D_B measure from which slit the electron passes, the deterministic evolution of the state-vector $|\psi\rangle$ is transformed into a probabilistic discontinuous state reduction R as explained by Penrose [2]. In the above thought experiment, it is said that the state-vector $|\psi\rangle = a|A\rangle + b|B\rangle$ evolving per the U process is reduced according to the R process from one describing a superposition of two observational outcomes to only one outcome either $|\psi\rangle = |A\rangle$ or $|\psi\rangle = |B\rangle$. The two processes U and R create a conflict in the formalism of quantum mechanics. Different ontologies have been proposed to interpret the strange combination of the deterministic continuous U process with the probabilistic discontinuous R process.

According to the Copenhagen interpretation [2, 3, 4], the state-vector $|\psi\rangle$ and the U and R processes should only be regarded as a description of the experimenter’s knowledge. There exist several other interpretations amongst which the Everett interpretation or what is more commonly known as the many-world interpretation [5, 6], according to which there is no wave function collapse and all measurement results exist but in different worlds. In line with this interpretation, Wallace [7] claims that when a measurement is conducted on an electron in the superposition state $a|A\rangle + b|B\rangle$, a deterministic branching takes place where on one branch detector A detects the electron while detector B doesn’t and at the same time but on the other branch, detector A doesn’t detect the electron while detector B does detect it. However, this interpretation poses some probabilistic as well as ontological problems. Kent [8] argues that the axioms of quantum mechanics say nothing about the existence of multiple physical worlds.

In this paper, it is intended to introduce an explanatory hypothesis that makes sense of the double-slit experiment as well as other features of quantum mechanics and in which the R process is replaced by an internal mechanism that does not come in conflict with the U process.

It is learned from the double-slit experiment that when no measurement is conducted, the state-vector is a bloc of two sub-states $|A\rangle$ and $|B\rangle$ whereas, an act of measurement reveals only one of these two sub-states. It can be reasonably conjectured that the bloc of two sub-states has an internal structure that inherently discriminates the two sub-states while permitting their coexistence at each instant of time. This reasonable conjecture infers a possible existence of an “internal dimension of time” different from the usual physical time such that the two sub-states may have a simultaneous existence with respect to the physical time but not with respect to the internal time. Thus, when no measurement is conducted, both sub-states $|A\rangle$ and $|B\rangle$ coexist simultaneously at each physical time instant while existing

separately at different internal time instants. The simultaneous coexistence of the two sub-states $|A\rangle$ and $|B\rangle$ is relative only with respect to the physical time. This relative-simultaneity of the two sub-states $|A\rangle$ and $|B\rangle$ explains the passage of an electron through slits A and B at the same physical time whereas, the inherent discriminating feature of the two sub-states $|A\rangle$ and $|B\rangle$ with respect to the internal time explains the bringing about of only one of these two sub-states by the act of measurement. This explanatory hypothesis is referred hereafter “internal-time-model”.

A “spatial” analogy to the above “temporal” model can be given by the structure of a coin. Indeed, a coin is a bloc of “two-overlapped-faces” thanks to a “separating edge” defined in a spatial dimension different from the spatial dimensions with respect to which are defined the two faces. The separating edge enables on one hand, the “overlapping coexistence” of the two faces which is a sort of a spatial analogy to temporal simultaneity while on the other hand, discriminates between the two faces by allowing them to coexist within two distinct parallel planes and thus, when the coin falls on the ground, only one of the two outcome faces is observed. The “internal time” plays simply the role of the coin’s separating edge.

Another peculiar feature of quantum mechanics is entanglement which occurs when the constituents of a system cannot be described independently. An example of an entangled system is one composed of two particles whose states are specified by two components of spin travelling in opposite directions and emanating from a single source. When a measurement of spin is made at either side, the outcome result is fixed for both particles.

Such phenomena were the subject of many papers and in particular, a paper by Einstein et al [9] describing what came to be known as the EPR paradox in which entanglement is considered to violate locality. However, Bell [10] proved the consistency of entanglement with the predictions of quantum mechanics. Entanglement was also verified experimentally by measuring the polarization or spin of entangled particles in different directions and the results were in agreement with Bell’s inequality [11, 12].

Again, an internal dimension of time may be considered as an entity that connects and synchronizes the states of the entangled particles giving a simple explanation of entanglement as shown hereafter.

More precisely, the “internal-time-model” considers time as having a certain “thickness” and proposes to define the evolution of each elementary physical system with respect to a three-dimensional-time presenting a thread-like-form, hereinafter referred to as “elementary-time-thread”. The “longitudinal direction” of the elementary-time-thread corresponds to the usual physical-time-axis where each point is specified by a physical-time-index. However, the “cross-section” of the elementary-time-thread is referred to as a “state-time-plane” where each point is specified by a “state-time-index” defined by a couple of state-time-coordinates. Finally, each point - referred to as “elementary-time-instant” - of the elementary-time-thread is specified by a triplet of time-coordinates (one physical-time-coordinate and two state-time-coordinates).

It is postulated that every elementary physical object has its proper elementary-time-thread in the same way as it has its proper spatial extension. Thus, a plurality of physical objects evolve with respect to their corresponding elementary-time-threads. However, the projection of their

respective elementary-time-threads onto an arbitrary physical-time-axis gives the illusion of an external time-axis with respect to which the different events are organized.

According to the proposed “internal-time-model”, a quantum system of an elementary physical object is defined by a fundamental-state-vector noted $|\psi\rangle_F$ composed of a set of sub-states $|\psi_i\rangle$. The fundamental-state-vector $|\psi\rangle_F = \{|\psi_i\rangle\}$ is governed by the usual deterministic unitary evolution U with respect to the physical-time-axis. However, at any given physical-time index, the fundamental-state-vector $|\psi\rangle_F$ is governed by an internal evolution with respect to a “state-time-line” contained in the state-time-plane of the quantum system’s elementary-time thread. The state-time-line with respect to which the evolution of the sub-states take place defines a sort of an “internal-state-time-clock” such that each ticking (defined at a specific state-time-index) is associated with only one sub-state such that at each physical-time-index, the set of sub-states does not occur at once. Thus, only one sub-state can exist at any given elementary-time-instant belonging to the elementary-time thread and therefore, the outcome of a measurement conducted at that given elementary-time-instant is naturally the sub-state that occurs at that instant.

In other words, when a measuring device conducts a measurement on an elementary physical object at a given elementary-time-instant, it will simply measure or “selects” the sub-state (out of a set of sub-states) that occurs at that instant. Once a sub-state has been selected, it will be trapped by the measuring device and thus, the quantum system can no more change its state.

Applying this model to the double-slit experiment leads to considering that at any given physical-time-index, the electron is at location A for a given state-time index and at location B for another state-time-index. Thus, at each physical-time-index, the electron “exists” at different locations and behaves as an extended object of a wave-like nature causing the interference pattern on the screen. However, when detectors are placed at the slits A and B, an electron may be detected either at A or at B. For example, if the electron is observed at A (i.e. the act of observation happened at an elementary-time-instant at which the electron was at A) then the transitions stop and the same electron will not be observed at B. Thus, the electron observed at A simply behaves as an individual particle causing a random impact on the screen as expected.

In the case of the entangled system of two particles, the state transition of both particles is synchronized by the same internal-state-clock. When a measurement is made at one side, the measured sub-state is trapped and thus, there are simply no other sub-states from which another state can be selected. The trapped state forces the internal-state-clock to stop ticking and thus, ends the transitions at the other side. Therefore, the action of measurement on any particle fixes the outcome result for both particles.

The proposed internal-time-model when put in the context of quantum mechanics replaces the reduction process R by an internal mechanism consistent with the U process and seems to imply and explain the observed quantum phenomena.

2. Formalism

2.1 Time-threads

The evolution of each quantum object is defined with respect to its proper “elementary-time-thread” which is a three-dimensional-time presenting a thread-like-manifold.

Each elementary-time-thread can be defined in a reference frame consisting of a three-dimensional coordinate system in $\mathbf{R-C}$ composed of the ordinary “physical-time-axis” ($t - axis$) along a real coordinate axis \mathbf{R} , a first “state-time-axis” ($\theta - axis$) and a second or “conjugate-state-time-axis” ($\eta - axis$) where the $\theta - axis$ and $\eta - axis$ define a complex plane \mathbf{C} . The elementary-time-thread has thus its cross-section comprised in a complex “state-time-plane” (θ, η) defined by the $\theta - axis$ and $\eta - axis$ and its longitudinal orientation defined along the real $t - axis$. Each “elementary-time-instant” of the elementary-time-thread is specified by a point $t_e = (t, \theta + i\eta)$ where t , θ and η are real numbers. For simplicity, each “elementary-time-instant” t_e of the elementary-time-thread is specified by the point $t_e = (t, v)$ where the physical-time-index t is a real number and the state-time-index v is a complex number of the form:

$$v = \theta + i\eta = |v|e^{i\varphi} \quad (1)$$

where $|v|$ and φ are the magnitude and argument of the state-time-index v .

2.2 Quantum features with respect to dynamical time-threads

Conventionally, a quantum system (e.g. spin of a particle) can be defined by a state-vector in an orthonormal eigenvector basis. For any observable Q , the state-vector $|\psi\rangle$ is defined by a superposition of vector projections in an eigenbasis $\{|\psi_i\rangle\}$. In other words, the state-vector $|\psi\rangle$ is defined as a linear combination of the different possible sub-states. The normalized conventional state-vector of the quantum system is expressed as follows:

$$|\psi\rangle = \sum_i c_i |\psi_i\rangle \quad (2)$$

where $|\psi_i\rangle$ are orthonormal sub-states of the quantum system verifying $\langle\psi_i|\psi_j\rangle = \delta_{ij}$ (Kronecker delta) and the coefficients c_i of the state-vector $|\psi\rangle$ define the probability amplitudes in the specific orthonormal eigenvector basis $\{|\psi_i\rangle\}$.

In contrast, according to the present internal-time-model, if Q is an observable and its spectrum of possible orthonormal vectors is $\{\psi_i\}$ where $\langle\psi_i|\psi_j\rangle = \delta_{ij}$, then the quantum system can be represented by a fundamental-state-vector noted $|\psi\rangle_F$ composed of a set of states (hereinafter called a set of sub-states) $\{|\psi_i\rangle\}$ in which the value of the observable Q is well defined.

The fundamental-state-vector $|\psi\rangle_F$ is governed by the deterministic unitary evolution U (based on the Schrödinger equation) with respect to the physical-time-axis ($t - axis$). However, at any given physical-time index t , the fundamental-state-vector $|\psi\rangle_F$ undergoes a transition process between the different states of the set of sub-states $\{|\psi_i\rangle\}$. The transition process is governed by an “internal mechanism” defined in the “state-time-plane” (θ, η) . The different points (i.e. the different state-time-indices $v(\theta_i, \eta_i)$ belonging to the state-time-plane) visited by the set of sub-states $\{|\psi_i(v)\rangle\}$ form a “state-time-line” denoted “ (v) ” whose “oriented-length” (i.e. “oriented-state-time-life”) denoted “ Δv ” is a vector defined by a complex number in the state-time-plane (θ, η) .

Thus, each sub-state $|\psi_i(v)\rangle$ defines the state of the quantum system at a corresponding state-time-index v along the state-time-line (v) . More precisely, at any given physical-time index t , a surjective function is defined from the set of state-time-indices $\{v(\theta_{ki}, \eta_{ki})\}$ belonging to the state-time-line (v) onto the set of sub-states $\{|\psi_k\rangle\}$ such that every sub-state $|\psi_k\rangle$ has at least one corresponding state-time-index $v(\theta_{ki}, \eta_{ki})$. The sub-set of state-time-indices that correspond to the same sub-state $|\psi_k\rangle$ is denoted $\{v_k\}$ whose “oriented-length” Δv_k represents the oriented-state-time-life of the corresponding sub-state $|\psi_k\rangle$ at any given physical-time index t .

In other words, the oriented-state-time-life Δv of the fundamental-state-vector $|\psi\rangle_F$ can be subdivided into different sets $\{\Delta v_k\}$ of oriented-state-time-lives (i.e. $\Delta v = \sum_k \Delta v_k$) each of which (i.e. each set Δv_k) is associated to a specific sub-state $|\psi_k(v)\rangle$. Moreover, each set Δv_k of oriented-state-time-lives simply concatenates all oriented-sub-state-time-lives Δv_{ki} scattered all over the oriented-state-time-life Δv and visited by the same sub-state $|\psi_k(v)\rangle$. That is $\Delta v_k = \sum_j^n \Delta v_{kj}$ where n is the number of times the sub-state $|\psi_k(v)\rangle$ is visited during the whole oriented-state-time-life Δv of the fundamental-state-vector $|\psi\rangle_F$ at a given physical-time index t . Specifically, the oriented-state-time-life Δv_k of a sub-state $|\psi_k\rangle$ corresponds to the sum of oriented-sub-state-time-lives $\{\Delta v_{kj}\}$ of all state-time-indices visited by the sub-state $|\psi_k\rangle$. Thus, at any given physical-time index t , the internal transition process should be defined by the transitions between the different sub-states $\{|\psi_k(v)\rangle\}$ as well as by their corresponding “oriented-sub-state-time-lives” $\{\Delta v_{kj}\}$.

The relation between the oriented-state-time-life Δv of the fundamental-state-vector $|\psi\rangle_F$ at a given physical-time index t and the oriented-state-time-lives $\{\Delta v_k\}$ of the corresponding set of sub-states $\{|\psi_i\rangle\}$ is:

$$\Delta v = \sum_k \Delta v_k \quad \text{and} \quad \Delta v_k = \Delta \theta_k + i\Delta \eta_k = |\Delta v_k| e^{i\varphi_k} \quad (3)$$

where $\Delta \theta_k$ and $\Delta \eta_k$ are the components of Δv_k along respectively the θ – axis and η – axis and where $|\Delta v_k|$ and φ_k are respectively the magnitude and argument of Δv_k in the state-plane (θ, η) . The magnitude or module $|\Delta v_k|$ represents the duration of the oriented-state-time-life and is thus simply called “state-time-life”. The argument φ_k represents the orientation of the oriented-state-time-life. Thus, the whole state-time-life $|\Delta v|$ of the fundamental-state-vector $|\psi\rangle_F$ at a given physical-time index t , is:

$$|\Delta v| = \sqrt{\sum_k (\Delta v_k)^2} \quad (4)$$

Schematically, the fundamental-state-vector $|\psi\rangle_F$ may be viewed as evolving with respect to a two-dimensional-time-manifold (i.e. a “time-surface” referred hereafter “2d-time-manifold”) embedded in the elementary-time-thread and made up of elementary-time-instants $t_e = (t, \theta, \eta) = (t, v)$ wherein, each state-time-index t is mapped to a corresponding state-time-line $(v)_t$ whose oriented-state-time-life is Δv_t . Schematically, a state-time-line $(v)_t$ belonging to a 2d-time-manifold is formed by the intersection between the 2d-time-manifold and the cross-section (i.e. the state-time-plane (θ, η)) of the elementary-time-thread at the physical-time-index t . In the following text, the suffix “t” is dropped as it is implicitly clear that any state-time-line (v) is defined with respect to a corresponding physical-time index t .

A given fundamental-state-vector $|\psi\rangle_F$ evolving with respect to its corresponding 2d-time-manifold (t, v) can be expressed in function of its sub-states as follows:

$$|\psi\rangle_F = \sum_i \delta_v(\Delta v_i(t)) |\psi_i(t, v)\rangle \quad (5)$$

where $\delta_v(\Delta v_i(t))$ is the Dirac measure (or indicator function) defined as follows:

$$\delta_v(\Delta v_i) = \begin{cases} 1 & \text{if } v \in \Delta v_i \\ 0 & \text{if not} \end{cases} \quad (6)$$

The above expressions (5, 6) define the fundamental-state-vector $|\psi\rangle_F$ as a step function made up of a combination of Dirac measures of state-time-lives visited by the different sub-states. Equation (5) indicates that at any current physical-time-index t , the different sub-states $|\psi_i(t, v)\rangle$ can be viewed as forming a “state-block” wherein, all potential sub-state-outcomes exist but do not occur at once with respect to the corresponding state-time-line (v) . The “state-block” forms a kind of a “state-history” labeled by a sequence of “state-dates”. Thus, at any given time-index t , the different sub-states do not occur simultaneously and can only be considered as partially simultaneous with respect to the physical-time only. In other words, equation (5) indicates that for a given physical-time-index t and for a given state-time-life $\Delta v_k(t)$, the fundamental-state-vector $|\psi\rangle_F$ should be in only one specific sub-state $|\psi_k(t, v)\rangle$. Thus, for a given physical-time-index t , the transitions and oriented-state-time-lives $\Delta v_i(t)$ relative to the different sub-states $|\psi_i(t, v)\rangle$ form an “internal-state-clock” that governs the evolution of the system with respect to the state-time-line (v) . It should be noted that the oriented-state-time lives $\Delta v_i(t)$ and $\Delta v(t)$ may a priori, depend on the physical-time-index t .

To find a relation between the fundamental-state-vector $|\psi\rangle_F$ of equation (5) and the conventional state-vector of equation (2), a sort of a summation process should be used over the state-time-line (v) at each physical-time-index t , so that the “resultant” state-vector does not explicitly depend on the state-time. Thus, it is proposed to calculate at each physical-time-index t , a “resultant-state-vector” $|\psi(t)\rangle_R$ that describes the resultant behavior of the set of different sub-states $\{|\psi_i(t, v)\rangle\}$ over the whole oriented-state-time-period $\Delta v(t)$ while taking into consideration the values of the corresponding oriented-state-time lives $\Delta v_i(t)$. It can be reasonably conjectured that the impact of each individual sub-state $|\psi_i(t, v)\rangle$ on the global behavior of the quantum system should depend on the value of its corresponding oriented-state-time-life $\Delta v_i(t)$.

Thus, the resultant-state-vector $|\psi(t)\rangle_R$ can be expressed as the sum of the different sub-states $|\psi_j(t, v)\rangle$ affected by coefficients that depend on the physical time-index as well as on the corresponding oriented-state-time-lives $\Delta v_j(t)$ as follows:

$$|\psi(t)\rangle_R = \sum_j a_j(t, \Delta v_j(t)) |\psi_j(t, v)\rangle \quad (7)$$

The information concerning the dynamics of sub-states and their oriented-state-time-lives Δv_i as given by the Dirac measures $\delta_v(\Delta v_i(t))$ of equation (5) is transferred to the coefficients a_j of equation (7).

By replacing $\Delta v_j(t) = |\Delta v_j(t)| e^{i\varphi_j(t)}$ of equation (3) into equation (7), the latter equation becomes:

$$|\psi(t)\rangle_R = \sum_j a_j(t, |\Delta v_j(t)| e^{i\varphi_j(t)}) |\psi_j(t, v)\rangle = \sum_j b_j(t, |\Delta v_j(t)|) f_j(e^{i\varphi_j(t)}) |\psi_j(t, v)\rangle \quad (8)$$

or may be expressed more simply as:

$$|\psi(t)\rangle_R = \sum_j c_j(t, |\Delta v_j(t)|) e^{i\varphi_j(t)} |\psi_j(t, v)\rangle \quad (9)$$

Moreover, the resultant-state-vector $|\psi(t)\rangle_R$ should be governed by the deterministic unitary evolution U according to Schrödinger equation and satisfying the normalisation property:

$$\| |\psi(t)\rangle_R \| = (\langle \psi(t) | \psi(t) \rangle)_R = 1 \quad (10)$$

Thus, the product $(\langle \psi(t) | \psi(t) \rangle)_R$ in the eigenbasis $\{|\psi_i\rangle\}$ of the normalised resultant-state-vector $|\psi(t)\rangle_R$ is:

$$(\langle \psi(t) | \psi(t) \rangle)_R = \sum_j |c_j(t, |\Delta v_j(t)|)|^2 = 1 \quad (11)$$

where each mod-square $|c_j(t, |\Delta v_j(t)|)|^2$ can thus be interpreted as the probability of occurrence $P(|\psi_j(t, v)\rangle)$ of the corresponding sub-state $|\psi_j(t)\rangle$.

On the other hand, at a given physical-time-index t , the probability of occurrence $P(|\psi_j(t, v)\rangle)$ of a sub-state $|\psi_j(t, v)\rangle$ which depends on the physical-time t as well as the state-time v can be straightforwardly conjectured to be proportional to the ratio of the state-time-life $|\Delta v_j(t)|$ of that sub-state $|\psi_j(t, v)\rangle$ to the total state-time-life $|\Delta v(t)|$ of all the sub-states:

$$P(|\psi_j(t, v)\rangle) = |c_j(t, |\Delta v_j(t)|)|^2 \equiv \frac{|\Delta v_j(t)|}{|\Delta v(t)|} \quad (12)$$

and thus for a fixed physical-time-index t , equation (9) becomes:

$$|\psi(t)\rangle_R = \sum_j c_j(t, |\Delta v_j(t)|) e^{i\varphi_j(t)} |\psi_j(t)\rangle = \sum_j \sqrt{\frac{|\Delta v_j(t)|}{|\Delta v(t)|}} e^{i\varphi_j(t)} |\psi_j(t)\rangle \quad (13)$$

For simplicity, the above equation is expressed as follows:

$$|\psi(t)\rangle_R = \sum_j d_j |\psi_j(t)\rangle \quad (14)$$

$$\text{where } d_j = \sqrt{\frac{|\Delta v_j(t)|}{|\Delta v(t)|}} e^{i\varphi_j(t)} \quad (15)$$

Equation (14) clearly indicates that the resultant-state-vector $|\psi(t)\rangle_R$ constructed out of the fundamental-state-vector $|\psi\rangle_F$ is similar to the conventional-state-vector of equation (2).

Equations (5) and (14 or 2) express the same quantum system from different point of views. The fundamental-state-vector $|\psi\rangle_F$ gives an internal and a more detailed view of the quantum system whereas, the resultant-state-vector $|\psi(t)\rangle_R$ (or the conventional-state-vector $|\psi\rangle$) gives an external and global view of the quantum system.

Equation (14) or (2) should be interpreted as a mere “resultant” or a sort of “average” of the sub-states at any given physical-time-index only intended to countervail our ignorance of the sub-states evolution with respect to the internal-time and should certainly not be misleadingly interpreted as a factual superposition of these sub-states. It gives a global picture of all the actual sub-states in abstraction of the underlying internal-time and is thus more practical to

use whenever probabilities, interactions or a global picture are desired but certainly not when fundamental details are required.

2.3 Measurement

Equation (5) defines the fundamental-state-vector as a set of different sub-states evolving with respect to a 2d-time-manifold corresponding to the succession of state-time-lines (v) along the physical-time t . It describes the fundamental internal evolution of the state-vector and can be used to interpret any phenomenon of the quantum system and should be used whenever a measurement is conducted on the quantum system. In contrast, the alternative equation (14) expresses the resultant-state-vector as a sum over the different sub-states affected by coefficients whose mod-squares yield the probabilities with which the possible results of the measurement will be obtained. The coefficients of equation (14) are derived from the state-time-lives of each sub-state at a given physical-time-index.

According to equation (5), a measurement can be regarded as an act of selecting one sub-state at a given elementary-time-instant (t_i, θ_i, η_i) and once a sub-state has been selected, there are simply no other sub-states to select and thus the transition operation comes to a halt. In other words, the act of measurement stops the internally transitional process of the fundamental-state-vector $|\psi\rangle_F$ corresponding to the chosen observable.

Let $|\phi\rangle$ denote the state-vector of a measuring device and $|\psi\rangle_F$ that of the quantum system under equation (5). Prior to any interaction (at $t=0$), the state-vector $|\Psi\rangle$ for the entire system (i.e. the measuring device and the quantum system) is denoted as follows:

$$|\Psi(t=0)\rangle = |\psi\rangle_F |\phi_0\rangle = (\sum_j \delta_v (\Delta v_j) |\psi_j(t, v)\rangle) |\phi_0\rangle \quad (16)$$

where $|\phi_0\rangle$ denotes the initial state of the measuring device.

Under the action of the Schrödinger evolution with respect to the physical-time, the state-vector $|\Psi\rangle$ for the entire system of the above expression evolves into the following entangled state:

$$|\Psi(t)\rangle = \sum_j \delta_v (\Delta v_j) |\psi_j(t, v)\phi_j\rangle = \begin{cases} |\psi_j(t, v)\phi_j\rangle & \text{if } v \in \Delta v_j \\ 0 & \text{if not} \end{cases} \quad (17)$$

where $|\psi_j(t, v)\phi_j\rangle$ represents the quantum system being in the sub-state $|\psi_j\rangle$ and the measuring device being in the state indicating the sub-state $|\psi_j\rangle$.

Equation (17) expresses the fact that if the interaction between the measuring device and the quantum system occurred during the oriented-state-time-life Δv_k of the sub-state $|\psi_k\rangle$, then the measuring device indicates that the measured quantum system is at the state $|\psi_k\rangle$.

It should be noted that once a sub-state of a quantum system has been detected, it will be trapped by the measuring device (or in other words entangled with the measuring device), and thus the fundamental-state-vector $|\psi\rangle_F$ of the quantum system can no more change its state along the state-time-line (v). In other words, the internal-state-clock associated to the state-time-line stops ticking and thus, the oriented-state-time-life Δv_k of the sub-state $|\psi_k\rangle$ becomes equal to the whole oriented-state-time-life Δv of the fundamental-state-vector $|\psi\rangle_F$.

The above interpretation can be enlightened by making an analogy to a fictive experiment of consecutive tossing of a “quantum” coin. At each time, the coin is thrown into the air letting it rotate several times before it lands on a table in order to measure the outcome. The whole process of flipping a coin could of course be modeled by classical laws of physics in function of its trajectory and its precession. However, without any physical foundation and only for mere analogical purposes, the spinning motion of the coin in the air is assimilated to a permutation mechanism of transitions between its two faces with respect to an imaginary “state-time-line” (v) whereas, the consecutive outcomes on the table is assimilated to the evolution of the coin along a physical-time-axis (t – *axis*).

The space of states is a set of two elements “head” and “tail”: $\{e_1 = H, e_2 = T\}$. According to the conventional interpretation of quantum mechanics, the normalized state-vector $|\psi\rangle$ of the coin while it is rotating in the air is considered to be in a “superposition” state of two sub-states $|H\rangle$ and $|T\rangle$ that can be expressed as follows:

$$|\psi\rangle = 1/\sqrt{2} |H\rangle + 1/\sqrt{2} |T\rangle \quad (18)$$

The probability to find the outcome $|H\rangle$ or $|T\rangle$ when a measurement is conducted is $1/2$. When the coin falls on the table, there is only a single outcome either head or tail and in line with the conventional quantum mechanics, this is interpreted as a collapse of the initial superposed state of the coin into a single sub-state. This misinterpretation comes from the fact that the face-up side of the coin cannot be observed while it is spinning in the air and the coin was supposed to be simultaneously in different states.

However, according to the present internal-time-model, the face-up side of the coin while it is spinning in the air is accounted for by assigning a “state-time-life” for each face-up side of the coin. Let Δv_H and Δv_T denote the state-time-lives of the two sub-states $|H\rangle$ and $|T\rangle$ respectively while the coin is spinning. In this case, the state of the flipping coin while it is in the air should be represented by a fundamental-state-vector $|\psi\rangle_F$ as follows:

$$|\psi\rangle_F = \delta_v(\Delta v_H) |H(v)\rangle + \delta_v(\Delta v_T) |T(v)\rangle \quad (19)$$

The outcome can be observed with respect to the physical-time-axis only when the table (i.e., the measuring device) stops the flipping mechanism of the coin. Indeed, only when the coin settles down on the table that can be said whether the outcome is head or tail.

Let $|\phi\rangle$ denote the state vector of the table and $|\psi\rangle_F$ that of the quantum coin according to equation (19). Prior to landing on the table (at $t=0$), the state vector $|\Psi\rangle$ for the entire system (i.e. the table and the quantum coin) is denoted as follows:

$$|\Psi(t=0)\rangle = |\psi\rangle_F |\phi_0\rangle = (\delta_v(\Delta v_H) |H(v)\rangle + \delta_v(\Delta v_T) |T(v)\rangle) |\phi_0\rangle \quad (20)$$

where $|\phi_0\rangle$ denotes the initial state of the table wherein, the coin did not land yet.

Once the coin lands on the table, the state vector $|\Psi\rangle$ for the entire system (table+coin) becomes:

$$|\Psi(t)\rangle = \delta_v(\Delta v_H) |H(v)\phi_H\rangle + \delta_v(\Delta v_T) |T(v)\phi_T\rangle = \begin{cases} |H(v)\phi_H\rangle & \text{if } v \in \Delta v_H \\ |T(v)\phi_T\rangle & \text{if } v \in \Delta v_T \end{cases} \quad (21)$$

When the coin lands on the table at a given physical-time index t_m and a given state-time index v_m , the transition process comes to an end and the measured outcome becomes either

“head” or “tail”. Indeed, if the state-time index v_m belongs to the state-time-life Δv_H , then the quantum coin on the table would be in the state $|H\rangle$ and the table would indicate that the face-up side of the coin is “head”. Otherwise, if v_m belongs to Δv_T , the table would indicate that the face-up side of the coin is “tail”.

Therefore, by using the fundamental-state-vector of equation (19) instead of the conventional state-vector of equation (18), no reduction process happens at all. When a measurement is conducted, the outcome is simply the existing state at the measuring elementary-time-instant. The reduction process is simply an illusion resulting from not considering the internal-time by interpreting the superposition principle of the conventional state vector as an underlying truth.

Had the conventional state-vector of equation (19) been used instead of the fundamental-state-vector of equation (18), then measurement should not be interpreted as a “collapse” (or “reduction”) of the state-vector but rather as a two-steps procedure consisting of “decomposing” (or “separating”) the state-vector into its constituent sub-states with respect to an internal state-time before selecting the sub-state present at the physical-time-index and state-time-index during which the act of measurement took place.

It should be noted that before measurement, at each physical-time index t , the fundamental-state-vector is a sequence of a finite or infinite number of sub-states along a state-time-line (v). Once a measurement is conducted at a given physical-time index t , the fundamental-state-vector becomes “frozen” at the sub-state it had at the state-time-index during which the interaction between the quantum system and the measuring device took place. Thus, measurement affects the pre-existing states of the quantum system in the sense that before measurement the state-time-line is visited by a series of different sub-states, during measurement one single sub-state is selected in function of the elementary-time-instant during which the interaction took place and after measurement the state-time-line is visited by the single state that has been measured. Thus, the condition of the quantum object is altered by the experience. The object before measurement can be considered as the quantum object in-itself (or the noumenal object). After measurement, it becomes a phenomenal object that depends on the measuring device (or observer).

2.4 Schrödinger’s Cat Paradox

According to this well-known paradox, a live cat is placed in a box comprising a mechanism coupled to a radioactive atom and a vial of poison. At the beginning, the state-vector of the combined system “cat + atom” corresponds to a live cat. According to conventional quantum mechanics, the state-vector of the combined system at a time t , is a superposition of the state for a live cat and that for a dead cat. Upon opening the box, the state of the cat would collapse into either a live cat or a dead cat [13].

However, by applying the fundamental-state-vector $|\psi\rangle_F$ of equation (5), the cat paradox becomes simple.

Let A stands for a non-decayed atom and D for a decayed atom. Then by applying equation (5), the fundamental-state-vector $|\psi\rangle_F$ of the atom is:

$$|\psi(t)\rangle_F = \delta_v(\Delta v_A(t))|A(t, v)\rangle + \delta_v(\Delta v_D)|D(t, v)\rangle = \begin{cases} |A(t, v)\rangle & \text{if } v \in \Delta v_A \\ |D(t, v)\rangle & \text{if } v \in \Delta v_D \end{cases} \quad (22)$$

However, in accordance to the present internal-time model, a disintegration of an atom should be logically considered as a binary feature in the sense that at any given elementary date $t_e = (t, v)$, an atom is either decayed (state D) or not decayed (state A). Thus, if an atom is in a decayed state at an elementary-time-date $t_{e0} = (t_0, v_0)$, then, at the physical-time-index t_0 , the atom is in the state D for $v \geq v_0$ and at the state A for $v < v_0$ (i.e. the atom is in the decayed state as well as in the non-decayed state only with respect to the physical-time-index). In accordance with the measurement principle of section 2.3 and the binary feature of decay, it is straightforwardly concluded that for all physical-time-indices smaller than the physical-time-index t_0 , the oriented-state-time-life Δv_A of the sub-state $|A(t, v)\rangle$ is entirely equal to the oriented-state-time-life Δv of the whole state-line (v) while for all physical-time-indices greater than t_0 , the oriented-state-time-life Δv_D of the sub-state $|D(t, v)\rangle$ is entirely equal to the state-oriented-time-life Δv of the state-line (v). This is expressed as follows:

$$\Delta v_A = \begin{cases} \Delta v & \text{if } 0 \leq t < t_0 \\ 0 & \text{if } t > t_0 \\ \Delta v_A & \text{if } t = t_0 \end{cases} \quad \text{and}$$

$$\Delta v_D = \begin{cases} 0 & \text{if } 0 \leq t < t_0 \\ \Delta v & \text{if } t > t_0 \\ \Delta v_D & \text{if } t = t_0 \end{cases} \quad (23)$$

Let $|\phi\rangle$ denotes the state-vector of the cat and $|\Psi\rangle$ the state vector for the combined system “cat + atom”. At the beginning ($t=0$), the state vector $|\Psi\rangle$ for the combined system is:

$$|\Psi(t=0)\rangle = |\psi\rangle_F |\phi_0\rangle = (\delta_v (\Delta v_A(0)) |A(0, v)\rangle + \delta_v (\Delta v_D(0)) |D(0, v)\rangle) |\phi(0)\rangle \quad (24)$$

By considering expression (23) and knowing that at the beginning the cat is alive (i.e. $|\phi(0)\rangle = |\phi_{live}\rangle$), equation (24) is reduced into:

$$|\Psi(t=0)\rangle = |A(0, v)\rangle |\phi_{live}\rangle \quad (25)$$

Under the action of the Schrödinger evolution with respect to the physical-time, the state vector $|\Psi\rangle$ for the combined system of the equation (24) evolves into the following entangled state:

$$|\Psi(t)\rangle = \delta_v (\Delta v_A(t)) |A(t, v)\phi_{live}\rangle + \delta_v (\Delta v_D) |D(t, v)\phi_{dead}\rangle = \begin{cases} |A\phi_{live}\rangle & \text{if } v \in \Delta v_A \\ |D\phi_{dead}\rangle & \text{if } v \in \Delta v_D \end{cases} \quad (26)$$

where $|A(t, v)\phi_{live}\rangle$ represents the state in which the atom is in the non-decayed state and the cat is alive and where $|D(t, v)\phi_{dead}\rangle$ represents the state in which the atom is in the decayed state and the cat is dead and thus, the cat cannot be in a superposition state live and dead.

By applying equation (23) into equation (26), the state vector $|\Psi(t)\rangle$ for the combined system can be expressed as follows:

$$|\Psi(t)\rangle = \begin{cases} |A\phi_{live}\rangle & \text{if } 0 \leq t < t_0 \\ |D\phi_{dead}\rangle & \text{if } t > t_0 \\ |A\phi_{live}\rangle \text{ and } |D\phi_{dead}\rangle & \text{if } t = t_0 \end{cases} \quad (27)$$

The above equation expresses the fact that there exists a physical time index t_0 before which the cat is alive and after which the cat is dead which is naturally logical. However, the cat is in

a “partially-simultaneous” state dead and alive only with respect to the physical-time-index t_0 but not with respect to the elementary-time-date t_{e0} . Therefore, by using the fundamental-state-vector of equation (22), the Schrödinger’s cat paradox becomes intelligible.

2.5 Uncertainty

At any given physical-time-index, the internal-time model implies that a quantum system may have a plurality of sub-states indexed by different state-time-indices v in the state-time-plane (θ, η) thus inferring uncertainty. Indeed, different fundamental-state-vectors $|\psi\rangle_F$ and $|\psi'\rangle_F$ related to different observables of the same quantum object may evolve along different 2d-time-manifolds (t, v) and (t, v') embedded in the same elementary-time-thread wherein, for each physical-time index t , the state-time-lines (v) and (v') in the state-time-plane (θ, η) are different. It may thus be reasonably conjectured that non-commuting observables should refer to different sets of sub-states evolving with respect to different state-time-lines whereas, commuting observables should refer to sets of sub-states indexed with respect to the same state-time-line.

Let Q and P be two non-commuting observables (i.e. $[Q, P] \neq 0$) associated with first and second fundamental-state-vectors $|\psi_Q\rangle_F$ and $|\psi_P\rangle_F$ respectively, expressed as follows:

$$|\psi_Q(t, v_Q)\rangle_F = \sum_i \delta_{v_Q} (\Delta v_{Qi}(t)) |\psi_{Qi}(t, v_{Qi})\rangle \quad (28)$$

$$|\psi_P(t, v_P)\rangle_F = \sum_i \delta_{v_P} (\Delta v_{Pi}(t)) |\psi_{Pi}(t, v_{Pi})\rangle \quad (29)$$

where $\Delta v_{Qi}(t)$ and $\Delta v_{Pi}(t)$ are first and second oriented-state-time-lives in function of the physical-time index t of the corresponding first and second sub-states $|\psi_{Qi}(t, v_{Qi})\rangle$ and $|\psi_{Pi}(t, v_{Pi})\rangle$ respectively. The first fundamental-state-vector $|\psi_Q\rangle_F$ of the quantum object evolves with respect to a first 2d-time-manifold (t, v_Q) while the second fundamental-state-vectors $|\psi_P\rangle_F$ of the same quantum object evolves with respect to a second 2d-time-manifold (t, v_P) . Each sub-state defined with respect to the first 2d-time-manifold does not necessarily have a simultaneous corresponding sub-state evolving with respect to the second 2d-time-manifold unless if the two manifolds intersects each other at an elementary date common to these two sub-states. Indeed, since the first observable A is defined with respect to a first state-time-line (v_Q) and the second observable B with respect to a second state-time-line (v_P) , there could be no elementary-time-date $t_e = (t, \theta, \eta) = (t, v)$ at which both observables can be measured simultaneously unless there exist points where both state-time-lines intersect each other. Thus, for a fixed physical-time-index t , the first and second fundamental-state-vectors evolve with respect to two different state-time-lines (v_Q) and (v_P) signifying uncertainty between the two observables Q and P .

For example, once the first observable Q is measured at a specific elementary-time-date $t_{eM} = (t_M, v_{QM})$, the value of the first fundamental-state-vector $|\psi_Q(t_M, v_{QM})\rangle_F$ becomes well-defined by a single sub-state given by:

$$|\psi_Q(t_M, v_{QM})\rangle_F = |\psi_{QM}(t_M, v_{QM})\rangle \quad (30)$$

However, at the same physical-time-component t_M of the elementary-time-date $t_{eM} = (t_M, v_{QM})$, the second fundamental-state-vector $|\psi_P(t_M, v_P)\rangle_F$ still has a multitude of sub-states:

$$|\psi_P(t_M, v_P)\rangle_F = \sum_i \delta_{v_P}(\Delta v_{Pi}(t_M)) |\psi_{Pi}(t_M, v_P)\rangle = \{|\psi_{P1}(t_M, v_{P1})\rangle, \dots, |\psi_{Pk}(t_M, v_{Pk})\rangle, \dots\} \quad (31)$$

Thus, at the common physical-time-component t_M , the first fundamental-state-vector $|\psi_Q(t_M, v_{QM})\rangle_F$ has a definite value, while the second fundamental-state-vector $|\psi_P(t_M, v_P)\rangle_F$ has a plurality of states evolving along the state-time-line (v_P) that has a length (i.e. state-time-life) equals to $|\Delta v_P(t_M)|$.

Thus, the uncertainty principle should be related to the thickness of time and hence, an estimation of the thickness of time may be evaluated out of the uncertainty principle as will be shown in section 2.8.

On the other hand, it should be noted that each 2d-time-manifold has its own relation order but no relation order can be established between two different manifolds. Nevertheless, the projection onto the physical-time-axis of different 2d-time-manifolds embedded in the same elementary-time-thread generates a relative physical-time order between the different elementary-time-dates. Similarly, the projection onto an arbitrary physical-time-axis of different 2d-time-manifolds (embedded in respectively different elementary-time-threads) with respect to which different quantum objects evolve generates a relative physical-time order between the different events associated to these quantum objects. This order gives the illusion that the different quantum systems are governed by an external time and that each physical-time-coordinate forms an event.

2.6 Double-slit experiment

In the case of the double-slit experiment, once an electron comes out of a slit, its position at any given physical-time-index can be at any place between and/or beyond both slits. However, for simplicity and without much loss of generality, the fundamental-state-vector $|\psi\rangle_F$ of an electron may be considered to be composed of only two sub-states $|A\rangle$ and $|B\rangle$ corresponding to the position of the particle being in the neighbourhood of slits A and B respectively at the same physical time index t . The sub-states $|A\rangle$ and $|B\rangle$ depend on the state-time-indices v , and the fundamental-state-vector $|\psi\rangle_F$ can thus, be expressed as:

$$|\psi(t, v)\rangle_F = \delta_v(\Delta v_A(t)) |A(t, v)\rangle + \delta_v(\Delta v_B(t)) |B(t, v)\rangle \quad (32)$$

where $\Delta v_A(t)$ and $\Delta v_B(t)$ are the oriented-state-time-lives in function of the physical-time-index t of the two sub-states $|A\rangle$ and $|B\rangle$ respectively. In other words, at a given physical-time index t , Δv_A and Δv_B are the oriented-state-time intervals during which the electron's position is in the neighbourhood of slits A and B respectively and wherein $\delta_v(\Delta v_A)$ is equal to 1 if the state-time index v is an element of Δv_A and 0 if not, and similarly, $\delta_v(\Delta v_B)$ is equal to 1 if v is an element of Δv_B and 0 if not. In other words, the electron is at A and B at a given physical time t but not at the same state-time v . This situation may be considered as a “relative-simultaneity” with respect to only the physical time index t , but is certainly not an

“absolute-simultaneity” because there isn’t any elementary-time triplet $t_e = (t, \theta, \eta)$ with respect to which the electron is at A and B.

This relative-simultaneity is responsible for considering the particle as an extended object of a wave-like nature when in fact it is simply a single particle having several positions (A and B) at different state-times v but at the same physical-time-index t . The duality particle-wave becomes an illusion resulting out of our ignorance of the internal time.

Nevertheless, for a given physical-time-index t , the particle can be at position A as well as position B. In that case, a resultant-state-vector $|\psi(t)\rangle_R$ can be used to illustrate the observed phenomenon on the screen at a given physical-time-index t as in the conventional superposed-state-vector.

In view of equation (7) the resultant-state-vector $|\psi(t)\rangle_R$ in the situation of the double-slit experiment is:

$$|\psi(t)\rangle_R = a_A(t, \Delta v_A(t))|A(t, v)\rangle + a_B(t, \Delta v_B(t))|B(t, v)\rangle \quad (33)$$

As in the conventional quantum mechanics, equation (33) obviously illustrates the same interference phenomenon on the screen.

Indeed, in view of equation (13) the resultant-state-vector $|\psi(t)\rangle_R$ can also be written as:

$$|\psi(t)\rangle_R = c_A(t, |\Delta v_A(t)|)e^{i\varphi_A(t)}|A(t, v)\rangle + c_B(t, |\Delta v_B(t)|)e^{i\varphi_B(t)}|B(t, v)\rangle \quad (34)$$

For simplicity, the dependencies on v are dropped and the coefficients are shortened such that the above equation becomes:

$$|\psi(t)\rangle_R = c_A e^{i\varphi_A(t)}|A(t)\rangle + c_B e^{i\varphi_B(t)}|B(t)\rangle \quad (35)$$

The probability P that an electron passing through slits A and B arrives at a given point on the screen is:

$$P = |c_A e^{i\varphi_A(t)} + c_B e^{i\varphi_B(t)}|^2 = c_A^2 + c_B^2 + 2c_A c_B \cos(\varphi_A - \varphi_B) \quad (36)$$

where $2c_A c_B \cos(\varphi_A - \varphi_B)$ is the interference term [14].

On the other hand, when detectors A and B are placed at slits A and B respectively, the fundamental representation as characterized by the fundamental-state-vector $|\psi(t, v)\rangle_F$ is used to illustrate the detection of the electron through slit A or slit B.

Let $|\phi_A\rangle$ and $|\phi_B\rangle$ denote the state-vectors of the detectors D_A and D_B at slits A and B respectively and let $|\psi\rangle_F$ be the fundamental-state-vector of the electron according to equation (32). Prior to any interaction (at $t=0$), the state vector $|\Psi\rangle$ for the entire system (i.e. the detectors and the electron) is denoted as follows:

$$|\Psi(t=0)\rangle = |\psi(0)\rangle_F |\phi_A(0)\rangle |\phi_B(0)\rangle \quad (37)$$

Thus,

$$|\Psi(t=0)\rangle = (\delta_v (\Delta v_A(0))|A(0, v)\rangle + \delta_v (\Delta v_B(0))|B(0, v)\rangle)|\phi_A(0)\rangle |\phi_B(0)\rangle \quad (38)$$

where the phase terms $e^{i\varphi_A(t)}$ and $e^{i\varphi_B(t)}$ are dropped for simplicity. At the beginning ($t=0$), detectors A and B do not detect any electron. Let $|\phi_{A0}\rangle$ and $|\phi_{B0}\rangle$ denote the state of the

detectors when they are not clicking (i.e. not detecting) and thus the above expression (38) is reduced into:

$$|\Psi(t = 0)\rangle = |A(0, v)\rangle|\phi_{A0}(0)\rangle + |B(0, v)\rangle|\phi_{B0}(0)\rangle \quad (39)$$

Equation (39) indicates that there is no electron at slit A and detector D_A is not clicking and that there is no electron at slit B and detector D_B is not clicking.

Under the action of the Schrödinger evolution with respect to the physical-time, the state vector $|\Psi\rangle$ evolves into the following entangled state:

$$|\Psi(t)\rangle = \delta_v(\Delta v_A(t))|A(t, v)\phi_A(t)\rangle + \delta_v(\Delta v_B)|B(t, v)\phi_B(t)\rangle \quad (40)$$

Equation (40) can be expressed as follows:

$$|\Psi(t)\rangle = \begin{cases} |A\phi_{A1}\rangle + |B\phi_{B0}\rangle & \text{if } v \in \Delta v_A \\ |A\phi_{A0}\rangle + |B\phi_{B1}\rangle & \text{if } v \in \Delta v_B \end{cases} \quad (41)$$

where $|A\phi_{A1}\rangle$ represents the state in which the electron is at the slit A and the detector D_A clicks and similarly $|B\phi_{B1}\rangle$ represents the state in which the electron is at the slit B and the detector D_B clicks. The above expression indicates that the electron is either on the one hand detected at slit A and not at slit B or on the other hand is not detected at slit A and detected at slit B. Therefore, no interference can take place and the electron simply behaves as a simple particle impacting the screen at a specific point.

2.7. Entanglement

Consider a quantum system composed of first and second entangled particles travelling in different directions. Suppose $\{|\lambda_k\rangle\}$ and $\{|\varphi_l\rangle\}$ are two eigenbasis of the first and second particles respectively. The composite state of the quantum system is defined by the tensor product which can be expressed as follows:

$$|\lambda_k\varphi_l\rangle = |\lambda_k\rangle \otimes |\varphi_l\rangle \quad (42)$$

Similarly to equation (5), the fundamental-state-vector $|\psi(t, v)\rangle_F$ in the composite space of states of the two particles can be expressed in function of the vector projections in the eigenbasis $\{|\lambda_k\varphi_l\rangle\}$ of an observable as follows:

$$|\psi(t, v)\rangle_F = \sum_{klj} \delta_v(\Delta v_j(t)) |\lambda_{kj}\varphi_{lj}\rangle = \begin{cases} |\lambda_{kj}\varphi_{lj}\rangle & \text{if } v \in \Delta v_j \\ 0 & \text{if not} \end{cases} \quad (43)$$

where as in equation (6), $\delta_v(\Delta v_j)$ is the Dirac measure that labels and orders the composite sub-states $|\lambda_k\varphi_l\rangle$ along the state-time-line (v) and where in general $j = k \times l$. The first and second parts of the ket represent the sub-states of the first and second particles respectively.

Equation (43) indicates that the composite sub-states $|\lambda_{kj}\varphi_{lj}\rangle$ do not occur at once and only one composite sub-state exists at each state-time-index v . The transition from one composite sub-state into another is governed by the internal-state-clock “materialized” by the corresponding state-time-line. Indeed, at each physical-time-index, the corresponding internal-state-clock governs the entangled system as a single entity thus, synchronizing the transitions of both particles. In other words, both particles are “connected” by the same 2-d-

manifold whatever is the spatial separation between them. Both particles can thus be considered as “connected” by a same “time-filament” belonging to the 2-d-manifold at each ticking of the internal-state-clock.

Suppose a measurement is to be made at the side of the first particle. Let $|\phi\rangle$ denotes the state-vector of the measuring device and $|\Psi\rangle$ the state vector for the combined system “measuring device + particle”. At the beginning ($t=0$) and before measurement, the state vector $|\Psi\rangle$ for the combined system is:

$$|\Psi(t=0)\rangle = |\psi\rangle_F |\phi_0\rangle = (\sum_{klj} \delta_v(\Delta v_j(t)) |\lambda_{kj}\varphi_{lj}\rangle) |\phi(0)\rangle \quad (44)$$

Under the action of the Schrödinger evolution with respect to the physical-time, the state vector $|\Psi\rangle$ for the combined system of the above expression evolves into the following entangled state:

$$|\Psi(t)\rangle = |\psi\rangle_F |\phi_t\rangle = \sum_{klj} \delta_v(\Delta v_j(t)) |\lambda_{kj}\varphi_{lj}\rangle |\lambda_{kj}\phi_j(t)\rangle \quad (45)$$

Thus:

$$|\Psi(t)\rangle = \begin{cases} |\lambda_{kj}\varphi_{lj}\rangle |\lambda_{kj}\phi_j(t)\rangle & \text{if } v \in \Delta v_j \\ 0 & \text{if not} \end{cases} \quad (46)$$

where $|\lambda_{kj}\varphi_{lj}\rangle |\lambda_{kj}\phi_j(t)\rangle$ represents on the one hand the state in which the first particle is in the state $|\lambda_{kj}\rangle$ and the measuring device being in the state indicating the state $|\lambda_{kj}\rangle$ and on the other hand the state in which the first particle is in the state $|\lambda_{kj}\rangle$ and the second particle is in the state $|\varphi_{lj}\rangle$.

This indicates that once the state of the first particle is observed to be at the state $|\lambda_{kj}\rangle$ (i.e. it has been selected by the measuring device when it was at the state $|\lambda_{kj}\rangle$), no more transitions can take place simply because the internal-state-clock along the state-line (v) stops ticking and therefore the state of the second particle is held-up at the corresponding state $|\varphi_{lj}\rangle$.

When at a specific physical-time-index, a measurement is made at either side of the entangled system, the internal-state-clock governing the fundamental-state-vector $|\psi(t,v)\rangle_F$ simply stops the transition of the entangled system into any other sub-state. In other words, the internal-state-clock synchronizing the transition of states of both particles stops ticking and thus, the action of measurement on any particle fixes the outcome result for both particles.

Consider for example an entangled system of two particles characterized by two spins specified by the z components travelling in opposite directions and emanating from a source midway between two detectors. The composite state of the two-spin system is a tensor product having the following basis vectors [15]:

$$|uu\rangle ; |ud\rangle ; |du\rangle ; |dd\rangle \quad (47)$$

where the u stands for spin “up” (i.e. an upward direction of spin with respect to a z-axis) and the d for spin “down” (i.e. a downward direction of spin with respect to the z-axis) and where the first and second parts of the ket represent the states of the first and second particles respectively.

Let the two-spin system be in a maximally entangled state corresponding to the singlet state $|sing\rangle$ conventionally expressed as follows:

$$|sing\rangle = c_1|ud\rangle + c_2|du\rangle \quad (48)$$

However, in accordance to equation (43), the fundamental-state-vector $|sing(t, v)\rangle_F$ is:

$$|sing(t, v)\rangle_F = \delta_v(\Delta v_1)|ud\rangle + \delta_v(\Delta v_2)|du\rangle = \begin{cases} |ud\rangle & \text{if } v \in \Delta v_1 \\ |du\rangle & \text{if } v \in \Delta v_2 \end{cases} \quad (49)$$

Equation (49) expresses the fact that both particles are “connected” by the same 2d-time-manifold composed of an ordered sequence of two “time-filaments” corresponding to sub-states $|ud\rangle$ and $|du\rangle$ respectively. In other words, the state transition of both particles is synchronized by the same internal-state-clock, and thus, when a measurement is made at either side, the internal-state-clock simply stops the transition into any other sub-state.

The following affirmations can be concluded out of the above formalism:

- Any physical object has a definite state at each elementary-time independently of our observing it.
- No hidden information is built-in the entangled particles at their source and their states are simply synchronized by a common state-time-line. Thus, before measurement, the two entangled particles are governed by a common cause that screens off the apparent correlation between them. This common cause is the internal-state-clock (i.e. the state-time-line and transition process) that synchronizes their states.
- After measurement, the act of measurement at one end stops the ticking of the internal-state-clock (i.e. stops the transition process along the state-time-line) and thus affects the transition process at the other end. The measured state of one particle at one end “instantaneously” with respect to the physical-time (i.e. at the same physical-time-index t) affects the state of the other particle at the other end whatever is the distance between them. However, this interaction between both particles takes place through the internal-state-time and instantaneously with respect to the physical-time without any transfer of information through space.

To summarize, there is a common cause (internal-state-clock) that synchronizes the entangled particles before measurement and the act of measurement on one particle instantaneously (by means of the state-time) affects the state of the other particle.

Thus, the model of internal-time seems to reconcile the local realism of Einstein with the predictions of quantum mechanics. Indeed, any physical system has pre-existing states independently of our observing them and the principle of locality is not transgressed because entangled particles that are space-like separated do not causally influence each other through space but through the internal state-time.

2.8. Estimation of Time's thickness and Testability

The state-time index v along the state-time-line (v) is considered as a simple index or parameter. However, in certain cases and in particular, when time is an internal variable (as is the case of the state-time index v), it can be considered as a time operator [16, 17].

Indeed, for a given quantum system at a given physical-time-index t , the internal state transitions of a given fundamental-state-vector $|\psi(t, v)\rangle_F$ evolve with respect to the state-time-line (v) . These transitions along the state-time-line (v) define an internal-state-clock inherent to that fundamental-state-vector $|\psi(t, v)\rangle_F$. The internal-state-clock is ticking in a domain limited to the oriented-state-time-life Δv of the state-time-line (v) and wherein the corresponding fundamental-state-vector $|\psi(t, v)\rangle_F$ evolves in the domain Δv and vanishes to zero outside this domain. In order to make a distinction between a simple parameter on the state-time-line and an inherent state-time-index, the latter is simply called a “state-time-date”. The state-time-date is intrinsically related to the elementary-time-thread of the system and may thus be advantageously considered as a state-time observable associated to a Hermitian operator N such that:

$$N|v\rangle = v|v\rangle \text{ for all } v \in \Delta v. \quad (50)$$

Every real number v_m in the domain Δv is an eigenvalue of N , and the corresponding eigenvectors are thus given by Dirac’s delta functions:

$$|v_m\rangle = \delta(v - v_m) \quad (51)$$

Thus, the Hermitian operator $N(v_m)$ is considered to be the state-time-operator at the state-time-date v_m . Hence, the action of the state-time-operator $N(v_m)$ on the fundamental-state-vector $|\psi(t, v)\rangle_F$ determines and selects the corresponding state-time-date and automatically yields the state associated to the selected state-time-date:

$$|\psi(v_m)\rangle = \int_{-\infty}^{+\infty} \delta(v - v_m) |\psi(t, v)\rangle_F dv \quad (52)$$

Advantageously, for any given physical-time-index t , the Hermitian operator $N(v_m)$ may be used to define the measurement of an observable at any state-time-date v_m along the state-time-line (v) .

A Hermitian energy operator H can thus be defined with respect to the state-time-line (v) and the uncertainty relation between energy E and the state-time-life $|\Delta v(t)|$ of the corresponding state-time-line (v) can be used to evaluate the thickness of time. Thus, a rough estimate value of the thickness of time $|\Delta v|$ for a particle having a well-defined energy E is:

$$|\Delta v| \sim h/E \quad (53)$$

This value is very small hence, the expression “thickness of time”. To be more precise, the above expression represents the lower bound estimate of the thickness of time knowing that the uncertainty relation is $|\Delta v|E \geq h$.

The double-slit experiment may be used to determine the upper bound estimate by increasing the distance between the slits until the interference disappears. Indeed, when the state-time needed for a particle to be at both slits exceeds the thickness of time, interference should not take place. This kind of experiment may be used as a test for the internal-time model as well as, a means to calculate the thickness of time.

Let the maximum distance between the slits be d_{max} above which no interference takes place. Thus, for a particle having a speed c , a rough upper bound estimate of the thickness of time may be given by:

$$|\Delta v| \leq d_{max} / c \quad (54)$$

It should be noted that time-thickness for macroscopic objects or phenomena is very small to be noticeable and can thus be neglected. Nevertheless, it becomes important and should be taken into consideration for short living phenomena such as the creation of virtual particles. Indeed, the time life of a newly created virtual particle should be given by:

$$t_{life} \sim \sqrt{t^2 + v^2} > t \quad (55)$$

Another example where time-thickness should not be neglected is the early stage of the universe where the expansion of the universe should have lasted more than what would be expected by an observer who traces the history only along the physical-time axis. This may explain a very early inflation of the universe.

2.9. Cosmic consequences of the internal-time model

The standard big bang theory explains the expansion of the Universe, the spectrum of the cosmic microwave background radiation as well as plenty other observations. However, it leaves some questions unanswered and seems to demand very carefully chosen initial conditions [18]. This feature, known as the horizon problem cannot be accounted for by the standard big bang theory. Another special feature, known as the flatness problem requires from the big bang to specify the mass density of the early Universe with extreme precision. These initial conditions were explained by Guth [19] by an inflation phenomenon at the early stage of the Universe.

The internal-time model can be used to give an alternative scenario for explaining these seemingly special initial conditions of the universe. In a microscopic universe (i.e. at the beginning of the universe), the thickness of time should not be neglected. Indeed, as the physical-time tends to zero, the state-time should have been comparable in magnitude to that of the physical-time. Therefore, in order to understand the phenomena that took place at the beginning of the universe, time-thickness should be taken into consideration.

The approach explained by Carroll [20] is used to derive an appropriate metric that takes into consideration the global-time-tube metric of equation (2). A spatially homogenous and isotropic Universe evolving within a global-time-tube can be represented at each point of the global-time-tube by spacelike three-dimensional slices such that each slice is maximally symmetric. Thus spacetime is considered to be $R^3 \Sigma$ where R^3 represents a three-dimensional time metric and Σ is a maximally symmetric three-dimensional space metric. The six-dimensional spacetime can thus be expressed by the following sort of Robertson-Walker metric:

$$ds^2 = -dt_G^2 + a^2(t_G)d\sigma^2 \quad (56)$$

where t_G is the global-time-tube, $a(t_G)$ is a dimensionless scale factor and $d\sigma^2$ is the metric on Σ .

The above metric of equation (43) obeys the following Friedman equations:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad (57)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad (58)$$

where a is the scale factor that stands for $a(t_G)$, H is the Hubble parameter, G is the gravitational constant, ρ is the energy density, k is the spatial curvature, and p is the pressure. Friedmann equation (44) can equivalently be written in the following form:

$$|\Omega - 1| = \frac{|k|}{a^2 H^2} \quad (59)$$

where Ω is the density parameter measuring the ratio between the density and the critical density and where a flat space is represented by $\Omega = 1$ [18, 21].

By taking into consideration the metric of equation (2), an element of the global-time-tube at the beginning of the big bang (i.e., t being very small) may be expressed as follows:

$$t_G^2 \sim t^2 + \theta^2 + \eta^2 = t^2 + v^2 \quad (60)$$

After introducing, the above element of the global-time-tube, the scale factor for a universe dominated by only one kind of energy density (which indeed should have been the case at the beginning of the universe) is given by the following relation:

$$a \propto t_G^{2/n} \sim (t^2 + v^2)^{1/n} \quad (61)$$

Differentiating the above expression with respect to the physical time t gives:

$$\dot{a} \propto \frac{2t}{n} (t^2 + v^2)^{1/n-1} \quad (62)$$

Differentiating again with respect to t gives:

$$\ddot{a} \propto \frac{2(t^2 + v^2)^{1/n-2}}{n} \left(v^2 - \frac{n-2}{n} t^2 \right) \quad (63)$$

The first term is positive and thus:

$$\ddot{a} > 0 \text{ when } t < \sqrt{\frac{n}{n-2}} v \quad (64)$$

$$\text{Let } \tau = \sqrt{\frac{n}{n-2}} v \quad (65)$$

where τ represents the physical time below which, the Universe was in an inflation-era.

Introducing the estimation of time-thickness of equation (53) into equation (65) gives:

$$\tau = h \sqrt{\frac{n}{n-2}} / E \quad (66)$$

At the beginning, the universe was radiation-dominated (i.e. $n=4$, $p = \rho/3$) and thus the inflation era is given by:

$$\tau = \sqrt{2} h / E \sim h / E \quad (67)$$

Equation (67) gives an estimation of the inflation era in function of the energy at the beginning of the Universe.

On the other hand, by substituting $\ddot{a} > 0$ into the Friedmann equations (57, 58), the following inequalities are derived :

$$p < \rho/3 \quad (68)$$

$$\frac{d(H^{-1}/a)}{dt} < 0 \quad (69)$$

In particular, relation (69) forces the value of Ω in equation (59) to 1 which solves the flatness problem. This scenario explains the observed flatness and isotropy of the Universe without the need to introduce any kind of “dark energy”.

However, according to this scenario, the space inflation seems to be more of an illusion and it simply results out of observing the inflation-era with respect to the very small scale ($t < \tau$) of the physical-time axis. Indeed, for $t < \tau$, the length of the global-time-tube t_G is greater than that of the physical time t ($t_G \sim \sqrt{t^2 + v^2} > t$) and thus, the real period of time is greater than that of the physical time. In other words, the expansion of the universe during the inflation era lasted more than what would be expected by an observer who traces the history only along the physical-time axis.

On the other hand, for $t \geq \tau$, $\ddot{a} \leq 0$ and thus, the inflation stops at $t = \tau$ and the acceleration decreases for $t > \tau$.

For $t \gg \tau$, the projections of the global-time-tube on the state-time-axis can be neglected as they are very small compared to that on the physical-time axis. The global-time-tube can simply be approximated by its projection on the physical-time axis (i.e. $t_G \sim t$). Thus, far from the inflation-era the global-time-tube behaves almost as the familiar physical time.

However, depending on the energy density of the universe, the curvature of spacetime should affect the dynamical relation between the different components of time. In other words, the global-time-tube becomes more or less curved and this in its turn should affect the apparent rate of expansion of the universe. For example, if the “length” of the global-time-tube is greater than its projection on the physical-time axis, then the universe would seem to be spatially inflating if the physical-time-axis is the only one to be considered and not the real “length” of the global-time. The present observations seem to indicate that the universe is inflating and this apparent inflation could be simply explained by the dynamic nature of time without the need to introduce any kind of dark energy.

3. Conclusion

This internal-time model defines an internal evolving mechanism embedded in the U-process. At each physical-time-index, the state of the quantum system evolves from one state into another with respect to the state-time-line. A measurement stops the internal evolving mechanism such that the quantum system will only be at the state associated to the elementary-time-date during which the interaction between the quantum system and the measuring apparatus took place. Unlike the conventional U and R processes, the internal-time

model does not seem to create any conflict between the internal mechanism and the U-process.

On the other hand, the internal-time model comes fully in terms with the fundamentals or principles of quantum mechanics. Yet, it seems to maximize simplicity for explaining the measurement problem, the particle-wave duality, and the uncertainty principle. It also gives a reasonable explanation of entanglement without transgressing the principle of locality. These simple explanations may be considered as indications of the veracity of this model. Nevertheless, the internal-time model is experimentally falsifiable and provides new features concerning for example the life duration of virtual particles which can be tested.

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