# The microeconomic maximisation of a ratio of profit : a restatement

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# THE MICROECONOMIC MAXIMISATION OF A RATIO OF PROFIT: A RESTATEMENT

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**ABSTRACT.** On the traditional microeconomic theory, firms are supposed to maximise the aggregate pure profit. We study other objective functions which are ratios profit over capital. We explore various combinations, proposing a typology of the ratios of profit, particularly the rate of profit (accounting profit over capital), the rate of gross profit (gross profit over capital), the rate of accounting profit (accounting profit over capital). The cases of monopoly with variable coefficient of capital, monopoly with fix coefficient of capital, competition with fix coefficient of capital, are studied. The solutions given by the maximisation of the aggregate pure profit and the maximisation of the rate of profit are compared. We conclude to a usually lower volume of production with the maximisation of the rate of profit. Thus, as firms are smaller, competition may be greater; however the clearing of the market must be studied; in competition at the very long term, with the rate of profit maximisation, the pure profit remain positive.

**KEYWORDS.** Maximisation, Profit, Rate of Profit, Microeconomics.

# INTRODUCTION

1) Maximisation of the aggregate profit is one of the foundations of microeconomic theory (and of the theory of industrial organisation). However, it is well accepted, its basis are not very much studied. In many handbook of microeconomic theory, the principle of the maximisation of the aggregate profit is asserted without justifications: "it goes without saying". In the other side, economic calculus (actualisation, comparison of projects of investment, etc.) required to think in terms of maximisation of profitability ratios. There is a gap between microeconomic theory and the practice. In fact, microeconomic theory has some special characteristics. For example, it considers a very special type of firm confused with the capitalist-shareholder (apart from the fact that this firm have no intern structure: interactions between individuals are not taken into account).

In one hand, we can say that in the pre-industrial age and from the beginning of the industrial revolution, capitalists (or landowners) often had no alternative possibility of investment than their own business (or property). The modern legal form of firms, like company, was not widespread and the financial market was non-existent. That is why the Classics authors, A. Smith et D. Ricardo principally, have adopted naturally the principle the more in accordance with the observed reality of their era, the maximisation of the aggregate profit: why maximise a ratio of profitability when your funds cannot be employed elsewhere (this profitability have no signification is this case)? In this point of view, capitalists (or landowners) maximise the aggregate profit, because they must invest all their funds in their own business (or property): as long as they had funds, they invested if the profit remains non negative; the not invested funds yield nothing.

In the other hand, we must consider the cases where there are alternative possibilities of investment with the concept of opportunity cost of the capital. The cost of the accounting capital [1] is an opportunity cost, because these free funds may be invested in another place at a normal interest rate: the accounting capital looks to come from an operation of renting similar to a loan, and its opportunity cost looks like an interest of which the rate is the average rate of the market (calculated with a standard alternative investment) [2]. This leads to calculate the pure

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[1] In the following, the words ***accounting capital*** refers to the funds advanced by the shareholders to the firm: there is nor obligatory pay back, nor obligatory payment of a remuneration, but it is the usual counterpart of a property right. This remuneration is free ex ante as ex post, and varies between zero and the amount of accounting profit (less the funds putted in reserve and taxes), depending on the policy of dividend of the firm. Accounting capital remains in the balance sheet of the firm and remains the property of the shareholder (to the contrary of a debt, which disappear of the balance sheet of the firm when paid back).

In the opposite, loans are paid back obligatorily at the settlement date (except if bankruptcy or at the dissolution of the firm) with obligatory payment of an interest to the moneylender, without giving to him a property right (the question of the control of the shareholders over the firm differs): interest is a cost.

Self-financing correspond to a withdraw over a reserve (a past profit or over an actual profit): it is comparable to a withdraw over the shareholder and then similar to an increase of accounting capital (self-financing plus accounting capital forms the own funds of the firm.). Also, the reselling of shares is only a transfer between shareholders and does not concern the firm directly (it determine only the future success of the operations of increasing the accounting capital).

The words ***accounting profit*** denotes the accounting profit able to remunerate the shareholders, that is to say before payment of dividends. It is not used to pay the eventual interest (yet paid). If accounting profit is positive, a dividend is optionally paid, equally to every shareholders, in order to remunerate this accounting capital. Dividend is not a cost. Then, accounting capital is free. In this paper, the costs corresponding to the accounting profits are the ***accounting costs***.

Note that capital gains over reselling of shares may constitute a remuneration of shareholders. We do not take this question into account.

[2] In [ de MESNARD 1992 ] the following arguments are given. The opportunity cost is due to an *abstinence*, as in Senior, or whether it is due to an expectation as in Marshall, or whether a *preference for the present* as in Böhm-Bawerk or an *impatience* as in Fisher. It is not easy to calculate and maximise the aggregate pure profit. Indeed, the alternative investment of the funds must be well defined, unique and known, in order to calculate the opportunity cost. The marshallian conception sees in the pure profit the salary of the manager. Another point of view (E.H. Knight, M. Allais) sees in the pure profit a conjectural yield, created by some errors of forecast; the remuneration of the capital is integrated in interests and then in the costs, including the insurable risks; the accidental phenomenon, the non insurable risks, create a temporary profit. More recently, Shih-Yen Wu argues as it follows [ WU 1989 ]: the remuneration of the capital included interests is fixed ex ante, and dividends fixed ex post, is a cost. This position seems to confuse between the cost to obtain the capital and the its remuneration; it supposes a perfect financial market, with a unique interest rate; it supposes that the funds obtained by increasing the accounting capital must be remunerated, what is contrary to the intrinsically risked (and linked to a property right) nature of the capital: the accounting capital may be remunerated.

profit (or *economical profit*): it is the profit obtained by the firm including every costs and opportunity costs, that is to say, including the normal remuneration of the accounting capital.

Consequently, in microeconomic theory, the firm invests as long as the pure profit is not negative, even it is very very small. A caricatural example is the construction of a plant for one billion dollars, because the pure profit will be one cent! Nobody do that for many reasons: the risk of investment, the cost of managing the investment. However, these two reasons may be taken into account in the pure profit: the risk have a cost, the management have a cost. There is a third reason: the sunk costs or the immobilisation of the funds over a certain period. It has also a cost in the actualisation theory, but we think that every man prefers to conserve the liquidity of the capital putting it in a remunerated account if the pure profit is nil (that is to say if the net remuneration of its funds including all the opportunity cost is equal to the remuneration in the financial market for liquid funds).

However, the evolution of firms leads toward modern companies, principally limited companies and other forms where the shareholders are distinct from the managers, and toward the financial market and the Stock Exchange. Capitalist become shareholders, with alternative possibilities of investment: the funds not invested in one business yield something in another business and the comparison between the profitability of businesses is based on ratios. Thus shareholders maximise the profitability of their funds, that is to say the rate of profit [3]. In the view point of the agency theory, the objective function of the firm depends on the objective function of the shareholders. If shareholders think in terms of profitability, the firm must think in terms of profitability. The theory of the choice of investment, the theory of demand of capital (Keynes, among others), the portfolio theory, etc., thinks into terms of profitability.

In addition, the calculating the pure profit is difficult because the opportunity cost are hard to know.

Note that the same behaviour, *take into account the available means to reach the objective*, is observed in any human activities, when a problem of choice exists. In some cases, human people do not respect this principle, for example when the objective is vital (high level sports competitions, military fight, etc.).

2) So, the question is: why the subtractive form of the function of profit (profit minus opportunity cost of the capital) prevailed over the "divisive" form of the function of profit (profit over capital)?

To decide if a firm produces or not, there are two possibilities. The firm produces if:

- $\text{(accounting profit)} - \text{(opportunity costs per unit of capital)} \,.\, \text{(capital)} \geq 0$
- or $\dfrac{\text{(accounting profit)}}{\text{(capital)}} \geq \text{(opportunity costs per unit of capital)}$ .

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[3] In any case, the amount of funds to be invested must be limited. If the amount is not limited, the aggregate pure profit may be maximised, even if there are alternative possibilities of investment (however, in this case, as the capital has no price, the opportunity cost of the accounting capital is nil: the pure profit is equal to the accounting profit).

In the first case, we compare the pure profit to zero. In the second case, we compare the rate of profit to the opportunity costs per unit of capital. These two possibilities are obviously equivalent. However, this indicates nothing about the optimal amount of production: we shall show that it is not the same thing to maximise the pure profit and to maximise the rate of profit. The two theories differs from this point and this shows that it is necessary to explore the alternative theory of the maximisation of the rate of profit.

So, following [ de MESNARD 1992 ], we want to link up microeconomic theory and practice, suggesting an alternative principle to the maximisation of the profit, *the maximisation of the rate of profit* (that is to say the ratio of the accounting profit over the whole capital engaged in the production), and making a distinction between shareholders and producers [4].

The maximisation of the rate of profit has not been studied, even if it is an old idea in the business world [5]. The maximisation of a rate of profit margin have been studied even it is less realistic than the maximisation of the rate of profit [6]. The model of financial theory introduce the rate of profit. However, there is not maximisation of the rate of profit.

For example, in the Lerner-Carleton model, the firm maximises its present actualised value, under a constraint of financing, which indicates the relation the rate of profit and the value of the capital [ LERNER and CARLETON 1966 ]. The present actualised value of the firm is the actualised sum of the future dividends: $V = \sum_{t=1}^{n} \frac{d_t}{(1+\tau)^t}$, where $\tau$ is the actualisation rate. The future dividends $d_t$ depends on $g$, the rate of growth of the firm: $d_t = (1-r)\,\Pi_t = (1-r)\,\Pi_0\,(1+g)^{t-1}$, where $\Pi$ is the profit and $r$ is the share of benefits kept by the firm. This gives the Gordon-Shapiro formula (when $n \to \infty$): $V = \frac{(1-r)\,\Pi_0}{\tau - g}$. And the rate of growth depends on the rate of profit $\pi$ by the trivial relation $g = \pi r$ . At this point, the financial constraint describes what level of the rate of profit is obtained with each level of capital: $\pi = \mathrm{f}(K)$ [7]. We obtain the present actualised value of the firm for each level of the capital: $V = \mathrm{f}(K)$.

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[4] Remark that asserting a distinction between shareholders and producers leads to a point of view near those of O. Lange [ LANGE 1936 ]or those of J. Tobin (and its "q") [ TOBIN 1969 ]or those of F. Modigliani and H.M. Miller [ MODIGLIANI and MILLER 1958 ], but also those of J. M. Keynes, for whom investment comes from the comparison between the anticipated profitability and the interest rate. These two categories of agents, the firms and the capitalists-shareholders have their own logic.

[5] For example R. Lantner in a short quotation in the educational review *DEES* [ LANTNER 1987, p. 71 ]:

*In fact, nobody invest one billion to obtain one billion and one franc finally: then it is the rate and not the aggregate profit the capitalist optimises.*

[6] In the case of a firm installed in an economic space, R. Lantner and J.-F. Thisse have envisaged the maximisation of a rate of profit margin calculated as the ration of the profit over the costs. They show that the marginal cost of production (which is, with the transportation cost, a component of the marginal cost of production), may be decreasing and that the optimum may be to the left of the point of the minimum cost of production, at the condition that the marginal transportation cost would be increasing [ LANTNER and THISSE 1974 ].

[7] Lerner and Carleton use a simple function of capital like a coefficient of capital indicating the amount of capital for each level of production: as the rate of profit is linked up to the amount of production by the demand function and by the cost function, this rate is linked up to the capital.

This approach introduces the rate of profit. However, $V$, the present actualised value of the firm, is maximised. Thus this model (and the financial models which maximise $V$) remains classical: basically, the present actualised value is homogeneous to a profit (with a nil rate of actualisation, $V$ is a part of a sum of profits). Then, the maximisation of $V$ is homogeneous to the maximisation of the profit $\Pi$.

3) In [ de MESNARD 1992 ], the accounting capital was putted at the denominator of the ratio of profit, that is to say did not comprise loans in the capital [8]: here we call this ratio *the rate of accounting profit*. It leaded to a non constant denominator if the financial structure varies, which make the economic calculus difficult: for a project, it is not easy to know the part of its funds financed by increasing accounting capital and the part financed by loans because, often, operations of increasing accounting capital are decided independently from operations of investment. Moreover, as the capital loaned is not taken into account, it is supposed that the accounting capital may increase independently from the loaned capital: for example, the first derivative of the function of accounting capital were supposed positive which is not sure, while it is more acceptable to suppose that the first derivative of the function of the whole capital is positive.

These arguments push the theory of the maximisation of the rate of accounting profit back to microeconomic theory and economic calculus.

To avoid this pitfall, after proposing a typology of the possible ratios of profit, we retain two other types in addition to the maximisation of the rate of accounting profit: a new ratio that we call ***the rate of profit, relying on the accounting profit at the numerator, and putting the whole capital at the denominator*** (and not only to the accounting capital), and another ratio ***the rate of gross profit, with the gross profit*** (accounting profit without debts) ***at the numerator and the whole capital at the denominator***. These ratios are more convenient regarding to the practice and these new versions of the theory of the maximisation of the rate of profit are more compatible with the microeconomic customs. The maximisation of the rate of profit introduces a financial structure, including both the loans and the increase of the accounting capital as a way to finance the investment; particularly, we study the case of a variable financial structure in a first time, and the more simple case of a constant financial structure in a second time.

In [ de MESNARD 1992 ], the problem of the fixity of the capital is treated by a distinction between long term (capital is variable) and short term (capital is fixed). We propose an alternative but not exclusive point of view: the calculus is made *ex ante*, where $K(Q)$ is freely fixed, while *ex post*, $K(Q)$ is fixed. However, we shall see that a condition of triviality is not only the fixity of the capital but also the fixity of the financial structure.

As in [ de MESNARD 1992 ], we shall not explore all the improvements brought to microeconomic theory by the authors: uncertainty and stochastic models, dynamic models, incomplete information, theory of the bounded rationality, theory of the agency, theory of imperfect markets and fixed prices equilibrium, financial theory, managerial and behaviourist theory of the firm, etc. These improvements are useful, and we want not call them into question.

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[8] The argument were: loans are included into costs and are not taken into account in the calculus of the opportunity cost of accounting capital.

The theory of the maximisation of the rate of profit is upstream from them. Moreover, the notion of profit itself will not be called into question: we remain into the logic of capitalism [9].

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After presenting a typology of the ratios of profit and recalling some simple results in a few classical situations (monopoly, competition), we shall study two ratios of profit: the rate of profit itself (accounting profit over capital) and the rate of gross profit (gross profit over capital). We shall focus on the problem of the aggregate pure profit: as the opportunity costs of the capital are taken into account in the pure profit, is it necessary to maximise a ratio of profit over capital? The answer will lead to a comparison of the optimal production when the aggregate pure profit is maximised and when the rate of profit is maximised.

# 1. VARIABLES AND NOTATIONS

We call ***argument of the pure profit*** the following argument of triviality of the maximisation of the rate of profit: the pure profit yet includes the remuneration of the accounting capital. To dismiss this argument, in the following, we shall introduce explicitly the pure profit, in order to show that the question is not to know if we must or not work with pure profit, but to know if we must work in term of profit maximisation or in terms of rate of profit maximisation.

The calculus of the accounting profit do not includes the remuneration of the accounting capital, while the calculus of the pure profit leads to remunerate this capital at the interest $t$ (as if it was loaned).

The different variables are:

- $p(Q)$ is the inverse of the curve of demand. $R(Q)=p(Q)Q$ is the receipt.
- $C(Q)$ the cost of production, excluding the interest of loans and the remuneration of the accounting capital. Introducing the pure profit, the cost $C(Q)$ is no more undetermined: it includes the whole of the costs of production, excluding the interest of eventual loans and the remuneration of the accounting capital subscribed to shareholders. The opportunity cost associated to this remuneration of the accounting capital will be introduced explicitly.
- $K(Q)$ the positive (financial) capital used in the production. By simplification, the capital is entirely dedicated to the actual production.
- $K_A(Q)$ the positive *accounting capital* used in the production (we confuse the accounting capital with the totality of the own funds excluding the long term debts).
- $K_L(Q)$ the *loaned capital* used in the production. We have by hypothesis $K(Q)=K_A(Q)+K_L(Q)$.

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[9] Note that, even it treats of the rate of profit, the theory must not be confused with some aspects of the marxian theory (like the Marx's law of the decreasing rate of profit).

- What happens when the financial structure varies? The more simple way is to suppose that the financial structure depends on $Q$. A functional parameter $\alpha(Q)$, depending on $Q$, is the part of the capital $K(Q)$ financed by the shareholders, that is to say it is the ratio $\alpha(Q)=\dfrac{K_A(Q)}{K(Q)}$: it is the part of the accounting capital in the total of the capital, varying between 0 and 1; we suppose that $\alpha(Q)$ is a derivable function. $(1-\alpha(Q))$ is the part of the capital $K(Q)$ financed by loans.

- $t$ the constant and positive *interest rate* in the financial market, valid for the borrower and the moneylender: the same rate is used for the loans of the firm and for the opportunity cost of the funds. The *accounting capital* is remunerated at the fix rate of interest $t$, giving an opportunity cost of: $C_K(Q)=t\ \alpha(Q)\ K(Q)$

- The *total cost* when calculating the accounting profit is $C_A(Q)=C(Q)+t\,[1-\alpha(Q)]\ K(Q)$.

- The *total cost* when calculating the pure profit includes the opportunity cost of the accounting capital $C_P(Q)=C_A(Q)+C_K(Q)=C(Q)+t\ K(Q)$.

- The *gross profit* is $\Pi_G(Q)=R(Q)-C(Q)$.

- The *accounting profit* is:

  $$\Pi_A(Q)=R(Q)-C_A(Q)=R(Q)-C(Q)-t\,(1-\alpha(Q))K(Q)=\Pi_G(Q)-t\,(1-\alpha(Q))K(Q).$$

- The *pure profit* is independent from $\alpha(Q)$:

  $$\Pi(Q)=R(Q)-C_P^T(Q)=R(Q)-C(Q)-t\ K(Q)=\Pi_G(Q)-t\ K(Q)$$

  and $\Pi(Q)=R(Q)-C_P^T(Q)=R(Q)-C_A^T(Q)-C^K(Q)=\Pi_A(Q)-t\ \alpha(Q)\ K(Q)$.

# 2. TYPOLOGY OF THE RATIOS OF PROFIT

## 2.1. The different types of ratios

Considering the gross profit $\Pi_G(Q)=R(Q)-C(Q)$, the accounting profit $\Pi_A(Q)=\Pi_G(Q)-t\,(1-\alpha(Q))K(Q)$ and the pure profit $\Pi(Q)=\Pi_G(Q)-t\ K(Q)$, considering the capital $K(Q)$ and the accounting capital $K_A(Q)$, there are six great types of rate of profit possible:

1. the rate of profit $\pi(Q)=\dfrac{\Pi_A(Q)}{K(Q)}=\dfrac{\Pi_G(Q)-t\,(1-\alpha(Q))K(Q)}{K(Q)}=\pi_G(Q)-t\,(1-\alpha(Q))$

2. the rate of accounting profit

$$\pi_A(Q) = \frac{\Pi_A(Q)}{K_A(Q)} = \frac{\Pi_A(Q)}{\alpha(Q)\,K(Q)} = \frac{\pi(Q)}{\alpha(Q)} = \frac{\pi_G(Q) - t\,(1-\alpha(Q))}{\alpha(Q)}$$

3. the rate of gross profit $\pi_G(Q) = \dfrac{\Pi_G(Q)}{K(Q)} = \pi(Q) + t\,(1-\alpha(Q))$ : it is independent of $\alpha(Q)$

4. the rate of "accounting gross profit"

$$\pi_{GA}(Q) = \frac{\Pi_G(Q)}{K_A(Q)} = \frac{\pi_G(Q)}{\alpha(Q)} = \frac{\pi(Q) + t\,(1-\alpha(Q))}{\alpha(Q)} = \pi_A(Q) + t\,\frac{1-\alpha(Q)}{\alpha(Q)}$$

5. the rate of pure profit $\pi_P(Q) = \dfrac{\Pi(Q)}{K(Q)} = \dfrac{\Pi_G(Q) - t\,K(Q)}{K(Q)} = \pi_G(Q) - t = \pi(Q) - t\,\alpha(Q)$ : it is independent of $\alpha(Q)$

6. the rate of "accounting pure profit" $\pi_{PA}(Q) = \dfrac{\Pi_G(Q) - t\,K(Q)}{K_A(Q)} = \dfrac{\pi(Q)}{\alpha(Q)} - t = \pi_A(Q) - t$ .

The optimum of the last two ratios are confused with the optimum of other ratios. The optimum of the rate of pure profit is identical to the optimum of the rate of gross profit; the optimum of the rate of "accounting pure profit" is identical to the optimum of the rate of accounting profit. Note that the rate of pure profit is the lower bound of the rate of profit and the rate of gross profit is the upper bound.

Then, regarding to the object of this paper, these two ratios may be pushed back. It remains the four first ratios. The rate of "accounting gross profit" is an hybrid ratio: is indicates how the accounting capital creates the gross profit, disregarding the role of the loaned capital. Thus, it may be dismissed.

It remains three ratios.

- The rate of profit $\pi(Q)$ takes into account the financial structure and seems to be operational.
- The rate of accounting profit $\pi_A(Q)$ studied in [ de MESNARD 1992 ], but with a fix financial structure.
- The rate of gross profit $\pi_G(Q)$ indicates how the whole capital creates the gross profit: it does not depends on the financial structure.

As the maximisation of the rate of gross profit is a particular case of the maximisation of the rate of profit ( $\alpha(Q) = 1$ for every $Q$ ), we shall study only the more general case: the maximisation of the rate of profit.

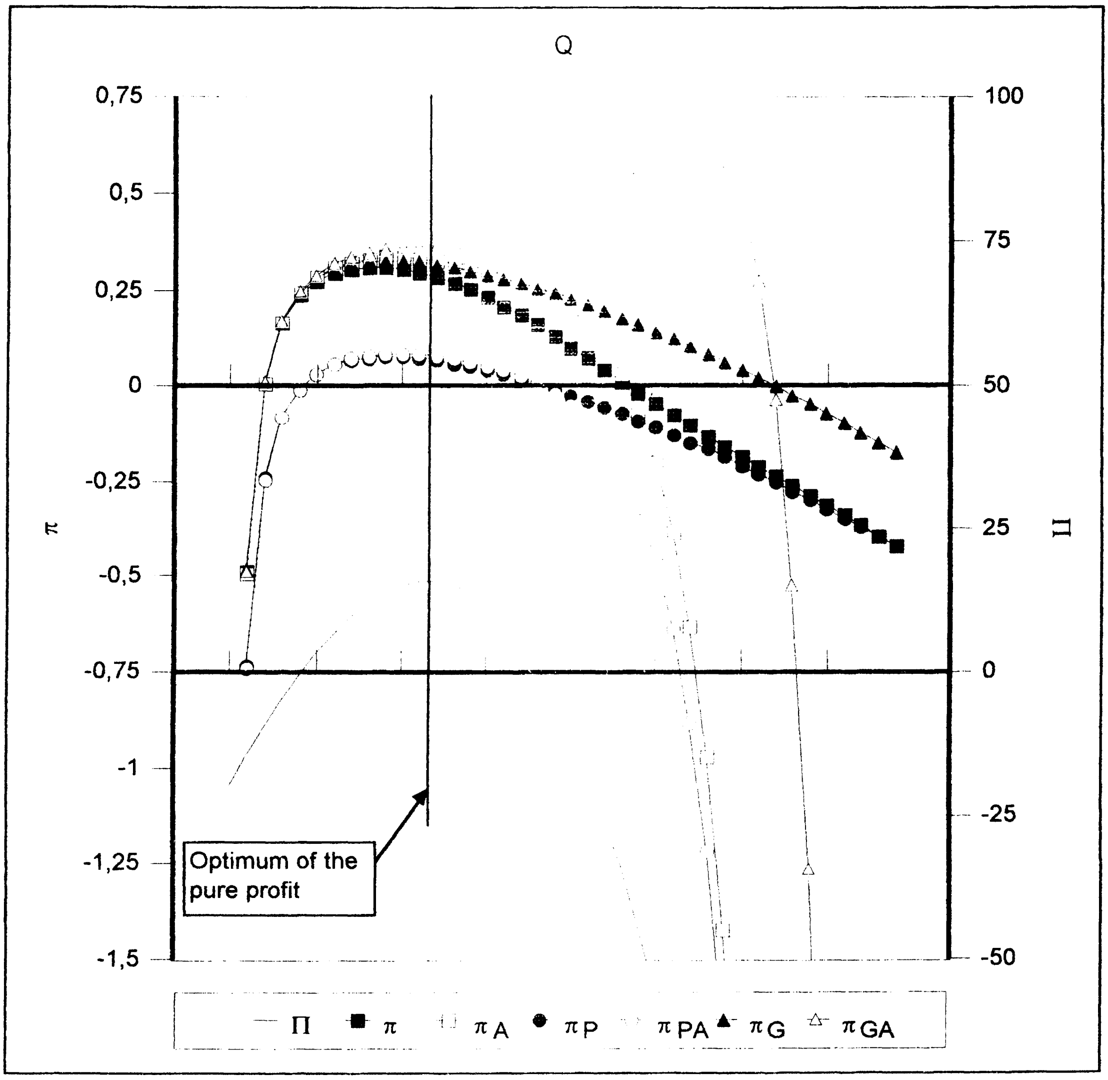
## 2.2. Example

We keep the same example as in [ de MESNARD 1992 ], adapted:

$C(Q)=5\,Q^3+10\,Q^2-5\,Q+20$, $p(Q)=-5\;Q+100$, $R(Q)=p\,Q$, $K(Q)=200\,Q$ and $t=0{,}25$ .

$\alpha(Q)=\dfrac{1}{1+0.1^{2-Q}}$: decreasing part of shareholders with a logistical function [10] (inflexion point at 2);

[10] A logistical function is well adapted to such a phenomenon: a slow decrease, followed by a fast decrease and a slow decrease. When a firm is developing, it has some difficulties to find loaned funds in the beginning; after, things are more easy; at the end, the part of loans must not exceed 1.




**The different ratios of profit**

## 2.3. Optimum of the maximisation of a general ratio of profit

In [ de MESNARD 1992 ] three cases of maximisation of the rate of accounting profit were studied.

Consider the more general case, the monopoly with function of capital, and suppose that the first derivative of the function of capital is positive (that is to say $e_{K(Q)/Q} > 0$ ). Consider a ratio of profit: $\pi_U(Q) = \dfrac{\Pi_U(Q)}{K(Q)}$ where $\Pi_U(Q) = R(Q) - C_U(Q)$ is an undefined expression of the profit and $\pi_U(Q)$ is the corresponding ratio of profit. The maximisation of this ratio of profit gives:

$$\Pi'_U(Q) = \Pi_U(Q)\frac{K'(Q)}{K(Q)} \Leftrightarrow \Pi'_U(Q) = e_{K(Q)/Q}\,\overline{\Pi}_U(Q) \Leftrightarrow \Pi'_U(Q) = \pi(Q)\,K'(Q)$$

where $e_{K(Q)/Q} = \frac{dK(Q)/K(Q)}{dQ/Q} = \frac{K'(Q)}{K(Q)}Q$ is the relative elasticity of the capital to the production and $\overline{\Pi}_U(Q) = \frac{\Pi_U(Q)}{Q}$ is the average profit.

***The firm produces up to the point where the marginal*** *accounting profit* ***be equal to the average*** *accounting profit* ***multiplied by the relative elasticity of the capital to the production*** $e_{K(Q)/Q}$.

At the optimum, the relative elasticity of the capital to the production is equal to 1. Indeed, the above result could be written $\frac{\Pi'_U(Q)/\Pi_U(Q)}{K'(Q)/K(Q)} = 1 \Leftrightarrow e_{P(Q)/K(Q)} = 1$.

As a particular case, [ de MESNARD 1992 ] studied the monopoly with fixed coefficient of capital: $K(Q) = k\,Q$ and $e_{K(Q)/Q} = 1$. Thus $\Pi'_U(Q) = \overline{\Pi}_U(Q)$: the firm produces up to the point where *the marginal accounting profit is equal to the average accounting profit* (or to the point where the average profit is maximal). As said in [ de MESNARD 1992 ], this corresponds to an observed behaviour of the firms: a firm produces and invest as much as the profit brought by a supplementary unit is superior or at least equal to the average profit brought by other units. And this point is not the point of long term equilibrium of the traditional monopoly ($R'(Q) = C'_{LT}(Q)$ at a long term) and do not correspond to the point of the minimum of the average cost. If the marginal accounting profit is decreasing (condition of diminishing returns, in order to obtain a maximum of the accounting profit), the optimum is to the left to the point which maximises the accounting profit [11]; however, this does not take into account the question of the pure profit and then it does not proof that, always, the optimum of the maximisation of the rate of profit is to the left of the classic microeconomic optimum (maximisation of the pure profit): it must be done further.

As another particular case [ de MESNARD 1992 ] studied the case of perfect competition with fixed coefficient of capital: $p(Q) = p$ and $e_{K(Q)/Q} = 1$. This gives $C'_U(Q) = \overline{C}_U(Q)$. The competitive firm produces up to the ***minimum point of the average accounting cost***. In [ de MESNARD 1992 ] two consequences are brought out.

- Not surprisingly, the optimum is identical to the point of long term equilibrium of the firm;. However, the profit is here the accounting profit and not the pure profit; and, not said in the quoted paper, ***the profit is not nil at the optimum***.

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[11] This result corresponds to those pointed out by (but not demonstrated) R. Lantner in this short quotation [ LANTNER 1987, p. 71 ]:

*...under some simple hypothesis, it is easy to show that the maximisation of the rate of profit leads to invest and produce less the maximisation of the aggregate profit does.*

[12] See the discussion farther.

- ***The optimum does not depend on the price***: the production does not vary if the price vary, what corresponds to an observed behaviour. As the price is no more an indicator for the volume of production of the firm, ***the optimum does not automatically clears the market as supposed by the neo-classical theory***. And there is an *equilibrium with rationing* because the marginal cost is not equal to the price.

# 3. THE RATE OF PROFIT

## 3.1. Comparison of the optimal levels of production

It is not sufficient to say that the optimum point of the maximisation of a ratio of profit is to the left of the optimum of the maximisation of the corresponding profit. It is necessary to compare that the first optimum with the classical optimum of the microeconomic theory, the maximisation of the pure profit. Taking into account the pure profit, and introducing a variable financial structure (under the form of a derivable function), we obtain the following fundamental theorem:

Theorem 1. In the more general case of the monopoly with a function of capital, with $K'(Q) > 0, \forall Q$, compared to the profit maximisation, the maximisation of the rate of profit gives ***a lower optimal quantity*** (respectively higher) ***when the compensated rate of profit*** (the rate of profit divided by one plus the relative elasticity $e_{\alpha(Q)/K(Q)}$ ***of*** $\alpha(Q)$ facing to $K(Q)$ or divided by the relative elasticity $e_{K_A(Q)/K(Q)}$ of $K_A(Q)$ facing to $K(Q)$), ***is higher*** (respectively lower) ***than the interest rate***:

1) $$\frac{\pi(Q_\pi^*)}{\left[\alpha(Q_\pi^*)\right]\left[1 + e_{\alpha(Q)/K(Q)}\Big|_{Q=Q_\pi^*}\right]} > t \Rightarrow \max \pi \prec \max \Pi$$

or $$\frac{\pi_A(Q_\pi^*)}{e_{K_A(Q)/K(Q)}\Big|_{Q=Q_\pi^*}} > t \Rightarrow \max \pi \prec \max \Pi$$

2) $$\frac{\pi(Q_\pi^*)}{\left[\alpha(Q_\pi^*)\right]\left[1 + e_{\alpha(Q)/K(Q)}\Big|_{Q=Q_\pi^*}\right]} < t \Rightarrow \max \pi \succ \max \Pi$$

or $$\frac{\pi_A(Q_\pi^*)}{e_{K_A(Q)/K(Q)}\Big|_{Q=Q_\pi^*}} < t \Rightarrow \max \pi \succ \max \Pi$$

where $Q_\pi^*$ is the optimal quantity which maximises the rate of profit $\pi(Q)$.

Comments.

1. This theorem does not indicate the relative position of optimums in general: it requires the calculus of the optimal rate of profit. However, it is useful, because it allows a manager to know, when he has calculated the optimum of the maximisation of the rate of profit if the maximisation of the aggregate pure profit provides an optimum before or after the first optimum.

2. The first case $\dfrac{\pi(Q_\pi^*)}{\left[\alpha(Q_\pi^*)\right]\left[1+\mathrm{e}_{\alpha(Q)/K(Q)}\Big|_{Q=Q_\pi^*}\right]} > t \Leftrightarrow \dfrac{\pi_A(Q_\pi^*)}{\mathrm{e}_{K_A(Q)/K(Q)}\Big|_{Q=Q_\pi^*}} > t$ corresponds to the "standard" case because generally the more realistic case is $\mathrm{e}_{\alpha(Q)/K(Q)}\Big|_{Q=Q_\pi^*} < 0$ (the part of the accounting capital decreases when capital increases) thus $\left[\alpha(Q_\pi^*)\right]\left[1+\mathrm{e}_{\alpha(Q)/K(Q)}\Big|_{Q=Q_\pi^*}\right] < 1$. Then, ***the standard case corresponds to a lower size of firms***. This is very important.

3. If $\dfrac{\pi(Q_\pi^*)}{\left[\alpha(Q_\pi^*)\right]\left[1+\mathrm{e}_{\alpha(Q)/K(Q)}\Big|_{Q=Q_\pi^*}\right]} = t \Leftrightarrow \dfrac{\pi_A(Q_\pi^*)}{\mathrm{e}_{K_A(Q)/K(Q)}\Big|_{Q=Q_\pi^*}} = t$

   the two points $Q_P^*$ and $Q_\pi^*$ coincide.

4. Even if the capital $K(Q)$ is fixed ($K'(Q)=0\,,\forall Q$), the two points $Q_P^*$ and $Q_\pi^*$ may not coincide, because in this case $\Pi'(Q)=\Pi'_A(Q)-t\;\alpha'(Q)\,K(Q)$ and the optimum of the rate of profit $\pi(Q)$ is given by $\Pi'_A(Q)=0$. Thus, in this case, the two theories are identical if the financial structure is fixed also: $\alpha'(Q)=0$.

5. The hypothesis $K'(Q)>0\,,\forall Q$ is more acceptable than the hypothesis $K'_A(Q)>0\,,\forall Q$ made in [ de MESNARD 1992 ] (see in annexe). Indeed, the sign of $\mathrm{e}_{\alpha(Q)/K(Q)}$ is undetermined, because the part of the shareholders may increase or decrease when the amount of capital increases. In very short firms, loans often are very low and accounting capital comes from family of friends: $\alpha(Q)$ is near to 1. When its size increases, the firm may loan to banks but it cannot go to the Stock Exchange: $\alpha(Q)$ decreases quickly. Big firms may issue bonds and may go to the Stock Exchange: $\alpha(Q)$ may go on decrease or may increase [12].

[12] See the discussion farther.

6. A large part of the argument of the pure profit is refuted: the maximisation of the rate of profit is not identical to the maximisation of the aggregate pure profit.

Proof. Before, we have: $1+e_{\alpha(Q)/K(Q)}=e_{K_A(Q)/K(Q)}$ and remember that $\pi_A(Q)=\dfrac{\pi(Q)}{\alpha(Q)}$.

Next, the solution for the maximisation of the aggregate pure profit $\Pi(Q)$ is obtained for $\Pi'_A(Q)=t\,\alpha'(Q)\,K(Q)+t\,\alpha(Q)\,K'(Q)$.

Consider the curve $\Pi'(Q)=\Pi'_A(Q)-t\,\alpha'(Q)\,K(Q)-t\,\alpha(Q)\,K'(Q)$.

The solution for the maximisation of the rate of profit $\pi(Q)$ is obtained for $\Pi'_A(Q)=\pi(Q)\,K'(Q)$. Consider the curve $f_\pi(Q)=\Pi'_A(Q)-\pi(Q)\,K'(Q)$.

Let us compare the respective positions of $\Pi'(Q)$ and of $f_\pi(Q)$. Suppose that $\Pi'(Q)$ is decreasing ( $\Pi''(Q)<0$ ), so that the optimum of the aggregate pure profit $\Pi(Q)$ would be a maximum. Let us write $\Pi'(Q)$ as a function of $f_\pi(Q)$ :

$$\Pi'(Q)=\Pi'_A(Q)-\pi(Q)\,K'(Q)+\left[\pi(Q)-t\,\alpha(Q)-t\,\alpha'(Q)\frac{K(Q)}{K'(Q)}\right]K'(Q)$$

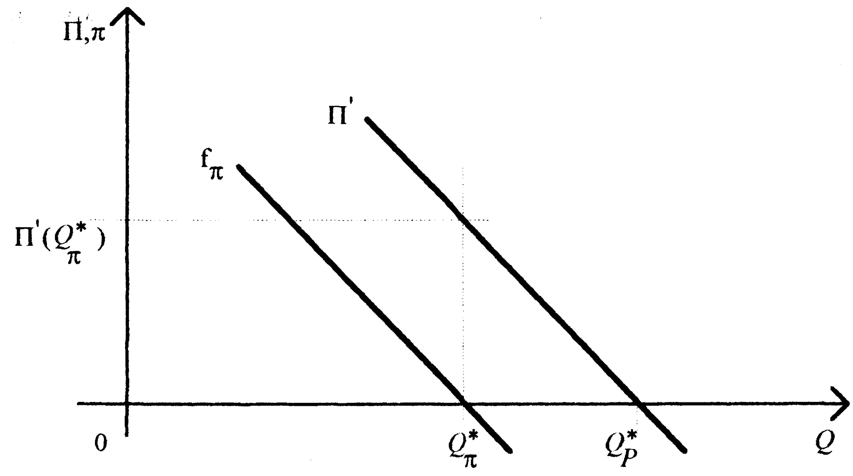
$$=f_\pi(Q)+\left[\frac{\pi(Q)}{\alpha(Q)}-t\left(1+\frac{\alpha'(Q)}{\alpha(Q)}\frac{K(Q)}{K'(Q)}\right)\right]\alpha(Q)\,K'(Q)$$

$$=f_\pi(Q)+\left[\frac{\pi(Q)}{\alpha(Q)}-t\left(1+e_{\alpha(Q)/K(Q)}\right)\right]\alpha(Q)\,K'(Q).$$

Denote $Q_P^*$ the point of maximisation of the aggregate pure profit, such as $\Pi'(Q_P^*)=0$. Denote $Q_\pi^*$ the solution of the maximisation of the rate of profit, such as $f_\pi(Q_\pi^*)=0$.

1. Suppose that $\dfrac{\pi(Q_\pi^*)}{\left[\alpha(Q_\pi^*)\right]\left[1+e_{\alpha(Q)/K(Q)}\Big|_{Q=Q_\pi^*}\right]}>t\Leftrightarrow\dfrac{\pi_A(Q_\pi^*)}{e_{K_A(Q)/K(Q)}\Big|_{Q=Q_\pi^*}}>t.$

   Then, $\Pi'(Q_\pi^*)>f_\pi(Q_\pi^*)$. Then, near to $Q_\pi^*$, the curve $f_\pi(Q)$ is under the curve $\Pi'(Q)$. Thus, $Q_P^*$ must be higher than $Q_\pi^*$. At the limit, if $\Pi'(Q)$ is vertical near to $Q_P^*$, $\Pi'(Q)$ will cut the horizontal axis in $Q_\pi^*$ and then $Q_P^*=Q_\pi^*$.




2. Suppose that $\dfrac{\pi(Q_\pi^*)}{\left[\alpha(Q_\pi^*)\right]\left[1+\mathrm{e}_{\alpha(Q)/K(Q)}\big|_{Q=Q_\pi^*}\right]} < t \Leftrightarrow \dfrac{\pi_A(Q_\pi^*)}{\mathrm{e}_{K_A(Q)/K(Q)}\big|_{Q=Q_\pi^*}} < t$.

   Then, $\Pi'(Q_\pi^*) < \mathrm{f}_\pi(Q_\pi^*)$. Then, near to $Q_\pi^*$, the curve $\mathrm{f}_\pi(Q)$ is over the curve $\Pi'(Q)$. Thus, $Q_P^*$ must be lower than $Q_\pi^*$.

Remarks about the proof.

1. If $K''(Q_\pi^*) = \dfrac{\Pi''_A(Q_\pi^*)}{t\,\alpha(Q_\pi^*)}$, $\Pi'(Q)$ is horizontal near to $Q_\pi^*$ and $Q_P^*$ is rejected to infinite.

2. Even if the optimum of the pure profit is unique, the optimum of the rate of profit may be non unique: it would be necessary to study the case of the curves $\Pi'(Q)$ and $\mathrm{f}_\pi(Q)$ with multiples zeros. •

Remark. 1. If the elasticity of the capital regarding to the production is negative, the result are inverse: this case is not realistic, because it implies that less capital is used when production increases. If the elasticity is nil, the two solutions are identical.

## 3.2. Special case: fixity of the financial structure

Here, the parameter $\alpha$ is constant; thus $\mathrm{e}_{\alpha(Q)/K(Q)} = 0$ and $\pi(Q) = \pi_G(Q) - t\,(1-\alpha)$. Theorem 1 becomes:

Theorem 1'. Compared to the profit maximisation, the maximisation of the rate of profit gives ***a lower optimal quantity*** (respectively higher) ***when the rate of profit is higher*** (respectively lower) ***than the interest rate***:

1. $\frac{\pi(Q_\pi^*)}{\alpha} > t \Rightarrow \max \pi \prec \max \Pi$ or $\pi_A(Q_\pi^*) > t \Rightarrow \max \pi \prec \max \Pi$

2. $\frac{\pi(Q_\pi^*)}{\alpha} < t \Rightarrow \max \pi \succ \max \Pi$ or $\pi_A(Q_\pi^*) < t \Rightarrow \max \pi \succ \max \Pi$

where $Q_\pi^*$ is the optimal quantity which maximises the rate of profit $\pi(Q)$.

Comments.

1. As in [ de MESNARD 1992 ]the first case correspond to a standard case (only when the financial structure is fixed) because the condition $\frac{\pi(Q_\pi^*)}{\alpha} > t \Leftrightarrow \pi_A(Q_\pi^*) > t$ is normally respected (if there exists some alternative investments): if it is not, an alternative investment would have be chosen. Thus, the ***maximisation of the rate of profit normally leads toward a lower optimal production than the maximisation of the aggregate pure profit does***. Another large part of the argument of pure profit is out: ***the maximisation of the aggregate pure profit is not the more careful strategy because the maximisation of the rate of profit normally leads toward a lower investment***.

   The consequences of this result is that, in this normal case compared to the maximisation of the pure profit, the firms are smaller, there are more firms; however, the price is higher.

2. If $\frac{\pi(Q_\pi^*)}{\alpha} = t \Leftrightarrow \pi_A(Q_\pi^*) = t$, that is to say, if the rate of profit is equal to the rate of interest, the two solutions coincide.

3. The results of [ de MESNARD 1992 ] are retrieved for $\alpha = 1$ (except for the theorem 1bis p. 112-113): see annexe.

Result 1. The optimum is at the same place whatever be $\alpha$ and $t$. Thus, if the condition $\frac{\pi(Q_\pi^*)}{\alpha} > t \Leftrightarrow \pi_A(Q_\pi^*) > t$ is valid for one value of $\alpha$ (respectively $t$), it is valid for any other value of $\alpha$ (respectively $t$).

Comment. This result shows that taking into account of the financial structure is a little trivial if this structure remains fixed. The interesting result is obtained when the financial structure is variable.

Proof. It is obvious: $\pi'(Q) = \pi'_G(Q)$ is independent of $\alpha$ and $t$.

## 3.3. Example

We keep the same example as above:

$C(Q) = 5\,Q^3 + 10\,Q^2 - 5\,Q + 20,\ p(Q) = -5\ Q + 100,\ R(Q) = p\,Q,\ K(Q) = 200\,Q$

The curves are:

$\Pi 1$: aggregate pure profit when $t$ = 0,25;

$\Pi 2$: aggregate pure profit when $t$ = 0,40;

$\pi 0$: rate of profit when $\alpha = 1$ (it is independent of $t$ and corresponds to those of [ de MESNARD 1992 ]);

$\pi 7$: $t = 0.25,\ \alpha = 0$; $\pi 8$: $t = 0.40,\ \alpha = 0$ (rate of pure profit);

$\pi 1$: $t = 0.25,\ \alpha(Q) = \dfrac{1}{1 + 0.1^{2-Q}}$ and $\pi 2$: $t = 0.40,\ \alpha(Q) = \dfrac{1}{1 + 0.1^{2-Q}}$: decreasing part of shareholders with a logistical function [13] (inflexion point at 2);

$\pi 3$: $t = 0.25$ and $\alpha(Q)$ not depending of a function but never increasing;

$\pi 4$: $t = 0.25,\ K(Q) = 20\ \forall Q$ (constant accounting capital );

$\pi 5$: $t = 0.25,\ \alpha(Q) = \dfrac{1}{1 + 0.1^{Q-2}}$ and $\pi 6$: $t = 0.40,\ \alpha(Q) = \dfrac{1}{1 + 0.1^{Q-2}}$: increasing part of shareholders[14] with a logistical function (inflexion point at 2).

The values of $\pi$ are given in percent.

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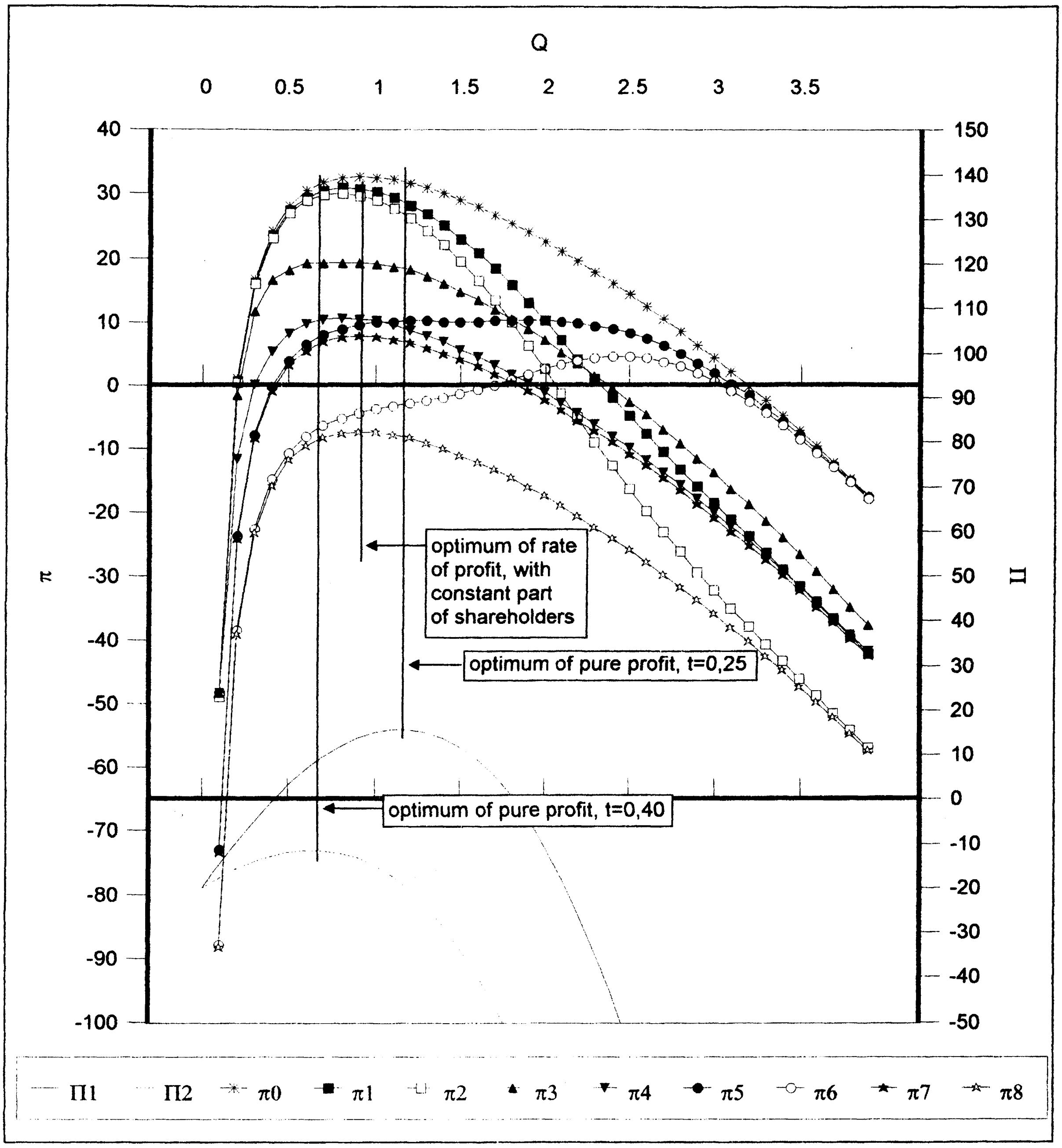
[13] A logistical function is well adapted to such a phenomenon: a slow decrease, followed by a fast decrease and a slow decrease: when a firm is developing, it has some difficulties to find loaned funds in the beginning; after, things are more easy; at the end, the part of loans must not exceed 1.

[14] An increasing part of shareholders is less realistic. However, it is temporally possible and the case is show in a didactic way.




**Maximisation of the pure profit compared to the maximisation of the rate of profit**

We see that the optimum of $\pi 1$ and $\pi 3$ are to the left of the optimum of the pure profit maximisation; the optimum of $\pi 2$ and $\pi 4$ are to the right; the optimum of $\pi 5$ and $\pi 6$ also are to the right, but the optimum of $\pi 5$ is quasi degenerated because the curve is quasi horizontal in a wide part (and a local optimum); this is the effect of the point of inflexion; the important fact is that we obtain a quasi horizontal optimal section of curve for the objective function without supposing that the cost curve is itself horizontal (U curve) as it is made in the classical literature.

The optimum of $\pi 3$ is degenerated: an horizontal optimal section for the objective curve is possible if the part of shareholders increases but also if it decreases.

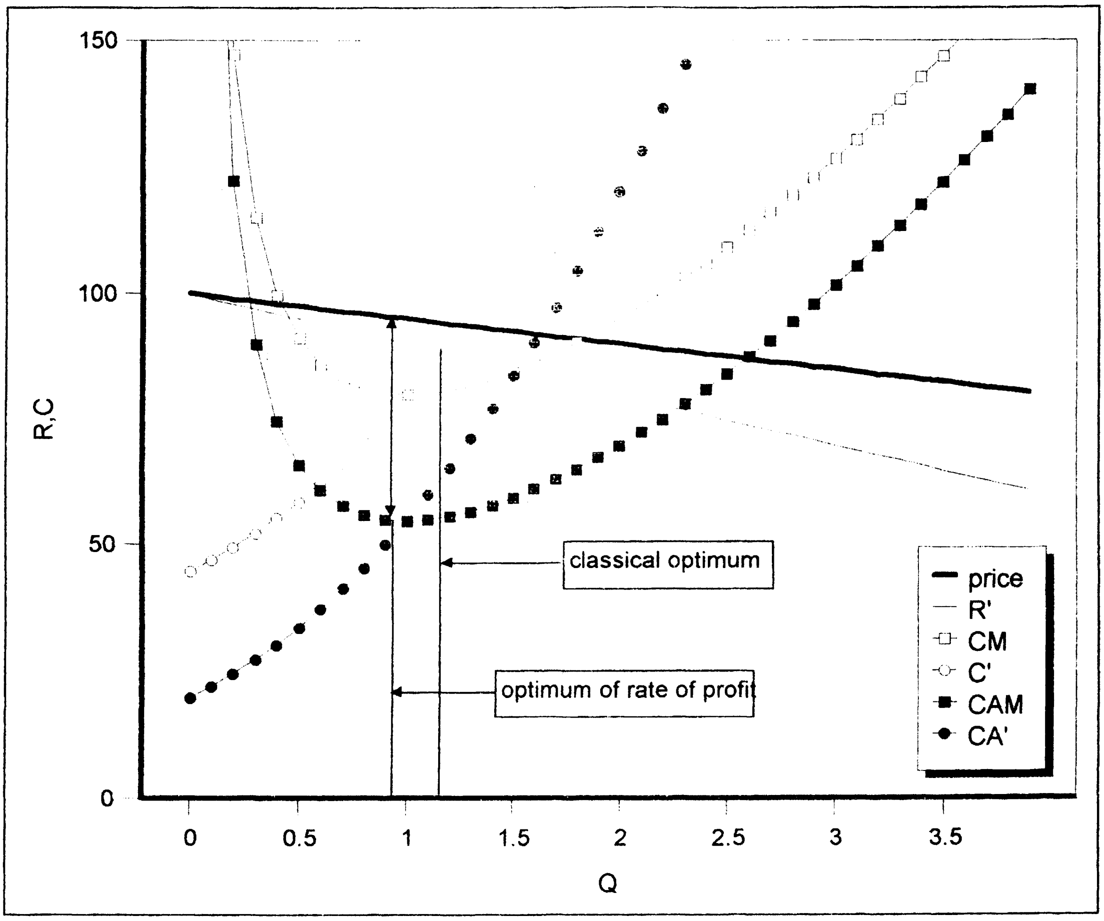
Note that the curves $\pi 0$, $\pi 7$ and $\pi 8$ are the limit curves.

For a constant part of shareholders and a variable interest rate, the optimum of the maximisation of the rate of profit is unchanged ($\pi 0, \pi 7, \pi 8$) when the interest rate increases; this optimum goes toward the left if the part of shareholders is decreasing ($\pi 1$ and $\pi 2$), and goes to the right if the part of shareholders is increasing ($\pi 5$ and $\pi 6$).

When $t =$ 0,40 the aggregate pure profit is negative, while the rate of profit is sometimes positive (curves $\pi 0, \pi 2, \pi 6$).

The optimum of curves $\pi 5$ and $\pi 6$ correspond to a large production but with a low rate of profit. The optimum of curves $\pi 1$ and $\pi 2$ correspond to a large rate of profit but with a low production. Note that the optimum of the rate of profit may be realisable even if the optimum of the pure profit is not realisable (curves $\pi 2, \pi 6$ and $\Pi 2$).

The following graph shows the classical curves when in the case of fixed coefficient of capital and constant part of shareholders (for other cases as variable coefficient of capital or variable part of shareholders, the graph may be not so simple):




**Classical curves:**
**monopoly with fixed coefficient of capital and constant part of shareholders**

CM is the average cost, CAM is the average accounting cost and CA' is the marginal accounting cost.

## 3.4. Dynamics

The question of dynamics is: does the optimum of the rate of profit tends toward the optimum of the profit? In the above example, starting from the optimum of the rate of profit, the rate of profit is decrasing when the production increases [15]. Thus, the firm may not tend by itself to increase is scale of its production.

In fact, a dynamical evolution from one optimum to another point requires to introduce another reflections. For example, the demand function moves in a long term basis or another firms enter the sector.

[15] Note that the rate of profit is not always decreasing when the production is increasing, as in the Lerner-Carleton model.

### 3.4.1. The entry point in the sector

As in the classical microeconomic theory, even if there is maximisation, the optimum may be unacceptable. In the classical case the optimum of pure profit may be negative. In the theory of the maximisation of a ratio of profit, the optimum may be inferior to the interest rate. The point were the optimum becomes acceptable is the entry point in the sector.

If the rate of gross profit is inferior to the interest rate, the firm will never enter in the sector whatever be its financial structure (the pure profit is negative). If $\pi_G(Q) > t$, the firm may enter, depending on the ratio it choose as an objective function. If the firm chooses the rate of profit, it enter if $\pi(Q) > t$; if the firm chooses the rate of accounting profit, it enters if $\pi_A(Q) > t$. However, as we have $\pi_A(Q) = \pi_G(Q)$ if and only if $\pi_G(Q) = t$ , the condition $\pi_G(Q) > t$ is equivalent to the condition $\pi_A(Q) > t$ . As $\pi(Q) \leq \pi_G(Q)$, the entry point when the firm maximises the rate of profit is to the left of the entry point when the firm maximises the rate of gross profit or the rate of accounting profit: the pure profit is not necessary nil.

Nevertheless, the rate of profit is not a good indicator of entry in the sector. Suppose that $\pi(Q) < t$ with a positive accounting profit and $\pi_G(Q) > t$ ; then the firm does not enter in the sector; however the pure profit is positive and an inferior positive profit may be realised by lending the accounting capital at the monetary market; thus it is better for the firm to enter even if $\pi(Q) < t$. This proves that the good indicators of entry are the rate of gross profit or the rate of accounting profit, even it may be used for the maximisation.

### 3.4.2. The very long term

The model of maximisation of a ratio of profit is a long term model because capital varies. It leads to consider a very long term, where the number of firms varies (this very long term is analogous to the long term of the neoclassical microeconomic theory).

In perferct competition, the classical reasoning is that if the pure profit is positive, other firms may want to enter in the sector; thus the pure profit of each firm decreases until zero where the number of firms is maximal; this point corresponds to the minimum point of the curve of long term cost [16]. In monopoly, the demand function moves to the point where the profit is nil (tangency between the demand function and the average cost function)..

Analogously, when the rate of profit is superior to the interest rate, another firms may want to enter in the sector because profitability is higher there; thus the rate of profit of each firm decreases toward the interest rate where the number of firms is maximal. The entry point is the point of very long term equilibrium. Thus,

- As seen before, when firms maximise the rate of accounting profit, the point of very long term equilibrium $\pi_A(Q_\pi^*) \to t$ corresponds to the point where the pure profit is nil and

[16] Note that this reasoning seems contradictory with the general equilibrium theory where the number of agents is given.

maximum. This result was not clearly stated in [ de MESNARD 1992 ] and it is the same with the rate of gross profit.

- However, when firms maximise the rate of profit, it must be pointed out that the point of very long term equilibrium $\pi\left(Q_\pi^*\right) \to t$ does not correspond to the point where the pure profit is nil. And when the financial structure is variable, this point does not coincide with the point of maximum of pure profit, because $\left[\alpha\left(Q_\pi^*\right)\right]\left[1+\mathrm{e}_{\alpha(Q)/K(Q)}\Big|_{Q=Q_\pi^*}\right] \neq 1$ generally. A necessary condition for $\left[\alpha\left(Q_\pi^*\right)\right]\left[1+\mathrm{e}_{\alpha(Q)/K(Q)}\Big|_{Q=Q_\pi^*}\right] = 1$ is $\mathrm{e}_{\alpha(Q)/K(Q)}\Big|_{Q=Q_\pi^*} > 0$ (remember $\alpha\left(Q_\pi^*\right) \leq 1$) which is not the more realistic case; thus generally $\left[\alpha\left(Q_\pi^*\right)\right]\left[1+\mathrm{e}_{\alpha(Q)/K(Q)}\Big|_{Q=Q_\pi^*}\right] < 1$ and the very long term optimum is to the left of the classical long term optimum. Thus, the question which remains is: does the market clears?

# 4. THE RATE OF ACCOUNTING PROFIT WITH VARIABLE FINANCIAL STRUCTURE

The case studied in [ de MESNARD 1992 ], the optimum of the maximisation of the rate of accounting profit, is formally retrieved putting $\alpha = 1$. However, we give the general solution of the maximisation of the rate of profit.

The solution for the maximisation of the aggregate pure profit $\Pi(Q)$ is obtained for:

$$\Pi'_A(Q) = t\ \alpha'(Q)\ K(Q) + t\ \alpha(Q)\ K'(Q) = t\ \alpha(Q)\ K(Q) \frac{t\ \alpha'(Q)\ K(Q) + t\ \alpha(Q)\ K'(Q)}{\alpha(Q)\ K(Q)}$$

$$\Leftrightarrow \Pi'_A(Q) = t\ \alpha(Q)\ K(Q)\left(\frac{\alpha'(Q)}{\alpha(Q)} + \frac{K'(Q)}{K(Q)}\right).$$

Consider the curve $\Pi'(Q) = \Pi'_A(Q) - t\ \alpha(Q)\ K(Q)\left[\frac{\alpha'(Q)}{\alpha(Q)} + \frac{K'(Q)}{K(Q)}\right]$.

The solution for the maximisation of the rate of accounting profit $\pi_A(Q)$ is obtained for: $\Pi'_A(Q) = \mathrm{e}_{K_A(Q)/Q}\ \overline{\Pi}_A(Q) \Leftrightarrow \Pi'_A(Q) = \Pi_A(Q)\frac{K'_A(Q)}{K_A(Q)}$

$$= \Pi_A(Q)\frac{\alpha'(Q)\ K(Q) + \alpha(Q)\ K'(Q)}{\alpha(Q)\ K(Q)} = \Pi_A(Q)\left(\frac{\alpha'(Q)}{\alpha(Q)} + \frac{K'(Q)}{K(Q)}\right).$$

Consider the curve $f_\pi(Q) = \Pi'_A(Q) - \Pi_A(Q)\left[\frac{\alpha'(Q)}{\alpha(Q)} + \frac{K'(Q)}{K(Q)}\right]$.

Let us write $\Pi'(Q)$ as a function of $f_\pi(Q)$ :

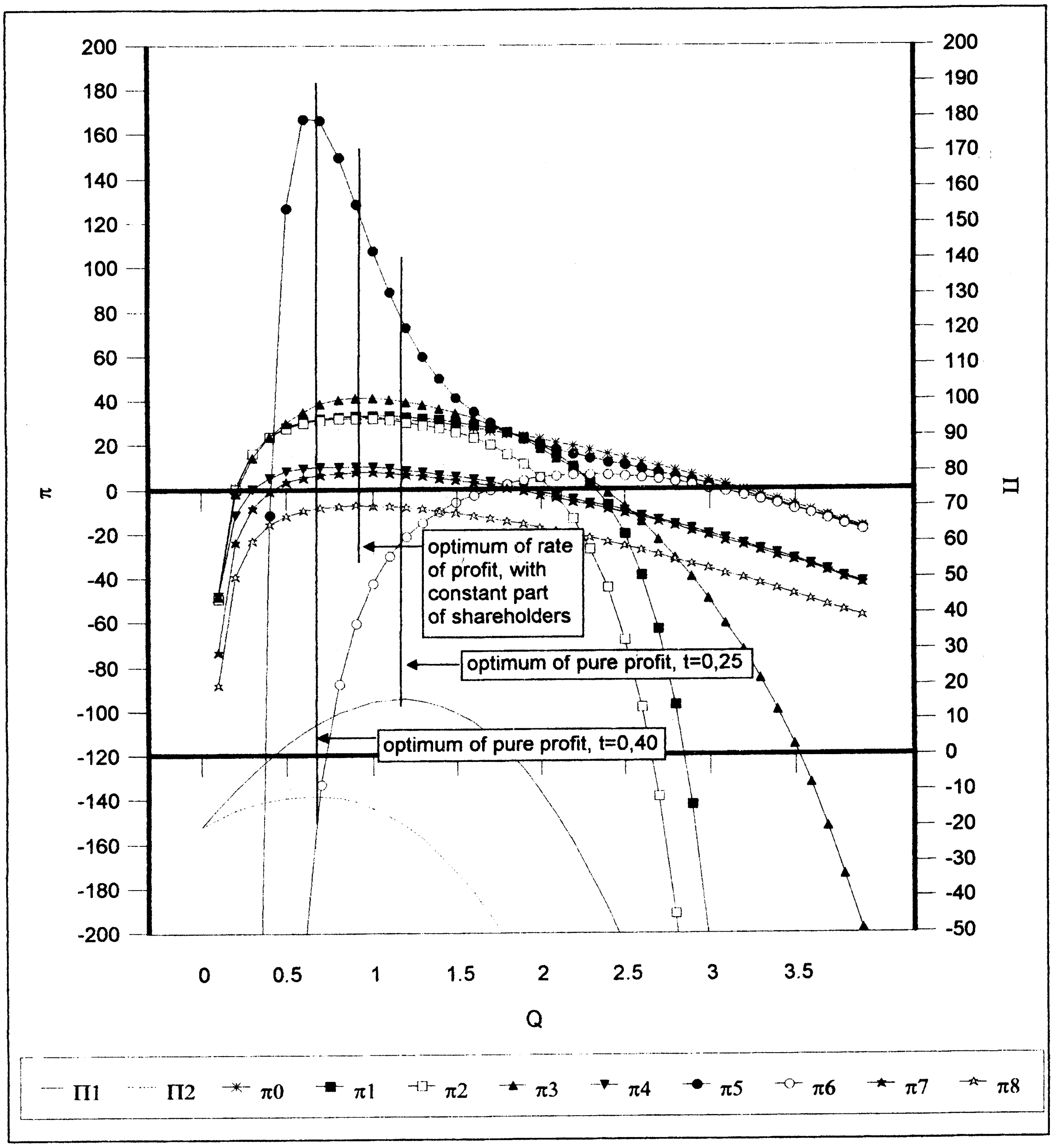
$$\Pi'(Q) = f_\pi(Q) + \left[\Pi_A(Q) - t\ \alpha(Q)\ K(Q)\right]\left[\frac{\alpha'(Q)}{\alpha(Q)} + \frac{K'(Q)}{K(Q)}\right]$$

$$\Leftrightarrow \Pi'(Q) = f_\pi(Q) + \left[\frac{\Pi_A(Q)}{\alpha(Q)\ K(Q)} - t\right]\left[\alpha'(Q)\ K(Q) + \alpha(Q)\ K'(Q)\right]$$

$$\Leftrightarrow \Pi'(Q) = f_\pi(Q) + \left[\pi_A(Q) - t\right] K'_A(Q)$$

Thus, to study the respective position of the optimum of the maximisation of the pure profit and the maximisation of the rate of accounting profit we must compare $\pi_A(Q)$ and $t$, depending on the sign of $K'_A(Q)$; this is the condition of [ de MESNARD 1992 ]. However, the sign of $K'_A(Q)$ or $\left[\frac{\alpha'(Q)}{\alpha(Q)} + \frac{K'(Q)}{K(Q)}\right]$ is not determined clearly when there is a financial structure. Thus, the optimum of the maximisation of the rate of profit is to the left of the optimum of the maximisation of the aggregate pure profit if $\pi_A(Q) > t$ and $K'_A(Q) > 0$, or $\pi_A(Q) < t$ and $K'_A(Q) < 0$, and it is to the right if $\pi_A(Q) > t$ and $K'_A(Q) < 0$ or $\pi_A(Q) < t$ and $K'_A(Q) > 0$.

**Example** (the same):




**Maximisation of the pure profit compared to the maximisation of the rate of accounting profit**

## 5. DISCUSSION UPON THE OBJECTIVE FUNCTION OF THE FIRM

The objective function of the firm, the maximisation of the rate of profit, is realistic, because it corresponds to an economical idea of the capital: that is why we have logically supposed $K'(Q) > 0$. And the firm knows how many funds it engaged in the capital: the accounting capital plus the reserved plus the long term debts.

However, there are some problems. Consider the following reasoning. When the part of shareholders is taken as an increasing parameter (that is to say when $Q$ is given), the rate of profit increases: if $\alpha = 0$ then $\pi = \pi_G - t$ and if $\alpha = 1$ then $\pi = \pi_G$. Thus the firm may want to increase the part of shareholders in order to increase the rate of profit. This strategy decreases the rate of accounting profit: if $\alpha = 0$ then $\pi_A \to \infty$ and if $\alpha = 1$ then $\pi_A = \pi_G$ [17]. Thus, there is an incompatibility between the objective function of the firm and the objective function of the shareholders [18].

This reasoning does not take into account one parameter: the variability of $Q$. To change the part of shareholders $\alpha(Q)$, it may be necessary to change the optimal level of production $Q$. Thus $\pi_G(Q)$ may vary and the comparison is not possible. In the above example, the curve $\pi 5$ shows an increasing part of shareholders which leads to a quasi non increasing rate of profit, and it shows a considerable increase of the production to go from a low level to a high level of $\alpha(Q)$ (from $\pi 7$ to $\pi 0$ in the following example).

A second argument is that it may be relatively difficult to find shareholders: it requires to go at the Stock Exchange. For many firms it is impossible because they are too much small (curve $\pi 4$ of the above example, giving a decreasing part of shareholders); even for big firms, it is not always possible to issue a big amount of shares or to make repeated issues. That is why many firms loan, by bonds or with banks.

A third argument is that the loans were supposed to be perpetual implicitly. If not, shareholders must be paid back. This pay back is a deduction over the cash and then on the accounting profit distributed to shareholders (and appears only in the balance sheet). Thus the effective rate of remuneration of the accounting capital may be lower than the maximum possible: often, managers decide to pay a dividend giving a normal remuneration inferior to the rate of accounting profit (for example, a rate equal to the rate of profit), and they use the rest of the accounting profit to pay back loans.

These simple financial and accounting arguments show that the theory of the maximisation of the rate of profit must be understood in that way. It is not a financial theory; it is a microeconomic theory. The part of shareholders $\alpha(Q)$, that is to say the financial structure, must be seen as either a parameter, either an endogenous variable, but not as an exogenous variable to be optimised. To find the optimal value of the production which maximises the rate of profit, we

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[17] The contrary and reasonable strategy, decrease the part of shareholder to increase the rate of accounting profit, is the well known financial shift effect.

[18] This problem does not occur in [ de MESNARD 1992 ] because the objective function was the maximisation of the rate of accounting profit.

must fix the value of the parameter $\alpha$ then see how the optimal production changes if $\alpha$ changes, or we must suppose a law of evolution for $\alpha(Q)$ then see how the optimal production changes at the inflexion point (or the law itself) changes, and we must not try to find the optimal value of $\alpha(Q)$ which maximises the rate of profit for a given level of production.

Obviously, the theory of the maximisation of the rate of gross profit does not bring about these problems because it does not depend on the financial structure. The theory of the maximisation of the rate of accounting profit directly take into account the profitability of the accounting capital and reflects the influence of the shareholders over the firm.

# CONCLUSION

In this paper, the question is: what is the objective function of the firm? To answer, we proposed a typology of the ratios of profit. Taking into account the financial structure (by a coefficient of financial structure: the accounting capital over the whole capital), in addition to the rate of accounting profit (seen in [ de MESNARD 1992 ]) we studied the rate of gross profit (gross profit over capital) and the rate of profit (accounting profit over capital).

The main result concerns the optimal production when the rate of profit is maximised instead of the aggregate pure profit: it is lower when the *compensated rate of profit* (the ratio of profit over accounting capital, divided by the relative elasticity of the accounting capital with regard to the whole capital) is higher than the interest rate. This result may be particularise to the case where the financial structure is fixed (the production obtained with the rate of profit maximisation instead of the maximisation of the aggregate pure profit is lower when the *rate of profit* is higher than the interest rate) and it generalises the preceding results.

One of the more interesting consequences of the theory of the maximisation of the rate of profit is that, as each firm is smaller often, there are more firms; thus the interaction between firms is lower and competition is greater. Moreover, the clearing of the market must be proved again.

An argument in favour of the theory of the maximisation of a ratio of profit is that its thought processes are near the operational economic calculus. Within firms, managers thinks in terms of ratio of profit, that is to say in terms of profitability, more than in terms of opportunity costs and more than in terms of maximisation of an aggregate profit, even it is a pure profit. The question rests into the choice among the three ratios studied: rate of accounting profit, rate of profit, rate of gross profit. However, the result of the maximisation of a ratio of profit remains different to the result of the maximisation of the aggregate pure profit.

This may provide a program of research with the following questions. What become the classical models when a ratio of profit is chosen instead of the aggregate pure profit, whatever be this ratio? What ratio of profit must be chosen? What happens when one firm choose one ratio, and one another firm choose another ratio?

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