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► **To cite this version:**

Louis de Mesnard. Coordination and profit rate maximization. [Research Report] Laboratoire d'analyse et de techniques économiques(LATEC). 1995, 24 p., Figure, ref. bib.: 6 ref. hal-01526973

**HAL Id: hal-01526973**

**<https://hal.science/hal-01526973v1>**

Submitted on 23 May 2017

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Pôle d'Economie et de Gestion

2, boulevard Gabriel - 21000 DIJON - Tél. 03 80 39 54 30 - Fax 03 80 39 54 43

ISSN : 1260-8556

**n° 9503**

**Coordination and profit rate maximization**

**Louis de MESNARD**

**February 1995**

# Coordination and profit rate maximization

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## **ABSTRACT.**

We study the effects over the coordination between firms of the choice of another objective function for the firm: the profit rate maximization which seems to be closer to business practice than the classical pure profit maximization. After recalling the effect of the new objective function over the optimum (the optimal production is lower), two cases are considered : perfect competition (the price signal disappear), and duopoly-oligopoly (the functions of reactions may vanish).

## **RESUME.**

Nous étudions les effets sur la coordination inter-firmes du choix d'une autre fonction objectif pour la firme: la maximisation du taux de profit qui semble être plus proche de la pratique des affaires que la classique maximisation du profit pur. Après avoir rappelé l'effet de la nouvelle fonction objectif sur l'optimum (la production optimale est plus faible), deux cas sont considéré : la concurrence parfaite (le signal prix disparaît), et le duopole-oligopole (les fonctions de réactions peuvent disparaître).

## **KEYWORDS.**

Coordination, Profit, Profit rate, Maximization, Microeconomics.

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<sup>1</sup> Faculty of Economics,  
University of Dijon  
4 Bd Gabriel  
21000 Dijon, FRANCE  
80.58.25.58  
E-mail : lmesnard@satie.u-bourgogne.fr

## I. Introduction

Coordination between firms is one of the major theme of economics : it is the central idea of both signal theory and games theory.

- In perfect competition, the firm equalize price to marginal cost in order to maximize its pure profit. The price signal allows coordination between the firm and the market.
- In imperfect competition with a little number of firms, like duopoly-oligopy, firms build reactions functions where the conjectural variations, the terms  $v^i = \frac{\partial \tilde{Q}^i}{\partial Q^i}$  with  $Q^i$  as the production of firm  $i$  and  $\tilde{Q}^i$  as the production of other firms, play the central role, indicating the reaction of the firm(s) other than  $i$  regarding to the action of the firm  $i$ . The intersect of the functions of reaction determine the optimal production of both firms. It is one of the starting point of games theory.

We shall explore the consequences over the coordination between firms of the abandoning of the neoclassical objective function, pure profit maximization, to the benefit of another objective function, profit rate maximization, which looks to be closer to business practice.

## II. Preliminaries

### A. *The objective functions of the firm*

#### 1. Two alternative functions

Leaving aside deliberately the improvements made to the neoclassical theory of the firm, like incomplete information, theory of the bounded rationality, theory of the agency, theory of contracts and incentive, managerial and behaviorist theory of the firm, etc., there are two

possibilities for a firm : *to maximize the absolute satisfaction* or *to maximize the relative satisfaction*.

In the following,

- The words ***accounting profit*** denotes the accounting profit able to remunerate the shareholders, that is to say before payment of dividends. It is not used to pay the eventual interest (yet paid). If accounting profit is positive, a dividend is optionally paid, equally to every shareholders, in order to remunerate this authorized capital. In this paper, the costs corresponding to the accounting profits are the ***accounting costs***.
- The words ***authorized capital*** refers to the funds advanced by the shareholders to the firm: there is nor obligatory pay back, nor obligatory payment of a remuneration, but it is the usual counterpart of a property right. This remuneration is free *ex ante* as *ex post*, and varies between zero and the amount of accounting profit (less the funds putted in reserve and taxes), depending on the policy of dividend of the firm. Authorized capital remains in the balance sheet of the firm and remains the property of the shareholder (to the contrary of a debt, which disappear of the balance sheet of the firm when paid back).

The cost of the authorized capital is an opportunity cost, because these free funds may be invested in another place at a normal interest rate: the authorized capital looks to come from an operation of renting similar to a borrowing, and its opportunity cost looks like an interest of which the rate is the average rate of the market (calculated with a standard alternative investment).

In the opposite, borrowing are paid back obligatorily at the settlement date (except if bankruptcy or at the dissolution of the firm) with obligatory payment of an interest to the moneylender, without giving to him a property right (the question of the control of the shareholders over the firm differs) : interest is a cost. Self-financing correspond to a withdraw over a reserve (a past profit or over an actual profit): it is

comparable to a withdraw over the shareholder and then it is similar to an increase of authorized capital.

Remark. The reselling of shares is only a transfer between shareholders and does not concern the firm directly (it determines only the future success of the operations of increasing the authorized capital); it may constitute a remuneration of shareholders, but only of the ancient one, the seller, not of the new one, the purchaser.

## 2. Entry in the sector

The conditions of entry in the sector for the firm are the following :

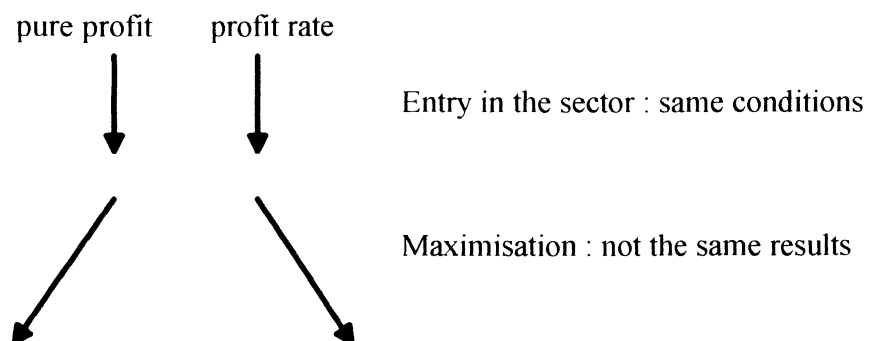
- if the pure profit is positive or nil, that is to say if:

$$(\text{accounting profit}) - [ (\text{opportunity costs per unit of authorized capital}) \times (\text{authorized capital}) ] \geq 0 \quad (1)$$

- or if the ratio of profit is superior or equal to the opportunity costs per unit of capital, i.e. if:

$$\frac{(\text{accounting profit})}{(\text{authorized capital})} \geq (\text{opportunity costs per unit of authorized capital}) . \quad (1')$$

These two conditions are obviously equivalent to decide if the firm must or must not produce. However, the classical microeconomic theory choose the first possibility. Moreover, we shall see that the results diverge when we want to maximize the pure profit or the profit rate.



### 3. Historical aspects

So, the question is: why the subtractive form of the function of profit (profit minus opportunity cost of the capital) prevailed over the "divisive" form of the function of profit (profit over capital)? Yet, nor the first nor the second possibility of calculation indicate anything about the question: what is the optimal production? We shall see that these two objective functions differ on this point.

Why did one historically choose to maximize the pure profit? We may propose the following conjecture.

In the pre-industrial age and from the beginning of the industrial revolution, capitalists (or landowners) often had no alternative possibility of investment than their own business (or property). The modern legal form of firms, like company, was not widespread and the financial market was non-existent. Thus the opportunity costs were disregarded and one choose the maximisation of the simple profit. That is why the pre-classic authors, like Bacon or Hobes, have adopted naturally the principle the more in accordance with the observed reality of their era, the maximization of the satisfaction, that is to say the maximization of the earnings, later modernized as the maximization of the profit.

Even if the classic authors insist on the profit rate (Smith, Ricardo, etc.), they consider, like Marx later, the social profit rate and the profit rate of the sectors (see the law of equalization of the profit rate of sectors), more than the profit rate of the firm, and they do not think in terms of profit rate maximization.

Later, the neoclassic authors have conserved the principle of profit maximization, introducing the opportunity costs and the pure profit. In the General Equilibrium, the profit of each firm is the revenue of this productive agent. Thus, it seems natural to maximize the revenue of the agent. The notion of pure profit was introduced to consider the opportunity costs of the social capital, however keeping the profit maximisation and rejecting other forms as the maximisation of a ratio, in order to not modify too much habits of reasoning.



Now, it is clear that, in the real world, firm maximizes a ratio of profitability, that is to say a profit rate. And the financial theory does not allow to leave out profit rate maximization (see Annex).

This proves that it is necessary to explore the alternative theory of the profit rate maximization. The microeconomic consequences of it has not been studied, even if it is an old idea in the business world.

### Remarks.

- Many theories think into terms of profitability: the theory of the choice of investment (with the rate of actualization), the theory of demand of capital (Keynes, among others, with the marginal efficiency of the capital), etc.
- This behavior, *take into account the available means to reach the objective*, is observed in any human activities, when a problem of choice exists. In some cases, human people do not respect this principle, for example when the objective is vital (high level sports competitions, military fight, etc.).

## **B. Variables and notations**

The variables that we shall introduce are:

- $p(Q)$ , the inverse of the curve of demand.
- $R(Q) = p(Q) Q$  the receipt.
- $C_p(Q)$ , the *cost of production*. It includes all the costs (direct costs and opportunity costs) of variable factors, like labor, plus renting costs and writing off of the fixed capital). It excludes the interest of borrowings and the remuneration of the authorized capital. The opportunity cost associated to the remuneration of the authorized capital will be introduced explicitly.
- $K_T(Q)$ , the positive *capital* (it is monetary, not physical) used in the production.

Obviously,  $K_T(Q)$  is linked to  $C_P(Q)$ , because it serves to pay the costs, and generally  $K_T(Q) \geq C_P(Q)$ .

However, the link between the cost of production and the capital funds is not so important because we find the costs of production both in the maximization of profit and in the maximization of a profit rate.

- $t$ , the constant and positive interest rate in the financial market, valid for the borrower and the moneylender: as the financial market is assumed to be perfect, the same rate  $t$  is used for the borrowing of the firm and for the opportunity cost of the funds.
- $K(Q)$ , the positive *authorized capital* used in the production

We confuse the authorized capital with the totality of the own funds of the firm. It is given by the shareholders and recovered by them only at the end of the process of production. There is an opportunity cost endured at each period at the rate  $t$  when the firm uses it.

The *authorized capital* is remunerated at the fixed interest rate  $t$ , giving an opportunity cost,  $t K(Q)$ .

- $K_B(Q)$ , the *borrowed capital* used in the production.

An interest is paid by the firm at each period :  $t K_B(Q)$ .

We have by hypothesis :  $K_T(Q) = K(Q) + K_B(Q)$ .

- $C(Q) = C_P(Q) + t K_B(Q) = C_P(Q) - t [K_T(Q) - K(Q)]$ , the *accounting cost*.
- $\Pi(Q) = R(Q) - C(Q)$ , the *accounting profit*.
- $\Pi(Q) = \Pi_A(Q) - t K(Q)$ , the *pure profit*.
- $\pi(Q) = \frac{\Pi_A(Q)}{K(Q)}$ , the *profit rate*<sup>2</sup>.

<sup>2</sup> In fact, we may define many other types of profit ratios. This one correspond to a financial profit rate. However, it is the more in accordance with the business practice, and we use it, calling it "profit rate" in order to lighter the notations.

$$\text{We have : } \pi(Q) = \frac{R(Q) - C_P(Q) - t [K_T(Q) - K(Q)]}{K(Q)} = \frac{\Pi(Q)}{K(Q)} + t .$$

### C. About the long-term

In the short period, the classical microeconomics assume that the capital is fixed. In this case, obviously,  $\{\text{Max profit}\}$  and  $\left\{\text{Max } \frac{\text{profit}}{\text{capital}}\right\}$  are equivalent. However it is not these formulas that we compare. We compare the classical pure profit maximization  $\{\text{Max } \Pi_{.1} - t K\}$  to the profit rate maximization  $\left\{\text{Max } \frac{\Pi_{.1}}{K}\right\}$  : at once in the pure profit maximization and in the profit rate maximisation, the authorized capital is considered, by its opportunity costs in the first case, directly by a ratio in the second case.

Anyway, the classical theory says : at short-term, the capital is fixed, at long-term the capital varies. It is not correct, because, at short-term, if the capital is fixed, the scale of production is given; thus it is necessary to introduce this constraint in the classical calculation of the short-term equilibrium : the program is  $\text{Max}_Q \Pi(Q)$  under  $Q \in [\check{Q}, \hat{Q}]$ , where  $\check{Q}$  and  $\hat{Q}$  are the bounds of  $Q$  due to the physical capital invested, which generates the fixed costs.

This is why, in the following, we will not consider a long period where the capital is variable (as in [ de Mesnard, 1992 ] ) because it is confusing, but a short period with an *ex ante* calculation, where the capital is freely fixed (while *ex post*, the capital is fixed).

### D. General case : monopoly

Proposition 1. Consider the monopoly with function of capital : it is the more general case. The firm produces up to the point where the marginal profit is equal to the average profit multiplied by the relative elasticity of the capital to the production.

Proof. Consider the profit rate  $\pi(Q) = \frac{\Pi_{.1}(Q)}{K(Q)}$ . Its maximization gives:

$$\Pi'_{.1}(Q) = \Pi_{.1}(Q) \frac{K'_1(Q)}{K(Q)} \Leftrightarrow \Pi'_{.1}(Q) = e_{K/Q} \bar{\Pi}_{.1}(Q) \Leftrightarrow \Pi'_{.1}(Q) = \pi(Q) K'(Q)$$

where  $e_{K/Q} = \frac{dK(Q)}{K(Q)} = \frac{K'(Q)}{K(Q)} Q$  is the relative elasticity of the capital to the production

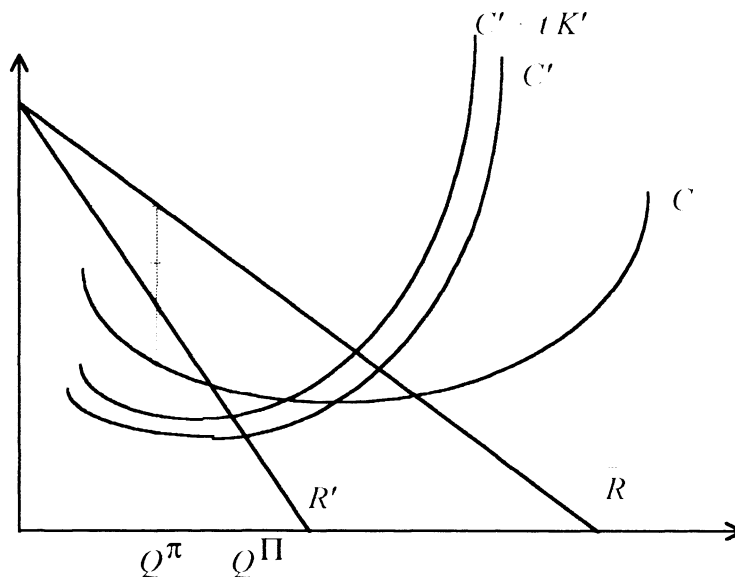
and  $\bar{\Pi}_i(Q) = \frac{\Pi_i(Q)}{Q}$  is the average profit ■

**Proposition 1'.** The monopoly with fixed coefficient of capital produces up to the point where the marginal profit is equal to the average profit (or to the point where the average profit is maximal).

**Proof.** We have :  $K(Q) = k Q$  with  $k > 0$  , and then  $e_{K/Q} = 1$ . Hence,  $\Pi'_i(Q) = \bar{\Pi}_i(Q)$  . ■

As said in [ de MESNARD 1992 ], this corresponds to an observed behavior of the firms: a firm produces and invest as much as the profit brought by a supplementary unit is superior or at least equal to the average profit brought by other units.

Denoting  $Q^\Pi$  as the classical equilibrium and  $Q^\pi$  is the profit rate equilibrium, we obtain the following graph :



**Figure 1. Short-term monopoly**

Proposition 2. The optimum is independant to the level of the fixed costs and to the constant term of demand.

Proof. Suppose  $C = C_F + f(Q)$  and  $D = D_0 + g(Q)$ . Then  $\Pi'_{,t}(Q)$  does not include the terms  $C_F$  and  $D_0$ . Thus, if the demand increases uniformly, or if the fixed costs increases, a profit rate maximizing monopoly conserves an unchanged optimum.

Remark. The equilibrium is not the point of long period equilibrium of the traditional monopoly (that is to say  $R'(Q) = C'_{LT}(Q) + t K'_{LT}(Q)$ ) at a long period) and do not correspond to the point of the minimum of the average cost.

### **E. Profit rate versus pure profit**

We may compare the optimum of the profit rate with the classical microeconomic optimum (pure profit); we obtain the following fundamental theorem.

Theorem. Consider the more probable case  $K'(Q^\pi) > 0$ , where  $Q^\pi$  is the optimal quantity which maximizes the profit rate  $\pi(Q)$ .

Compared to the pure profit maximization, the profit rate maximization gives a **lower optimal production** (respectively higher) **when the profit rate is higher** (respectively lower) **than the interest rate at the optimum of the profit rate. The results are the opposite if  $K'(Q^\pi) < 0$ .**

- $\pi(Q^\pi) > t \Rightarrow (\max \pi)$  is before  $(\max \Pi)$
- $\pi(Q^\pi) < t \Rightarrow (\max \pi)$  is after  $(\max \Pi)$

This is the condition of [ de MESNARD 1992 ].

Note that these results are made at the optimum: it is necessary to calculate optimum of a profit rate to know if it will be before or after the pure profit.

Proof. The solution for the maximization of the pure profit  $\Pi(Q)$  is obtained for :

$$\Pi'(Q) = \Pi'_{,t}(Q) - t K'(Q) = 0 .$$

The solution for the maximization of the profit rate  $\pi(Q)$  is obtained for :

$\Pi'_i(Q) - \pi(Q) K'(Q) = 0$  . Consider the curve  $f_\pi(Q) = \Pi'_i(Q) - \pi(Q) K'(Q)$  .

Let us compare the respective positions of  $\Pi'(Q)$  and of  $f_\pi(Q)$  .

Suppose that  $\Pi'(Q)$  is decreasing (  $\Pi''(Q) < 0$  ), so that the optimum of the pure profit  $\Pi(Q)$  would be a maximum.

Let us write  $\Pi'(Q)$  as a function of  $f_\pi(Q)$  :  $\Pi'(Q) = f_\pi(Q) + [\pi(Q) - t] K'(Q)$  .

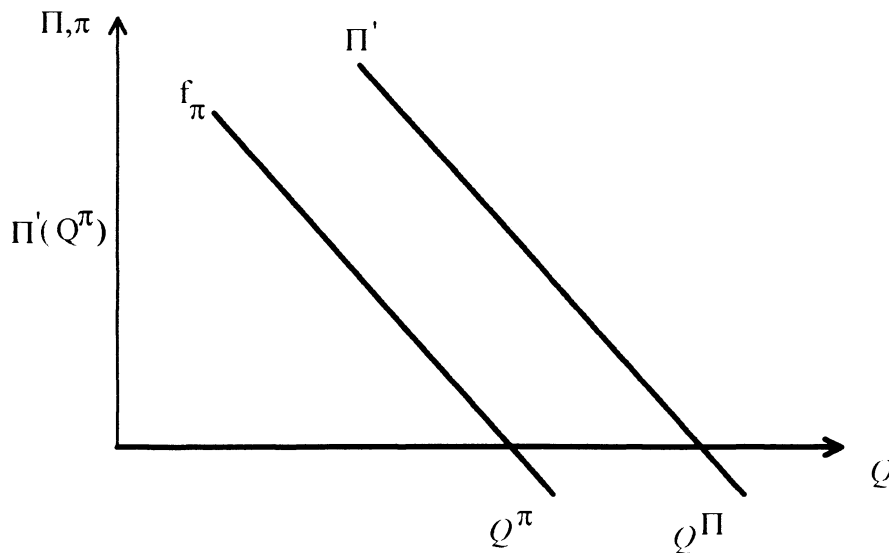
Denote  $Q^{II}$  the point of maximization of the pure profit, such as  $\Pi'(Q^{II}) = 0$  .

Denote  $Q^\pi$  the solution of the maximization of the profit rate, such as  $f_\pi(Q^\pi) = 0$  .

1) Suppose that  $K'(Q) > 0$  and  $\pi(Q^\pi) > t$ . Then,  $\Pi'(Q^\pi) > f_\pi(Q^\pi) = 0$  .

Near to  $Q^{II}$  , the curve  $f_\pi(Q)$  is under the curve  $\Pi'(Q)$  . Thus,  $Q^{II}$  must be higher than  $Q^\pi$ .

At the limit, if  $\Pi'(Q)$  is vertical near to  $Q^{II}$ ,  $\Pi'(Q)$  will cut the horizontal axis in  $Q^\pi$  and then  $Q^{II} = Q^\pi$  .



**Figure 2. Mathematical comparison of optimums**

2) Suppose that  $K'(Q) > 0$  and  $\pi(Q^\pi) < t$ . Then,  $\Pi'(Q^\pi) < f_\pi(Q) = 0$  .

Then, near to  $Q^\pi$  , the curve  $f_\pi(Q)$  is over the curve  $\Pi'(Q)$  .

Thus  $Q^{II}$  must be lower than  $Q^\pi$  . ■

### III. Coordination

#### A. Perfect competition

##### 1. Short-term

Consider one firm, in short-run perfect competition among a market (the number of firms is given). The above result obtained for monopoly with function of capital is applied here, assuming that the price is independent of  $Q$ :  $p(Q) = p$ . At the optimum, we have:  $p - C'(Q) = e_{K/Q} (p - \bar{C}(Q))$ .

Proposition 3. In perfect competition with fixed coefficient of capital, the firm produces up to the point of minimum cost.

Proof. We have:  $e_{K/Q} = 1$ . Thus  $C'(Q) = \bar{C}(Q)$  at optimum. ■

**The optimum does not depend on the price;** thus, the production does not vary if the price vary. It corresponds to an observed behavior.

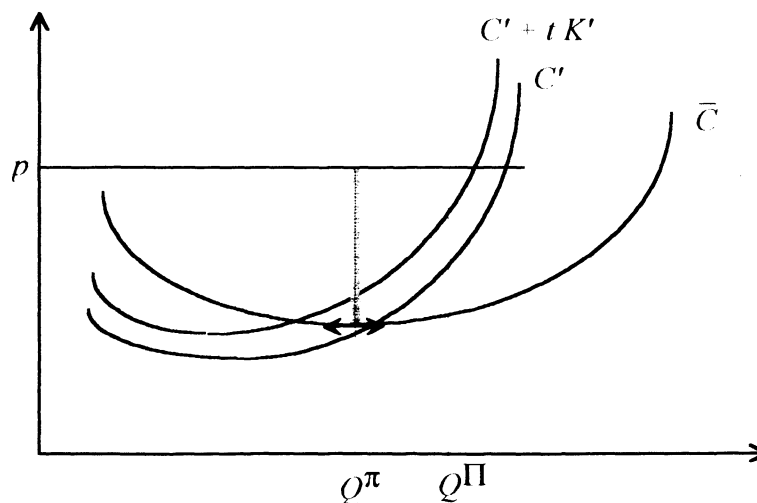
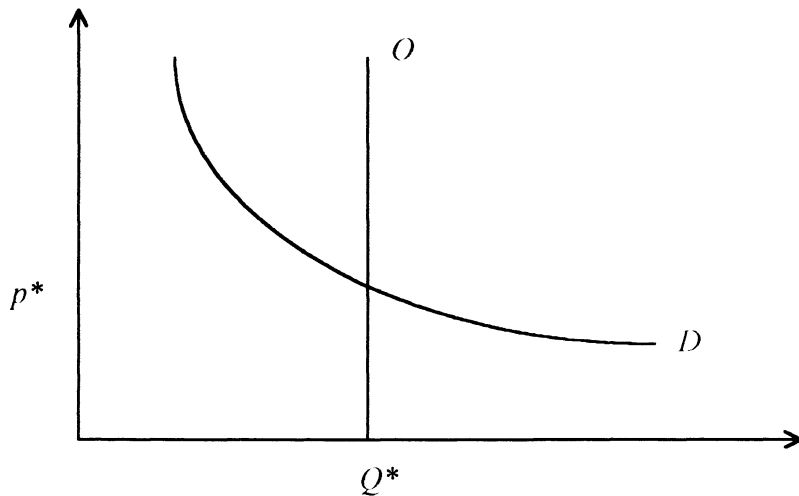


Figure 3. Short-term equilibrium of the firm in perfect competition

- Moreover, **there is no individual supply curve**, even in this case of perfect competition. However, there is a global supply curve, completely inelastic and there exists a global equilibrium of the market, following a classic scheme <sup>3</sup> :



**Figure 4. Global equilibrium in short-term perfect competition**

- In perfect competition, firms are smaller, there could be more firms; thus interaction between firms could be lower and competition could be greater.

Remark. The optimum of pure profit is given by  $p = C'(Q) + t K'(Q)$  .

Remark. The optimum seems to be identical to the classical point of long period equilibrium of the firm. However, do not confuse : we are not in long-term, but only *ex ante*, as said before. The number of firms remains fixed in our calculation, whereas the number of firms is variable in the long-run.

<sup>3</sup> In [ de MESNARD 1992 ], we assert that , as the price is no more a signal for the volume of production of the firm, the optimum does not automatically clears the market in the short period as supposed by the neoclassical theory. It is false : there is a clearing of the market in the sense that price is fully adapting, the quantity remaining fixed.



## 2. Long-term

Denote  $Q_{LT}^{\Pi}$  as the long-term optimum of pure profit and  $Q_{LT}^{\pi}$  as the long-term optimum of profit rate. These points are confused :

$$Q_{LT}^{\pi} = Q_{LT}^{\Pi} .$$

To prove it, denote  $\bar{K}_{LT}(Q)$  as the mean capital at long-term. At long-term,

- the pure profit maximizing firm moves to the point  $Q_{LT}^{\Pi}$  such as

$$\Pi_{LT}(Q_{LT}^{\Pi}) = p - [\bar{C}_{LT}(Q_{LT}^{\Pi}) + t \bar{K}_{LT}(Q_{LT}^{\Pi})] = 0 ;$$

- the profit rate maximizing firm moves to the point  $Q_{LT}^{\pi}$  such as

$$\pi_{LT}(Q_{LT}^{\pi}) = \frac{p - \bar{C}_{LT}(Q_{LT}^{\pi})}{\bar{K}_{LT}(Q_{LT}^{\pi})} = t .$$

These formulas are obviously identical : the conditions of entry in the sector are the same.

## B. Imperfect competition

### 1. Duopoly and functions of reactions

The demand for the sector with two firms 1 and 2 is  $D(Q^1 + Q^2)$  .

The inverse demand is  $p = D^{-1}(Q^1 + Q^2)$  , assumed to be decreasing.

The total revenue of each firm are:

$$R^1(Q^1 + Q^2) = Q^1 D^{-1}(Q^1 + Q^2) \text{ and } R^2(Q^1 + Q^2) = Q^2 D^{-1}(Q^1 + Q^2) .$$

The cost of each firm are  $C^1(Q^1)$  and  $C^2(Q^2)$  .

The profits of each firm are :

$$\Pi_1^1(Q^1, Q^2) = Q^1 D^{-1}(Q^1 + Q^2) - C^1(Q^1) \text{ and } \Pi_1^2(Q^1, Q^2) = Q^2 D^{-1}(Q^1 + Q^2) - C^2(Q^2) .$$

### i. The classical duopoly

For the classical maximisation of the pure profit, we solve the system :

$$\frac{\partial \Pi_1(Q^1, Q^2)}{\partial Q^1} = 0 \text{ and } \frac{\partial \Pi_2(Q^1, Q^2)}{\partial Q^2} = 0 .$$

That is to say, for the firm 1 :

$$\left\{ \begin{array}{l} \frac{\partial D^{-1}(Q^1 + Q^2)}{\partial Q^1} Q^1 + D^{-1}(Q^1 + Q^2) - C'^1(Q^1) - t K'(Q^1) = 0 \\ \frac{\partial D^{-1}(Q^1 + Q^2)}{\partial Q^2} Q^2 + D^{-1}(Q^1 + Q^2) - C'^2(Q^2) - t K'(Q^2) = 0 \end{array} \right\} .$$

Even for a very simple linear function of demand, the system has two functions of reaction : the solution is found by the intersect of two curves (look to the example later).

Denote  $v^1 = \frac{\partial Q^2}{\partial Q^1}$  and  $v^2 = \frac{\partial Q^1}{\partial Q^2}$  as the *conjectural variations*.

In the Cournot-Nash duopoly, when each firm modifies its production, it is supposed that the other firm would not react and change its own production. Thus  $v^1 = 0$  and  $v^2 = 0$  and the quantity  $\frac{\partial D^{-1}(Q^1 + Q^2)}{\partial Q^1}$  is replaced by  $\frac{d D^{-1}(Q^1 + Q^2)}{d Q^1}$ . However, functions of reaction remain.

### ii. The profit rate maximizing duopoly

Now, we assume that each firm maximizes its profit rate :

$$\left\{ \begin{array}{l} \pi^1(Q^1, Q^2) = \frac{Q^1 D^{-1}(Q^1 + Q^2) - C^1(Q^1)}{K^1(Q^1)} \\ \pi^2(Q^1, Q^2) = \frac{Q^2 D^{-1}(Q^1 + Q^2) - C^2(Q^2)}{K^2(Q^2)} \end{array} \right\}$$

The necessary conditions of optimality are,

$$\frac{\partial \pi^1(Q^1, Q^2)}{\partial Q^1} = 0 \text{ and } \frac{\partial \pi^2(Q^1, Q^2)}{\partial Q^2} = 0 .$$

For the firm 1, we obtain :

$$\Leftrightarrow \left\{ \begin{array}{l} \left[ \frac{\partial R^1(Q^1, Q^2)}{\partial Q^1} - \frac{dC^1(Q^1)}{dQ^1} \right] K^1(Q^1) - [Q^1 D^{-1}(Q^1 + Q^2) - C^1(Q^1)] \frac{dK^1(Q^1)}{dQ^1} = 0 \\ \left[ \frac{\partial R^2(Q^1, Q^2)}{\partial Q^2} - \frac{dC^2(Q^2)}{dQ^2} \right] K^2(Q^2) - [Q^2 D^{-1}(Q^1 + Q^2) - C^2(Q^2)] \frac{dK^2(Q^2)}{dQ^2} = 0 \end{array} \right\}$$

$$\Rightarrow \left\{ \begin{array}{l} \frac{\partial D^{-1}(Q^1 + Q^2)}{\partial Q^1} Q^1 + D^{-1}(Q^1 + Q^2) (1 - e_{K^1/Q^1}) = C'^{-1}(Q^1) - \bar{C}^1(Q^1) e_{K^1/Q^1} \\ \frac{\partial D^{-1}(Q^1 + Q^2)}{\partial Q^2} Q^2 + D^{-1}(Q^1 + Q^2) (1 - e_{K^2/Q^2}) = C'^{-2}(Q^2) - \bar{C}^2(Q^2) e_{K^2/Q^2} \end{array} \right\}$$

$$\text{where } \bar{C}^i(Q^i) = \frac{C^i(Q^i)}{Q^i}, \text{ and } C'^{-i}(Q^i) = \frac{dC^i(Q^i)}{dQ^i}, \text{ and } e_{K^i/Q^i} = \frac{\frac{dK^i(Q^i)}{K^i(Q^i)}}{\frac{dQ^i}{Q^i}} .$$

To solve the system, the classical process is : the firm 1 announces that it will produce the quantity  $Q_{(t-1)}^1$  .

The firm 2 calculates the level of its own production  $Q_{(t)}^2$  which maximizes its profit, taking the production of the firm 1 as fixed, i.e.  $dQ_{(t-1)}^1 = 0$  .

Then, the firm 1 recalculates  $Q_{(t+1)}^1$  maximizing its profit, with  $Q_{(t)}^2$  fixed, i.e.  $dQ_{(t)}^2 = 0$  .

The following system to be solved is the expression of the two reaction functions :

$$\left\{ \begin{array}{l} \frac{\partial \pi^1(Q_{(t+1)}^1, Q_{(t)}^2)}{\partial Q_{(t+1)}^1} = 0 \\ \frac{\partial \pi^2(Q_{(t-1)}^1, Q_{(t)}^2)}{\partial Q_{(t)}^2} = 0 \end{array} \right\}$$

It is easy to find some counter-examples where the reaction functions disappear with the profit rate maximisation, while they exists with the pure profit maximisation. Generally, the reaction function disappear if the first equation does not include  $Q^2$  , and the second equation does not include  $Q^1$  .

Proposition 4. The deletion of reaction functions depends only on the parameters of the function of demand and on the elasticity of authorized capital regarding to the production, but it does not depend on the function of costs.

Proof. In the last system of equations, the right member of the equation of equilibrium of firm  $i$  does not depend on the production  $Q^j$  of the other firm  $j$ , but the left member depends on it. ■

Proposition 5. In the duopoly in prices, the functions of reactions are deleted if :

- the inverse function of demand is linear :  $D^{-1}(Q^1, Q^2) = d - a Q^1 - b Q^2$
- and the elasticity of the authorized capital according to the production is equal to 1 for each firm :  $e_{K^i/Q^i} = 1$  for all  $i$  (this occurs when there is a coefficient of capital :  $K^i(Q^i) = k Q^i$  for all  $i$ ).

Proof. With these conditions, the system of equations becomes :

$$\left\{ \begin{array}{l} -a Q^1 - b v^1 Q^1 = C'^{-1}(Q^1) - \bar{C}^1(Q^1) e_{K^1/Q^1} \\ -a Q^2 - b v^2 Q^2 = C'^{-2}(Q^2) - \bar{C}^2(Q^2) e_{K^2/Q^2} \end{array} \right\}.$$

Thus,  $Q^1$  does not depend on  $Q^2$ . Reciprocally,  $Q^2$  is independent from  $Q^1$ . ■

**The solution is not found by the intersect of two functions of reaction:** in fact, each firm acts independently to the other and finds immediately the good solution; each firm considers only its cost to determine its production of equilibrium: only the parameters of cost and play a role; and are not equal. It is known that the duopoly have not always a stable solution. However, there are reaction function in all other models of duopoly even if dynamics may be complicated or chaotic [ RAND 1978 ] [ PIATECKI 1994 ].

Example. The above proposition corresponds to a very popular case, as the example of J.-C. Milleron [ MILLERON 1979, pp. 117-122 ], adapted to introduce the pure profit.

$$p = d - (Q^1 + Q^2)$$

$$R^1(Q^1 + Q^2) = d Q^1 - (Q^1)^2 - Q^1 Q^2 \text{ and } R^2(Q^1 + Q^2) = d Q^2 - Q^1 Q^2 - (Q^2)^2$$

$$\text{and, } C^1(Q^1) = c^1 + (Q^1)^2, C^2(Q^2) = c^2 + (Q^2)^2$$

and,  $K^1(Q^1) = k Q^1$  ,  $K^2(Q^2) = k Q^2$

Then,

$$\Pi_A^1(Q^1, Q^2) = -c^1 + d Q^1 - 2 (Q^1)^2 - Q^1 Q^2 \quad , \quad \Pi_A^2(Q^1, Q^2) = -c^2 + d Q^2 - Q^1 Q^2 - 2 (Q^2)^2 .$$

Thus, with the profit rate maximization :

- For the general duopoly in prices, the system to be solved is :

$$\left\{ \begin{array}{l} -Q_{(t+1)}^1 (1 + v^1) = Q_{(t+1)}^1 - \frac{c^1}{Q_{(t+1)}^1} \Rightarrow Q_*^1 = \sqrt{\frac{c^1}{2 + v^1}} \\ -Q_{(t)}^2 (1 + v^2) = Q_{(t)}^2 - \frac{c^2}{Q_{(t)}^2} \Rightarrow Q_*^2 = \sqrt{\frac{c^2}{2 + v^2}} \end{array} \right\}$$

which could be compare to the equivalent solution of the pure profit maximizing general duopoly :

$$\left\{ \begin{array}{l} (4 + v^1) Q_{(t+1)}^1 + Q_{(t)}^2 = d - t k \\ Q_{(t-1)}^1 + (4 + v^2) Q_{(t)}^2 = d - t k \end{array} \right\} \Rightarrow$$

$$Q_*^1 = \frac{3(d - t k) + (d - t k) v^2}{15 + 4v^1 + 4v^2 + v^1 v^2} \quad \text{and} \quad Q_*^2 = \frac{3(d - t k) + (d - t k) v^1}{15 + 4v^1 + 4v^2 + v^1 v^2}$$

- For the Cournot-Nash duopoly, the system to be solved is <sup>4</sup> :

$$\left\{ \begin{array}{l} -Q_{(t+1)}^1 = Q_{(t+1)}^1 - \frac{c^1}{Q_{(t+1)}^1} \Rightarrow Q_*^1 = \sqrt{\frac{c^1}{2}} \\ -Q_{(t)}^2 = Q_{(t)}^2 - \frac{c^2}{Q_{(t)}^2} \Rightarrow Q_*^2 = \sqrt{\frac{c^2}{2}} \end{array} \right\}$$

which could be compare to the equivalent solution of the pure profit maximizing duopoly of Cournot-Nash :

$$\left\{ \begin{array}{l} 4 Q_{(t+1)}^1 + Q_{(t)}^2 = d - t k \\ Q_{(t-1)}^1 + 4 Q_{(t)}^2 = d - t k \end{array} \right\} \Rightarrow Q_*^1 = Q_*^2 = \frac{d - t k}{5} .$$

<sup>4</sup> This result was initially set up in : "The new theory of the microeconomic maximization of profit rate : a typological and applied study", roneo, Dijon, march 1994.

Proposition 6. The functions of reactions disappear in the duopoly in prices if :

- the inverse function of demand is simply quadratic :  $D^{-1}(Q^1, Q^2) = d - a Q^1 - Q^2$
- and the elasticity of the authorized capital according to the production is equal to 2 for each firm :  $e_{K_i^c/Q^i} = 2$  for all  $i$

Proof. With the first condition, the left member of the equation of the firm 1 becomes :

$$-2 a Q^1 Q^2 - a v^1 (Q^1)^2 + d - d e + e a Q^1 - Q^2$$

The terms  $Q^2$  are off if  $e_{K_i^c/Q^i} = 2$  . ■

It is false to say that the condition about the elasticity seems to be linked strictly with the degree of the demand function.

Proposition 7. The functions of reaction does not disappear if the quadratic inverse function of demand includes a linear term :  $D^{-1}(Q^1, Q^2) = d - a Q^1 - b Q^2 - f Q^1 Q^2$  .

Proof. With the first condition, the terms including  $Q^2$  in the left member of the equation of the firm 1 are :

$$-f Q^1 Q^2 (2 - e_{K_i^c/Q^i}) - b Q^2 (1 - e_{K_i^c/Q^i}) .$$

This does not vanish whatever  $e_{K_i^c/Q^i}$  . ■

## 2. Oligopoly

The "model of Mason" reflects the paradigm Structure-Conduct-Performance. However ancient, it serves as a start point to the new economics of Industrial Organization. K. Cowling and M. Waterson present a theoretical version of this classical model [ COWLING and WATERSON 1976 ] .

Consider  $N$  firms producing at the same unique price  $p$ , the quantity  $Q^i$  at the cost  $C^i$ ; the number of firms is given (there is no entry in the sector).

We have  $Q = \sum_{i=1}^N Q^i$ . The demand is  $p = D^{-1}\left(\sum_{i=1}^n Q^i\right)$ .

Assume that the financial capital is proportional to the production for each firm:  $K_{.i}^i = k^i Q^i$  where the coefficient of capital  $k_i$  is constant.

We assume that each firm maximizes the profit rate :

$$\pi^i(Q^i) = \frac{\Pi_{.i}^i(Q^i)}{K_{.i}^i(Q^i)} = \frac{D^{-1}\left(\sum_{i=1}^n Q^i\right) Q^i - C^i}{K_{.i}^i(Q^i)} \quad \text{and} \quad \pi(Q) = \frac{\Pi_{.i}(Q)}{K_{.i}(Q)}$$

$$\text{Thus: } \frac{\partial \pi^i(Q^i)}{\partial Q^i} = 0$$

$$\Rightarrow \frac{\partial D^{-1}\left(\sum_{i=1}^n Q^i\right)}{\partial Q^i} Q^i + D^{-1}\left(\sum_{i=1}^n Q^i\right) \left(1 - e_{K_{.i}^i/Q^i}\right) = C^i{}'(Q^i) - \bar{C}^i(Q^i) e_{K_{.i}^i/Q^i} .$$

Again, the right member of this equation does not depend on the production  $\tilde{Q}^i$  of the other firms, but the left member depends on it : functions of reactions vanish depending on the the parameters of the function of demand and on the elasticity of the authorized capital.

#### IV. Conclusion

We proved that, when the function of profit of the firm is changed, maximisation of the rate of profit instead of the pure profit, these may be no coordination because each firm acts independently to the other, in some special cases :

- in competition with fixed coefficient of capital, the price signal plays no role : firms are not coordinated even if they are smaller than in the classical model; the market may not clear in the short-term.

- in imperfect competition, duopoly or oligopoly, the functions of reaction may disappear in some popular cases, like a linear inverse function of demand with a coefficient of capital : there is no more game between firms. These simple cases may be seen as counter-examples of no coordination (in the classical model, there are functions of reactions even in these cases).

These results are not a simple curiosity : the profit rate maximization is a credible alternative to the pure profit maximization because its looks not so far to practice and financial theory (see Annex).

## V. Annex : profit rate and financial theory

### A. *Static model*

#### 1. Value of the firm

We will study a simple version of the financial theory. The simple financial theory consider a financial ratio of profit  $\pi$ :

$$\pi = \frac{\Pi_{.t}}{K_{.t}}$$

where  $\Pi_{.t}$  is the accounting profit after taxes and  $K_{.t}$  is the authorized capital (plus accumulated funds).

When they control the firm, shareholders want to maximize their wealth. Shareholders must maximize the (Stock Exchange) value  $V$  of the firm (at this step, the rate  $\pi$  is not supposed to be maximized) :

$$V = \frac{K_{.t} \pi}{\pi^*} = \frac{\Pi_{.t}}{\pi^*} \text{ where } \pi^* \text{ is the minimal profitability required by shareholders.}$$

The required minimal profitability  $\pi^*$  tends to be equal to the interest rate (without risk) of the financial market,  $t$ , if the financial market is perfect (each firm has no individual impact on this market). Thus:  $V = \frac{\Pi_{.t}}{t}$ .



As  $t$  is independent of the firm, it is constant for it, then the maximization of  $V$  is identical to the maximization of  $\Pi_{.t}$ .

$V$  becomes homogeneous to an absolute value. At this point, the maximization of  $V$  is near to the maximization of another absolute value, the authorized profit (even if the authorized capital  $K_{.t}$  performs no role in the maximization)

## 2. Profit by dollar

For the financial theory, the justification of that behavior (maximize the value of the firm) is: the shareholders want to maximize the profit by share in order to see if a supplementary investment of the firm is opportune. If shareholders have only one firm to choose, it is acceptable to say that they do it. However, if there are many firms to choose, the shareholders cannot think in terms of profit by share. They must think in terms of profit by dollar, dividing the profit by share by the value of the share.

Unfortunately, it is easy to prove that, in this model with no time, this rate is fixed and equal to the interest rate ( $N$  is the number of shares):

$$\frac{\text{profit by share}}{\text{quotation}} = \frac{\frac{\Pi_{.t}}{N}}{\frac{V}{N}} = t.$$

Thus this rate, despite its homogeneity with  $\pi$ , is trivial when there is no time.

The objective function in the static model rest homogeneous to the maximisation of  $\Pi_{.t}$ . That is why it is necessary to introduce the time and an actualization rate.

## B. Dynamic model

In this case, classically the firm maximizes its present actualized value, that is to say the actualized sum of the future dividends  $D(T)$ :

$$V = \sum_{T=1}^n \frac{D(T)}{(1+a)^T}, \text{ where } a \text{ is the actualization rate.}$$

In the Lerner-Carleton model, under a constraint of financing, which indicates the relation the rate of profit and the value of the capital [ LERNER and CARLETON 1966 ], the future dividends  $D_T$  depends on  $g$ , the growth rate of the firm:

$$D(T) = (1 - s) \Pi_A(T) = (1 - s) \Pi_A(0) (1 + g)^T,$$

where  $\Pi_A$  is the profit and  $s$  is the share of benefits kept by the firm.

This gives the Gordon-Shapiro formula (when  $n \rightarrow \infty$ ):  $V = \frac{(1 - s) \Pi_A(1)}{a - g}$ .

$V$  is homogeneous to an absolute value; *ceteris paribus*, maximize  $V$  is identical to maximize the profit  $\Pi_A(1)$ .

However, as the rate of growth depends on the profit rate  $\pi$  (supposed to be invariable)

by the trivial relation  $g = s \pi$ ,  $V$  become a function of  $\pi$ :

$$V = \frac{(1 - s) \Pi_A(1)}{a - s \pi}.$$

If  $s$  and  $a$  are constant,  $V$  is an increasing function of the profit  $\Pi_A(1)$  and of the profit rate  $\pi$ .

Finally, introducing the Lerner-Carleton model, we can say that the maximization of the present value  $V$  is close to the maximization of the financial profit rate  $\pi$ . These developments show that, if firm maximizes profitability, in fact it maximizes the rate of growth, that is to say the financial profit rate, if the ratio of retention of profits is fixed.

**Remark.** A rational evaluation of  $a$  is  $a = \pi^*$ . If  $s = 0$  we retrieve the simple formula :  $V = \frac{\Pi_A}{\pi^*}$ . If the financial market is perfect,  $a = t$ . Thus, the profitability to be maximized becomes :

$$\frac{\text{profit by share}}{\text{quotation}} = \frac{\frac{\Pi_A(1)}{N(1)}}{\frac{V}{N(1)}} = \frac{t - \pi s}{1 - s} = \frac{t - g}{1 - s}.$$

**Remark.** The Price Earning Ratio (ratio of quotation of the share over profit by share) is the inverse of this rate  $PER = \frac{1 - s}{t - g}$ . We must have  $t > g$  to obtain a positive profitability (or a positive PER). In the time model this ratio shows only the possibilities of growth and the policy of retention of profits, even if in the no-time model the profitability (or the PER) is

trivial. If there are some alternative opportunities of investment, the financial theory says that the shareholders choose the firm with the best possibility of growth.

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